V.O. Melnyk[1], V.P. Sylovanyuk[1], D.V. Rudavskyi[2]
[1]Karpenko Physico-Mechanical Institute of NAS of Ukraine
[2]Lviv Polytechnic National University

BOUNDARY EQUILIBRIUM OF A TRANSVERSELY ISOTROPIC BODY WITH A HEALED DISK-SHAPED CRACK

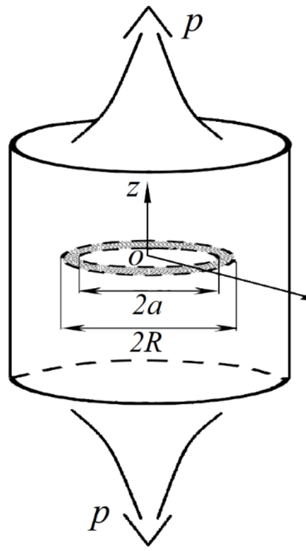

Fig. 1. Penny-shaped crack in an elastic-plastic body.

We consider a transversely isotropic three-dimensional body with a circular planar crack of radius a in the plane of isotropy (Fig. 1). Tensile loads are applied at points far from the crack, inducing a uniform stress p in the crack plane in the absence of the crack. As shown in [1], the strength of such a body, calculated based on the δ_c-model [2], is given by:

$$p_* = \sigma_0(2(1 - a_*/2a)(a_*/a))^{1/2}, \quad \text{for } a \geq a_*$$
$$p_* = \sigma_0, \quad \text{for } a < a_*. \tag{1}$$

Here, σ_0 is the tensile strength along the axis of isotropy; δ_c is the critical crack opening; $a_* = \frac{\pi\delta_c B}{4\sigma_0}$; $B = \frac{A_{44}(1+m_1)(1+m_2)(\sqrt{\nu_2}-\sqrt{\nu_1})}{m_2-m_1}$; ν_1, ν_2 are roots of a characteristic equation; m_1, m_2 are defined by:

$$m_i = \frac{A_{11}\nu_i - A_{44}}{A_{13} + A_{44}}, \quad i = 1,2, \tag{2}$$

and the constants $A_{44}, A_{11}, A_{33}, A_{13}$ are elastic moduli of the transversely isotropic body.

Let the crack surfaces be bonded by a thin layer of injection material as a result of applying injection technology. We aim to evaluate the effectiveness of such crack healing in the body.

The response of the filler layer to tensile loading is described using the Winkler foundation model. To derive the following integral equation, we consider an axially symmetric boundary value problem for a transversely isotropic half-space above the crack plane ($z = 0$). The symmetry allows the domain to be reduced to $z \geq 0$, with shear stress $\sigma_{rz} = 0$ along the interface. The normal stress $\sigma_{zz}(r,0)$ is defined piecewise: $-p + \sigma_{zz}^*(r)$ on the crack surface ($0 \leq r \leq a$) and $-p + \sigma_0$ within the surrounding damage zone ($a < r \leq R$), while a zero-displacement condition $u_z = 0$ is imposed for $r > R$.

Under these mixed boundary conditions, the solution is formulated using Hankel transforms, leading to a system of dual integral equations. Applying Sneddon's approach [3], this system is reduced to the following single integral equation:

$$u_z(r) = \frac{2}{\pi B}\int_r^R \frac{dt}{\sqrt{t^2 - r^2}}\int_0^t \frac{r_1 p_1(r_1)\, dr_1}{\sqrt{t^2 - r_1^2}}, \quad 0 \leq r \leq R,$$
$$p_1(r) = \begin{cases} -p + (u_z + u_z^0)Eh^{-1}, & 0 \leq r \leq a \\ -p + \sigma_0, & a < r \leq R \end{cases}, \tag{3}$$
$$u_z^0 = \frac{(A_{11} + A_{12})h}{A_{33}(A_{11} + A_{12}) - 2A_{13}^2}$$

Numerical simulations were performed using the finite element method in the Code Aster environment. A finite element model of the body with a healed crack was built (Fig. 2) to evaluate stress in both elastic and elastic-plastic media. Analysis of the data (Fig. 3) shows that stress in the layer remains close to uniform, with maximum deviations not exceeding 3%. This confirms the correctness of the analytical assumption about the spheroidal inclusion and justifies the application of the Winkler model to describe the interaction of the injection material with the base body.

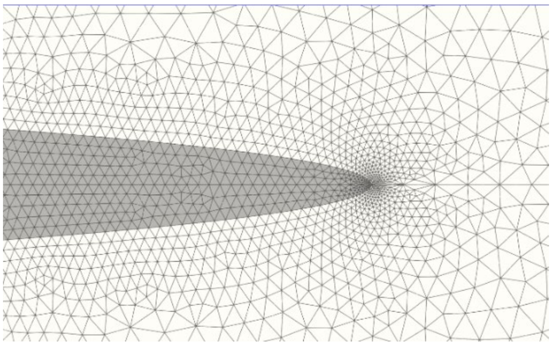

Fig. 2. Finite element mesh scheme of the body with the filled crack.

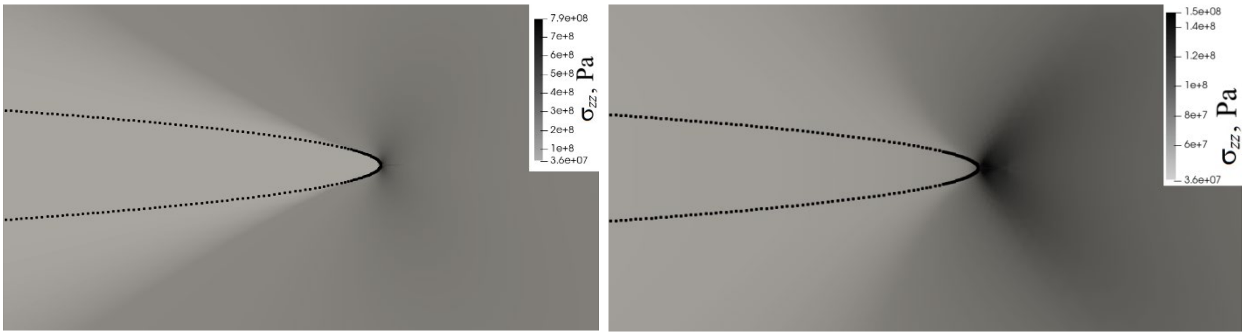

Fig. 3. Stress intensity in the base and filler materials (FEM): 1) elastic; 2) elastic-plastic

Based on the assumption, a closed-form analytical solution of the integral equation was obtained. Further application of the δ_c-criterion allowed us to express the critical load p_* as follows:

$$p_* = \frac{\sigma_0\left(1-\left(\frac{2a_*}{a}-1\right)^2\right)}{A+\sqrt{A^2-\left(\left(\frac{2a_*}{a}-1\right)^2-1\right)(1-2A)}}, a > 2a_*/1+\sqrt{1+\frac{A^2}{1-2A}} \qquad (5)$$

It is shown that with the proper stiffness of the injection material, the strength of the body can be fully restored ($p_* = \sigma_0$), even if the crack is close to the critical size.

An engineering formula for assessing the effectiveness of crack injection has been obtained. The finite element method confirmed the analytical assumptions and provided additional evaluation of the influence of the mechanical properties of the injection layer. The results can be used to assess the strength of reinforced structures and to optimize injection parameters.

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[3] I. N. Sneddon, Mixed Boundary Value Problems in Potential Theory, North-Holland Publishing Company, 1966, p. 283.