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Dr. engineering Stanislav Noga

Rzeszow University of Technology

INVERSE KINEMATICS OF A TWO WHEELED MOBILE ROBOT

In this paper the inverse kinematics problem of the two-wheeled mobile robot is considered. The sinusoidal and the circular trajectory is analyzed. The kinematic equations for arbitrary chosen point of the system are given by using classical equations of the theoretical mechanics. The kinematic parameters of motion from the solution of inverse kinematics problem are obtained.

Станіслав Нога

Технологічний університет, Жешов, Польща

ОБЕРНЕНА КІНЕМАТИКА ДВОКОЛІСНОГО МОБІЛЬНОГО РОБОТА

Розглянуто обернену задачу кінематики двоколісного мобільного робота. Проаналізовано синусоїдальну і кругову траєкторії. Подано кінематичні рівняння для довільно вибраної точки системи з використанням класичних рівнянь теоретичної механіки. Одержано кінематичні параметри руху від розв'язку оберненої задачі кінематики.

1. Introduction

Nonholonomic mobile robot is the object of study in this article. Problems concerning kinematics and dynamics of wheeled mobile robots gain more and more attention. It results mainly from the fact, that mobile robots are very often used in industry, especially where contribution of an operator in some technological processes is not recommended. The mobile robot is a system with nonholonomic constrains. Fundamental theory of nonholonomic systems is richly documented by work ranging from Nejmark and Fufajev's comprehensive book [2] and Gutowski's monograph [1] to more recent developments (see for example, [5, 7]). The kinematic modeling of mobile robots is investigated by many researchers. The problem of kinematics of a two wheeled mobile robot are analyzed in works [3, 4, 7, 8, 9]. In references [5, 7, 8] a natural orthogonal complement is applied to a two-wheeled mobile robot. The problem of motion along the sinusoidal trajectory of the mobile robot is discussed in paper [9], excluding the kinematics of bracket and caster wheel. In works [3, 4] kinematics of this system using classical equations of mechanics are described including the kinematics of bracket and caster wheel. This work continues the recent author's investigations concerning kinematics and dynamics of mobile robots [3, 4, 9].

The present paper is organized as follows. Section two describes kinematics of the mobile robot system under study. In section three the problem of inverse kinematics for the circular and sinusoidal trajectory is studied. Section four presents some simulation results. Section five gives some concluding remarks.

2. Kinematics of the two wheeled mobile robot

A made an assumption model of a two-wheeled mobile robot is presented in Fig. 1. The basic units of the model are: frame 4, driving and steering wheels 1 and 2, and bracket with the caster wheel 3. The wheels 1 and 2, mounted on the shafts that are driven separately by two electric motors. These wheels rotate around their axes, which positions are invariable in relation to the frame. The third wheel is mounted on a rotary bracket.

The work [6] presents the specifications and controls of the mobile robot, presented above. The coordinates α_1, α_2, and α_3 are the angles of rotation of the wheels 1, 2, and 3 respectively. The angle β is the angle of instantaneous angular displacement of the frame 4. The angle ψ is the angle between the chassis longitudinal symmetry axis and the bracket longitudinal symmetry axis. The following elements: l, l_1, l_2, l_3, l_4, r_1, r_2, r_3 are the dimensions defined in Fig. 1.

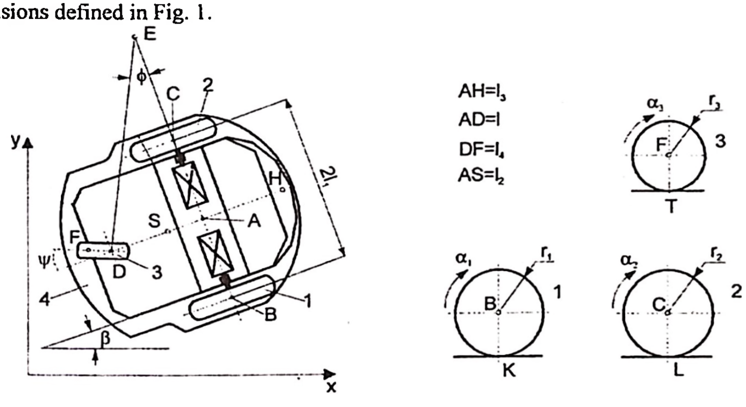

Figure 1 – The two-wheeled mobile robot.

The point A is the characteristic point of the frame. This is the point of intersection of the frame longitudinal symmetry axis with the axis of rotation of the wheels 1, and 2. The points B, C, and F are the centres of masses of the wheels 1, 2, and 3 respectively. The point S is the centre of mass of the chassis 4. The point D is the connection point of the bracket 3 and the frame 4.

The position and orientation of the mobile robot is described by seven coordinates: x_A, y_A – the Cartesian coordinates of the point A, α_1, α_2, α_3, β, ψ – the angle coordinates.

Under the no – slipping and no – sideway motion assumptions, the mobile robot has two degrees of freedom only. One can choose the angles of driving wheels as generalized coordinates for the convenience of control design of the mobile robot. The objective of the kinematic analysis of the mobile robot is to derive the kinematic parameters of the mobile platform in terms of generalized coordinates.

The considered robot is moved on the plane. The chassis and the bracket with the caster wheel undergo planar motion. The velocity distribution of characteristics points of the system is displayed in Fig. 2.

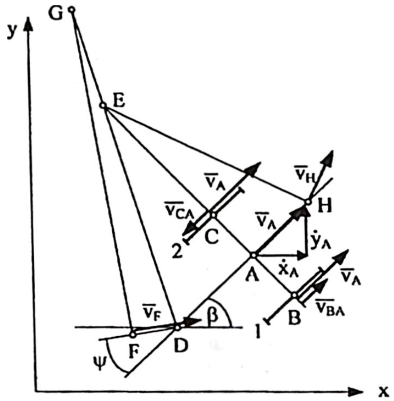

Figure 2 – The velocity distribution of the system.

The point E is the instantaneous centre of chassis 4, and the point G is the instantaneous centre of the bracket. According to the nonoholonomic constraints and no – slipping condition, the robot must move in the direction of the axis of symmetry, i.e.:

$$\begin{aligned} \dot{x}_A &= v_A \cos\beta, & \dot{x}_F &= v_F \cos(\beta-\psi), \\ \dot{y}_A &= v_A \sin\beta, & \dot{y}_F &= v_F \sin(\beta-\psi), \end{aligned} \tag{1}$$

where (x_A, y_A) are the coordinates of the point A, v_A is the velocity of the point A, v_F is the velocity of the point F (see Fig. 1 and Fig. 2), and (x_F, y_F) are the coordinates of the point F and are given directly by:

$$x_F = x_A - l\cos\beta - l_4\cos(\beta-\psi), \quad y_F = y_A - l\sin\beta - l_4\sin(\beta-\psi). \tag{2}$$

From the equations (1), (2), the angular velocity of the bracket with respect to the chassis is determined by [4]:

$$\dot{\psi} = \frac{1}{l_4}\left[\dot{\beta}\left(l\cos\psi + l_4\right) - v_A\sin\psi\right]. \tag{3}$$

Taking into account kinematics equations for the points B, C, and F, the angular velocities of the wheels 1, 2, and 3 can be written as [4]:

$$\dot{\alpha}_1 = \frac{v_A}{r} + \dot{\beta}\frac{l_1}{r},\ \dot{\alpha}_2 = \frac{v_A}{r} - \dot{\beta}\frac{l_1}{r},\ \dot{\alpha}_3 = \frac{v_A}{r_3}\cos\psi + \dot{\beta}\frac{l}{r_3}\sin\psi \tag{4}$$

3. Assigned trajectory of the chosen point of the system

A kinematic analysis of this system is involved with the problems of inverse kinematics. In kinematic description of any point of the system it is advantageous to define so called equation of kinematics. Many authors describe kinematics of these systems using a natural orthogonal complement [5, 7]. In this paper, the kinematic equations for arbitrary chosen point of the system are given by using classical methods. In the case when mobile robot moves over a lever floor, the frame and the bracket remain in the translatory or plane motion. In this work the circular and sinusoidal trajectory of the arbitrary chosen point of the system is assumed.

At first it is taken that the point H (see Fig. 1 and Fig. 2), moves along the circular trajectory of the radius R, i.e.:

$$x_H = R\sin\theta,\ y_H = R(1-\cos\theta), \tag{5}$$

where (,) are the coordinates of the point H, and is the angle of rotation on the path. The value of the velocity vector of point A is constant. Then the equation of kinematics can be written in the following appropriate form, as [4]:

$$\begin{bmatrix} 1 & 0 & 0 & -R\cos\theta \\ 0 & 1 & 0 & -R\sin\theta \\ 0 & 0 & l_3\sin\beta & R\cos\theta \\ 0 & 0 & -l_3\cos\beta & R\sin\theta \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_H \\ \dot{y}_H \\ \dot{\beta} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ v_A\cos\beta \\ v_A\sin\beta \end{bmatrix} \tag{6}$$

In the next case it is assumed that the point H moves along sinusoidal trajectory of the amplitude A_0 and period L, i.e.:

$$y_H = A_0 \sin\omega x_H, \tag{7}$$

where $\omega = 2\pi/L$. The equation of kinematics can be written in the following appropriate form, as [4]:

$$\begin{bmatrix} 1 & 0 & l_3\sin\beta \\ 0 & 1 & -l_3\cos\beta \\ A_0\omega\cos\omega x_H & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_H \\ \dot{y}_H \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} v_A\cos\beta \\ v_A\sin\beta \\ 0 \end{bmatrix} \tag{8}$$

4. Simulation

In this section, simulation results of inverse kinematics problem are presented. Computer simulation makes it possible to find the solution for a system of equations (3), (4), (6), and (8), i.e. to determine the interesting angular velocities and respective angles. The motion phase, as: take – off run, steady motion, and braking are analyzed. The velocity of point A is assumed to be constant $v_A = const$. The parameters characterizing the two wheeled mobile robot used in calculations are shown in Tab. 1.

Table 1 – Parameters characterizing the two wheeled mobile robot

$l\,[m]$	$l_1\,[m]$	$l_3\,[m]$	$l_4\,[m]$	$l_5\,[m]$	$r\,[m]$	$r_3\,[m]$
0.217	0.163	0.133	0.025	0.27	0.0825	0.035

The velocity of point A $v_A = 0.3[m/s]$ is used in the calculations.

As mentioned earlier the circular path of radius $R = 1[m]$ is assumed along which the point H moves. Parameters of the trajectory are selected in the experimental way.

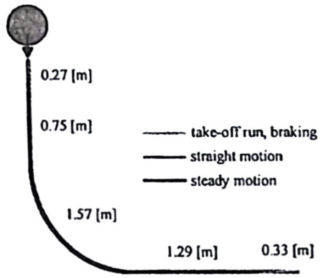

Figure 3 – The assigned trajectory of the characteristic point of the system.

Due to space limitations, the principal results of simulation are only presented. The reader interested in further details is referred to ref. [4]. The time histories of the parameters of motion, including angular velocities and angles of rotation of the wheels 1, 2, and 3 and angular velocity and angle of rotation of the chassis 4, and angular velocity and angle of rotation of the bracket with respect to the chassis are obtained by using a computer simulation. Parameters of motion are displayed below.

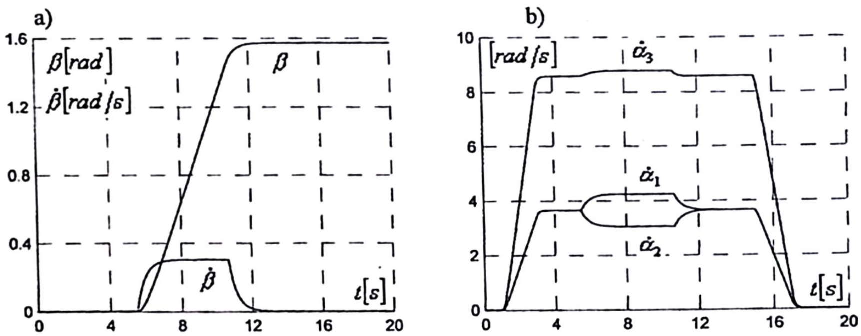

Figure 4 – Time histories of parameters of motion.

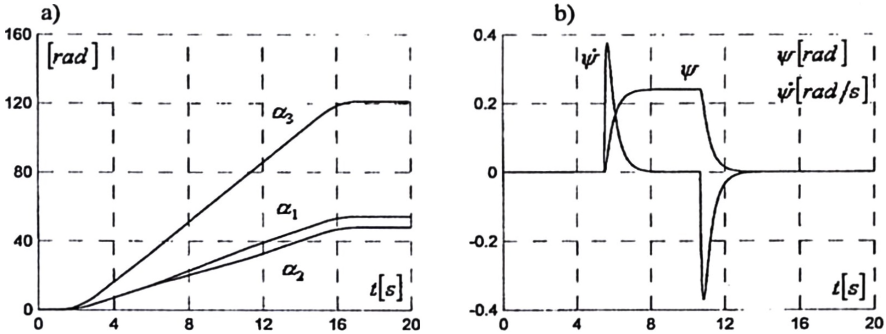

Figure 5 – Time histories of parameters of motion.

In the next case the sinusoidal path of amplitude $A_0 = 1[m]$, and period $L = 2.45[m]$ is assumed along which the point H moves.

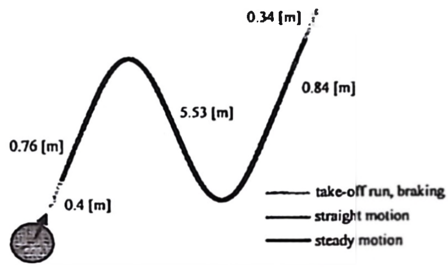

Figure 6 – The assigned trajectory of the characteristic point of the system.

Likewise parameters of the trajectory are selected in the experimental way. Simulation results are presented below.

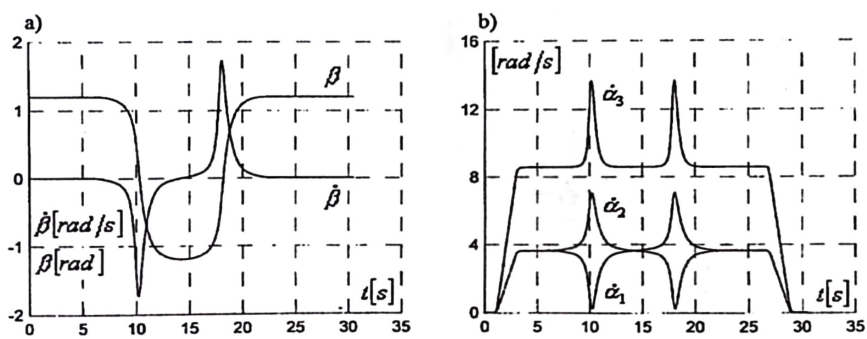

Figure 7 – Time histories of parameters of motion of the dragging system.

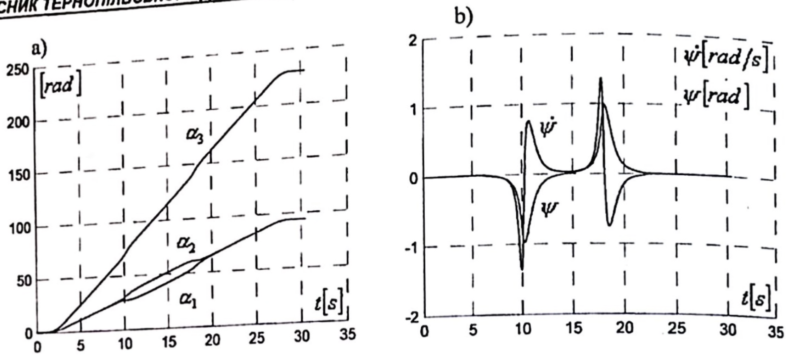

Figure 8 – Time histories of parameters of motion of the dragging system.

Computer simulation results show that the considerable qualitative and quantitative difference between the first and second presented results are appeared. Parameters of motion is needed to solve the problem of the inverse dynamics, i.e. to find the driving torques of wheels 1 and 2, respectively.

6. Conclusions

Kinematic analysis of the system is involved with problems of inverse kinematics. Motion of the analyzed system using classical equations of mechanics is described including the kinematics of the bracket and the caster wheel. The author is not aware of examples in the literature of using this method to analyze problems of inverse kinematics two wheeled mobile robot that include the bracket and the caster wheel.

In the present paper, the circular and sinusoidal trajectory of the point H of the mobile robot is considered. The trajectory planing is the important problem in the process of identification in real time. Such a trajectory should have a persistency of excitation (PE) condition of the above mentioned mobile robot. In other hand the needed trajectory should be realized by the robot.

Time histories of parameters of motion can be utilized to plan the movement trajectory system in the joint variables or to solve the problem of the inverse dynamics. Simulation results show that the considerable qualitative and quantitative difference between the first and second presented results are emerged. This consideration can be applied for kinematic description of any wheeled mobile robot model, independently of the number of wheels.

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