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DEVELOPMENT OF SLIP LINE OF PLASTICITY LOCALIZATION IN CONSTRUCTION MATERIALS

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Abstract. *Uniaxial dynamic problem for a rod made with a material with yielding peak is formulated. The model used for formulating the problem takes into account an experimental observation that during the process of yielding a slow wave exists which divides a specimen into domains of elastic and plastic behavior. Process of yielding is described assuming the yielding peak and softening behavior is connected to the process of dislocation release. Discrepancies created during this process in the form of Lüders strips are described as shear lines between structural layers (e.g. ferrite-pearlite boundary, interatomic lattice) under an applied loading in a state of plastic yielding.*

Key words: *stress, strain, yielding, plasticity, localization, shear strips, Yoffe problem.*

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1. INTRODUCTION

Experiments in [1, 2, 3, 4] show the process of localization of plastic deformation in solids in a form of Lüders strips. In empirical data [5, 6] localized band is shown to progress along the solid body at a constant rate V. However this works do not discuss the velocity this process occurs at. The solution [7] proposes analytical solution to the problem of plasticity localization, yet it is applicable to certain type of materials with strain-softening behavior under stresses that exceed elastic limit and require the segment of negative slope on the stress-strain curve.

Localized plasticity bands in soft steels are developed as the lines of internal slip between structural layers (e.g. crystal lattice, pearlite-ferrite boundary). A number of properties such as displacement discontinuity v and shear stresses at the edges plastic bands share with mode II cracks [8]. At the same time an ability to «heal» due to the reversible slip set the localized bands apart from the cracks. Steady crack propagation problem in given in [9]. The solution describes mode I crack and healing does not correspond to experimental observations in this case.

This work describes development of localization slip at a constant rate that continues research done in [10, 11]. The model proposed assumes slip to occur at the tip of the line and healing process at the end. The model exhibit properties typical for moving dislocations and allows characterizing physical processes in them.

2. PROBLEM DESCRIPTION

For modeling of development of Lüders bands in soft steel that consists of ferrite with inclusions of sustainably more solid pearlite slip η ($\eta = \gamma d$, where η is shear strain, d is characteristic size of domain under consideration) of two neighboring grains of pearlite and ferrite under shear force T ($T = \tau d$, where τ is shear stress) is considered. Considering of elastic-plastic model for the ferrite grain and brittle collapse for the pearlite grain dependency of shear stress τ between two grains from shear deformation γ can be expressed in a form of relation that generalize model of crack mode II in an elastic solid by Novozhilov [12]

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$$\tau = G(\gamma/\gamma_C)\exp(-\gamma/\gamma_C) + \tau_0(1 + a\gamma/\gamma_C)\left[1 - \exp(-\gamma/\gamma_C)\right], \quad (1)$$

where γ_C is deformation for ultimate shear stress τ_C

$$\tau_C = G\exp(-1) + \tau_0(1 + a)\left[1 - \exp(-1)\right]. \quad (2)$$

The terms in (1) correspond to disruption of pearlite grain and elastic-plastic deformation of ferrite respectively.

It follows from (1) that system of two grains may exist in three equilibrium states denoted as 1, 2 and 3 on the stress-strain curve $\tau \sim \gamma$ (Fig. 1). First state stands for an ascending slope of the stress-strain curve $\tau \sim \gamma$; second state stands for the descending slope and the third state stands for hardening. Points 2, 3 are states of stable equilibrium, while 1 is unstable. A pair of grains interacting according to a descending segment of the stress-strain curve $\tau \sim \gamma$ inevitably transits to hardening state at point 3. If all pair of grains of two contiguous layers crossings a body transformed to such state, then the whole body passed to the state of ideal plasticity. Thus, elastic body being in a state of stable elastic deformation, interaction defined by the law of descending area of stress-strain curve $\tau \sim \gamma$ may exist only locally. Hereupon it is possible to describe similar areas as the lines of displacement discontinuity in solid body or slip bands. All grains are in a state of stable interaction described by the law of ascending stress-strain curve $\tau \sim \gamma$ around these lines, thus there is no displacement discontinuities

3. MODEL DESCRIPTION

During theoretical research of equilibrium deformations of elastic-plastic bodies it is always possible to interpret a body as a continuous environment using the methods of the plasticity theory. However, it is possible to take into account not only the forms of equilibrium, when all grains interact according to the law of ascending (stable) stress-strain curve $\tau \sim \gamma$ but also the forms with displacement discontinuities with interaction occurring according to the law of descending stress-strain curve $\tau \sim \gamma$ between its edges (Fig. 1). The form and dimensions of these bands are unknown beforehand. They can be obtained from relations of elasticity theory describing the edge of each displacement band for corresponding boundary conditions, following from (1), at $\gamma > \gamma_C$

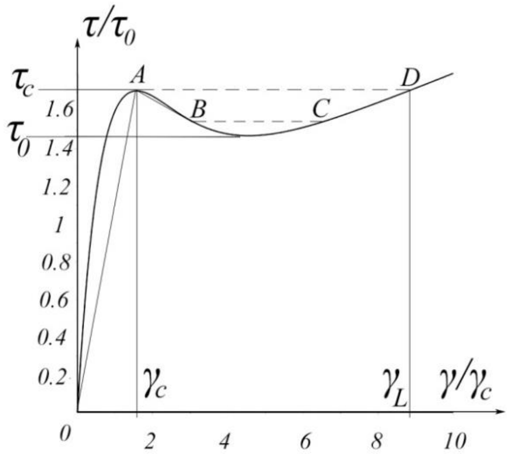

Figure 1. Stress-strain curve $\tau \sim \gamma$ according to equation (1)

Approximations according to the following assumptions are proposed since strictly formulating and solving of this non-linear problem impose certain inconveniences:

1) relation between stresses and strains on ascending (stable) segment ($\gamma < \gamma_L$), i.e. in the volume where solidity remains, is the linear Hooke's law;

2) the problem is treated as geometrically linear;

3) descending segment of the $\tau \sim \gamma$ relation is approximated in the simplest way

$$\tau = \tau_C H\left(\gamma_1 - \gamma_*\right), \left(\gamma_* = \gamma - \gamma_C, \gamma_* > 0\right), \quad (3)$$

where $H\left(x\right)$ is Heaviside step function.

Parameter γ_1 can be determined from the best approximation of stress-strain curve $\tau \sim \gamma$ as $\gamma > \gamma_C$. Determine γ_1 from the condition

$$\tau_C \gamma_1 = u, \quad (4)$$

demanding the area of an approximating curve (Fig. 1) to be zero in an interval $\gamma_C \le \gamma \le \gamma_L$. This condition is equivalent to the requirement for the approximating dependency to give the same value surface energy density. Accepted simplifications lead to linearization of all equations of the problems and enable to get its approximate solution.

The model of slip bands formation described reminds of the model dislocation motion, when the «quantum» of plastic slip is defined as the displacement of the dislocation on distance equal to length of the Burgers vector b_0. In the model it is accepted for polycrystalline material that plastic deformation develops due to the plastic slips of separate grains (crystallites) and the «quantum» of plastic strain is defined as a slip within the limits of a pair of grains δ.

Displacement discontinuity line is shown on Fig. 2. In spite of cracking processes the edges do not diverge there, thus the displacement *u* not *v* is shown. Energy independent healing on the tailing side is considered. Considering Coulomb friction and Barenblatt process domain the linearity of the problem is well imposed.

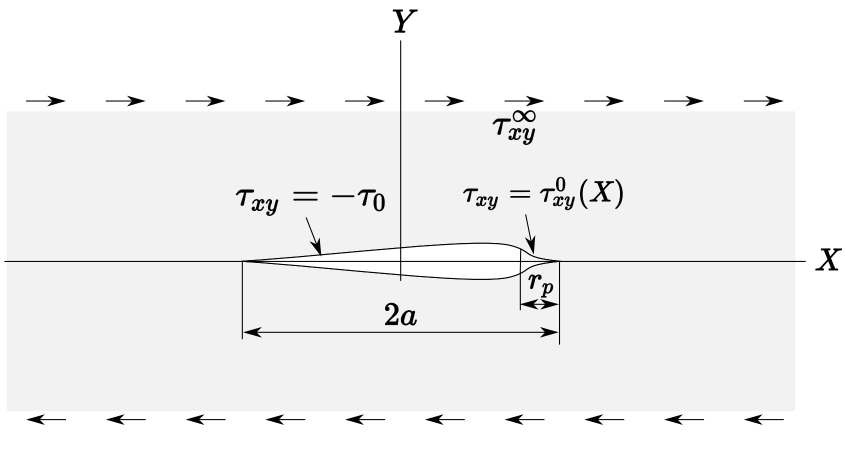

Figure 2. Yoffe problem for mode II crack

Solution for propagating slip band is determined from the static solution multiplied by the Yoffe function parameter $Y_{II}\left(\beta\right)$. Parameter β is a ratio between slip band propagation velocity and elastic wave velocity in the material $\beta = V / c_P$. It follows from the superposition principle that the shear tress reaction $\tau_{xy} = \tau_{xy}^0(x)$ on $Y = 0$ is determined from the static solution from the influence from shear stresses $\tau_{xy} = \tau_{xy}^0(x)$ multiplied by $Y_{II}(\beta)$.

In follows from model on displacement jump surface

$$\tau_{xy} = \tau_0 . \quad (5)$$

Since the equations remain the same, the solution in this case can be acquired directly from the solutions of slip problem [9]. Thus the solutions presented in [9] change to

$$\tau_{xy} = \left(\tau_{xy}^{\infty} - \tau_0\right)\sqrt{\frac{X+a}{X-a}} + \tau_0 \quad \text{for} \quad |X| > a \quad (6)$$

$$\frac{\partial u_+}{\partial x} = \frac{\left(\tau_{xy}^{\infty} - \tau_0\right) Y_{II}(\beta)}{2\left(1-k^2\right)\mu}\sqrt{\frac{a+X}{a-X}} \quad \text{for} \quad |X| < a \quad (7)$$

$$\Delta = \frac{\pi\left(\tau_{xy}^{\infty} - \tau_0\right) a Y_{II}(\beta)}{2\left(1-k^2\right)\mu} \quad (8)$$

Here static solution [13,14] is multiplied by Yoffe function $Y_{II}(\beta)$

$$Y_{II}(\beta) = \frac{2k\left(1-k^2\right)\beta^2\sqrt{k^2-\beta^2}}{R(\beta)} \quad (9)$$

Model is suitable for the description of both plane strain and plane stress. Model type is chosen by the parameter k, that is introduced as the ratio between the velocities of S and P waves:

$$k^2 = \begin{cases} (1-2\nu)/2/(1-\nu) & \text{for plane strain} \\ (1-\nu)/2 & \text{for plane stress} \end{cases} \quad (10)$$

where $R(\beta)$ is Rayleigh function

$$R(\beta) = 4k^3\sqrt{1-\beta^2}\sqrt{k^2-\beta^2} - \left(2k^2-\beta^2\right)^2 = k^4 R\left(a_P, a_S\right) \quad (11)$$

where a_S and a_P are dimensionless velocity parameters for S and P waves

$$a_P = \sqrt{1-\beta^2}, \quad a_P > 0 , \quad (12)$$

$$a_S = \sqrt{1-\beta^2 / k^2}, \quad a_S > 0. \tag{13}$$

In this case

$$\begin{aligned}\mu\gamma_L h / (\tau_0 a) &= \gamma_L / \gamma \cdot h / a \\ &= \pi\left(\tau_{xy}^{\infty} / \tau_0 - 1\right)\frac{k\beta^2\sqrt{k^2-\beta^2}}{2k^3\sqrt{1-\beta^2}\sqrt{k^2-\beta^2}-\left(2k^2-\beta^2\right)^2}\end{aligned} \tag{14}$$

It concludes from (6) that

$$K_{II} = 2\left(\tau_{xy}^{\infty} - \tau_0\right)\sqrt{\pi a} \tag{15}$$

Introduction of Barenblatt model and Barenblatt process was made for the slow propagation of mode II crack [5]. The acquired results will be used further for dynamic pre-Rayleigh case. Crack length (including the process domain) is denoted as *2a*, and shear stress at $Y = 0$:

$$\tau_{xy} = \begin{cases} \tau_0 & \text{for} \quad -a < X < a - r_P \\ \tau_{xy}^0(X) & \text{for} \quad a - r_P < X < a \end{cases} \tag{16}$$

where r_P is crack development domain length, $\tau_{xy}^0(X)$ is a value in case of smooth closing

$$\frac{1}{\pi}\int_{a-r_P}^{a}\frac{\left|\tau_{xy}^0(\xi)-\tau_0\right|d\xi}{\sqrt{a^2-\xi^2}} = \tau_{xy}^{\infty}(\xi) - \tau_0 \tag{17}$$

For dynamic pre-Rayleigh case usage of static solution [8, 9], that corresponds to neutral healing energy (so that there is no process on the tailing) concludes

$$\begin{aligned}\tau_{xy} &= \frac{X\sqrt{X^2-a^2}}{\pi|X|}\int_{-a}^{a}\frac{\tau_{xy}(\xi)d\xi}{\sqrt{a^2-\xi^2}(X-\xi)} \\ &= \frac{X\sqrt{X^2-a^2}}{\pi|X|}\int_{a-r_P}^{a}\frac{\left[\tau_{xy}^0(\xi)-\tau_0\right]d\xi}{\sqrt{a^2-\xi^2}(X-\xi)} + \tau_f \quad \text{для} \quad |X| > a\end{aligned} \tag{18}$$

$$\frac{\partial u_+}{\partial x} = \frac{\sqrt{a^2-X^2}\,Y_{II}(\beta)}{2\pi\left(1-k^2\right)\mu}\int_{a-r_P}^{a}\frac{\left[\tau_{xy}^0(\xi)-\tau_0\right]}{\sqrt{a^2-\xi^2}(X-\xi)} \quad \text{для} \quad |X| < a \tag{19}$$

for $Y = 0$, and $r_P << a$ slip on each crack side is

$$\Delta \approx \frac{T_{II}Y_{II}(\beta)}{2\left(1-k^2\right)\mu}\sqrt{\frac{a}{2}} \tag{20}$$

Equality becomes strict on the limit $r_p / a \to 0$ and then corresponds to (7), as T_{II} is Barenblatt adhesive module for mode II crack

$$T_{II} = \sqrt{\frac{\pi}{2}} K_{II} = \pi \left(\tau_{xy}^{\infty} - \tau_0 \right) \sqrt{2a} \tag{21}$$

Values T_{II} and K_{II} during dynamic crack propagation depend on β and on the history of the crack propagation

It follows from (21) that the length of slip band

$$2a = \frac{T_{II}^2}{\pi^2 \left(\tau_{xy}^{\infty} - \tau_0 \right)^2} \tag{22}$$

and slip on the crack edges

$$\Delta = \frac{T_{II}^2 Y_{II}(\beta)}{4\pi \left(1 - k^2\right) \mu \left(\tau_{xy}^{\infty} - \tau_0 \right)} \tag{23}$$

This equations depend on adhesive modulus T_{II}, over-stress $\tau_{xy}^{\infty} - \tau_0$ and the velocity. During slow slip band propagation its length is determined unambiguously when T_{II} and $\tau_{xy}^{\infty} - \tau_0$ are known. But in dynamic case even when the ratio between T_{II} and the velocity is unambiguous and known, the equations above do not lead to determination of the velocity. Band length and accumulated slip cannot be determined from the dynamic solution from the ratio of stable state only: the history of the loading and the stable state must be introduced.

Energy dissipation on the leading edge is calculated in the same way as in the case of slow crack growth, but full energy dissipation also include friction energy along the whole slip band domain. Thus energy dissipation on the unity length along the interface after the uni-axial slip impulse is moved through the domain is

$$\frac{dW}{dS} \approx 2\tau_f \Delta + \frac{T_{II}^2 Y_{II}(\beta)}{2\pi \left(1 - k^2\right) \mu} \tag{24}$$

It should be denoted as the equation does not depend on r_P. The approximation turn to equality in the limit $r_p / a \to 0$. It will be shown further that corresponding equation for energy dissipation for pre-elastic wave crack velocity depends on r_P / a and disappears at the limit $r_p / a \to 0$.

The simple result is acquired for the ratio w_{pr} between the energy dissipation in the slip process domain (heading edge of the slip band) and full energy dissipation

$$w_{pr} = \left(\tau_{xy}^{\infty} - \tau_0 \right) / \tau_0 \tag{25}$$

In case of non-uniform material slip where weak interaction between band edges is considered, healing modulus H_{II} should not be omitted. Changes for non-zero value H_{II} are not drastic [15,16]. Thus the substitution $T_{II} \to \left(T_{II} - H_{II}\right)$ should be made in equations (20) and (22), and substitution $T_{II}^{2} \to \left(T_{II}^{2} - H_{II}^{2}\right)$ in (23) and (24). However (25) remains the same. Equation (21) should be replaced with

$$T_{II} = \sqrt{\frac{\pi}{2}} K_{II} \tag{26}$$

$$T_{II} - H_{II} = \pi\left(\tau_{xy}^{\infty} - \tau_{0}\right)\sqrt{2a} \tag{27}$$

The described approach and achieved solutions line allow describe for development of localization slip lines in plasticity states of materials with yielding and strain-softening. The results might be used to estimate stability and loss of structural stability for structures that use this kind of materials.

4. CONCLUSIONS

1. Solution of the dynamic problem for a material with yielding peak was determined. The model takes into account existence of a slow wave that divides material into domains of elastic and plastic behavior. Estimates for the velocity of the slow wave are noted.

2. Solutions for slip bands propagation in complex shift conditions are proposed. The solution is built by solving a static problem and then transforming the resulting equations by functions of the special kind that introduce time parameter into them.

3. The method under consideration can be applied to a variety of problems of localization bands dynamics and problems of slip bands propagation under other external conditions can be solved.

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РОЗВИТОК ЛІНІЇ ПЛАСТИЧНОЇ ЛОКАЛІЗАЦІЇ У КОНСТРУКЦІЙНИХ МАТЕРІАЛАХ

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Резюме. *Розглянуто особливості поведінки конструкційних матеріалів з майданчиком плинності в умовах пластичної деформації. З експериментів відомо, що перехід від пружного стану до стану пластичності відбувається в усьому об'ємі зразка поступово. При цьому в матеріалі виникають дві області з різними механічними властивостями та певною границею між ними, яка розповсюджується з визначеною швидкістю. Для моделювання такої поведінки сформульовано задачу динаміки для матеріалу, що має пік-зуб на діаграмі одновісного розтягу, та побудовано її розв'язок. Прийнято гіпотезу про зв'язок поведінки матеріалу на піку-зубі на початку пластичної течії та подальшого різкого падіння напружень із вивільненням дислокацій. Неоднорідності, які при цьому виникають у вигляді смуг Людерса, розглядаються як смуги зсуву структурних шарів (наприклад, зерен фериту та перліту для сталей, або шарів кристалічних ґраток у загальному випадку) під впливом зовнішнього навантаження в стані пластичності. Поведінка смуги зсуву при пластичній деформації є подібною до розвитку тріщини, а розв'язок поставленої задачі має особливості, подібні до поведінки рухомих тріщин у неоднорідних матеріалах, що описується моделлю Йоффе. Як і рухомі тріщини в моделі Йоффе, смуга локалізації також має передній фронт, де відбувається процес відривання, та фронт, де відбувається процес закриття («загоєння»). Для спрощення моделі використано особливості поведінки смуг зсуву, що є відмінними від процесів у тріщинах. Зокрема, процес «загоєння» лінії зсуву відрізняється від процесу розриву тим, що енергія при «загоєнні» вивільнюється, а не розсіюється. Однак кількість випромінюваної енергії набагато менше енергії розсіювання на передньому фронті. Отже, значенням випромінюваної енергії можна знехтувати, тоді процес «загоєння» стає однозначно визначеним. Отриманий розв'язок дозволяє зробити висновок про існування повільної хвилі, що визначає рух фронту пластичної деформації, та оцінити швидкість цієї хвилі. Запропонований підхід дозволяє описувати поведінку конструкційних елементів різної форми, які виготовлені з матеріалів з майданчиком плинності, та оцінювати їхнє подальше зміцнення й прогнозувати втрату стійкості.*

Ключові слова: *напруження, деформації, текучість, пластичність, локалізація, смуги зсуву, задача Йоффе.*

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