

# Mathematical, algorithmic and software support for signals wavelet detection in electronic communications

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## Abstract

In the work based on wavelet processing of signals in the Mexican HAT mother base, a mathematical (method) and algorithmic support for wavelet detection of useful signals in electronic communications against the background of noises was developed. 3D and 2D (averaged 3D) wavelet spectra, taking into account scale and shift indicators, are used as wavelet features for signal detection. Wavelet software for detecting useful signals in electronic communications has been developed in the Matlab environment. It is established that the developed software provides reliable wavelet detection of signals in electronic communications by wavelet spectra with the Mexican HAT basis function, which quantitatively and visually reflect the presence/absence of useful signals in electronic communications with noises.

## Keywords

Signal, mathematical and algorithmic support, software, wavelet detection, electronic communication.

## 1. Introduction

Signals (carriers of useful information) that are transmitted and received through electronic communications are always subject to various noises. This can lead to a complete distortion of signals, rendering them unrecognizable [1-9]. The processing of such distorted signals or the absence of a useful component in their structure (significant predominance of noises power over the level of the useful signal) can lead to incorrectness and unreliability of further processed results and, as a result, decisions made. Therefore, an important task for scientists in the field of electronic communications research is the process of reliable detection of useful signals as the first stage of further processing of the signal with an accurate determination of the useful signal in its composition. Such detection is necessary

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*CITI'2024: 2nd International Workshop on Computer Information Technologies in Industry 4.0, June 12–14, 2024, Ternopil, Ukraine*

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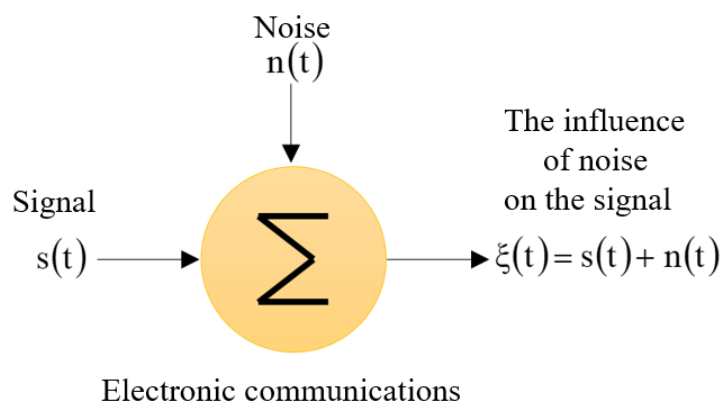
to avoid the process of further processing of the detected signal in case of absence of a useful component in its composition.

Fundamental researchers in the field of signal detection are scientists, in particular L. Rabiner, B. Gould, V.A. Kotelnikova, V.I. Tikhonov and others.

To detect signals, computer tools are used, in particular their software, which is implemented on mathematical support (processing methods), in particular: correlation (V.I. Tikhonov), cross-correlation (V. Kulakova), statistical (Zubakov V.D., Vaishtein L.A.), neural networks (Tsybaliuk I., Horbatiy I.) [10], statistical method (Myasnikov E.N., Zaboronkova T.M., Kogan, L.P.) [11], synphase/component (Khvostivska L.V., Khvostivskiy M.O., Dediv I.Yu., Koval L.M. [12-15]), wavelet processing in the Morlet basis (Khvostivska L.V., Kazmiriv V.V., Remez A.V. [16]). All of the mentioned methods, except for wavelets, do not make it possible to investigate fluctuation processes in the structure of the studied signals, which is relevant for the detection and subsequent recognition of useful signals in electronic communications with noises when taking into account the parameters of time variability and time shifts.

## 2. Mathematical support of signal detection in electronic communications

In electronic communications of the linear type, signals are described by an additive image, which is shown in Figure 1.

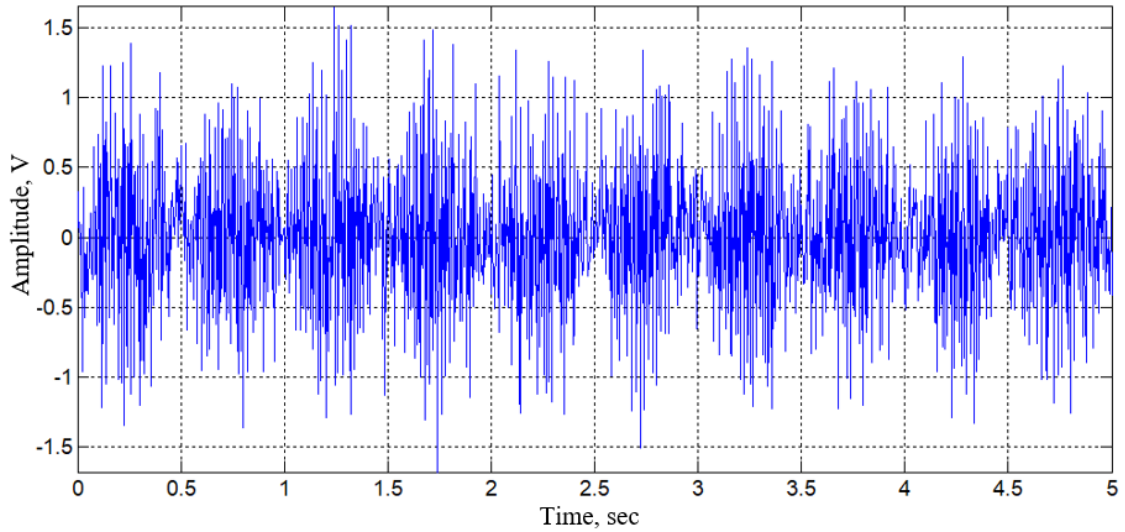


**Figure 1:** Additive signal model in electronic communications with noises.

The image of the signal model (Figure 1) provides a description of such signals in electronic communications  $\xi(t)$  through the summation of noises  $n(t)$  that are formed by external/internal (hardware) influences.

To detail the parameters of signals in distorted environments, an analytical method was used in the study of variations in the values of real signals [21, 22]. This allows the correct mathematical description of signals as a basis for the development of an effective method of detecting useful signals in electronic communications with noises.

Figure 2 shows an amplitude modulated (AM) signal with noise.



**Figure 2:** AM signal with noise.

The AM signal is characterized by randomness due to noises and repeatability of the modulation process during transmission. The amplitude values of the signal vary over time with the value of a certain deviation in relation to the average value of the useful signal, which leads to a fluctuation phenomenon, as can be seen in Figure 2. If you study the fluctuation phenomenon of the signal in time space, then, accordingly, you can quickly detect useful signals in an interfering environment.

The signal detection method should constructively take into account the fluctuation indicators of the studied signals in different time scales. These requirements are met by methods of wavelet transformation with different bases, which constructively provide the process of studying fluctuations in the signal structure at different time scales.

During wavelet processing, a scaling parameter is used to cover the short waves of the entire time range. This allows you to analyze fluctuations in the signals and detect changes in the time shift within the parent basis function.

The core of the wavelet signal detection method by its wavelet processing is based on the following expression [17]:

$$C(a,b) = \frac{1}{\sqrt{a}} \sum_{t=0}^{t_{\max}} \xi(t) \psi(t, a, b), \quad (1)$$

where  $b$  – shift;  $a$  – scale;  $\frac{1}{\sqrt{a}}$  – normalizing coefficient;  $\psi(t, a, b)$  – the mother basis function, which provides the definition and study of the fluctuation process in the signal, which is extremely relevant for the detection of a useful signal in electronic communications with noises (wavelet detection procedure).

Well-known basic wavelet functions, including Gauss, Haar, Mexican HAT, Morle, and a number of others [3, 18-20], are quite often used for continuous wavelet processing of

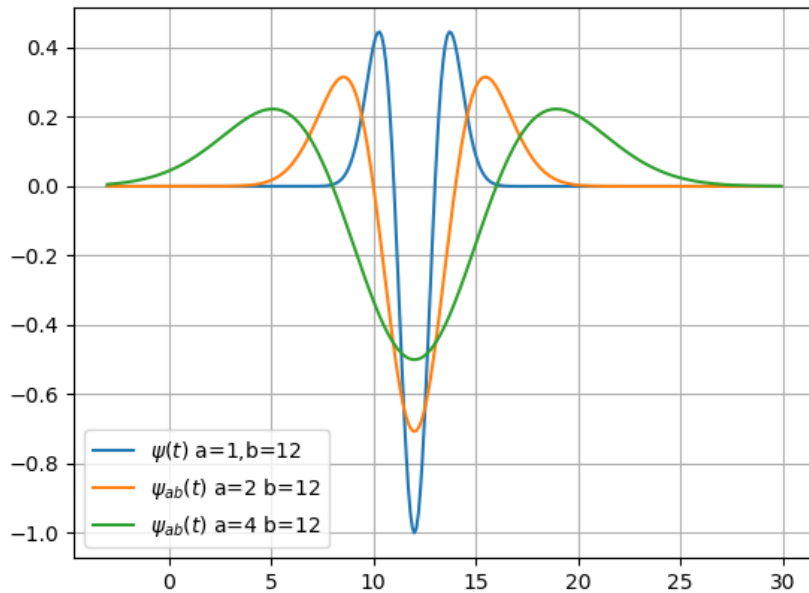
various signals. Mexican HAT and Morlet as parent functions are structurally similar to real modulated signals, taking into account the indicators, so the use of these functions for correlation from signals is reasonable.

When choosing the parent function of signal processing for the purpose of obtaining wavelet estimates for their detection in electronic communications with noises, preference is given to the use of the basic Mexican HAT parent function. The process of variability of the selectivity of the Mexican HAT base in the frequency domain guarantees the presence of dominant frequencies in the signals, which is subject to modulation.

The Mexican HAT function is formed by double differentiation of the Gaussian function:

$$\psi(t) = \left(1 - \left(\frac{t-a}{b}\right)^2\right) e^{-\frac{(t-a)^2}{2}}. \quad (2)$$

An example of the implementation of the Mexican HAT is shown in Figure 3.

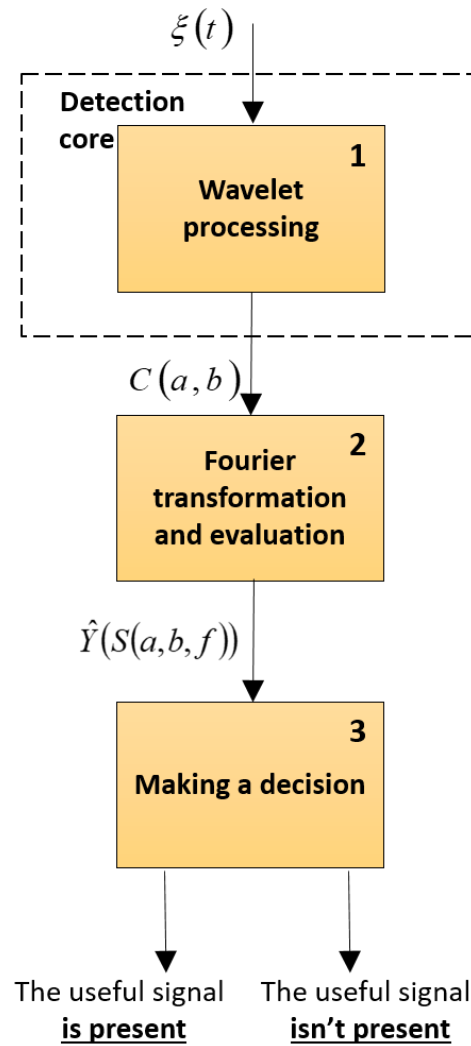


**Figure 3:** View of the mother function of the basis of the Mexican hat (Mexican HAT).

Mexican HAT is structurally similar to modulated signals, so their correlation is relevant for wavelet detection.

### 3. Algorithmic support for signal detection in electronic communications

Figure 4 shows the wavelet algorithm for detecting signals in electronic communications with noises based on the wavelet transform.



**Figure 4:** The structure of wavelet signal detection.

The following operations are present in the structure shown in Figure 4:

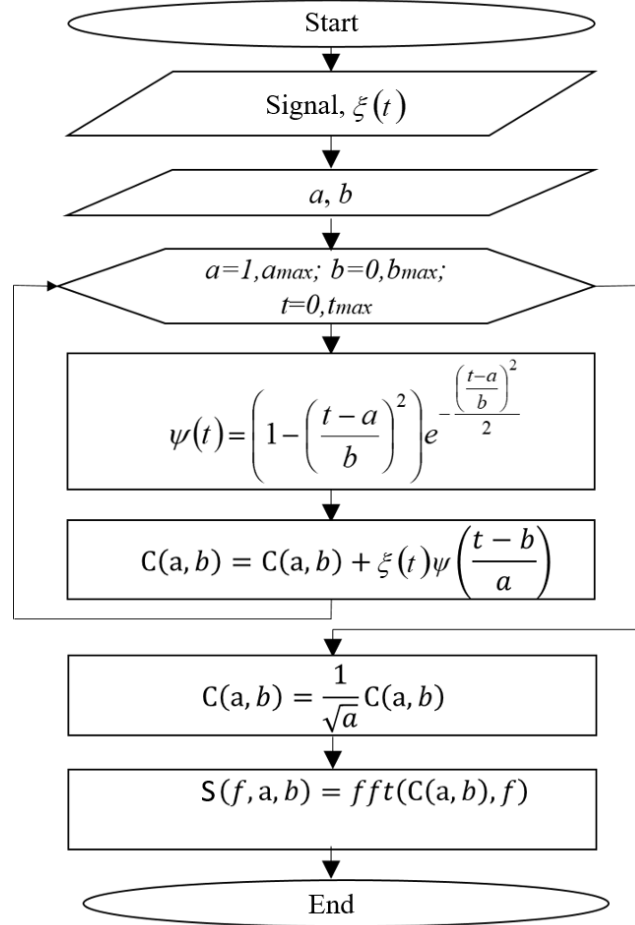
- Wavelet processing of signals in interfering electronic communications is carried out according to expressions (1-2) in order to calculate features of wavelet detection  $C(a,b)$ , which characterize fluctuating processes in signals of different time scales.
- Calculation of the Fourier transform (transition to the invariant region)  $S(a,b,f)$  and evaluation of features of wavelet detection of signals  $\hat{Y}(S(a,b,f))$ ;
- The process of making decisions based on morphological analysis or visual analysis regarding the presence of a useful signal in electronic communications with noises.

When estimating values of wavelet spectra of signals  $S(a,b,f)$ , the procedure of averaging values by time shifts was applied, using the expression

$$\hat{Y}(S(a,b,f)) = M_b \{S(a,b,f)\}. \quad (3)$$

where  $M_b$  - operator of averaging on shift  $b$ .

When using wavelet processing expressions in the Mexican HAT base, an algorithmic support for signal processing has been developed for its detection in the form of an algorithm, which is shown in Figure 5.



**Figure 5:** Wavelet signal detection algorithm with Mexican HAT base function

The wavelet signal detection algorithm (Figure 5) includes the following stages:

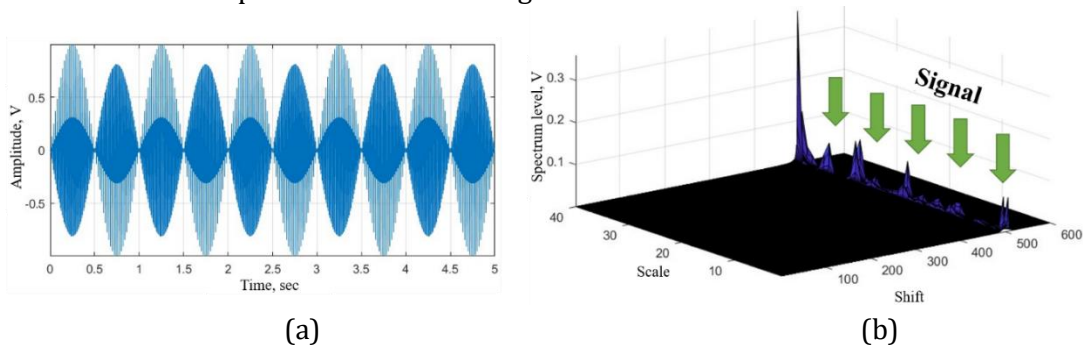
1. Loading the signal  $\xi(t)$ .
2. Enter the values of scale coefficients  $a=1, a_{max}$ , time shift  $b=1, b_{max}$  and time range limits given through the numerical sequence  $t=0, t_{max}$ .
3. Calculation of frequency  $\omega$  and values of mother function Mexican HAT  $\psi(t)$ .
4. Calculation in a cycle of coefficients-wavelet  $C(a, b)$  with values  $a, b, t$ .
5. Application of Fourier to process wavelet coefficients for the purpose of transition to the frequency space  $S(a, b, f)$ .

This process obtains wavelet coefficients for given signal parameters and uses a Fourier tool for frequency analysis.

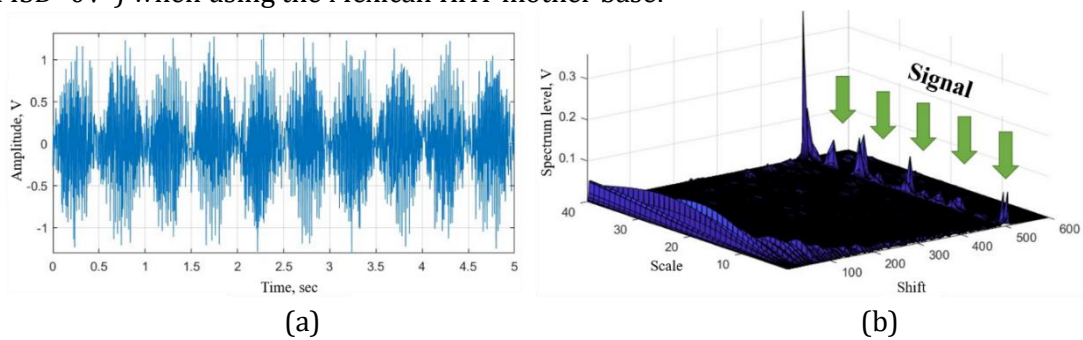
The detection algorithm provides the signal processing process when applying the Mexican HAT mother function. This makes it possible to study signal fluctuations in time and frequency spaces in 3D projection during its wavelet detection. This approach reproduces all the structural variations of the signal in the conditions of interfering electronic communications, which serve as an indicator of the detection of a useful signal among various noises.

#### 4. Wavelet detection results of signal detection

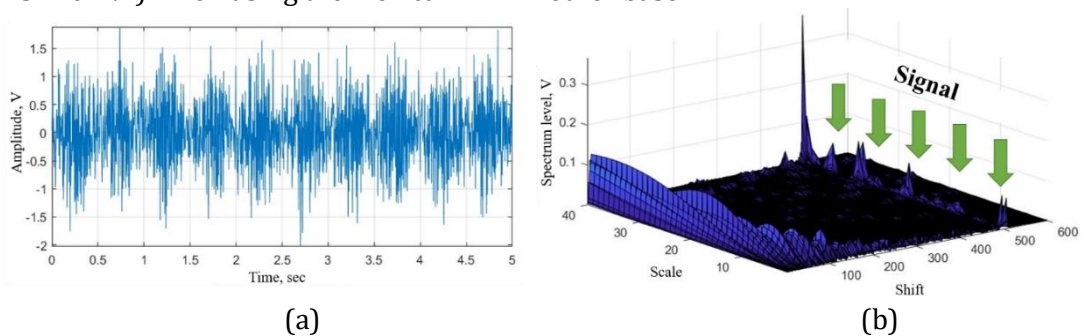
The results of wavelet detection of signals in the mother in the base of the Mexican HAT under the influence of noises with root mean square deviation (RMSD) at the levels of 0-0.6V<sup>2</sup> in the form of spectra are shown in Figure 6-9.



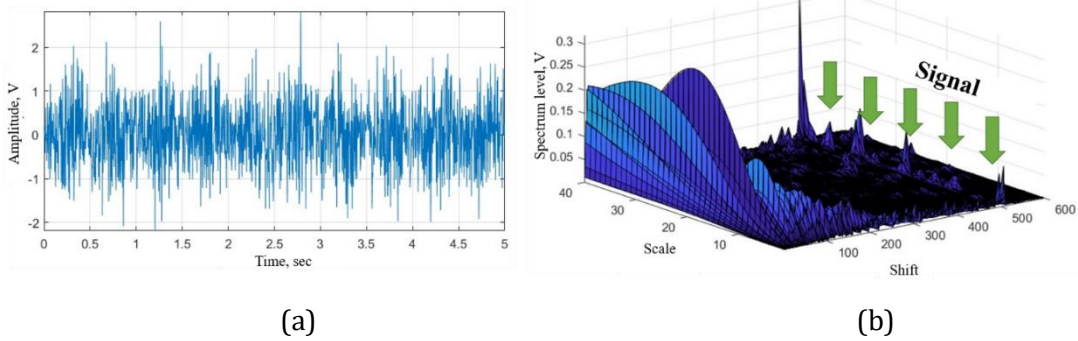
**Figure 6:** 3D spectra (b) wavelet of signal detection (a) (without the influence of noise with RMSD=0V<sup>2</sup>) when using the Mexican HAT mother base.



**Figure 7:** 3D spectra (b) wavelet of signal detection (a) (influence of noise with RMSD=0.2V<sup>2</sup>) when using the Mexican HAT mother base.



**Figure 8:** 3D spectra (b) wavelet of signal detection (a) (influence of noise with RMSD=0.4V<sup>2</sup>) when using the Mexican HAT mother base

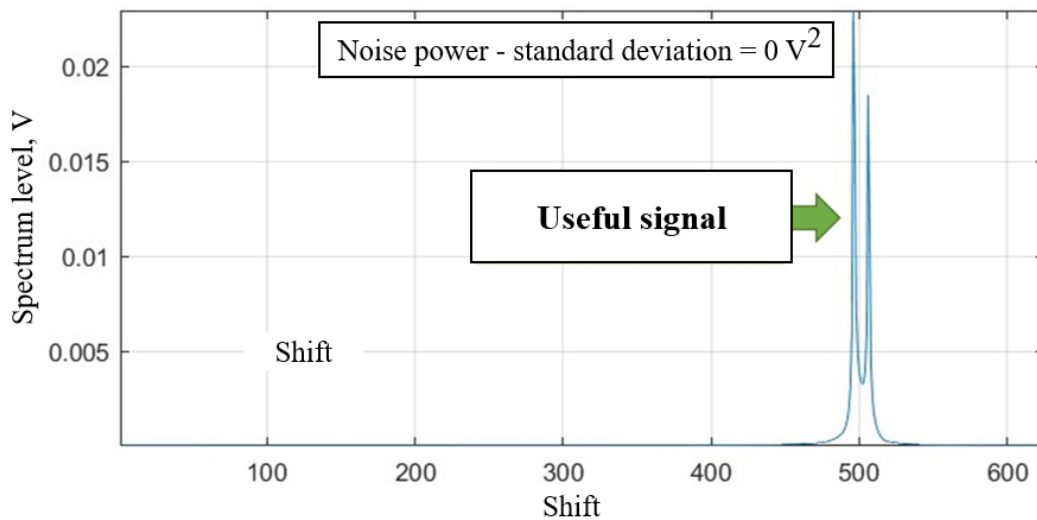


**Figure 9:** 3D spectra (b) wavelet of signal detection (a) (influence of noise with  $RMSD=0.6V^2$ ) when using the Mexican HAT mother base

Based on the results of wavelet detection (Figure 6-9), it can be determined that the component of the wavelet spectrum of the useful signal is clearly localized in electronic communications with noises with different levels of RMSD of noises (from 0.2 to  $0.6 V^2$ ) per 500 displacement (shift) on all numerical scales.

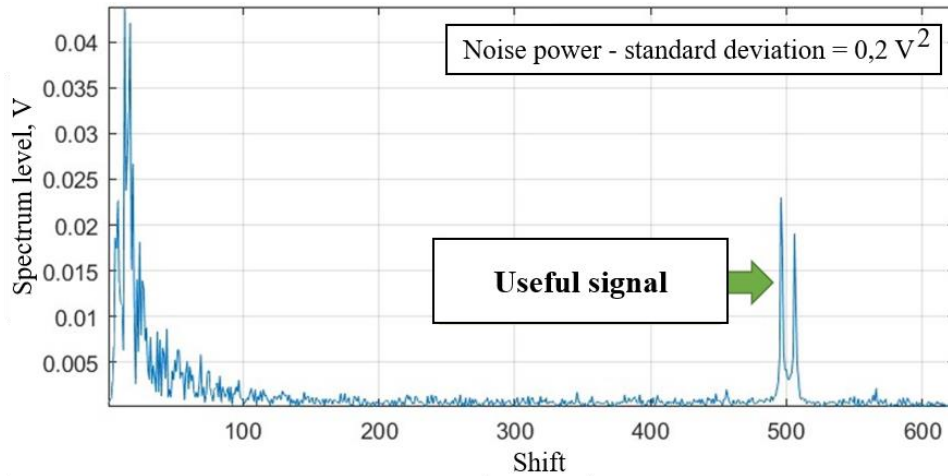
The calculated spectra numerically and visually make it possible to draw a conclusion about the presence or absence of a signal in electronic communications based on a clearly localized spectrum of the useful signal.

When evaluating the 3D presentation of the  $S(a,b,f)$  signal spectra, statistical processing was applied to them, in particular, the averaging procedure by time shifts (shifts). Statistically evaluated 3D representation of the signal spectra at the RMSD level of noises 0- $0.6 V^2$  is shown in Figure 10-13.

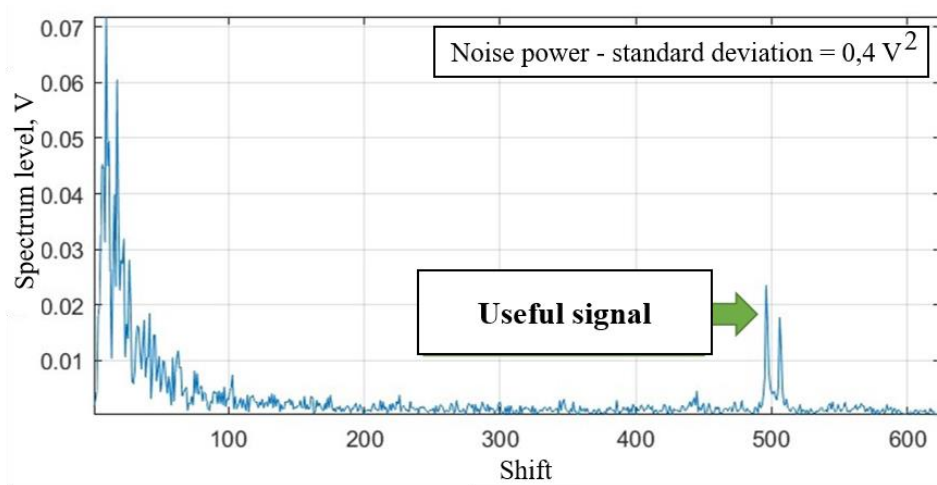


**Figure 10:** Average values of the 3D signal spectrum (noise power -  $0 V^2$ ).

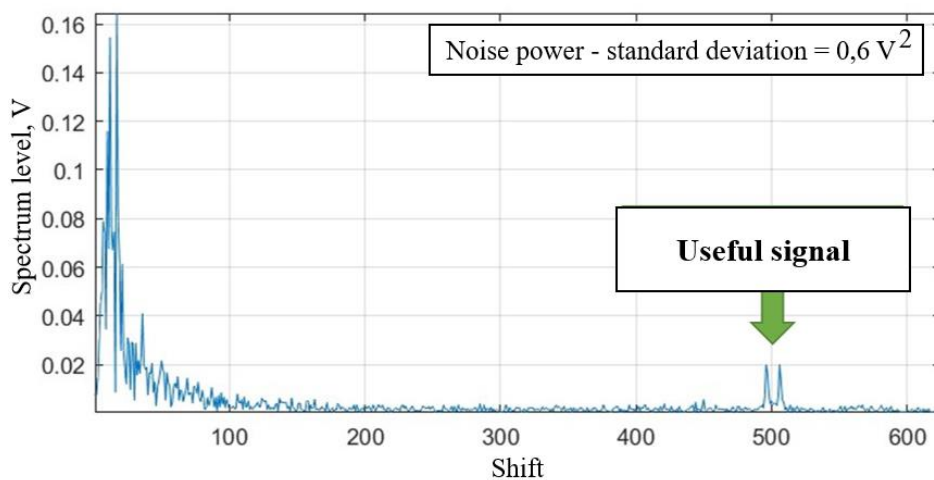




**Figure 11:** Average values of the 3D signal spectrum (noise power – 0.2 V<sup>2</sup>).



**Figure 12:** Average values of the 3D signal spectrum (noise power – 0.4 V<sup>2</sup>).



**Figure 13:** Average values of the 3D signal spectrum (noise power – 0.6 V<sup>2</sup>).

The averaged realizations of wavelet spectra (Figure 10-13) allow for a more detailed comparative analysis of data in comparison with 3D representations of spectra. This guarantees the effectiveness of the reliable wavelet detection procedure in signals in electronic communications with noises.

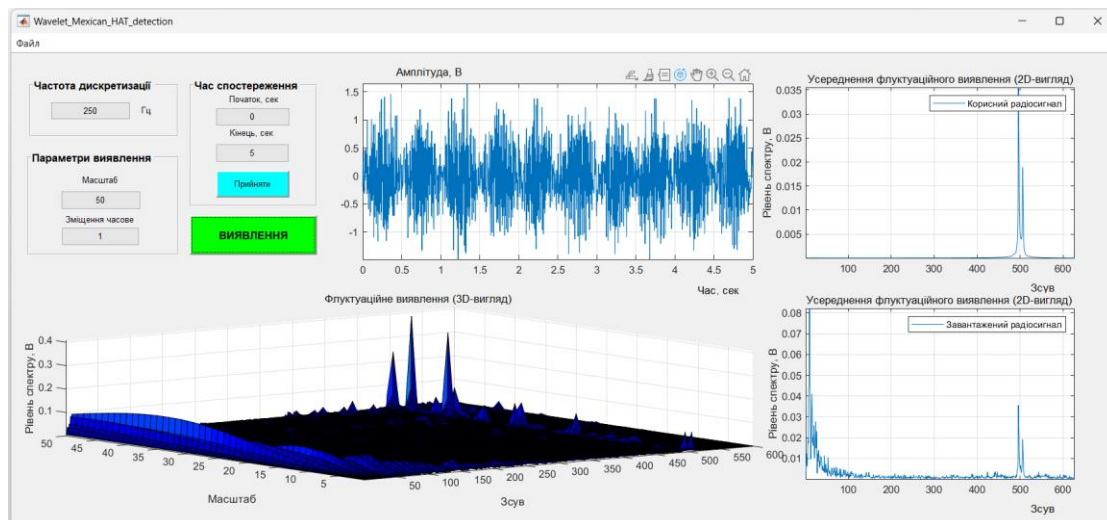
The averaged spectra (Figure 10-13) (2D view) allow for a more detailed comparative characterization of the calculated data in comparison with 3D views of the spectra, which ensures the effectiveness of the procedure for reliable and accurate wavelet detection of signals in electronic communications with noises.

The reliability indicator is provided due to non-shifting (non-displacement) and stability of the spectrum structure for different levels of noise RMSD. This testifies to their invariance and informativeness, which confirms the suitability of new features for wavelet detection of signals in electronic communications with noises.

The developed software enables reliable and accurate wavelet detection of signals in interfering electronic communications by 3D and 2D representations of spectra calculated as a result of wavelet processing with the Mexican HAT basis function.

## 5. Automated wavelet signal detection software in electronic communications

The use of the GUIDE utility in the MATLAB environment made it possible to develop automated wavelet signal detection software with noises in electronic communications. The software interface and the detection result are shown in Figure 14.



**Figure 14:** The interface of the automated software for detecting jammed signals in electronic communications and the result of its operation.

The developed software provides a correct, reliable and accurate process of automated wavelet detection of a useful signal in electronic communications with noise.

## 6. Conclusions

The implemented mathematical, algorithmic and software for wavelet detection of useful signals in interfering electronic communications based on wavelet processing with the mother base function Mexican HAT (Mexican hat) made it possible to increase the number of features for detecting the type of wavelet spectra of 3D and 2D types that reflect the fluctuating processes of signals and visually and quantitatively indicate the presence/absence of useful signals against the background of noise of different power.

## References

- [1] Melvin W L and Scheer J A 2013 Principles of Modern Radar: Advanced Techniques (New York:SciTech Publishing, IET, Edison). P. 846.
- [2] Richards M A, Scheer J A and Holm W A 2010 Principles of Modern Radar: Basic Principles (New York: SciTech Publishing, IET, Edison). P. 924.
- [3] Kolumbán G., Krébesz T. Chaotic Communications with Autocorrelation Receiver: Modeling, Theory and Performance Limits. Intelligent Computing Based on Chaos, 2009, SCI 184, pp. 121–143. doi: 10.1007/978-3-540-95972-4\_6.
- [4] Laptiev O., Polovinkin I., Vitalii S., Stefurak O., Barabash O. and Zelikovska O. The Method of Improving the Signal Detection Quality by Accounting for Interference. 2020 IEEE 2nd International Conference on Advanced Trends in Information Theory (ATIT), 2020, pp. 172-175. doi: 10.1109/ATIT50783.2020.9349259.
- [5] Yu.É. Korchagin, K.D. Titov Detection of an Ultra-Wideband Quasi Radio Signal with Unknown Duration Against the Background of White Noise. Radiophysics and Quantum Electronics volume 61, pp.853–866 (2019). DOI: 10.1007/s11141-019-09942-5.
- [6] Trifonov A.P., Korchagin Yu.É., Trifonov M.V.. Detection of Radio Signals with Unknown Duration, Amplitude, and Initial Phase. Radiophysics and Quantum Electronics, Volume 58, Issue 5, pp.361-372. October 2015. doi: 10.1007/s11141-015-9610-5.
- [7] Xue Y, Li X, Xu L and Ren Y 2012 Research on position differential method of dual-satellitesTDOA and FDOA in passivelocation system. IEEE International Frequency Control Symposiumpp 1–5.
- [8] Proakis J G and Manolakis D G 2007 Digital Signal Processing: Principles, Algorithms and Applications (New Jersey: Pearson Prentice Hall) p. 1084.
- [9] Wu R, Zhang Y, Huang Y, Xiong J and Deng Z 2018 A Novel Long-Time Accumulation Methodfor Double-Satellite TDOA/FDOA Interference Localization. Radio Science 53 129–142.
- [10] Tsybaliuk, I., & Horbatiy, I. (2023). Approach to processing radio signals with amplitude modulation of many components using one-dimensional convolutional neural network. *Eastern-European Journal of Enterprise Technologies*, 6(9 (126), 14–22. <https://doi.org/10.15587/1729-4061.2023.292854>.
- [11] Myasnikov, E.N., Zaboronkova, T.M., & Kogan, L.P. (2021). ZaboronkovaL Kogan. Detection of radio signals against the background of strongelectromagnetic noise in transport. *Journal of Physics: Conference Series Journal of Physics Conference Series*. 2131(5):052046. DOI: 10.1088/1742-6596/2131/5/052046.
- [12] Khvostivska L.V., Koval L.M. Detection of useful radio signals as periodically correlated stochastic processes under conditions of a priori uncertainty. Materials of the 4th All-Ukrainian scientific and practical Internet conference of students, postgraduates and young scientists on the topic "Modern computer systems and networks in

management": a collection of scientific works / Edited by G.O. Rayko. Kherson: Publishing House FOP Vyshemirskiy V.S., 2021. P.133.

- [13] Khvostivska L., Khvostivskyy M., Dunets V., Dediv I. Mathematical and Algorithmic Support of Detection Useful Radiosignals in Telecommunication Networks. Proceedings of the 2nd International Workshop on Information Technologies: Theoretical and Applied Problems (ITTAP 2022). Ternopil, Ukraine, November 22-24, 2022. P.314-318. ISSN 1613-0073.
- [14] Khvostivska L., Khvostivskyy M., Dunets V., Dediv I. (2023) Mathematical, algorithmic and software support of synphase detection of radio signals in electronic communication networks with noises. Scientific Journal of TNTU (Tern.), vol 111, no 3, pp. 48-57. ISSN 2522-4433.
- [15] Khvostivska L., Khvostivskyy M., Dediv I., Yatskiv V., Palaniza Y. Method, Algorithm and Computer Tool for Synphase Detection of Radio Signals in Telecommunication Networks with Noises. Proceedings of the 1st International Workshop on Computer Information Technologies in Industry 4.0 (CITI 2023). CEUR Workshop Proceedings. Ternopil, Ukraine, June 14-16, 2023. P.173-180. ISSN 1613-0073.
- [16] Khvostivska L.V., Kazmiriv V.V., Remez A.V. (2022) Wavelet processing of radiosignals for the problem of their detection against the background of interferences [Вейвлет обробка радіосигналів для задачі їх виявлення на фоні завад]. XI International scientific and practical conference of young researchers and students «Current issues in modern technologies» (Tern., 7-8 December 2022), pp. 119-120. [in Ukrainian].
- [17] Ali N. Akansu, Richard A. Haddad. Chapter 6 - Wavelet Transform. Editor(s): Ali N. Akansu, Richard A. Haddad. Multiresolution Signal Decomposition (Second Edition). Academic Press, 2001. Pages 391-442. ISBN 9780120471416. <https://doi.org/10.1016/B978-012047141-6/50006-9>.
- [18] Singh, A., Rawat, A., Raghuthaman, N. (2022). Mexican Hat Wavelet Transform and Its Applications. In: Singh, J., Dutta, H., Kumar, D., Baleanu, D., Hristov, J. (eds) Methods of Mathematical Modelling and Computation for Complex Systems. Studies in Systems, Decision and Control, vol 373. Springer, Cham. [https://doi.org/10.1007/978-3-030-77169-0\\_12](https://doi.org/10.1007/978-3-030-77169-0_12).
- [19] Mohammed A. Abdala. Genetically Based Wavenets for System Modeling. (2008). Journal of Engineering and Sustainable Development, 12(3), 120-132. <https://jeasd.uomustansiriyah.edu.iq/index.php/jeasd/article/view/1685>.
- [20] Yan, Y., Zhou, M., Zhang, D. et al. Scale-invariant Mexican Hat wavelet descriptor for non-rigid shape similarity measurement. Sci Rep 13, 2518 (2023). <https://doi.org/10.1038/s41598-023-29047-4>.
- [21] Martsenyuk, V., Sverstiuk, A., Klos-Witkowska, Kozodii, N., Bagriy-Zayats, O., Zubenko, I. Numerical analysis of results simulation of cyber-physical biosensor systems. CEUR Workshop Proceedings, 2019, 2516, pp. 149-164.
- [22] Martsenyuk, V., Klos-Witkowska, A., Sverstiuk, A. Stability Investigation of Biosensor Model Based on Finite Lattice Difference Equations. Springer Proceedings in Mathematics and Statistics, 2020, 312, pp. 297-321. [https://doi.org/10.1007/978-3-030-35502-9\\_13](https://doi.org/10.1007/978-3-030-35502-9_13).