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# INFLUENCE OF MATERIAL MICROSTRUCTURE ON FRACTURE DEVELOPMENT IN DEFORMABLE BODIES

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***Summary.*** *The problem of predicting the location and direction of local fracture and estimating the ultimate load of composite material is one of the practically important problems of fracture mechanics. With a wide variety of composite materials, it is natural to expect the presence of a large number of failure criteria, each of which should be valid for a certain class of materials and reflect most completely its physical and mechanical properties, structural defects, operating conditions, etc. The choice of effective assessment of the bearing capacity of modern composite material under extreme conditions is at present the main task of researchers. In this paper, the criterion of fracture of composite materials based on simple and quite effective theory of macrostresses by M.Ya. Leonov and K.M. Rusynka is preferred. The macrostress theory effectiveness is shown in the papers by V.V. Panasiuk, L.T. Berezhnytskyi, S.Ya. Yarema, L.V. Ratych, and M.H. Stashchuk. In our paper, we clarify and extend the advantages of macrostress concept criterion in comparison with other fracture criteria in deformable bodies. Examples of crack propagation in the process of body tension are given in this paper.*

***Key words:*** *Strength, macrostress theory, crack propagation resistance in the material ($K_{1c}$), macrostress concentration coefficient $K_М$, average technical strength of the material ($\sigma_в$), Poisson's ratio ($\nu$), structural parameter ($\rho$) and crack (defect) length (2l).*

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**Statement of the problem.** In modern conditions, while improving methods for calculating real structural materials and products made of them, it is necessary to take into account the most completely and adequately their physical and mechanical properties and microstructure as well. All real materials have certain structural heterogeneity (defects and imperfections in the crystal lattice, microcracks, pores, microinclusions, scratches, marks, etc.). The existing inhomogeneities of the solid deformable body are of different origin. For example, technological defects in the structure occur during the process of manufacturing of both the material and the product. Various microdefects occur during the structure operation under the environment influence (exposure to active liquids and gases, radioactive irradiation, thermal effects, etc.) There are structural defects (macro defects) in products and parts. This is a purposeful sharp change in the shape of the body (for example, cuts, darts, protrusions, fillets, holes, channels, etc.). The microinhomogeneities available in the body are randomly distributed and, similarly to structural stress concentrators, also cause disturbances in the stress field. Therefore, in some materials (e.g., gray cast iron, concrete) with corresponding level of structural inhomogeneity, artificially created concentrators of a certain sharpness, as well as defects such as cracks and sharp-edged inclusions, can have little or almost no effect on strength. Thus, taking into account the geometry of macrodefects (various constructive, artificially created stress concentrators, etc.), it is necessary at the same time to take into account the real structure (macrodefects and microinhomogeneities) of the matrix material while studying the limit state of solid deformed bodies.

**Analysis of available investigation results.** One of the options which takes into account the micro-inhomogeneities of the material structure is based on the application of relatively simple and sufficiently effective theory of macrostresses by M. Ya. Leonov and K. M. Rusynka. According to this theory, a body is called macrohomogeneous in a certain area, if the mechanical properties of any elementary volume, conditionally cut out of the specified area, are the same. According to this model, a solid body is considered to be continuous medium, around each point of which it is possible to distinguish such minimal volume $V_0$, which still has (on the basis of statistical data) mechanical properties determined during the usual investigations of macrobodies. The sphere with radius $\rho$, which is taken as the material structural parameter, is chosen as such volume $V_0$ [1–3]. Its value depends on the structural microinhomogeneities of the material, their size, type and distribution density. For materials with more coarser-grained and heterogeneous structure, the value of the structural parameter $\rho$ will be larger, and vice versa, for materials with smaller microinhomogeneities, the value $\rho$ will be smaller. According to the macrostresses theory, the elastically deformed state of the body is determined by the macroelongation $\varepsilon_M$ and macroextension $\varepsilon_C$, averaged over the middle of the sphere with radius $\rho$, and their corresponding macrostresses $S$ according to Hooke's law. The effectiveness of the macrostress theory is shown in the papers by V. V. Panasyuk, L. T. Berezhnytskyi, S. Ya. Yarema, L. V. Ratych, M. H. Stashchuk [4–7]. The concept of macrostresses was used in the development of the analytical method for determining the effectiveness of stress concentration factors in papers [4–6], where cases with stress concentrators under symmetrical loading were considered. The theory of macrostresses was used to solve various problems under symmetrical body loading.

**Statement of the task.** Let us use the theory of macrostresses to the deformed solid body being in complex tense state. Suppose the unlimited elastic body is under plane deformation and contains a crack or sharp-edged inclusion (a set of defects that do not interact with each other) with $2l$ length. At infinity, this body is loaded by mutually perpendicular principal stresses $p$, $q$, with the defect oriented at angle $\alpha$ relatively to the principal stress $p$. Based on the well-known concept of macrostresses [1–3, 7–8], we accept the following hypothesis: the local fracture close to the top of defect occurs along the plane of maximum tensile stresses $S = Sm$, where

$$S = 2\mu\varepsilon_M + \frac{2\nu\mu}{1-2\nu}\varepsilon_C . \quad (1)$$

Here $\varepsilon_M=\varepsilon_M(\beta, \rho/l,\nu,\alpha,\eta)=\frac{1}{2\rho}[W_A - W_B]$ is macroelongation (scheme Fig. 1); $\varepsilon_c = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \varepsilon_c\left(\beta, \frac{\rho}{l}, \nu, \alpha, \eta\right)$ is macroextension, averaged of certain sphere with radius $\rho$ centered at point $(1+\rho\cos\beta, \rho\sin\beta, 0)$, where $\rho$ is so-called [1–3] structural parameter of the material; $U$ and $V$ –are components of the vector of elastic displacements in Cartesian coordinates; $\rho$, $\beta$ are polar coordinates with the origin at the top of the defect; $\mu$ is shear modulus; $\nu$ is Poisson's ratio; $\eta = \frac{q}{p}$ is ratio of principal stresses at infinity.

Let us note that, according to the macrostress theory, the relative displacement of two points with the distance between them exceeding $2\rho$, is determined by the relations of the linear elasticity theory. From (1), it is evident that macrostress in the vicinity of the defect tip is the following function

$$S = S\left(\beta, \frac{\rho}{l}, \nu, \alpha, p, \eta\right). \quad (2)$$

Thus, according to the accepted hypothesis, we get the following criterion correlations:

$$\frac{\partial S}{\partial \beta}_{\beta=\beta_*} = 0; \tag{3}$$

$$S_m\left(\beta_*, \frac{\rho}{l}, \nu, \alpha, p, \eta\right) = \sigma_B, \tag{4}$$

where $\sigma_B$ is technical strength of the matrix material; β*=β*(ρ/l,ν,α,signp,η) is the angle determining the direction along which the maximum tensile macro-stresses $S_m$ act.

Note that the criterion relations (3) and (4) at fixed parameter value $\rho$ make it possible to consider the conditions of initial propagation of micro- and macro-cracks from the same position, and at fixed defect length $l$ to observe the formation of local fracture close to the defect tip for materials with different microstructures $\rho$. Here, defects should be understood not only as cracks with extremely sharp tips, but also as cavities with finite radii of curvature, inclusions, etc.

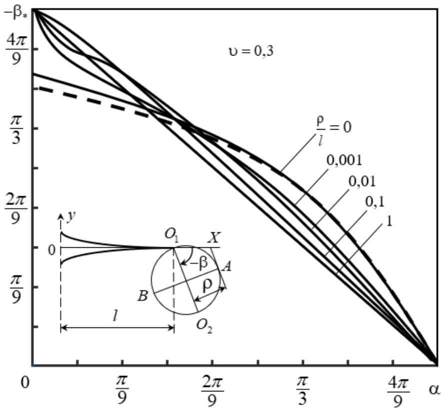
**Crack propagation in the process of tension of microhomogeneous body.** Let us apply [7] criterion relations (3) and (4) to evaluate the limit equilibrium of the body with arbitrarily oriented crack in the case of uniaxial $(p \neq 0, q = 0, \eta = 0)$ tension (scheme in Fig. 2). We select Cartesian coordinate system with the origin at the crack center and axis Ox passing through one of its vertices. Due to the apparatus of the theory of elasticity [9], we represent macrodeformation in direction $BA$ (scheme in Fig. 1) in the following form:

$$\varepsilon_M = \frac{1}{2\rho}\left[W_A - W_B\right]. \tag{5}$$

Macroextension is as follows:

$$\varepsilon_c = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \frac{æ-1}{\mu} R_e \Phi(l+\rho e^{i\beta}). \tag{6}$$

Here $æ = \begin{cases} 3-4\nu \ \textit{for plane strain}, \\ \dfrac{3-\nu}{1+\nu} \ \textit{for the generalized plane stress state.} \end{cases}$



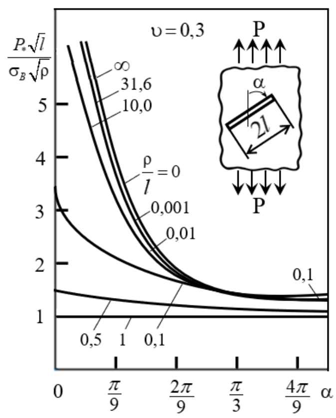



**Figure 1.** The dependence of the direction of local destruction (-β*) on the orientation of the defect-crack at $\nu = 0,3$

**Figure 2.** Dependence of the limit load p* on the orientation $(\alpha)$ of the defect-crack at $\nu = 0,3$ and different values of the parameter p/l

Complex Kolossov-Muskhelishvili potentials [10] are of the following form:

$$\Phi(z) = \frac{p}{4}\left[\frac{z\left(1-e^{2i\alpha}\right)}{\sqrt{(z-1)(z+1)}} + e^{2i\alpha}\right]:$$
$$\varphi(z) = \frac{p}{4}\left[\left(1-e^{2i\alpha}\right)\sqrt{(z-l)(z+l)} + ze^{2i\alpha}\right]; \tag{7}$$
$$\omega_1(z) = \frac{p}{4}\left[\left(1-e^{2i\alpha}\right)\sqrt{(z-l)(z+l)} - ze^{2i\alpha}\right].$$

Displacement $W$ in direction $BA$ is equal to (scheme in Fig. 1):

$$W = -U\sin\beta + V\cos\beta. \tag{8}$$

Then the increment of the vector of elastic displacements based on (8) is written as follows:

$$W_A - W_B = \left(-U_A + U_B\right)\sin\beta + \left(V_A - V_B\right)\cos\beta = R_e\left\{\left\{\left[U+iV\right]_A - \left[U+iV\right]_B\right\}(-i)e^{-i\beta}\right\}. \tag{9}$$

By means of (5) and (9), the expression of macrodeformation in direction $BA$ is:

$$\varepsilon_M = \frac{1}{2\rho} I_m\left\{\left[U+iV\right]_B^A e^{-i\beta}\right\}, \tag{10}$$

where $[\ ]_B^A$ is the increment of the vector of elastic displacements during transition from point B to point A.

Let us note that indices $A$, $B$ in formulas (5) and (9) denote the value of this expression atpoint $A$ or $B$. Point $A$ has coordinates (Fig. 1): $x_A$= l+ ρ(cosβ – sinβ), $y_A$= ρ(cosβ + sinβ) and point $B$, $x_B$=l+ρ(cosβ + sinβ), $y_B$= ρ(sinβ – cosβ) respectively. The ratio $U+iV$ is found by formula:

$$2\mu\left(U+iV\right) = æ\varphi(z) - \omega_1\left(\bar{z}\right) - \left(z-\bar{z}\right)\overline{\Phi}(z) + const. \tag{11}$$

Based on formulas (1), (6)–(11), in the case of unilateral tension, we get $\left(p>0, q=\eta=0\right)$, after some transformations, the expression for macrostress is:

$$S = p\sqrt{\frac{l}{2\rho}}\ [a_0 + a_1(\frac{\rho}{l})^{1/2} + a_2\frac{\rho}{l}], \tag{12}$$

where

$$\begin{aligned} a_0 = {} & \frac{1}{2}\sin\alpha\{T_1[\cos(\frac{3\beta}{2} - Q_1 - \alpha) - æ\cos(\frac{\beta}{2} + Q_1 - \alpha)] - \\ & -T_2[\cos(\frac{3\beta}{2} - Q_2 - \alpha) - æ\cos(\frac{\beta}{2} + Q_2 - \alpha)] - \frac{\sin\beta + \cos\beta}{T1}\sin(\frac{\beta}{2} + Q_1 + \alpha) + \\ & + \frac{\sin\beta - \cos\beta}{T2}\sin(\frac{\beta}{2} + Q_2 + \alpha) - \frac{4\sqrt{2}}{T3}\nu\sin(\frac{\beta}{2} - Q_3 - \alpha)\}, \\ a_1 = {} & \sqrt{2}\cos2\alpha[\frac{æ+1}{4} - \cos^2\beta + \nu];\ a_2 = -\frac{1}{2}\sin\alpha\{\frac{\sin\beta + \cos\beta}{T1}[\cos(\frac{3\beta}{2} + Q_1 + \alpha) + \end{aligned} \tag{13}$$

$$+\sin(\tfrac{3\beta}{2} + Q_1 + \alpha)] + \frac{\sin\beta - \cos\beta}{T2}[\cos(\tfrac{3\beta}{2} + Q_2 + \alpha) - \sin(\tfrac{3\beta}{2} + Q_2 + \alpha)] -$$
$$-\frac{4\sqrt{2}}{T3}\nu\sin(\tfrac{\beta}{2} + Q_3 + \alpha)\}.$$

Here

$$Q_1 = -\tfrac{1}{2}\mathrm{arctg}\,\frac{1+\rho/l\cos\beta}{1-\rho/l\sin\beta};\quad T_1 = \{2 + 2\,\rho/l(\cos\beta - \sin\beta\} + (\tfrac{\rho}{l})^2\}^{1/4};$$
$$Q_2 = \tfrac{1}{2}\mathrm{arctg}\,\frac{1+\rho/l\cos\beta}{1+\rho/l\sin\beta};\quad T_2 = \{2 + 2\,\rho/l(\cos\beta + \sin\beta) + (\tfrac{\rho}{l})^2\}^{1/4}; \qquad (14)$$
$$Q_3 = -\tfrac{1}{2}\mathrm{arctg}\,\frac{\rho/l\sin\beta}{2+\rho/l\cos\beta};\quad T_3 = \{4 + 4\,\rho/l\cos\beta + (\tfrac{\rho}{l})^2\}^{1/4}.$$

Calculations for the following values of parameter ρ/l: 0; 0.001; 0.002; 0.01; 0.05; 0.1; 0.5; 1.0 were carried out according to the criterion relations (3) and (4). In the calculations, it was assumed that the conditions of plane deformation are met (the results of calculations under plane stress state differ only slightly). Fig. 1 shows graphs of the change in the angle ($-\beta_*$) depending on orientation $(\alpha)$ of the crack defect at $\nu = 0,3$. Angles ($-\beta_*$) characterize the directions perpendicular to which the maximum macrostresses act, i.e., the directions along which, according to the accepted hypothesis, local fracture starts. In the framework of the macrostress theory, local fracture is defined as:

1) local propagation of the initial crack in direction $O_1O_2$ ($-\beta_*$);

2) the occurrence along the direction $O_1O_2$ ($-\beta_*$) of one or more collinear (quasi-collinear) initial microcracks, the interaction of which with each other and the initial defect results in the propagation of fracture process.

The second option is obviously possible in the case of materials with large microinhomogeneities or small initial defect (0,05≤ρ/l≤1). Dots in Fig. 1 show the results obtained on the basis of maximum intensity macrostress hypothesis $\sigma_\beta$ $[11-12]$, according to which the initial direction of macrocrack propagation does not depend on any elastic characteristics and material structure. It is evident from Fig. 4 that the results obtained according to different hypotheses are in good agreement in the wide range of angle $\alpha\left(\frac{\pi}{6} \le \alpha \le \frac{\pi}{2}\right)$ changes at (0.05≤ρ/l≤1). If the crack is oriented approximately along the line of action of the main stress $\left(0 \le \alpha < \frac{\pi}{6}\right)$, the hypothesis of $\sigma_\beta$ maximum intensity stresses results in some qualitative and quantitative deviations, and therefore it is reasonable to use the hypothesis of maximum macrostresses, since the directions of local fracture propagation predicted on its basis better aligned with 3 known experiments $[12-15]$. A slightly better coincidence with the experimental data is observed while using criteria $W_{\Pi}^{\min}, W_{\Phi}^{\min}$ (Fig. 4). However, it should not be forgotten that the stress criteria $\sigma_\beta$ of maximum intensity, $W_{\Pi}^{\min}, W_{\Phi}^{\min}$ and others are valid only in the case of macro defects and make it impossible to take into account the size of the initial defect on local fracture. Experimental data on the direction of crack propagation depending on the defect orientation under uniaxial tension of the body are shown in Fig. 3 and Fig. 4. These typical results for the given problem are taken from Williams and Eving work [15].

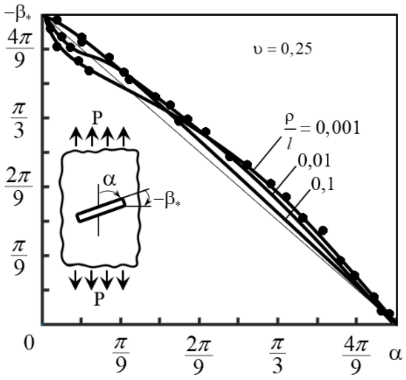



**Figure 3.** Dependencies of the direction of local destruction (-β*) on the orientation of the defect-crack for different values of the parameter p/l for the case of a plane stress state

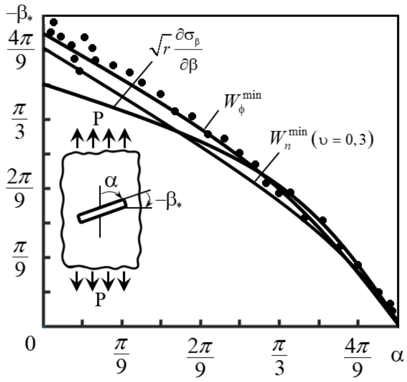



**Figure 4.** Dependencies of the direction of local destruction (-β*) on the orientation $(\alpha)$ of the defect-crack according to various destruction criteria

The experiments were carried out on thin polymethyl methacrylate plates at 20°C temperature and 50% relative humidity with strain rate 0.508 cm/min. The samples were 0.317 cm thick, 15.2 cm wide and 30.4 cm long. In the geometric center of the samples, central cracks were applied in a certain direction relatively to the applied load. The orientation of such cracks varied from zero to 90 degrees (by 5 degrees) with 2%accuracy. In the experiment, four crack lengths were chosen, namely 0.762 cm, 1.270 cm, 1.778 cm, and 2.54 cm. The accuracy of the length measurement was about 10%. According to paper [7], the value of the structural parameter of polymethyl methacrylate (plexiglass) was ρ = 0.159 cm. Thus, if we accept this value, then the experiments correspond to the following values of the parameter ρ/l: 0.0208; 0.0125; 0.0089; 0.0063. Changing the lengths of the initial defect in the experiments carried out in [15] slightly affected the value of the direction of initial fracture $\beta_*$. In order to obtain significant differences in $\beta_*$values, it was necessary to change the length of the initial crack (and at the same time the ratio ρ/l) in a wider range. For clearer comparison of theoretical and experimental results, only the experimental $\beta_*$ values corresponding to the initial crack with a half-length 1.778 cm (or ρ/l = 0.0089) are shown in Fig. 3.

It is shown in Fig. that there is a good consistency between the theoretical predictions of the initial direction of crack development based on the macrostress concept (under the conditions of plane stress state) and the actual ones obtained experimentally.

In their paper [15] Williams and Eving investigated the reason for the discrepancy between theoretical predictions of the direction of initial crack movement (especially at $\left(0 \le \alpha < \frac{\pi}{6}\right)$) using the hypothesis of maximum intensity stresses $\sigma_\beta$. The authors showed that for better correlation between theoretical and experimental values, it was necessary to take into account the second component in the stress $\sigma_\beta$ in addition to the singular term in the series expansion of the stress distribution in the vicinity of the crack tip. In this case, to match theoretical and experimental data (without any substantiation), the stress state of the test body was estimated at distance $r = 0,002$ mm from the crack tip on its extension. Such distance will probably be different for different materials, stress states of the investigated body, environment, etc. This approach should be considered empirical, since the rule for determining the critical distance $r$ is not specified.

It should be noted that if the initial crack becomes commensurate with the material structure (0.1< ρ/l≤1), the fracture process is oriented in the direction perpendicular to the applied tensile load. Such result should have been expected, since in this case we are dealing with quasi-homogeneous and quasi-isotropic material. These conclusions are confirmed by the change in the limiting value of the external load $p_*$, at which local destruction of the body occurs. The dependence $\frac{P_*\sqrt{l}}{\sigma_B\sqrt{\rho}}$ on the orientation of the defect $(\alpha)$ at different values of the parameter ρ/l and $\nu = 0,3$ is shown in Fig. 2. It is characteristic here that the strength of the body with microcracks is less subjected to orientation dependence compared to the strength of the body weakened by macrocrack. It is interesting that within the framework of macrostresses theory, we obtain the final $p_*$ values (which depend on ρ/l and $\nu$) at $\alpha \to 0$, which is not follow from previous hypotheses [11, 12, 16].

In the case of macrocrack (ρ/l→0) under plane deformation, according to [2, 3], the following relationship between the structural parameter $\rho$ and density of effective fracture energy $\gamma$ is observed:

$$\rho = \frac{\left[4\nu\sqrt{1+\sqrt{2}} + (3-4\nu)\sqrt{2}-1\right]^2}{4\pi\left(1+\sqrt{2}\right)\left(1-\nu 2\right)} \frac{E\gamma}{\sigma_b{}^2}. \tag{15}$$

On the basis of paper [7], we obtain:

$$B(\nu) = \frac{\left[4\nu\sqrt{1+\sqrt{2}} + (3-4\nu)\sqrt{2}-1\right]}{2\sqrt{2\left(1+\sqrt{2}\right)}}, \tag{16}$$

$$K_{1c} = \sigma_B\sqrt{\rho}\frac{1}{B(\nu)}. \tag{17}$$

It follows from formula (17) that the value $K_{1c}$ (crack resistance) is complex characteristic of the material and depends both on its microstructure and strength properties, in particular, on the ability to undergo plastic deformation. Value 1/B(ν) = 1.35 at $\nu = 0$; 1.29 at $\nu = 0,3$; 1.25 at $\nu = 0,5$. In the case of plane stress state

$$K_c = \sigma_B\sqrt{\rho}\frac{1}{B*(\nu)}, \tag{18}$$

where

$$B_*(\nu) = \frac{4\nu\sqrt{1+\sqrt{2}} + (3-\nu)\sqrt{2} - (1+\nu)}{2\sqrt{2(1+\sqrt{2})(1+\nu)}}.$$

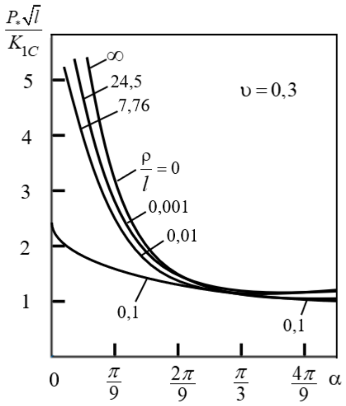



**Figure 5.** Dependence of the ultimate load p* on the orientation $(\alpha)$ of the defect-crack for sufficiently inhomogeneous bodies

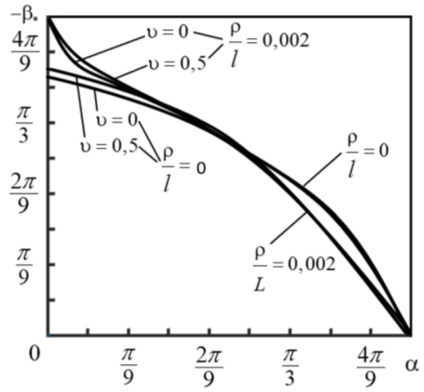



**Figure 6.** Dependencies of the direction of local destruction (-β*) on the Poisson ratio ν for p/l=0 and p/l=0.002

Value $1/B_*(\nu)$ = 1.35 at $\nu = 0$; 1.15 at $\nu = 0,3$; 1.11 at $\nu = 0,5$. Ratios (15), (17), (18) are derived on the basis of comparison of the limit values of external load of Griffiths problem, which are calculated using the concept of macrostresses and Irwin’s approach, respectively. In general case of defects wiyh any length, the material crack resistance $K_{1c}$ is equal to:

$$\check{K}_{1c}=\sigma_{B}\sqrt{\rho}/B_1(\nu,\rho/l), \tag{19}$$

where

$$B_1(\nu,\rho/l)=\frac{2\nu}{\sqrt{2+\rho/l}}(1+\rho/l)-\frac{1}{2}(2+\rho/l+(\rho/l)^2)^{-1/4}\left[(1+\rho/l)\sqrt{1+1/\sqrt{1+(1+\frac{\rho}{l})^2}}+\rho/l\sqrt{1-1/\sqrt{1+(1+\frac{\rho}{l})^2}}\right]+\frac{1+æ}{2}(2+2\rho/l+(\rho/l)^2)^{1/4}\sqrt{1-1/\sqrt{1+(1+\frac{\rho}{l})^2}}. \tag{20}$$

Hence

$$\check{K}_{1c}/K_{1c}=\frac{B(\nu)}{B1(\nu,\rho/l)}. \tag{21}$$

The graph of the change in value $\frac{\check{K}_{1c}}{K_{1c}}$ depending on $\frac{\rho}{1}$ is shown in Fig. 8 in semilogarithmic coordinates at $\nu = 0,3$.

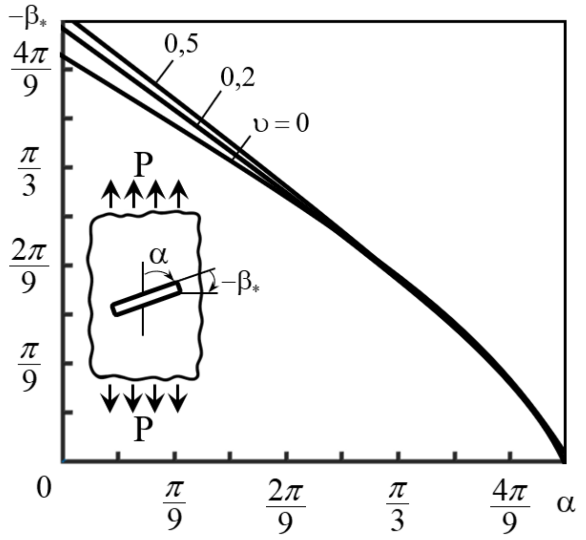



**Figure 7.** Dependences of the direction of local destruction (-β*) on the orientation $(\alpha)$ of the defect-crack for different values of the Poisson ratio according to the criterion of the minimum energy density of the shape change ($W_{\Phi}^{\min}$)

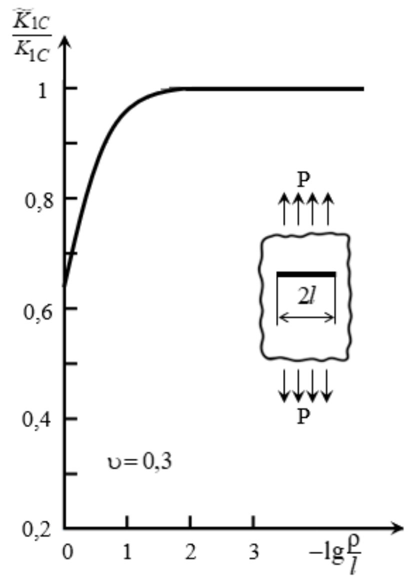



**Figure 8.** Dependence of the change in the value Ǩ1s/K1s on the parameter ρ/l at ν =0.3

We can see that formula (18) remains valid for sufficient number of microhomogeneous $\left(\frac{\rho}{1} \leq 0,01\right)$ materials. On the other hand, if we fix $\rho$, and change $l$, we notice that not all initial crack lengths are suitable for determining the crack resistance of the given material with structural parameter $\rho$. By formula (18), it is possible to determine the value of the structural parameter for a number of structural materials [6] if the values of $K_{1c}$, $\sigma_B$, and ν of the given material are known.

**Conclusions.** Changes in value $p*\frac{\sqrt{l}}{K_{1c}}$ depending on the orientation of defect $(\alpha)$ at $\nu = 0,3$ for some values ρ/l are shown in Fig. 5. An important conclusion that can be drawn from the graphs in this Figure is that the fracture criteria obtained for homogeneous bodies with macrocracks provide the estimate of the strength of sufficient number of heterogeneous bodies (within the range of angles $\left(\frac{\pi}{6} \leq \alpha \leq \frac{\pi}{2}\right)$ parameter ρ/l can vary within the range 0≤ρ/l ≤0.01).

In contrast to the results of criterion Ci [16], where Poisson's ratio is the determining parameter in defining the angles of crack propagation and critical load, the values – $\beta_*$ and $p_*$, which are determined by the theory of macrostresses, have little effect on the change in $\nu$. The idea of the influence of Poisson's ratio on the direction of local fracture is given by the graphs shown in Fig. 6. Thus, according to the theory of macrostresses, the main parameter that determines the nature of fracture in the crack vicinity (except for strength properties) is the material structure, and its elastic characteristics play a secondary role. Let us remind that, according to the criterion of minimum shape change energy density ($W_{\Phi}^{\min}$) [16], elastic properties of the material also have little effect on the initial direction of development of arbitrarily oriented crack in the case of plane deformation (Fig. 7) and no effect in the case of plane stress state.

Based on the concept of macrostresses by M. Y. Leonov and K. M. Rusynka [1–3], first applied by us [4–7, 10–12] for the case of complex stress state, we estimate the local fracture of bodies with cracks or inclusions of various sizes, taking into account microinhomogeneities in the structure of the binder material.

The advantages of the macrostress concept criterion in comparison with other fracture criteria in deformable bodies are clarified and extended. Examples for crack propagation in the process of body tension are given.

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# ВПЛИВ МІКРОСТРУКТУРИ МАТЕРІАЛУ НА РОЗВИТОК РУЙНУВАННЯ В ДЕФОРМУЮЧИХ ТІЛАХ

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***Резюме.*** *Проблема прогнозування місця, напрямку локального руйнування й оцінювання величини граничного навантаження складеного (композиційного) матеріалу є однією з практично важливих задач механіки руйнування. При значній різновидності композиційних матеріалів природно очікувати наявність великої кількості критеріїв руйнування, кожен з яких повинен бути справедливим для певного класу матеріалів і найповніше відображати його фізико-механічні властивості, дефектність структури, умови експлуатації і т. п. Вибір ефективного оцінювання несучої здатності сучасного композиційного матеріалу, який перебуває в екстремальних умовах, є головним завданням дослідників. Надано перевагу критерію руйнування композиційних матеріалів на основі простої й досить ефективної теорії макронапружень. Ефективність теорії макронапружень показано в роботах В. В. Панасюка, Л. Т. Бережницького, С. Я. Яреми, Л. В. Ратича, М. Г. Стащука. В нашій роботі уточнено й розширено переваги критерію концепції макронапружень порівняно з іншими критеріями руйнування в деформуючих тілах. Наведено приклади поширення тріщин у процесі розтягу тіла.*

***Ключові слова:*** *Міцність, теорія макронапружень, опір поширення тріщини в матеріалі ($K_{1c}$), коефіцієнт концентрації макронапружень Км , середня технічна міцність матеріалу ($\sigma_в$), коефіцієнт Пуассона ($\nu$), структурний параметр ($\rho$) і довжина тріщини (дефекта) (2l).*

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