UDC 539.3

FEA OF STRESS-STRAIN STATE AND VIBRATIONS OF A THREE-LAYER PLATE

Iaroslav Lavrenko; Maksym Sushchenko

National Technical University of Ukraine «Ihor Sikorsky Kyiv Polytechnic Institute», Kyiv, Ukraine

Summary. *Solar panels are considered as three-layer plates with a thick, rigid outer layer and a thin, soft inner layer. The model for anti-sandwich plates was used to describe the mechanical behavior of the plates in the example of a solar panel. The literature review includes scientific articles describing models for analytical and numerical calculations of three-layer plates. During the scientific study of the mechanical behavior of the solar plate under the influence of external factors, the finite element analysis method for multilayer plates was used. The shell elements were used to calculate and model the natural waveforms of three-layer plates. The paper presents scientific research under static loading under various conditions of influence, analyzes the natural frequencies, and vibration forms, and investigates the stress-strain state depending on the vibration frequencies of the three-layer plate. As part of the scientific work, a mechanical model of a thin solar panel was studied using finite element analysis in the ANSYS program, taking into account various temperature conditions and comparing the results with existing studies.*

Key words: *solar panel, sandwich, frequencies, stress, strain, finite element method, ANSYS.*

Formulation of the problem. In our time, there is a rapid development of renewable energy sources, and one of the most common among them is solar energy. Solar panels are subjected to significant loads due to temperature changes, wind, and precipitation. The mentioned effects lead to dynamic loads, potentially causing breaches in the seal between layers, crack formation in brittle materials, and consequently, a decrease in efficiency. Therefore, the study of plate vibrations, using solar panels as an example, is important and necessary for designing more robust solar panels.

Modeling and investigating the behavior of both individual layers and the entire plate are essential and highly relevant today [1, 2]. In scientific research, solar panels are considered as multilayered plates, and their study is conducted using classical theories.

The paper presents the results of determining natural frequencies and vibration modes for a three-layered plate, taking into account temperature factors. These results provide information about the shape the structure will take if a force with a specific frequency, equal to the corresponding natural frequency of vibrations, is applied to it.

Analysis of the available research results. Several studies have explored the dynamic behavior of composite sandwich panels with different focuses and methodologies. For instance, [3, 4] utilized a sandwich panel with laminate faces to analyze free vibrations. They employed the Shear Deformation Theory and finite element analysis in ANSYS Workbench to investigate the effects of design parameters on vibration responses. Another study [5] presented an exact mathematical model for three-layered sandwich plates, providing a time-efficient alternative to finite element methods.

A multiscale projection approach for analyzing photovoltaic modules, combining global and local scales is proposed in [6]. The modeling and design of composite multilayered

structures using a solid-shell finite element model, coupled with a response surface method for optimization investigated in [7].

A new formulation for solid-shell finite elements considering large displacements and different transverse shear strains approximations introduced in paper [8]. Authors [9] evaluated the application of the first-order shear deformation (FSDT) theory to analyze laminated glasses and photovoltaic panels.

A comprehensive analysis of functionally graded sandwich plates, focusing on deflection and stresses conducted in [10]. In paper [11] authors analyzed laminated glass beams for photovoltaic applications, assessing mechanical properties and relationships between deflection and applied force.

In work [12] proposed a simple equivalent plate model for the dynamic bending stiffness of three-layer sandwich panels, providing a computationally efficient solution. Authors in paper [13] explored the free vibration of a three-layer sandwich plate with a viscoelastic core modeled with fractional theory.

Three-dimensional vibration analysis of laminated composite plates performed in [14], while in work [15] investigated thermal mechanical bending responses in functionally graded material plates. Combining these approaches could enhance the understanding of dynamic behavior in structures with varying material properties.

A finite element model for analyzing the vibration and damping of sandwich plates under different boundary conditions is shown in [16]. Also modal analysis of a sandwich panel with composite laminated faces, studying the effect of design parameters on vibration response and identifying length sensitivity to dynamic wind loads conducted in [17].

The models for evaluating PV-module temperatures is shown in [18], and higher-order structural theories for laminated composite panels explored in [19]. Integrating these models could improve the design and performance prediction of photovoltaic structures.

An alternative determination of transverse shear stiffness's for sandwich and laminated plates, contributing to equivalent single-layer theories and enhancing structural analysis accuracy proposed by authors in. Experimental and numerical studies on the flexural behavior of fiber-reinforced polymer composite sandwich panels, assessing the effects of various parameters on flexural stiffness and strength demonstrated in. Study advance knowledge in structural mechanics, materials science, and engineering design, providing a comprehensive understanding of dynamic response, mechanical properties, and optimization in composite structures across diverse engineering applications.

The diverse range of topics, methodologies, and applications addressed in these works reflects the multidisciplinary nature of research in composite materials, structural analysis, and mechanical behavior. The integration of analytical, numerical, and experimental approaches provides a comprehensive understanding of the dynamic response, mechanical properties, and optimization of composite structures in various engineering applications.

These studies are particularly relevant to the solar energy sector, offering crucial insights into optimizing materials and structural designs for solar panels. Their multidisciplinary approach ensures a comprehensive understanding of how these components perform, emphasizing lightweight design and energy efficiency-key factors for enhancing solar panel efficiency. The proposed synergies and alternative methods contribute directly to refining models and innovating solutions tailored to the unique challenges in solar panel design. In summary, these studies play a pivotal role in advancing solar panel technologies, contributing to more effective and sustainable solar energy systems.

Objective of the research. The main goal of the research is to determine the natural frequencies and vibration modes of a three-layered plate using the finite element method, with the example model being a solar panel and considering the influence of temperature. The investigation also includes assessing the convergence of discretization to analyze the accuracy of the obtained results. Also, the study of the stress-strain state of the three-layered plate.

Statement of the problem. The object of the study is a three-layered plate subjected to various types of loading, using solar panels as an example. The typical structure of solar elements consists of outer glass layers and inner soft polymer layers, serving a protective function for very thin and brittle silicon solar panels (Fig. 1) [3–5].
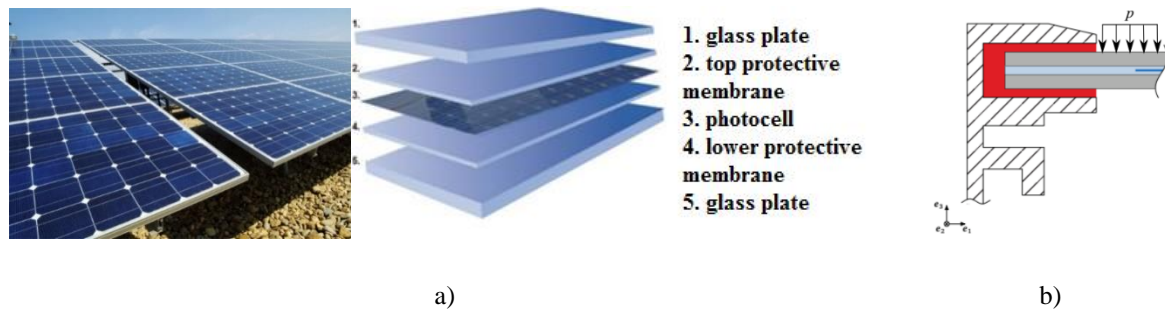

1. glass plate
2. top protective membrane
3. photocell
4. lower protective membrane
5. glass plate

a)   b)

Figure 1. a) Structure of a solar panel and b) boundary conditions [2]

As mentioned earlier, this mechanical model describes the typical construction of solar panels, which consists of glass and soft polymer materials. This type of construction serves a protective function for very thin and brittle silicon solar panels.

Scientific articles often lack information about the type of attachment of solar panels. In this case, analytical calculation is conducted in two boundary cases of attachment: rigid attachment and free attachment, allowing relative displacement of layers.

Shell elements used in classical theories do not account for transverse shear, which makes it impossible to model the real behavior of a solar panel in the thickness direction.

1. Geometry Modeling

The investigation was carried out for a three-layered thin anti-sandwich panel, where the outer layers are made of glass, and the inner layer is made of soft polymer EVA. The geometric characteristics of this model are as follows:

1) Planar dimensions are significantly larger than the overall thickness: $L_{1,2} > L_3$ (Fig. 2).

2) Thickness of the middle layer is very small compared to the thickness of the outer layers: $h^K < h^O, h^U$ (Fig. 2, Table 1).
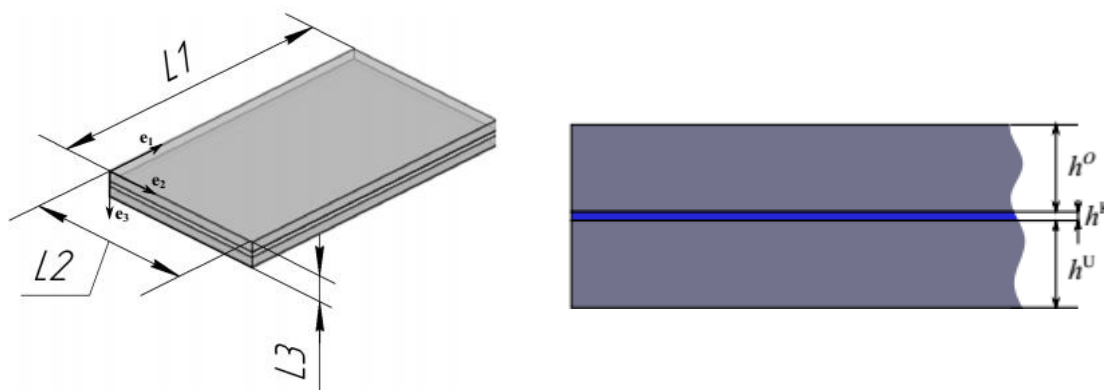

Figure 2. Plate Structure

Table 1

Geometric data [8]

Parameter	Value
L_1 [mm]	1620
L_2 [mm]	810
h^O [mm]	3,2
h^U [mm]	3,2
h^K [mm]	1,0

2. Material selection

As mentioned earlier, the outer layer of solar modules is protected by glass. Inside, there is a protective polymer layer, which is typically made from Ethylene Vinyl Acetate (EVA). It compensates for various thermal and mechanical deformations and serves as a sealing layer for the photovoltaic cells [9].

An important assumption is that each layer is considered isotropic. Additionally, the mechanical characteristics of the outer layers significantly outweigh the corresponding values of the inner layer: $E_U = E_O > E_K$, $G_U = G_O > G_K$. Material properties is provided in Table 2.

Table 2

Material properties [8]

Parameter	Glass	EVA
E [N/mm^2]	73000	7,9
ν [-]	0,3	0,41
P [kg/mm^3]	$25 \cdot 10^{-7}$	$960 \cdot 10^{-9}$
G [N/mm^2]	28076,9	2,80

Since the goal is to determine the mechanical behavior of the sandwich structure under real weather conditions, it is important to investigate the impact of different temperatures. The study was conducted at the following temperatures: under normal conditions (+23°C), under elevated conditions (+80°C), and under reduced conditions (-40°C). It is assumed that the properties of the outer layers do not change with temperature, and only the mechanical characteristics of the material in the middle layer vary. The corresponding values of mechanical properties are provided in Table 3.

Table 3

Material data for the middle layer at different temperatures [9]

T [°C]	-40	+23	+80
E [N/mm^2]	1019,04	7,9	0,52
ν [-]	0,41	0,41	0,41
G [N/mm^2]	361,36	2,80	0,18

3. Boundary Conditions and Loadings

Solar modules are typically embedded in an aluminum profile. Since the outer layers are made of glass, which is a relatively brittle material, direct contact between glass and metal is undesirable. This is because deformation would lead to high stress concentrations, resulting

in crack formation and failure. Therefore, a special buffer layer is used between the glass and the aluminum frame, absorbing deformation differences and compensating for stress (Fig. 1 b).

As detailed information about the buffer layer and its functions is lacking, the assumption in this work is that all layers of the sandwich panel can freely rotate at the edges [9, 11]. Hence, it is reasonable to choose fixed displacement constraints for each layer at the edges in the thickness direction, set to zero.

To simplify the problem, a uniform static load was applied in the form of pressure evenly distributed over the entire surface of the sandwich, orthogonal to the surface plane. When choosing the magnitude of the load, it was considered that the problem is geometrically linear, therefore the relationship should hold $w_{max}/h < 0,5$, where w_{max} – maximum deflection, h – the overall thickness of the plate. According to [1], the magnitude of the distributed load may vary within the range $\pm 2,4 \cdot 10^{-3}$ and $\pm 5,4 \cdot 10^{-3}$ N/mm^2. In this problem, it was decided to apply the load $0.5 \cdot 10^{-3}$ N/mm^2.

4. Finite Element Meshing

The finite element mesh was constructed in ANSYS Workbench using spatial shell elements. Only one element was used through the thickness of each layer. The aspect ratio of each element in the plane directions is set to one: $AR = \dfrac{h_1^e}{h_2^e} = 1$. Thus, the corresponding nodes at the boundaries between the layers coincide. Since the behavior of the element in the plane directions and in the thickness direction differs, it is important during discretization to specify the thickness direction for the elements. To do this, it is necessary to select the upper and lower surfaces. The geometric model with the finite element mesh is shown in Fig. 3, where the lower surface is indicated in blue.
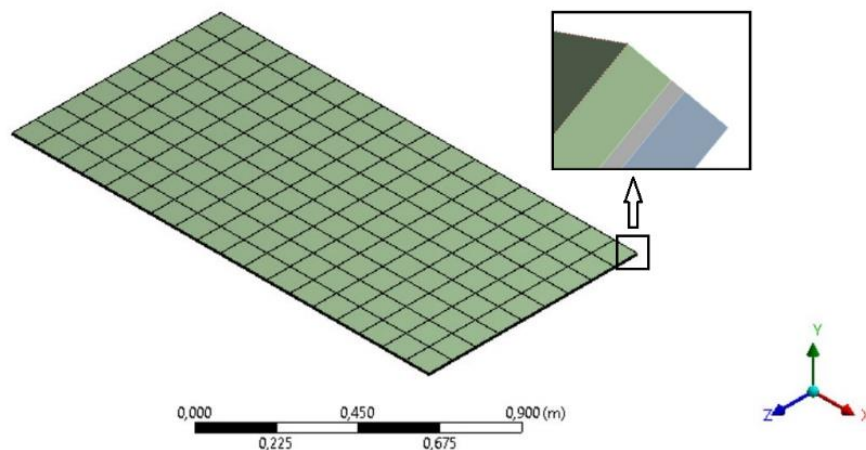

**Figure 3.** Spatial discretization of the plate

5. Investigation of Convergence of Results

The investigation of the convergence of results was conducted to check the accuracy of the finite element calculation. The h-method was used for this purpose, involving an increase in the number of elements. Several meshes with different numbers of elements were generated, and the maximum plate deflection values were calculated and compared.
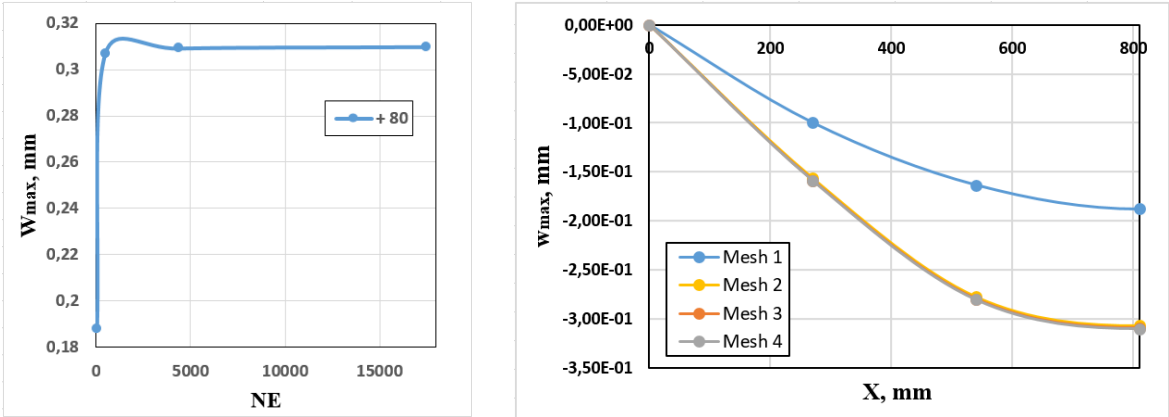
Since the goal is to determine the mechanical behavior of the sandwich panel under real weather conditions, it is important to study it at different temperatures. Numerical modeling was carried out using the ANSYS Workbench software package. The simulation results are presented in the figures (Fig. 4).
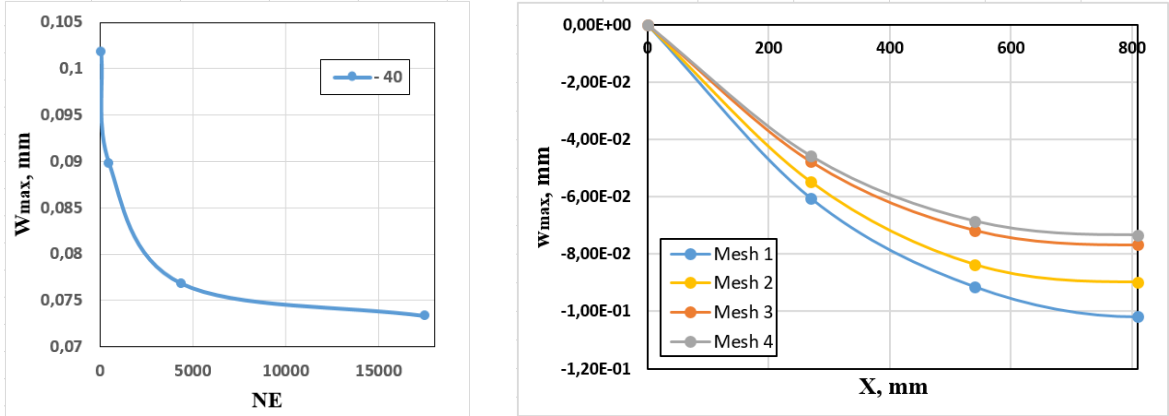
a)
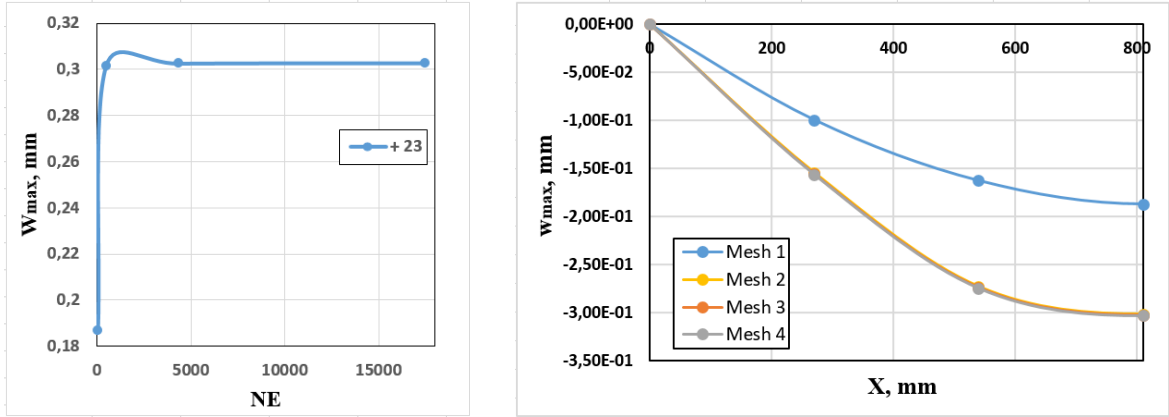
b)

c)

Figure 4. Maximum plate deflection values calculated using different
numbers of elements at various temperatures:
a) -40 °C, b) +23 °C, c) +80 °C.

Numerical values of the results depicting the dependence of the maximum plate deflection on the number of elements are provided in Table 4.

Table 4

Main results of calculations

№	NE	w_{max} (T=+23°C)	w_{max} (T=+80°C)	w_{max} (T=-40°C)
1	54	0,18676	0,18789	0,10186
2	486	0,3014	0,30679	0,08977
3	4374	0,30252	0,30931	0,076833
4	17496	0,30271	0,30995	0,073334

From the figures and the table, it is clear that the deviations between the deflections on the last two meshes (№ 3 and № 4) are very small (less than 5%). This indicates that the solutions converge, and therefore, for further calculations, it is sufficient to use mesh 3.

As evident from Fig. 4 and Table 4, the maximum deflection increases with an increase in the number of elements, which corresponds to the physical situation where a plate becomes less rigid with more elements.

In contrast, at a temperature of -40°C, the convergence shows physically correct results, meaning that with mesh refinement, the deflection decreases.

The deflection distribution is symmetrical (Fig. 5), with the same trend, i.e., the deflection increases with an increase in temperature.
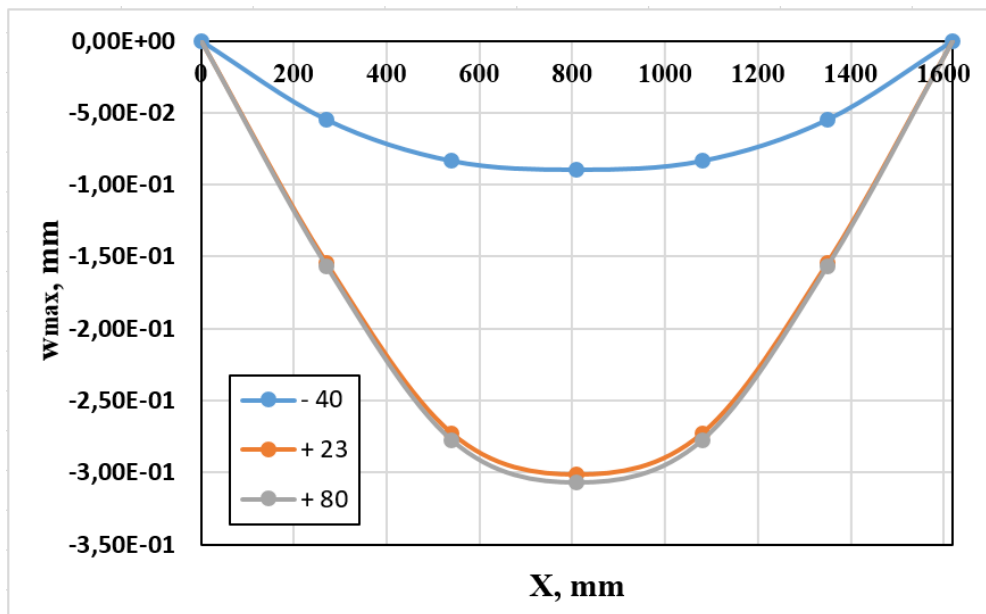

Figure 5. Plate deflections in the thickness direction
at different temperatures

Results of the research. As a result of modeling the static analysis of the stress-strain state of a three-layer plate, displacements, strains, normal and shear stresses were determined, taking into account the temperature effect (Figs. 6, 7, 8).
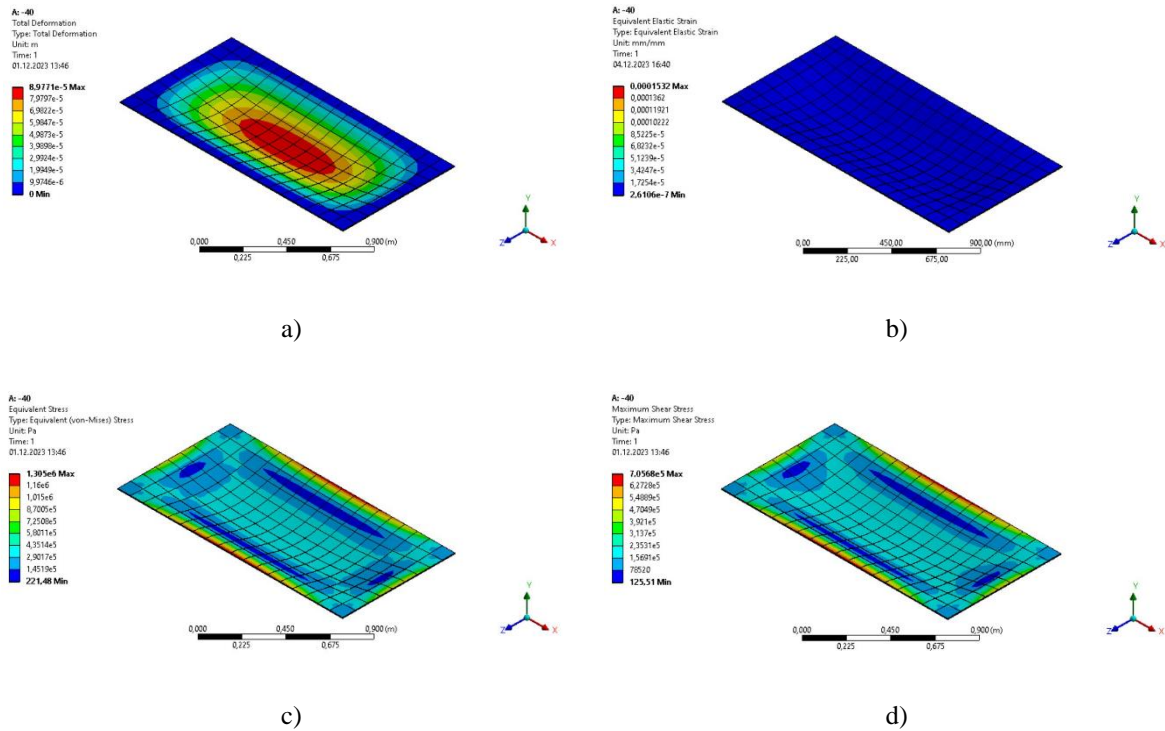
Figure 6. Results of static analysis:
a) deflection, b) equivalent strains, c) equivalent stresses (von Mises),
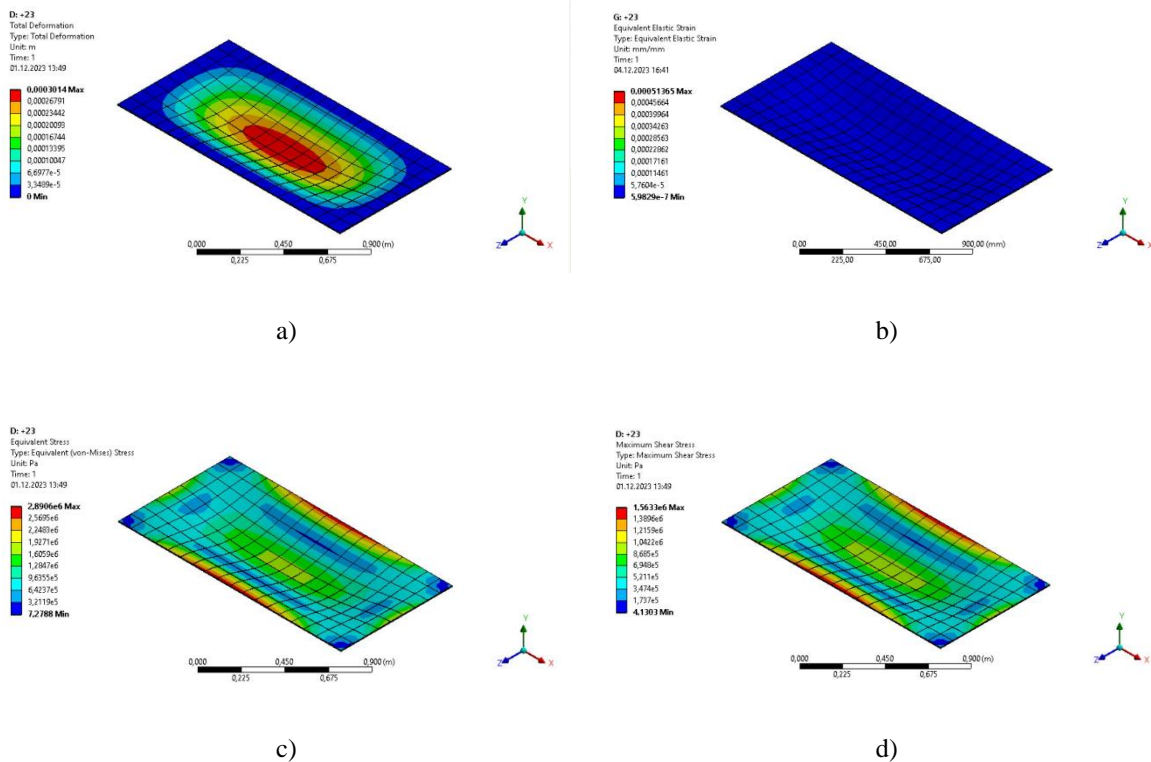d) maximum shear stresses occurring in the three-layer plate at a temperature of -40°C

Figure 7. Results of static analysis:
a) deflection, b) equivalent strains, c) equivalent stresses (von Mises),
d) maximum shear stresses occurring in the three-layer plate at a temperature of +23°C
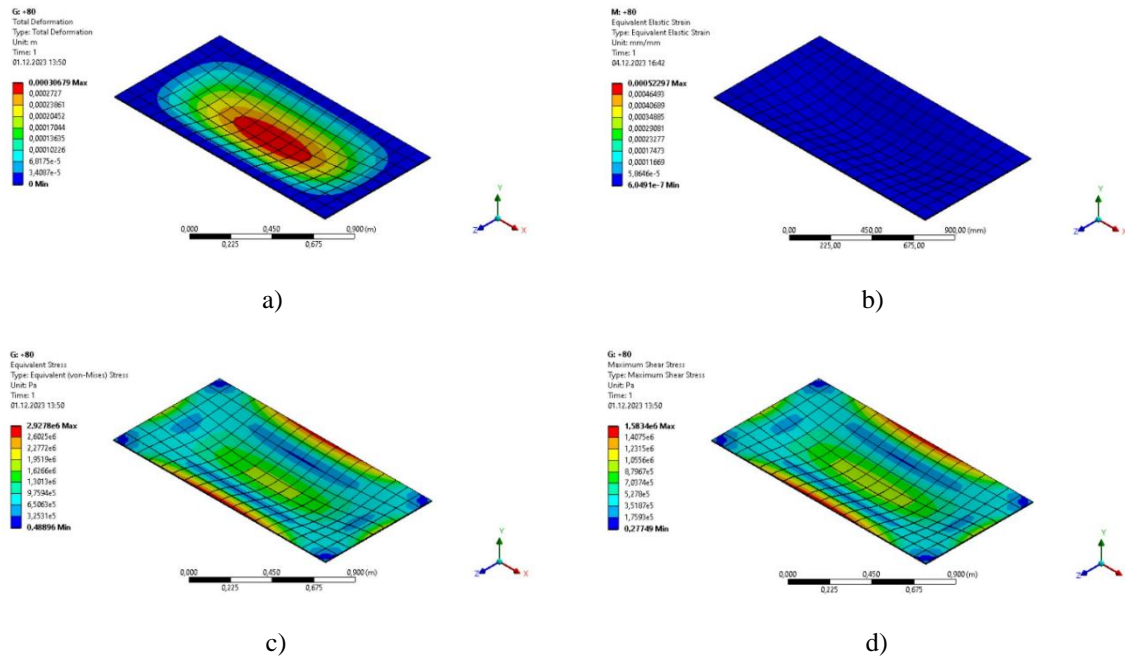
a)

b)

c)

d)

Figure 8. Results of static analysis: a) deformation, b) equivalent strains, c) equivalent stresses (von Mises),
d) maximum shear stresses occurring in the three-layer plate at a temperature of +80°C

The results of the calculation of the corresponding deformations, strains, and stresses are shown in Table 5.

Table 5

Main results of calculations

Temperature	Deformation, mm	Strain, mm/mm	Equivalent stress (von Mises), MPa	Shear stress, MPa
-40°C	0,089771	0,000153	1,305	0,7057
+23°C	0,3014	0,000514	2,8906	1,5633
+80°C	0,30679	0,000523	2,9278	1,5834

Modal response of the three-layer plate. The modal analysis method is used to construct modes and find natural frequencies of vibrations and can be solved using the appropriate FEA software. Harmonic analysis is performed to find the amplitudes of displacements and stresses corresponding to each found natural frequency of the structure under exciting loads. The analysis was carried out using ANSYS Workbench. The calculation was limited to the tenth mode of vibration (Table 6).

Table 6

Eigenfrequencies for plate

№	Frequency, Hz (-40°C)	Frequency, Hz (+23°C)	Frequency, Hz (+80°C)
1	2	3	4
1	110,13	61,56	61,04
2	138,85	79,68	79,07
3	181,63	111,98	111,3
4	230,37	158,38	157,66
5	236,5	159,81	159,17
6	252,19	177,76	177,09

The end of the table

1	2	3	4
7	288,17	208,92	208,22
8	302,87	218,27	217,52
9	338,06	254,18	253,46
10	381,01	291,48	290,71

Eigenmodes of vibration of the three-layer plate with consideration of temperature influence obtained from modal analysis are shown in Fig. 9, 10, 11.
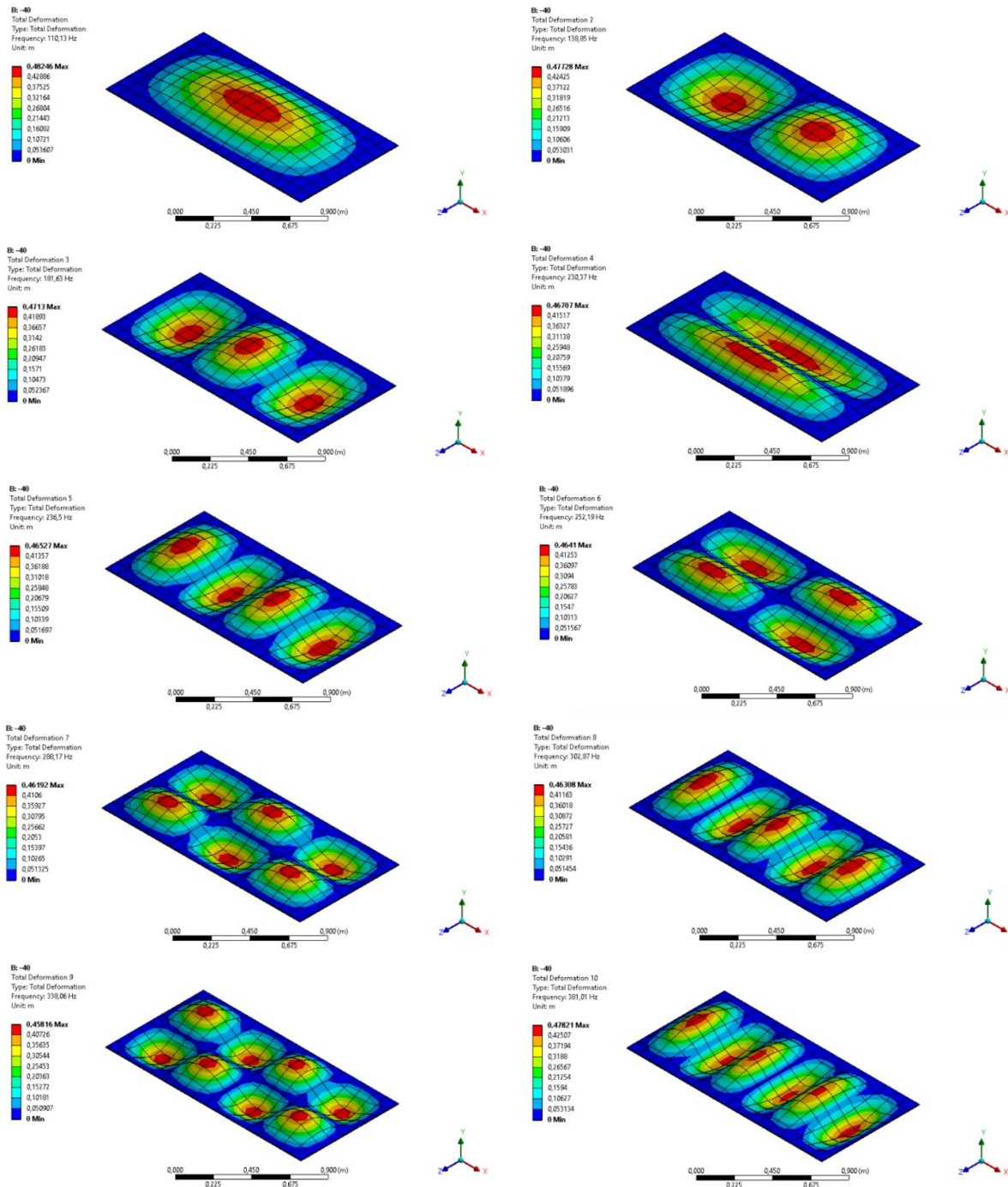

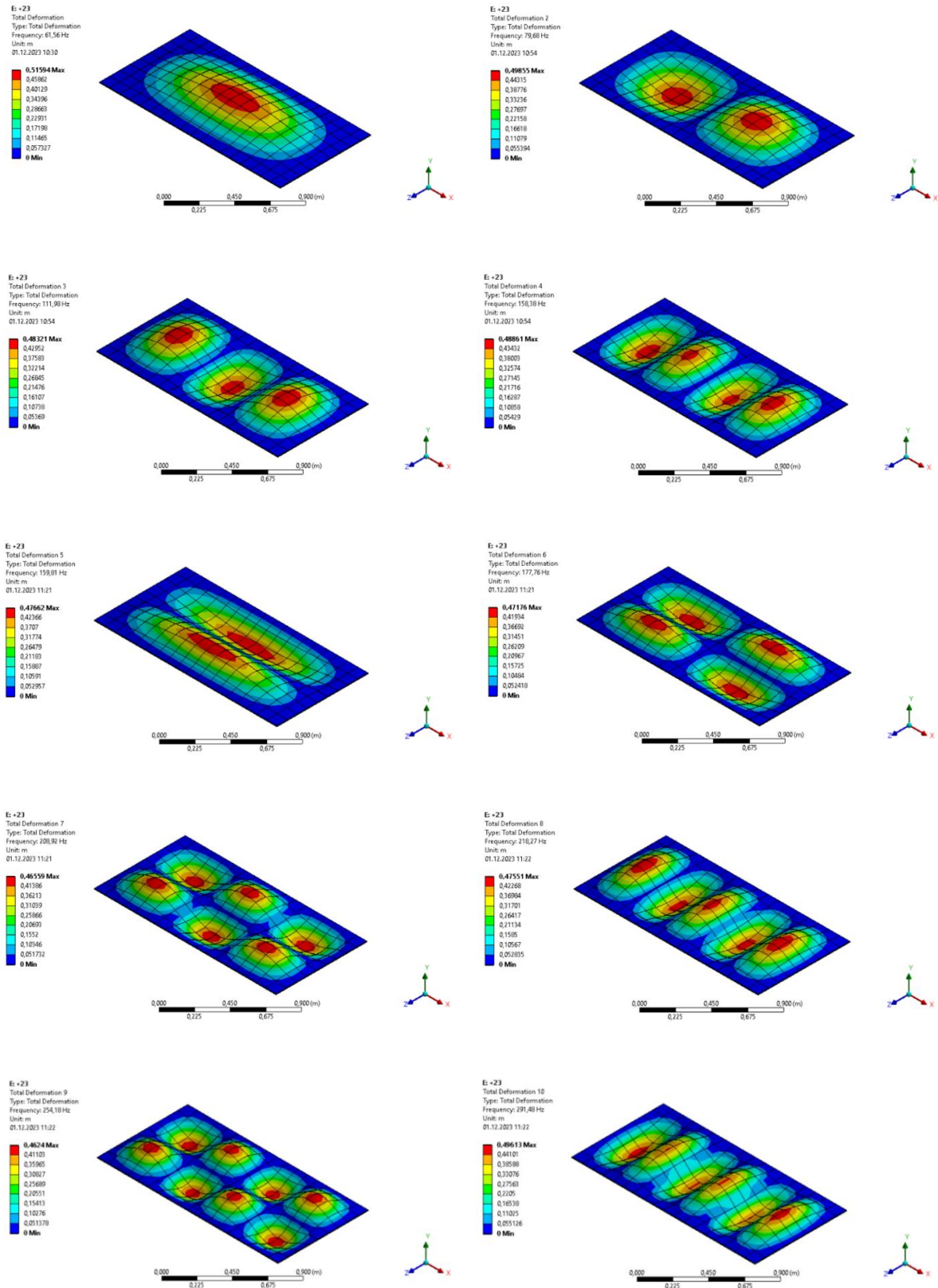**Figure 9.** Total deformation of the plate from Mode 1–10 using ANSYS (T = - 40°C)

Figure 10. Total deformation of the plate from
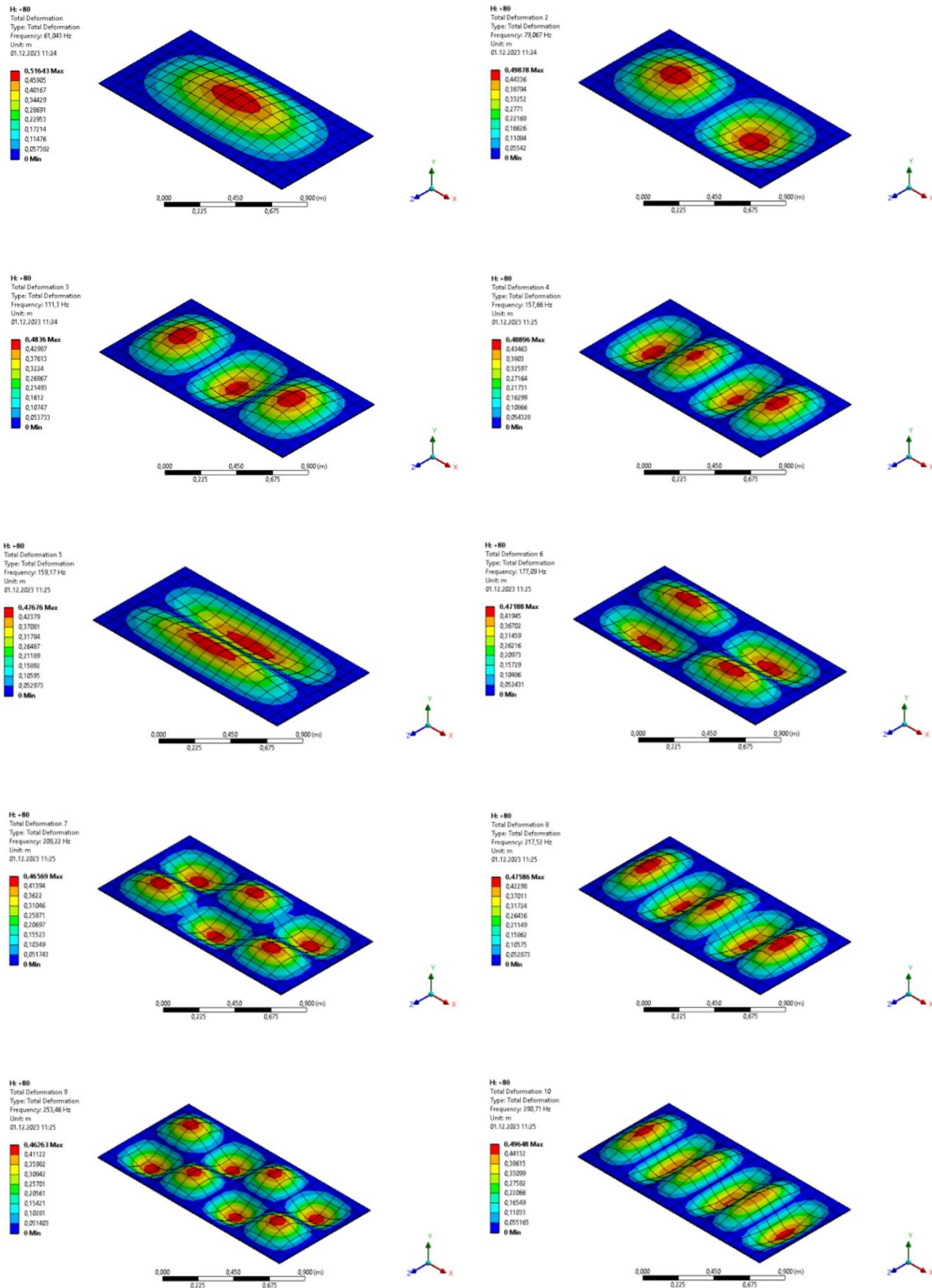Mode 1–10 using ANSYS (T = + 23°C)

Figure 11. Total deformation of the plate from Mode 1–10 using ANSYS (T = + 80 °C)

The results of the obtained natural frequencies values depending on the corresponding mode during the modeling process are presented in the form of a histogram in Fig. 12.
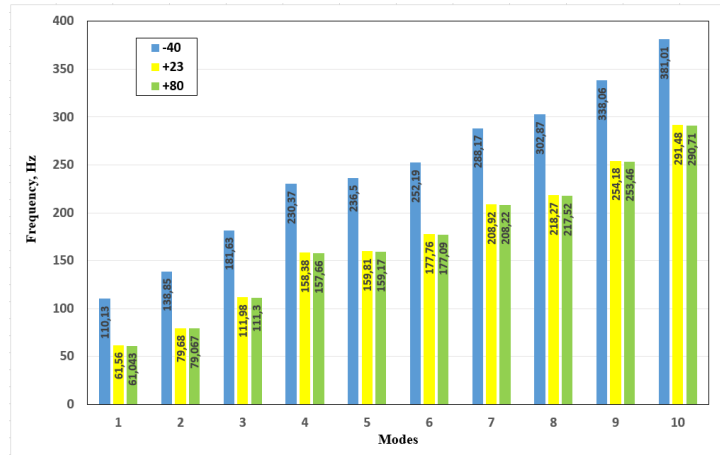
Figure 12. Frequency (Hz) v/s Modes

Harmonic response of the three-layer plate. Harmonic analysis for the three-layer plate, exemplified by a solar panel, was conducted with the application of external loading in the form of a distributed force. The graphical distribution of deformations and strains in different plate directions is shown in Fig. 13 and Fig. 14, respectively. Harmonic analysis is essential for determining the frequency and phase characteristics of the satellite shaft in the operating frequency range, solved through modal analysis. For the current investigation, the frequency range varies from 0 to 400 Hz.

In the amplitude-frequency plot (mm vs. Hz), a significant reduction is observed in the frequency range from 50 Hz to 100 Hz. The maximum displacement (2.62 mm) is observed in the Y-direction, while the minimum is observed in the X-direction (Fig. 13). The amplitude-frequency plot (Pa vs. Hz) shows that the maximum stress (155.31 Pa) is observed in the Y-direction compared to other directions (Fig. 15). Similar dependencies are observed in the harmonic analysis of the three-layer plate, exemplified by a solar panel, under different temperature influences.

The provided plots of deformation, strain, and stress amplitudes allow for the determination of maximum values in each direction.
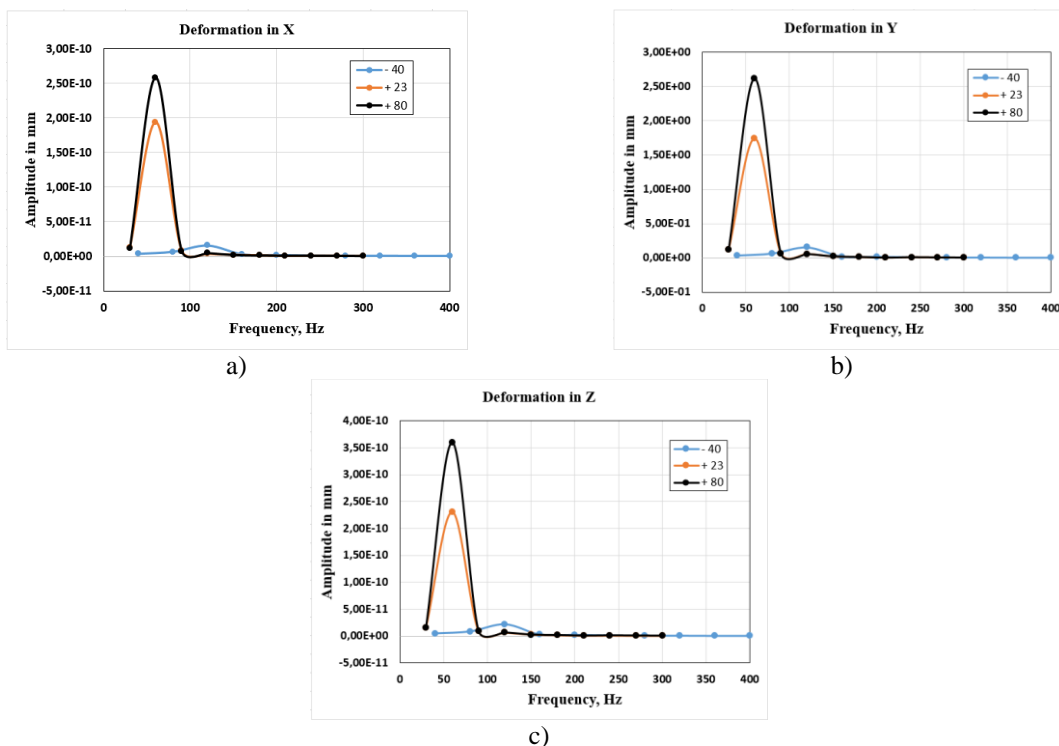

a)

b)

c)

Figure 13. Deformation Amplitude (mm) v/s Frequency (Hz): X – direction, Y – direction, Z – direction
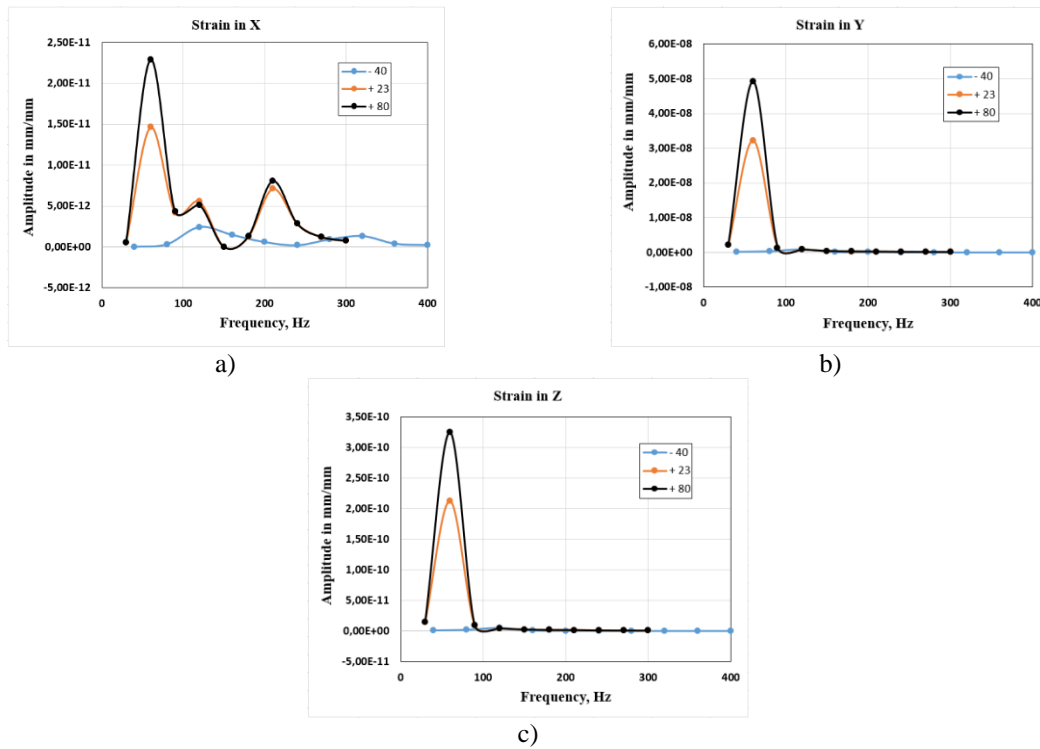
a)   b)

c)

Figure 14. Strain Amplitude (mm/mm) v/s Frequency (Hz): X – direction, Y – direction, Z – direction

a)   b)
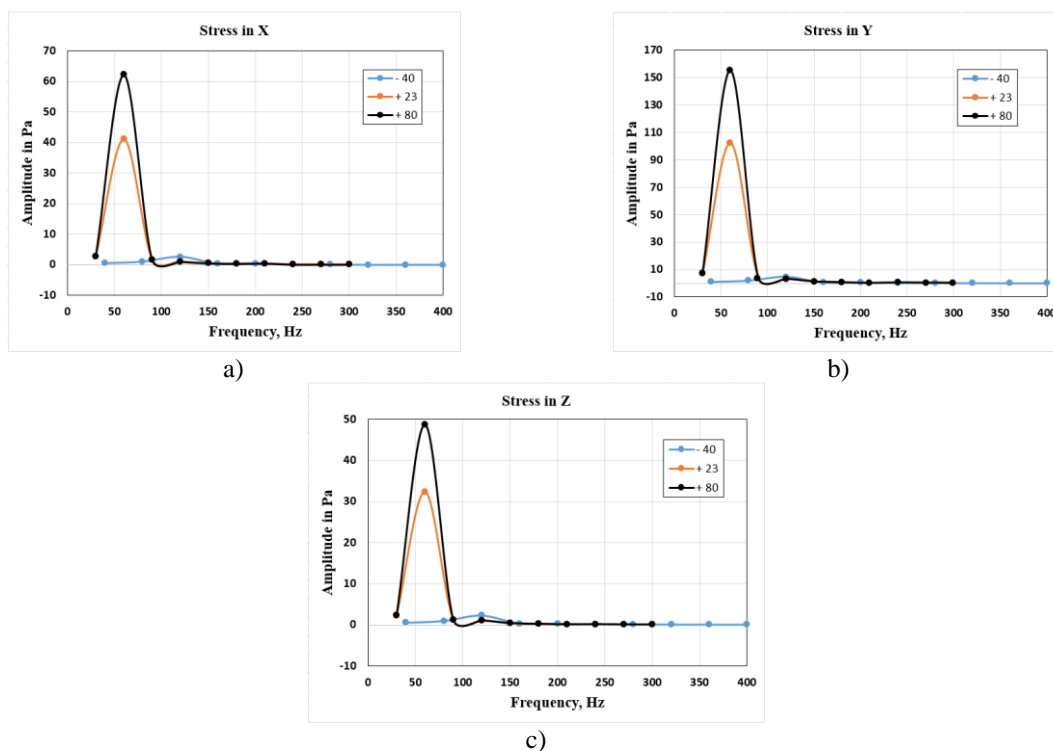
c)

Figure 15. Stress Amplitude (Pa) v/s Frequency (Hz): X – direction, Y – direction, Z – direction

Prospects for further research development. In the subsequent research works, there are plans to delve into the investigation of the influence of boundary conditions on the natural vibrations of the three-layer plate, exemplified by a solar panel. The aim is to determine equivalent mechanical characteristics such as Young's modulus and Poisson's ratio. This is crucial as understanding their impact on the dynamic characteristics of multilayer plates remains a pertinent task.

Conclusions. The article presents the results of investigating the behavior of a three-layer plate using the example of a solar panel through finite element analysis. Static calculations were performed to determine normal and shear stresses, as well as corresponding displacements and strains. To verify the adequacy of the calculations, a numerical convergence study was conducted for various mesh densities. A modal analysis of the three-layer plate was performed, allowing for the determination of natural frequencies and mode shapes at different temperatures.

The analysis also covered harmonic characteristics of the three-layer plate, considering the influence of temperature. The stress-strain state of the three-layer plate was explored in different planes and at various frequencies of oscillations using by ANSYS Workbench.

References

1. Lopez A., Roberts B., Heimiller D., Blair N., Porro G. (2012). U.S. Renewable Energy Technical Potentials: A GIS-Based Analysis. National Renewable Energy Laboratory Document 7, 1–40, ISBN: NREL/TP-6A20-51946. Doi: NREL/TP-6A20-51946. https://doi.org/10.2172/1219777
2. Stark W., Jaunich M., Investigation of Ethylene/Vinyl Acetate Copolymer (EVA) by thermal analysis DSC and DMA, Polymer Testing, vol. 30, 2011, pp. 236–242. https://doi.org/10.1016/j.polymertesting.2010.12.003
3. Kormanikova E., Kostrova K., Dynamic behavior of composite sandwich panel with CFRP outer layers, Wseas trasactions on applied and theoretical mechanics, vol. 17, 2022, pp. 263–269. https://doi.org/10.37394/232011.2022.17.32
4. Cecile Helfen, Stefan Diebels, A numerical homogenisation method for sandwich plates based on a plate theory with thickness change, Journal of applied mathematics and mechanics, Special Issue: Continuum Mechanics, vol. 93, issue 2–3, pp. 113–125. https://doi.org/10.1002/zamm.201100173
5. Foraboschi P., Three-layered sandwich plate: Exact mathematical model, Composites: Part B, vol. 45, 2013, pp. 1601–1612. https://doi.org/10.1016/j.compositesb.2012.08.003
6. Assmus M., Naumenko K., Altenbach H., A multiscale projection approach for the coupled global–local structural analysis of photovoltaic modules, Composite Structures, vol. 158, 2016, pp. 340–358. https://doi.org/10.1016/j.compstruct.2016.09.036
7. Naceur H., Shiri S., Coutellier D., Batoz J. L., On the modeling and design of composite multilayered structures using solid-shell finite element model, Finite Elements in Analysis and Design, vol. 70–71, 2013, pp. 1–14. https://doi.org/10.1016/j.finel.2013.02.004
8. Fernando G. Flores Liz G. Nallim, Sergio Oller, Formulation of solid-shell finite elements with large displacements considering different transverse shear strains approximations, Finite Elements in Analysis and Design, vol. 130, 2017, pp. 39–52. https://doi.org/10.1016/j.finel.2017.03.001
9. Eisentrager J., Naumenko K., Altenbach H., Köppe H., Application of the first-order shear deformation theory to the analysis of laminated glasses and photovoltaic panels, International Journal of Mechanical Sciences, vol. 96–97, 2015, pp. 163–171. https://doi.org/10.1016/j.ijmecsci.2015.03.012
10. Zenkour A. M., A comprehensive analysis of functionally graded sandwich plates: Part 1 – Deflection and stresses, International Journal of Solids and Structures, vol. 42, 2005, pp. 5224–5242. https://doi.org/10.1016/j.ijsolstr.2005.02.015
11. Stefan-H. Schulze, Matthias Pander, Konstantin Naumenko, Holm Altenbach, Analysis of laminated glass beams for photovoltaic applications, International Journal of Solids and Structures, vol. 49, 2012, pp. 2027–2036. https://doi.org/10.1016/j.ijsolstr.2012.03.028
12. Arasan U., Marchetti F., Chevillotte F., Jaouen L., Chronopoulos D., Gourdon E., A simple equivalent plate model for dynamic bending stiffness of three-layer sandwich panels with shearing core, Journal of Sound and Vibration, vol. 500, 2021, 116025. https://doi.org/10.1016/j.jsv.2021.116025
13. Mohammad R. Permoon, Touraj Farsadi, Free vibration of three-layer sandwich plate with viscoelastic core modelled with fractional theory, Mechanics Research Communications, vol. 116, 2021, 103766. https://doi.org/10.1016/j.mechrescom.2021.103766
14. Kai Xue, Wenhao Huang, Qiuhong Li, Three-Dimensional Vibration Analysis of Laminated Composite Rectangular Plate with Cutouts, Materials, 2020, vol. 13, 3113. https://doi.org/10.3390/ma13143113
15. Mengna Han, Zichan Li, Zhicheng Huang, Xingguo Wang and Wenjie Gao, Thermal Mechanical Bending Response of Symmetrical Functionally Graded Material Plates, Materials, 2023, vol. 16, 4683. https://doi.org/10.3390/ma16134683
16. Kerem Ege, N. B. Roozen, Quentin Leclere, R.G. Rinaldi. Assessment of the apparent bending stiffness and damping of multilayer plates; modelling and experiment. Journal of Sound and Vibration, 2018, vol. 426, pp.129–149. https://doi.org/10.1016/j.jsv.2018.04.013

17. Eva Kormanikova, Modal analysis of sandwich panel with composite laminated faces, Vibroengineering procedia, 2019, vol. 23, pp.105–109. https://doi.org/10.21595/vp.2019.20658
18. Michael Koehl, Markus Heck, Stefan Wiesmeier, Jochen Wirth, Modeling of the nominal operating cell temperature based on outdoor weathering, Solar Energy Materials & Solar Cells, vol. 95, 2011, pp. 1638–1646. https://doi.org/10.1016/j.solmat.2011.01.020

УДК 539.3

МСА НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ ТА КОЛИВАНЬ ТРИШАРОВОЇ ПЛАСТИНИ

Ярослав Лавренко; Максим Сущенко

Національний технічний університет України «Київський політехнічний інститут імені Ігоря Сікорського», Київ, Україна

Резюме. Сонячні панелі розглянуто як тришарові пластини з товстим, жорстким зовнішнім шаром і тонким, м'яким внутрішнім шаром. Для опису механічної поведінки пластин на прикладі сонячної панелі використано модель анти-сендвіч пластин. Огляд літератури включає наукові статті, що описують моделі для аналітичних та чисельних розрахунків тришарових пластин. Наведено експериментальні дані механічних характеристик матеріалів, які використовувалися для моделювання напружено-деформованого стану та дослідження коливань тришарової пластини. Результати чисельного моделювання наведено в даній статті. Під час наукового дослідження механічної поведінки сонячної пластини під впливом зовнішніх факторів, таких, як вплив температури на погодні умови, використано метод скінчено-елементного аналізу для багатошарових пластин. Оболонкові елементи використано для розрахунку та моделювання власних форм коливань тришарових пластин. Дослідження збіжності результатів проведено з метою перевірки точності скінчено-елементного розрахунку. Згенеровано чотири сітки з різною кількістю елементів, на яких розраховано максимальні значення прогинів пластини та порівнено між собою. Представлено наукові дослідження при статичному навантаженні за різних умов впливу. Проведений модальний та гармонічний аналіз, які дали змогу визначити й проаналізувати залежності власних коливань та форми коливань, досліджено напружено-деформований стан залежно від частот коливань тришарової пластини. Для дослідження температурного впливу побудовано графічні залежності переміщень, деформацій та напружень у різних площинках. У рамках наукової роботи досліджено механічну модель тонкої сонячної панелі за допомогою скінчено-елементного аналізу в програмі ANSYS з урахуванням різних температурних режимів та порівнянням результатів з існуючими дослідженнями.

Ключові слова: сонячна панель, сендвіч, частоти, напруження, деформація, метод скінчених елементів, ANSYS.