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СИНУСОЇДАЛЬНЕ ЗАЧЕПЛЕННЯ З ПІДВИЩЕНОЮ ЗНОСОСТІЙКІСТЮ ЗУБЦІВ

Анотація. Запропоновано спосіб удосконалювання важконавантажених синусоїдальних зубчастих передач вибором раціональних параметрів на стадії проєктування. Параметри вибираються з умови рівності максимальних питомих ковзань на зубцях шестерні і колеса.
Ключові слова: циліндричні прямозубі передачі, синусоїдальне зачеплення, важконавантажені передачі, зношування зубців

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SINUSOIDAL GEARING WITH INCREASED TEETH WEAR RESISTANCE

Abstract. The way of improvement of heavy duty sinusoidal gears in terms of wear resistance using the selection of rational parameters on the design stage is proposed. The parameters are selected based on the condition of equal maximum values of specific sliding on the pinion and gear.
Keywords: spur gears, sinusoidal gearing, heavy duty gears, tooth wear

Gear teeth wear is notably influenced by the magnitude of specific sliding. Enhancing this meshing characteristic is often achieved through addendum modification of involute teeth. To minimize wear the values of profile shift coefficients can be selected based on the condition of equality of the maximum values of specific sliding on the pinion η_1 and gear η_2. The condition can be presented as follows [1]:

$$|\eta_{1\max}| = |\eta_{2\max}|. \quad (1)$$

According to [1], a way of determination of the profile shift coefficients of the pinion x_1 and gear x_2 is the selection of their values from the tables developed based on the Eq. (1) by Central Design Bureau of Gearbox. The feature of such selection is the condition $x_2 = -x_1$, that is not optimal. The authors of [1] proposed the method for optimal design of gears based on the condition $|\eta_{1\max}| - |\eta_{2\max}| \to \min$. It was presented in [2] that solution of the equation allow to define the optimal values of the coefficients x_1 and x_2 that provide 18% longer service life of the pinion and 20% longer service life of the gear.

An alternative approach is the selection of non-involute gear tooth shape. An example of such approach is the shape proposed in the US Patent [3]. The author proposed the shape that allows to improve operation noise. According to [3], the shape can be generated by a rack-type tool with sine-curve basic profile.

The further research [4] was related to the gears with teeth shape defined by [3]. These gears were named by the author of [4] “sinusoidal”. The research [4] was focused on calculation of their geometrical parameters and meshing characteristics using graphical-analytical method. The author of [4] validated the main object of the invention [3] that sinusoidal gearing can provide lower noise in operation.

The latest studies [5,6] demonstrate that the gearing can also provide a larger loading capacity and higher operating indicators, in particular better wear resistance. All the advantages are due to the convex-concave contact of the active surfaces.

The mathematical model allowable for determination of meshing characteristics of sinusoidal gears is presented in [7]. The reference profile related to module $m = 1$ mm (Fig.) can be given in parametric form by the equation [4,7]:

$$x_p = a \sin \lambda ;\ y_p = \lambda / 2 , \tag{2}$$

where a is the radius of sine generating circle (amplitude) that equals h, which is a half of full depth H, and λ is the parameter (Fig.). The positive values of λ correspond to the generation process of the pinion addendum and gear dedendum whereas negative values of λ correspond to the generation process of the gear addendum and pinion dedendum.

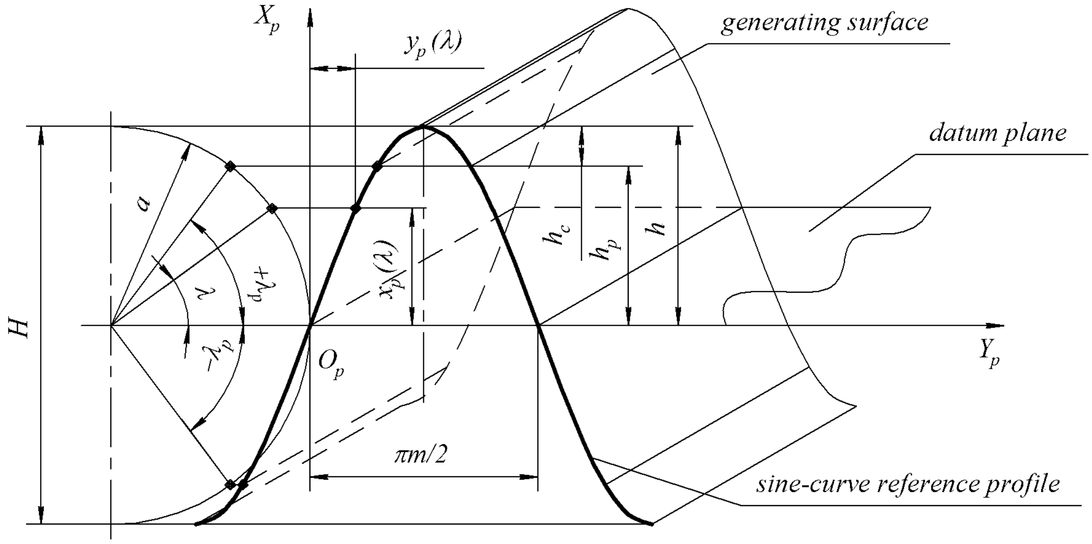

Fig. Generating surface for the teeth of sinusoidal gears

The teeth of the gears are generated by sinusoidal basic rack which has different signs of curvature on the addendum and dedendum segments. The fillet segment (height of h_c) of the basic rack and its flanks (height of h_p) are outlined by the same curve. The values $\lambda_p = \pm \arcsin h_p / h$ define the points between the flank and fillet of the tooth. The most common case is: $h_p = 1$ and $h = 1.25$, i.e. $h_p / h = 0.8$, and $\lambda_p = \pm 53.13°$.

In order to evaluate specific sliding of the teeth generated by the profile of Eq. (2) the formulas from [7] can be used in the form as follows:

$$\eta_1 = \frac{(u+1)}{u} \cdot \frac{\Omega(\lambda)}{r_1 + \Omega(\lambda)} ;\ \eta_2 = -\frac{(u+1)}{u} \cdot \frac{\Omega(\lambda)}{r_2 - \Omega(\lambda)} , \tag{3}$$

where $\Omega(\lambda) = h \sin \lambda (4h^2 \cos 2\lambda + 1)$; r_1 and r_2 are the radii of reference circles of the pinion and gear respectively; u is gear ratio.

The condition of equality of the maximum values of specific sliding on the pinion η_1 and gear η_2 in the case of sinusoidal gearing can be obtained using Eq. (3). The characteristics η_1 and η_2 take their maximum at the values $\tilde{\lambda}$ of the parameter λ, that fulfill the conditions $d\eta_1 / d\lambda = 0$ and $d\eta_2 / d\lambda = 0$, i.e. $\eta_{1\max} = \eta_1(\tilde{\lambda})$ and $\eta_{2\max} = \eta_2(\tilde{\lambda})$.

Let us differentiate the functions $\eta_1(\lambda)$ and $\eta_2(\lambda)$:

$$\frac{d\eta_1}{d\lambda} = \frac{(u+1)}{u} \cdot \frac{\Omega'(\lambda) \cdot r_1}{[r_1 + \Omega(\lambda)]^2} ;\ \frac{d\eta_2}{d\lambda} = -\frac{(u+1)}{u} \cdot \frac{\Omega'(\lambda) \cdot r_2}{[r_2 - \Omega(\lambda)]^2} , \tag{4}$$

Analysis of Eqs. (4) reveals that denominator equals infinity only at $r_2 = \infty$, i.e. for rack and pinion gearing. The function $\Omega(\lambda)$ at $\lambda = +\lambda_p ... -\lambda_p$ does not take infinite values. The values of $u+1$, r_1 or r_2 in the numerator may not be equal zero. Thus, the only case when $d\eta_1/d\lambda = 0$ and $d\eta_2/d\lambda = 0$ is $\Omega'(\lambda) = 0$. Let us define this derivative:

$$\Omega'(\lambda) = h\cos\lambda\left(4h^2\cos 2\lambda - 16h^2\sin 2\lambda + 1\right). \qquad (5)$$

Obviously, the right part of Eq. (5) equals zero at $\lambda = \pm 0.5\pi$ but this is not the case, because these points are out of tooth flank. The only equation

$$4h^2\cos 2\lambda - 16h^2\sin 2\lambda + 1 = 0 \qquad (6)$$

fulfill the condition $\Omega'(\lambda) = 0$. The solution of Eq. (6) is the required parameter $\tilde{\lambda}$:

$$\tilde{\lambda} = \pm\arcsin\left[\left(4h^2 + 1\right)/24h^2\right]^{0.5}. \qquad (7)$$

It can be inferred from the Eq. (7) that location of the point on tooth profile that corresponds to the maximum value of specific sliding does not depend on numbers of teeth, but depends only on tooth height coefficient.

Let us define the value of function $\tilde{\Omega} = \Omega(\tilde{\lambda})$ that correspond to $\eta_{1\max} = \eta_1(\tilde{\lambda})$ and $\eta_{2\max} = \eta_2(\tilde{\lambda})$ at $\tilde{\lambda}$ defined by Eq. (7):

$$\tilde{\Omega} = \pm 2\left[\left(4h^2 + 1\right)/6\right]^{1.5}, \qquad (8)$$

where the upper sign corresponds to “+” sign in Eq. (7), and the down sign corresponds to “–” sign in Eq. (7).

As it was shown in [5], the nature of η_1 and η_2 curves of sinusoidal gears differs fundamentally from the ones of involute gearings. They have two extremums (on addendum and dedendum) and one point of inflexion. Taking into account this fact, the condition (1) can be presented for four cases of the equality of specific sliding values as follows:

Case #1. On the dedendum of pinion tooth and the dedendum of gear tooth (the case corresponds to specific sliding fit in involute gearing):

$$\left|\eta_{1\max}\right|_{\lambda<0} = \left|\eta_{2\max}\right|_{\lambda>0}. \qquad (9)$$

Case #2. On the addendum of pinion tooth and the addendum of gear tooth:

$$\left|\eta_{1\max}\right|_{\lambda>0} = \left|\eta_{2\max}\right|_{\lambda<0}. \qquad (10)$$

Case #3. On the addendum of pinion tooth and the dedendum of gear tooth:

$$\left|\eta_{1\max}\right|_{\lambda>0} = \left|\eta_{2\max}\right|_{\lambda>0}. \qquad (11)$$

Case #4. On the dedendum of pinion tooth and the addendum of gear tooth:

$$\left|\eta_{1\max}\right|_{\lambda<0} = \left|\eta_{2\max}\right|_{\lambda<0}. \qquad (12)$$

As the signs of η_1 and η_2 are known from [5], the conditions (9) and (10) can be transformed at the appropriate values of λ to the form $\eta_{1\max} = \eta_{2\max}$. For the same reason the conditions (11) and (12) can be also presented at the appropriate values of λ without the absolute magnitude as follows: $\eta_{1\max} = -\eta_{2\max}$.

Thus, conditions (9) and (10) at $\tilde{\Omega}$ from Eq. (8) can be related to the equation:

$$\frac{(u+1)}{u}\cdot\frac{\mp 2\left[\left(4h^2+1\right)/6\right]^{1.5}}{r_1 \mp 2\left[\left(4h^2+1\right)/6\right]^{1.5}} = -\frac{(u+1)}{u}\cdot\frac{\pm 2\left[\left(4h^2+1\right)/6\right]^{1.5}}{r_2 \mp 2\left[\left(4h^2+1\right)/6\right]^{1.5}}, \qquad (13)$$

where upper and down signs correspond to Case #1 and Case #2 respectively.

Conditions (9) and (10) at $\tilde{\Omega}$ from Eq. (8) can be related to the equation:

$$\frac{(u+1)}{u}\cdot\frac{\pm 2\left[\left(4h^2+1\right)/6\right]^{1.5}}{r_1 \pm 2\left[\left(4h^2+1\right)/6\right]^{1.5}} = \frac{(u+1)}{u}\cdot\frac{\pm 2\left[\left(4h^2+1\right)/6\right]^{1.5}}{r_2 \mp 2\left[\left(4h^2+1\right)/6\right]^{1.5}}, \quad (14)$$

where upper and down signs correspond to Case #3 and Case #4 respectively.

The condition (13) can be fulfilled at $r_1 = r_2$, and the condition (14) can be fulfilled at

$$r_2 - r_1 = \pm 4\left[\left(4h^2+1\right)/6\right]^{1.5}, \quad (15)$$

where the upper and down signs correspond to Case #3 and Case #4 respectively. Obviously, that the upper sign corresponds to the reduction gearing, while the down sign corresponds to the multiplication gearing. The radii in Eq. (15) are defined by the known formulas: $r_1 = 0.5mz_1$; $r_2 = 0.5mz_2$. As the geometry of generating surface is related to module $m = 1$ mm, the followed ratios can be used: $r_1 = 0.5z_1$; $r_2 = 0.5z_2$. Thus, we obtain

$$z_2 = z_1 \pm 8\left[\left(4h^2+1\right)/6\right]^{1.5}, \quad (16)$$

where the upper and down signs correspond the reduction and multiplication gearings respectively.

The sinusoidal gear pair with numbers of teeth and teeth depth, that are close to fulfilling the condition (16), will have the equal values of the pinion and gear specific sliding. For the most common case of $h = 1.25$, the rational numbers of teeth from the condition (16) are $z_2 \approx z_1 \pm 11$. This requires as few pinion teeth in number as possible in order to provide gear ratio much more than $u = 1$. This is why the minimal number of teeth of sinusoidal gearing is to be defined in further research from the condition of undercutting prevention.

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