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DETERMINATION OF THE LINEAR AUTOMATIC SYSTEM PARAMETERS REGION, PROVIDING THE SET QUALITY PARAMETERS BY THE D-DECOMPOSITION METHOD

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In this paper a technique for determining the parameter range by deformation of the characteristic equation has been proposed. An example of determining the area in which the required quality of the control system is ensured by means of the proposed method is considered.

Keywords: D-decomposition, quality of control systems, root quality indicators.

As is known, the oscillations and the degree of stability of the developed automatic system can be determined from the roots of the characteristic equation. Moreover, the degree of stability is determined by the rightmost root of the characteristic equation, and the oscillations is determined by the minimum slope of the line passing through of the characteristic equation roots plane origin and drawn so that all the roots of the characteristic are on the left or lie on it. However, it is often necessary to solve the inverse problem: knowing the limitations on oscillations in the transient process and the degree of system stability, it is necessary to find the range of acceptable parameter values. The region of system parameters that satisfies a given degree of stability is easily obtained by the method of "ordinary" D-decomposition [1] by deforming the characteristic equation $D(p) = 0$ with replacing the variable $p = q + c$, and finding the stability region of the deformed characteristic equation $D(q + c) = 0$, by the usual rules. The resulting region of stability gives the region with a given degree of original system stability, which is required.

Similarly, it is possible to obtain the parameter range with a given oscillation – for this it is necessary to find the areas of stability of systems described by the characteristic equations:

$D(q \cdot (1 + jb)) = 0$ and $D(q \cdot (1 - jb)) = 0$

where b is some parameter determined by admissible oscillation, and find their intersection. However, simultaneous consideration of constraints on oscillation and degree of stability of the system greatly complicates the task and requires the construction of several regions, and then determination of their superposition – the area that satisfies all the above constraints. Such operations are difficult to carry out manually and it is practically impossible to automate with the help of mathematical packages even for low-order systems, and therefore the problem arises to find a simpler and more convenient method of at least an approximate method for obtaining the area of parameters change that satisfies the quality conditions of the control system.

It will be convenient to approximate the permissible region of placement of the roots of the characteristic equation on the complex plane so that it is most easily converted into the left half-plane when moving from the complex plane of the parameter p to the new complex plane of the parameter q. Theoretically, using the Schwarz-Christophel mapping [2], it is possible to accurately display the area of permissible root placement in the left half-plane and then analyze the obtained characteristic equation by the D-decomposition method. But, in general, the resulting characteristic equation will become non-algebraic.

In many cases, the approximation of the area of acceptable root placement can be carried out using a simpler area. For example, it is quite convenient to use as a given region the inside of a parabola, which lies entirely in the initial region and touches its boundary at 3 points (Fig. 1). This parabola can be described using the equation:

$P(t) = a \cdot t^2 + j\,t - d$;

where *j* is an imaginary unit, the real parameter *d* is determined by a given degree of stability, the real parameter *a* is determined by the touching condition of the parabola with the oscillation limit line, *t* – a parameter that changes from $-\infty$ to ∞. It is easy to prove that in this case the coordinate conversion

$p(q) = a \cdot q^2 + q - c$

maps the inside of parabola 2 to the left half-plane in the complex plane of the parameter *q*, and the exterior to the right half-plane, although of course such a mapping will no longer be mutually unambiguous.

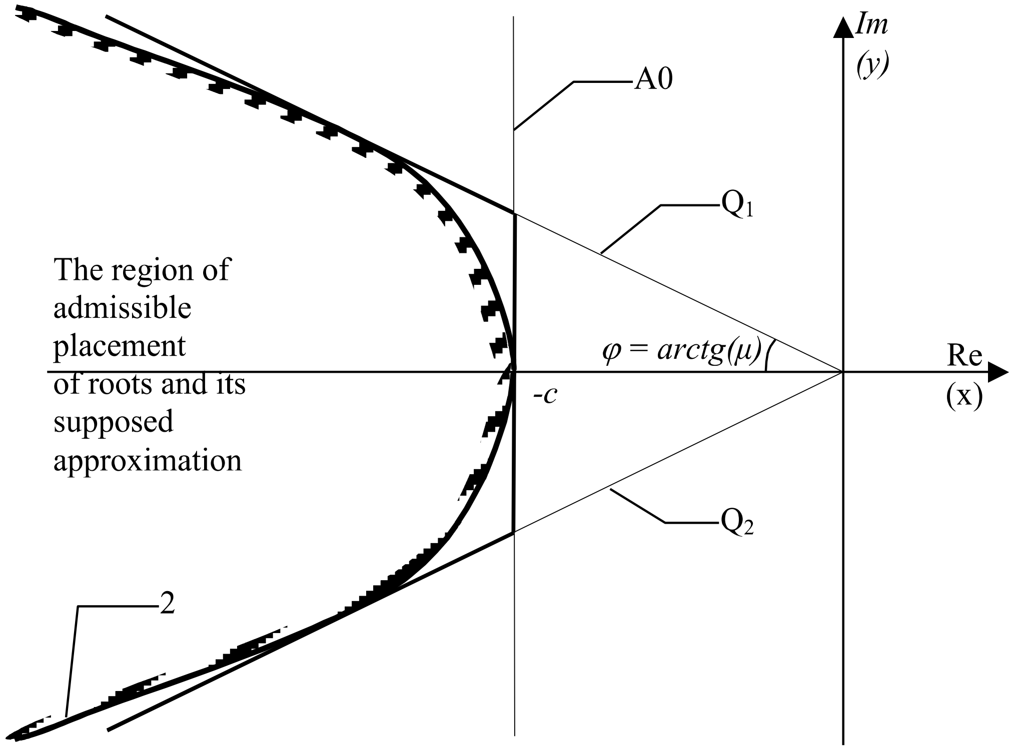

Fig. 1. The approximation of the permissible region of placement of the roots of the characteristic equation

It remains to find the dependence of the parameters c and a as the parameters of the root's admissible placement region. From the condition that the parabola must touch the line of the admissible degree of stability (A0) follow:

$d = c$

The formula for the line of limiting oscillation Q1 can be written as

$Q_1(v) = v - j\,tg(\phi)\,v$,

where v – a parameter that changes from $-\infty$ to ∞. From the condition of touching the line with the parabola follows:

$P(t) = Q_1(v)$ or $a \cdot t^2 + j\,t - c = v - j\,tg(\phi)\,v$.

From this follows the equation for the variable t.

$$a \cdot t^2 + \frac{t}{tg(\phi)} - c = 0$$

This is a quadratic equation for t that can have two real roots. In the case of a touch of a parabola and a line Q_1, the roots must coincide, therefore, a condition arises on the discriminant:

$$\frac{1}{tg^2(\phi)} + 4\,ac = 0$$

Whence it follows that:

$$a = \frac{1}{4\,c \cdot tg^2(\phi)} = \frac{1}{4\,c\,\mu^2}.$$

A touching between a parabola and another limiting line gives the same value for the parameter *a*. To illustrate the proposed method, we find the range of allowable values of the coefficient k of the characteristic equation

$$D(p) = p^3 + 16 \cdot p^2 + \left(85 + \frac{k}{5}\right) \cdot p + 148 + k$$

at a given degree of stability $c = 2$ and a given degree of oscillation $\mu = \sqrt{3}$. It is easy to see that the parameter $a = 1/24$, and $c = 2$. Substituting into the characteristic expression for $p(q) = a \cdot q^2 + q - c$ obtain

$$D(q) = \frac{1}{1382}q^6 + \frac{1}{19}q^5 + \frac{41}{28}q^4 + \frac{11}{6}q^3 + \left(\frac{91}{8} + \frac{k}{12}\right)q^2 + \left(33 + \frac{k}{5}\right)q + 34 + \frac{3k}{5}$$

Further, the stability region of the transformed characteristic equation in the plane of one parameter is built by the standard method - this will be the region with the given quality parameters, which was approximated.

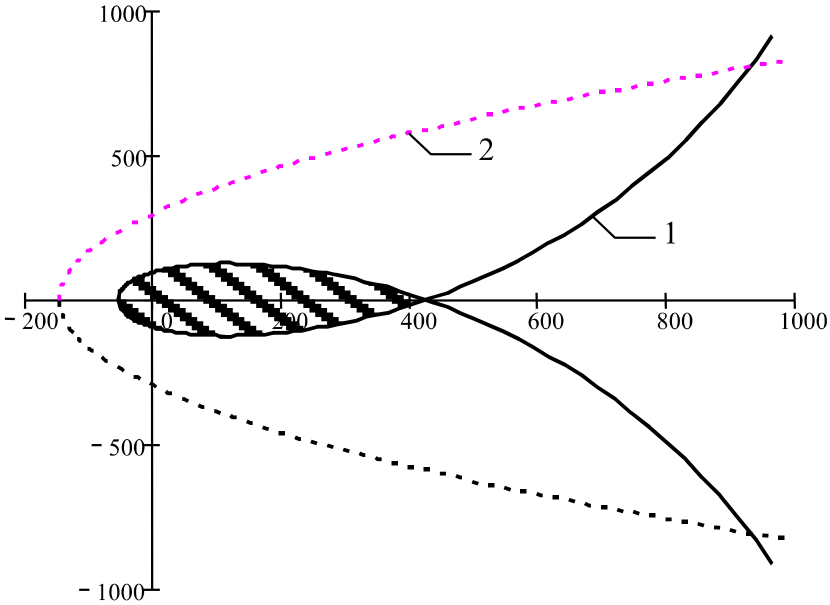

Fig 2. The quality area in the plane of the characteristic parameter (shaded)

It is shown in Figure 2 curve 1 corresponds to the D-decomposition curve, which highlights the area with a given quality, and curve 2 corresponds to D-decompositioning, which ensures the stability of a given system.

Conclusion. The described method with the help of D splitting makes it possible to obtain an approximation of the region of admissible parameters that provide a given oscillation and stability margin of the system, while the method itself is relatively simple and does not require increased computational costs.

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