

UDC 621.855

## MATHEMATICAL REPRESENTATION OF THE BRANCH KINEMATICS OF A TRANSMISSION WITH DISCRETE-FLEXIBLE CONNECTION

Ihor Lutsiv<sup>1</sup>; Taras Dubyniak<sup>1</sup>; Oleksandra Manziy<sup>2</sup>;  
Stanislav Andreichuk<sup>2</sup>

<sup>1</sup>*Ternopil Ivan Puluj National Technical University, Ternopil, Ukraine*

<sup>2</sup>*Lviv Polytechnic National University, Lviv, Ukraine*

**Summary.** The paper deals with the mathematical model development of the kinematical behavior of the flexible transmission branch exemplified. A typical example of transmission with discrete-flexible connection can be considered as the movement of the drive elements of the chain. The use of chain gears as a drive for a wide range of technological machines with high requirements in order to ensure a certain law of motion of the executive bodies is the task of studying changes in its kinetic characteristics during operation. It is established that random deviations of the chain step from the nominal are the result of manufacturing inaccuracy elements of transmission and wear during operation. The mathematical model of motion gives an idea of the real interpretation of the kinematics of the chain transmission taking into account the uneven dimensions of individual links. The model makes it possible to present the components of deviations of the transmission movement from the given in two groups: deviations created by the accumulated error of the chain section, and movement. Calculations based on the developed mathematical model show that when the hinges of the sleeve-roller chain are worn by 2.5%, the value of the coefficient of non-uniformity of the transmission increases threefold.

**Key words:** transmission, chain, roll, branch, kinematics, pitch.

[https://doi.org/10.33108/visnyk\\_tntu2022.02.005](https://doi.org/10.33108/visnyk_tntu2022.02.005)

Received 11.01.2022

Statement of the problem. The design of flexible links, the investigation of the improvement of their reliability and durability is one of the important tasks in the field of applied mechanical engineering [1]. Among flexible discrete gears, drive roller chains play an important role. In particular, the need of the national economy of Ukraine in such chains is about 50 million running meters [2]. Widespread use of chain gears in various fields of mechanical engineering is due to their undeniable advantages over other mechanical gears [3]. Acute shortages and lack of such chains hinder the growth of productivity and increase the level of automation of production. Therefore, determining the optimal ratios for the characteristics of chain transmissions is an important task [4]. In this regard, scientists both in Ukraine and abroad have conducted and are conducting multifaceted research in the theory of chain engagement, as well as the influence of various factors and parameters on the kinematic and dynamic performance of chain transmissions [5].

The study of discrete-flexible transmission can be reduced to the investigation of the movement of the drive elements of the chain. The use of chain gears as a drive for a wide range of technological machines with high requirements to ensure a certain law of motion of the executive bodies is the task of studying changes in its kinematic characteristics during operation.

Analysis of the known research results. While designing chains and chain transmissions, existing methods often do not take into account the changing conditions of the transmission, in this way the size of the chain is unreasonably choose and the effect of wear of its elements is

not taken into account. Thus this contributes to reduced characteristics of durability, reliability, vibration resistance and move irregularity [6].

During operation, the technical parameters of the chain acquire specific changes caused by wear and interaction of the working surfaces of the hinge parts [7]. Thus, during wear, the drive chains lose their efficiency due to the unacceptable increase in the actual contact step of the external links [8]. The wear of the chain hinges depends on many factors, including physical and mechanical and frictional characteristics of the hinges, operating conditions, load modes and others [9, 10].

One of the integral characteristics of a real chain in this negative plan is the variety of chain elements (particularly, roller). It results in the skew of the axes of its links, violation of the normal contact of the surface of the rollers with the teeth, the curvature of the axes of the hinges and other negative influences [11].

In general, the diversity of the chain dimensions affects almost all the processes that accompany the operation of the chain transmission [12]. Thus, during operation, the variety of the chain dimensions is constantly increasing, causing the decrease in the service life of the chain both in terms of wear of its hinges and in terms of fatigue strength of the chain [13].

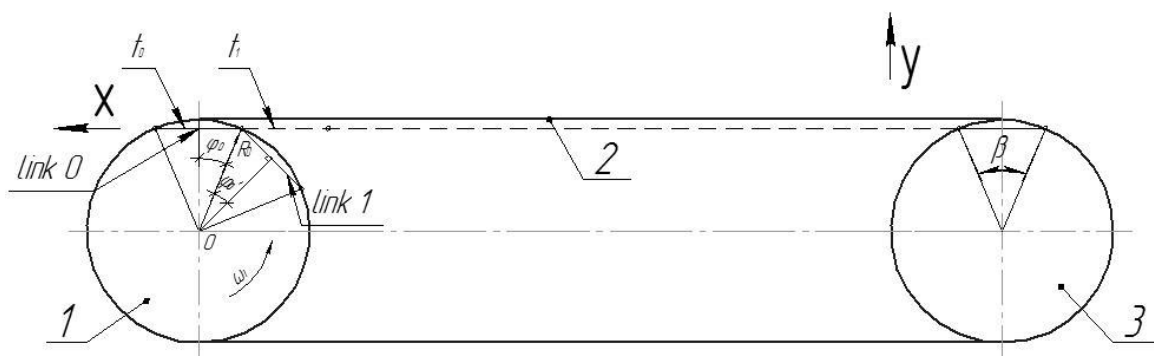
A separate aspect is the influence of the instantaneous dimension diversity of the chain on the increase in the move irregularity of the chain and the driven system and increase for this reason the vibrations of the branches and dynamic loads [14].

Accordingly, some researchers have paid considerable attention to the kinematic characteristics of the chain drive with roller pairs [15, 16].

Paper objective. Given the review of the literature, it can be argued that mathematical modeling of the kinematics of chain transmission in terms of taking into account the diversity of the chain to determine its impact on the uniformity of the transmission is an important scientific and applied problem.

Task setting. Mathematical model development. The characteristics proposed in [1] for estimating the move irregularity of the chain and driven system, which reflect the maximum change in speed of the latter over a period of time (single hinge parameter) to some extent characterize the transmission dynamics, but cannot be used as a measure to assess compliance movement of the executive body. In addition, they do not allow to assess the deviation of the true position of the executive body from the one set by law and its changes during operation due to chain wear.

The objective of this paper is to solve this problem. In order to solve it, we turn to Fig. 1, where 1 is the leading sprocket of the chain drive is rigidly connected to the drive system, which provides due to the high inertia and rigidity of the uniform movement of the latter; 2 is the leading link in the chain transmission; 3 is the driven sprocket, rigidly connected to the executive body.



**Figure 1.** General transmission scheme

While solving the problem, we assume that the deformation of the chain and the inertia of the driven system do not significantly affect the nature of their motion.

In order to simplify the solution, we assume that the necessary laws of motion of the executive bodies rigidly connected with the chain and the driven sprocket are described by the following equation:

$$\begin{aligned} S_p(\tau) &= h \cdot \tau; \\ \varphi_p(\tau) &= b \cdot \tau, \end{aligned} \quad (1)$$

where  $h$  and  $b$  are constant coefficients.

To ensure these laws it is necessary that the speed of the chain  $\mathcal{G}_m$  and the driven system  $\omega_2$  are equal to:

$$\begin{aligned} \mathcal{G}_o &= \omega_1 \cdot R_{1m} = h; \\ \omega_{2o} &= \omega_1 \cdot u = b, \end{aligned} \quad (2)$$

where  $\omega_1$  is angular velocity of rotation of the drive sprocket 1;

$R_{1m}$  is the average radius of the chain hinges in contact with the toothed sprockets.

$u$  is gear ratio of the chain transmission.

When using other variables, the course of the solution will not change if the laws of motion are given by other equations.

In [1, 2, 3, 4] it is shown that due to the kinematics of the chain hinges with sprocket teeth, as well as random deviations of the steps of the chain links from the nominal as a result of manufacturing inaccuracy and due to wear on their value to provide condition (2) during a certain period of time is impossible.

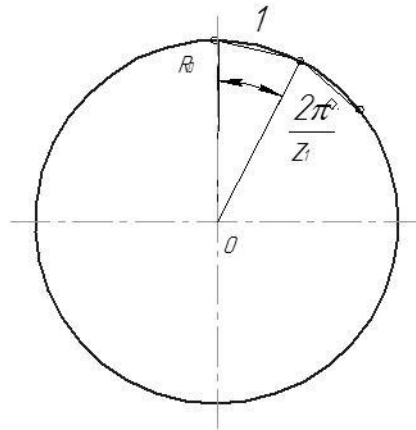
Therefore, in practice, these conditions are provided only with some approximation, considering it worse if the average values

$$\mathcal{G}_{om} = \mathcal{G}_o; \quad \omega_{2m} = \omega_{2o}$$

for a period of time equal to the mileage of one circuit.

To solve the problem, first we describe the law of motion of the chain when it impacts the leading sprocket. Suppose that at some point in time  $\tau = 0$ , the hinge of a certain zero link came into contact with the teeth, located at a certain radius  $R_0$  forming an angle with y-axis  $\varphi_0$ . The period of engagement of this hinge is called the time from the beginning of the entry into the engagement of the hinge of this link to the beginning of the contact of the next 1-st link. The movement of the chain during this period is to be described by the following equation:

$$\begin{aligned} S_0(\tau) &= R_0 \cdot \sin \varphi_0 - R_0 \cdot \sin(\varphi_0' - \omega_1 \tau); \\ \frac{\varphi_0 + \varphi_0'}{\omega_1} &\leq \tau \leq 0 \end{aligned} \quad (3)$$



**Figure 2.** The scheme of movement of the chain

The duration of this period is as follows:

$$\tau_0 = \frac{\varphi_0 + \varphi'_0}{\omega_1}$$

With the beginning of the engagement of the next 1st hinge, the movement of the chain is determined by the radius of its placement and the corresponding angles  $\varphi_1$  and  $\varphi'_1$ .

The movement of the chain is described by the equations:

$$S_1(\tau) = R_1 \cdot \sin \varphi_1 + R_1 \cdot \sin \varphi'_1 + R_2 \cdot \sin \varphi_2 - R_2 \cdot \sin(\varphi_1 + \varphi'_1 + \varphi_2 - \omega_1 \tau); \quad (4)$$

at

$$\frac{\varphi_1 + \varphi'_1}{\omega_1} < \tau < \frac{\varphi_1 + \varphi'_1 + \varphi_2 + \varphi'_2}{\omega_1},$$

where  $R_1, R_2$  are the radius vectors at points 1 and 2 of the chain hinge.

Given that

$$R_1 \cdot \sin \varphi'_1 + R_2 \cdot \sin \varphi_2 = t_2;$$

and

$$\varphi_1 + \varphi'_1 + \varphi_2 + \varphi'_2 = \frac{4\pi}{z},$$

for arbitrary k-hinge (and  $k > 1$ ) we write

$$S_k(\tau) = R_1 \cdot \sin \varphi_1 + \sum_{i=2}^k t_i - R_k \cdot \sin \left[ m \cdot \frac{4\pi}{z} - (-1)^k \cdot \varphi_k - \omega_1 \tau \right] \quad (5)$$

and when k is equal to

$k=3, 5, 7\dots$

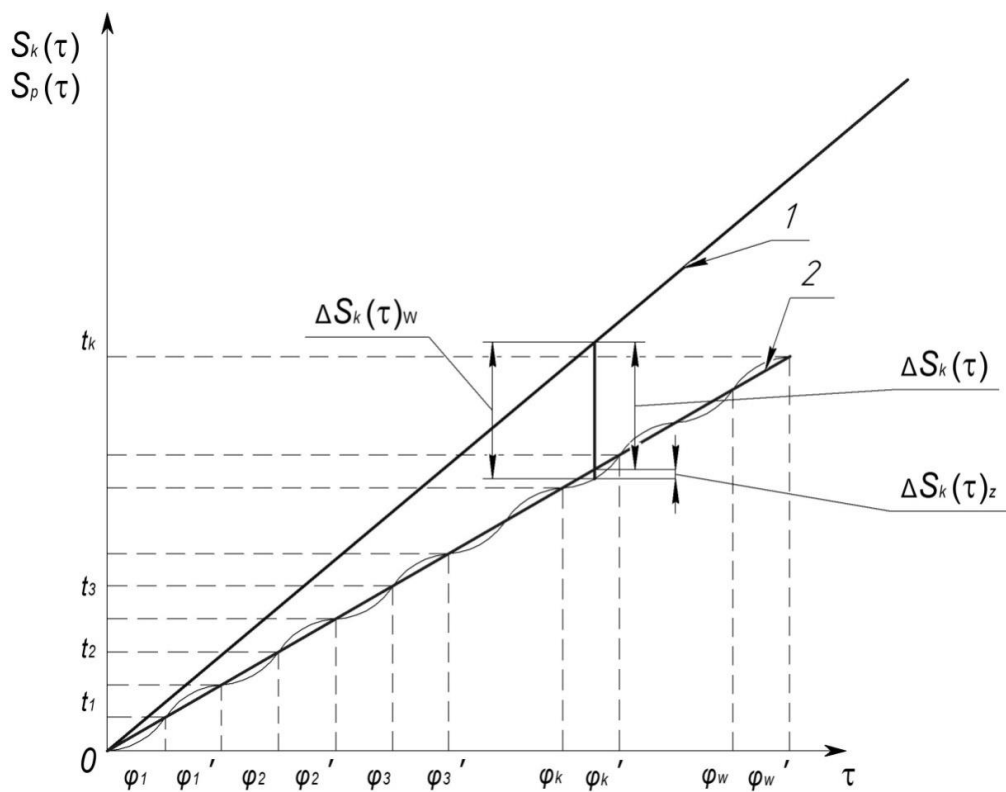
$$m = \frac{k-1}{2}; \varphi_k = \varphi_k; m \frac{4\pi}{z} \leq \omega_1 \tau \leq m \frac{4\pi}{z} + \varphi_k + \varphi_k'$$

$k=2, 4, 6\dots$

$$m = \frac{k}{2}; \varphi_k = \varphi_k; (m-1) \frac{4\pi}{z} + \varphi_k + \varphi_k' \leq \omega_1 \tau \leq m \frac{4\pi}{z}$$

Graphically, this law can be represented by curve 2 (Fig. 3). The line 1 in this figure shows the uniform motion:

$$S_p(\tau) = h \cdot \tau$$



**Figure 3.** Graphic of the chain move

As can be seen from Figure 3, the magnitude of the deviation of the chain movement over time  $\tau$  from a given (uniform) can be determined by the following formula:

$$\Delta S_k(\tau) = S_p(\tau) - S_k(\tau) = h \cdot \tau - S_k(\tau) \tag{6}$$

The angle of the line describing the given law of motion for our case can be determined from the following considerations.

With uniform rotation of the drive sprocket, during the one running contour

$$\tau_w = \frac{2\pi}{z_1} \cdot W / \omega_1$$

the chain passes the distance equal to the length of the contour

$$L = W \cdot t_m,$$

where  $W$  is the number of links in the chain;  
 $t_m$  is the average chain pitch.

Then

$$h = \frac{W \cdot t_m}{\frac{2\pi}{z_1} W / \omega_1} = \frac{z_1 \cdot \omega_1}{2\pi} \cdot t_m, \tag{7}$$

accordingly  $S_p(\tau) = t_m \frac{z_1 \cdot \omega_1}{2\pi} \tau.$

Hence equation (6) takes the following form

$$\Delta S_k(\tau) = t_m \cdot \frac{z_1 \cdot \omega_1}{2\pi} \tau - R_1 \sin \varphi_1 - \sum_{i=2}^k t_i + R_k \sin \left[ m \frac{4\pi}{z} - (-1)^k \varphi_k - \omega_1 \tau \right] \tag{8}$$

where  $z_1$  is the number of teeth of the drive sprocket.  
 $t_m$  is the average pitch of the contour defined by the length of the contour

$$t_{1m} = \frac{L}{W},$$

which provides the given law of motion.

In order to analyze the components of the value  $\Delta S_k(\tau)$ , we represent it in a slightly modified form:

$$\begin{aligned} \Delta S_k(\tau) &= \Delta S_1 + \Delta S_k(\tau)_w + \Delta S_k(\tau)_z = \\ &= \left\{ t_m \cdot \frac{z_1 \omega_1}{2\pi} \cdot \frac{\varphi_0}{\omega_1} - R_1 \sin \varphi_1 \right\} + \left\{ t_m \cdot \frac{z_1 \omega_1}{2\pi} \cdot m \cdot \frac{4\pi}{z_1 \omega_1} - \sum_{i=2}^k t_i \right\} + \\ &+ \left\{ t_m \cdot \frac{z_1 \omega_1}{2\pi} \cdot \left( \tau - \frac{\varphi_1}{\omega_1} - m \frac{4\pi}{z_1 \omega_1} \right) - R_k \sin \left[ m \frac{4\pi}{z} - (-1)^k \varphi_k - \omega_1 \tau \right] \right\}, \end{aligned} \tag{9}$$

where  $\Delta S_1$ ,  $\Delta S_k(\tau)_w$ ,  $\Delta S_k(\tau)_z$  – the corresponding components in curly brackets, which correspond to the deviation from the specified position of the 1st hinge;  
the deviation of the hinge in contact with the sprocket at time  $\tau$ .

This interpretation makes it possible to divide the corresponding components of the deviation into 2 groups: deviations created by the accumulated error in the length of the chain that runs on the star in time  $\tau$  and deviations formed during the entire time of engagement of the hinge, caused by the irregularity of its movement along the direction of movement during the engagement period

$$\varphi_k + \varphi'_k .$$

First let us consider the deviations formed during the engagement of two consecutive links. Moving the origin to the end of the node of the gear of the  $i$ -th hinge after simple transformations we get for the  $i$  hinge

$$\begin{aligned}
 &O \leq \omega_1 \tau \leq \varphi'_i ; \\
 &S_i(\tau) = R_i \cdot \sin \omega_1 \tau ; \\
 &S_i(\tau)_p = \frac{R_i \cdot \sin \omega_1 \tau}{\varphi_i} \omega_1 \tau ; \Delta S_i(\tau) = R_i \cdot \sin \omega_1 \tau - \frac{R_i \cdot \sin \omega_1 \tau}{\varphi'_i} \omega_1 \tau \quad (9 \text{ a})
 \end{aligned}$$

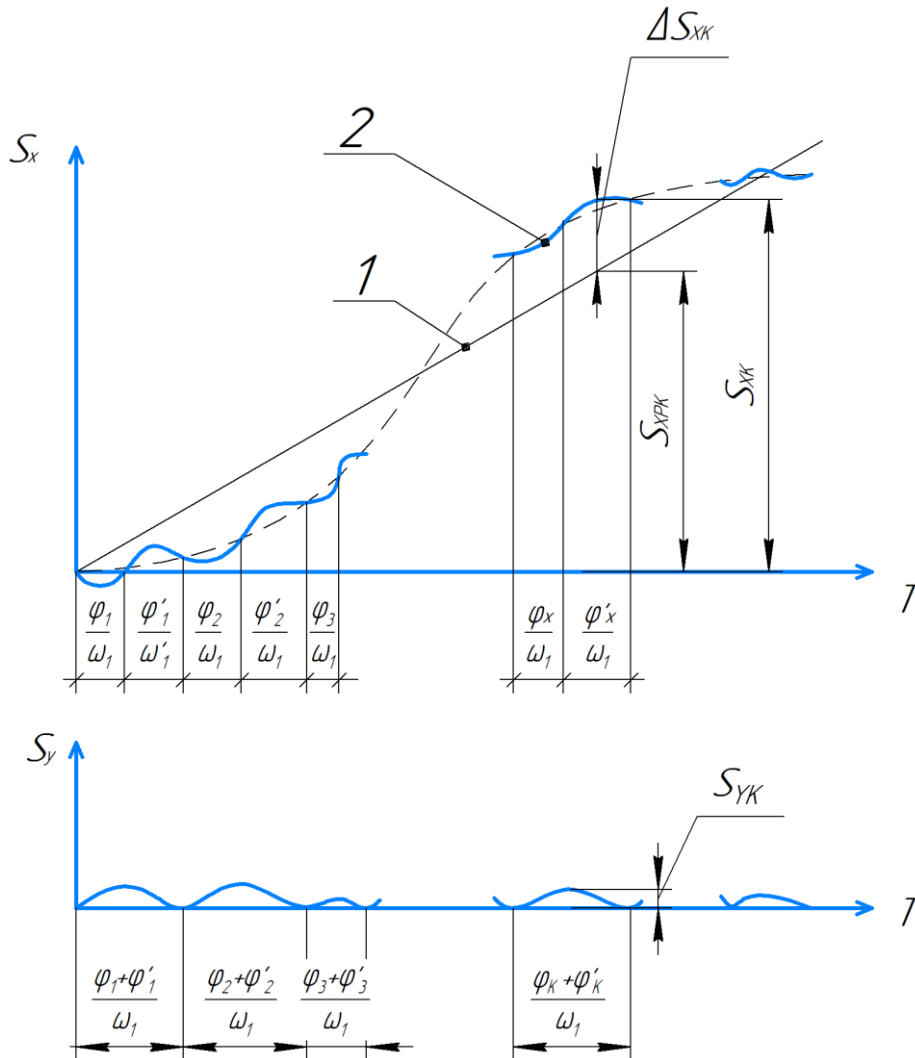
for  $i+1$ -st hinge:

$$\begin{aligned}
 &\varphi'_i \leq \omega_1 \tau \leq \varphi'_i + \varphi'_{i+1} + \varphi_{i+1} \\
 &S_{i+1}(\tau) = t_{i+1} + R_{i+1} \cdot \sin(\omega_1 \tau - \varphi'_i - \varphi_{i+1}) \\
 &S_{i+1}(\tau)_p = t_{i+1} + \frac{R_{i+1} \cdot \sin \varphi_{i+1}}{\varphi_{i+1}} (\omega_1 \tau - \varphi'_i - \varphi_{i+1}) \quad (9 \text{ b}) \\
 &\Delta S_{i+1}(\tau) = R_{i+1} \cdot \sin(\omega_1 \tau - \varphi'_i - \varphi_{i+1}) - \frac{R_{i+1} \cdot \sin \varphi_{i+1}}{\varphi_{i+1}} (\omega_1 \tau - \varphi'_i - \varphi_{i+1})
 \end{aligned}$$

for  $i+2$ -nd hinge:

$$\begin{aligned}
 &\varphi'_i + \varphi_{i+1} + \varphi'_{i+1} \leq \omega_1 \tau \leq \frac{4\pi}{z} \\
 &S_{i+2}(\tau) = t_{i+1} + t_{i+2} + R_{i+2} \cdot \sin(\omega_1 \tau - \frac{4\pi}{z}) \\
 &S_{i+2}(\tau)_p = t_{i+1} + t_{i+2} + \frac{R_{i+2} \cdot \sin \varphi'_i}{\varphi_{i+3}} (\omega_1 \tau - \frac{4\pi}{z}) \quad (9 \text{ c}) \\
 &\Delta S_{i+2}(\tau) = R_{i+2} \cdot \sin(\omega_1 \tau - \frac{4\pi}{z}) - \frac{R_{i+2} \cdot \sin \varphi_{i+2}}{\varphi_{i+2}} (\omega_1 \tau - \frac{4\pi}{z})
 \end{aligned}$$

**Analysis of the results.** The obtained results are the basis for the estimation of the move irregularity of the leading branch of the chain transmission. This can be represented graphically as shown in Fig. 4 illustrating displacements and errors, respectively, projected on the abscissa and ordinate.



**Figure 4.** The kinematic errors of the chain move

It should be noted that the deviation projected along x-axis depends mainly on the accumulated error in the length of the chain section. This approach makes it possible to determine the coefficient of non-uniformity of displacement as a criterion for estimation of the chain accuracy.

It can be concluded that the different size of the chain links due to the increase in the scattering of the size of the links due to wear results in the increase of the coefficient of non-uniformity of the chain.

So the calculations show that in the case when the wear of the bushing-roller chain DSTU 13568-2017 with the number of teeth of the drive sprocket 29 is 2.5%, the coefficient of unevenness increases up to three times compared to the case of unworn chain.

**Conclusions.** It is established that due to the peculiarities of the kinematics of gear transmission with discrete-flexible connection, random deviations of the pinch of such



connection from the nominal as a result of manufacturing inaccuracy elements of transmission and wear during operation to ensure a uniform move of transmission for a certain period of time is impossible.

To solve the problem of describing the motion of the transmission with discrete-flexible connection, the mathematical model of motion is developed on the example of the sleeve-roller chain, which gives an idea of the real interpretation of the kinematics of chain transmission taking into account the uneven dimensions of individual links. The specified mathematical model in the original setting makes it possible to present the components of deviations of the transmission movement from the given in two groups: deviations created by the accumulated error of the chain section, and movement.

Calculations and graphical representations based on the developed mathematical model show, for example, that when the hinges of the sleeve-roller chain are worn by 2.5%, the value of the coefficient of non-uniformity of the transmission increases threefold.

## References

1. Malashchenko V. O., Strilets' V. M., S trilets' O. R., Novits'kyi Ya. M. *Detali mashyn i pidiomno – transportne obladnannia*. Lviv: Novyi Svit, 2017. 347 p.
2. Dubyniak S., Kravuy P., Zubchenko I. Do pytannia optymizatsii velychyny partii detalei u rozpodilenomu v chasi seriinomu vyrobnytstvi pryvodnykh rolykovykh lantsiuhiv. *Visnyk Ternopil'skoho derzhavnogo tekhnichnogo universytetu*. Volume 4. Number 3. 14–18 p.
3. Pilipenko O. I. Friction and wear in groups of the chain drives from polymer composites. *International Symposium "Yarofri-91"*. Yaroslavl, 1991. 273–278 p.
4. Pilipenko O. I. Kontaknoe vzaymodeystviye elementov tsepnogo pryvoda izpolimernykh kompozytiv, tom 9, zecz 2, AGH, Polska, Krakov, 1990. P. 91–97.
5. Glushchenko I. P. *Osnovu proektyrovannya tsepnukh peredach s vtulochno- rolykovumy tsepiamy*. Lviv: Vid-vo L'viv's'kogo un-tu, 1964. 226 p.
6. Yaroshenko V. Do rozvytku konstruktivnykh lantsyuhovykh pryvodiv u sil's'kohospodars'kykh mashynakh. *Visnyk Ternopil's'koho derzhavnogo tekhnichnogo universytetu*. 1999. Tom 4. Chyslo 3. P. 58–62.
7. Dubyniak S. A., Zubchenko I. I., Shpak R. I. Pro vplyv sprats'ovuvannya lantsyuha na yoho zcheplennya iz zirochkooy. *Visnyk sil's'kohospodars'koyi nauky*. 1974. No. 1. P. 35–38.
8. Uzkglingis G. A. Metodu rascheta resursa rolykovykh tsepei po kryteriyu iznosostoikosty. 1988. 20. 28 p.
9. Balzers E. Povushenye nazhenosty vtulochnukh rolykovykh tsepei metodom sbalansyrovannoi iznosostoikosty. *Visnyk Ternopil'skoho derzhavnogo tekhnichnogo universytetu*. Volume 4. Number 3. 19–23 p.
10. Tomashuns I. A., Uzkglingis G. A., Dominiyeks L. A. Issledovaniye izmeneniya peredatochnogo chisla rolikovoy tsepnoy peredachi. *Tr. Latv. SKHA. Yelgava*, 1985. Vyp. 230. P. 52–61.
11. Kravuy P. D., Senyk A. A. *Konstruktorske-tekhnolohichne zabespechennia pidvyshchennia yakosti zghortnykh vtulok: monohrafiia*. 2019. 232 p.
12. Nazarevych O. Rozmirmi kharakterystyky pryvodnykh lantsyuhiv. *Visnyk Ternopil's'koho derzhavnogo tekhnichnogo universytetu*. 1999. Tom 4. Chyslo 3. P. 126–135.
13. Dubyniak S. A. 1966. O neravnomernosty vrashcheniya vedomoi systemy tsepnoi peredachy. *Vestnyk Lvovskoho polytekhnicheskoho ynstytuta. Rolikovyye tsepe peredachi*. No. 12. 1966. P. 68–78.
14. Dubyniak S. A. 1966. O vliyaniy raznorazmernosty shahov zvenev na dvyzhenye vedushchei vetvy tsepnoi peredachy. *Vestnyk Lvovskoho polytekhnicheskoho ynstytuta. Rolikovyye tsepe peredachi*. 1966. No. 12. P. 61–67.
15. Zubchenko I. I. Kinematyka i dynamika lantsyuhovykh peredach. *Vestnyk Lvovskoho polytekhnicheskoho ynstytuta*. 1972. 124 p.
16. Glushchenko I. P. O raznorazmernosti shagov zven'ev rolikovykh tsepey s pryamimi plastinami. *Vestnik Lvovskoho polytekhnicheskoho int-stituta. Rolikovyye tsepe peredachi*. No. 12. 1966. P. 5–16.

## Список використаних джерел

1. Малащенко В. О., Стрілець В. М., Стрілець О. Р., Новіцький Я. М. *Деталі машин і підйомно-транспортне обладнання*. Львів: Новий світ, 2017. 347 с.

2. Дубиняк С., Кривий П., Зубченко І. До питання оптимізації величини партії деталей у розподіленому в часі серійному виробництві приводних роликів ланцюгів. Вісник Тернопільського державного технічного університету. 1999. Том 4. Число 3. С. 14–18.
3. Pilipenko O. I. Friction and wear in groups of the chain drives from polymer composites. International Symposium «Yarofri-91». Yaroslavl, 1991. P. 273–278.
4. Пилипенко О. И. Контактное взаимодействие элементов цепного привода из полимерных композитов. *Mechanika*. Том 9, zesz 2, AGH. Polska, Krakow, 1990. P. 91–97.
5. Глушенко И. П. Основы проектирования цепных передач с втулочно-роликовыми цепями. Львів: Вид-во Львівського ун-ту, 1964. 226 с.
6. Ярошенко В. До розвитку конструкцій ланцюгових приводів у сільськогосподарських машинах. Вісник Тернопільського державного технічного університету. 1999. Том 4. Число 3. С. 58–62.
7. Дубиняк С. А., Зубченко І. І., Шпак Р. І. Про вплив спрацювання ланцюга на його зчеплення із зірочкою. Вісник сільськогосподарської науки. 1974. № 1. С. 35–38.
8. Узкглингис Г. А., Толмачев В. В., Томашунс И. А. Методы расчёта ресурса роликовых цепей по критерию износостойкости. Тр. Латв. СХА. Елгава. 1988. Вып. 252. С. 20–28.
9. Бальцерс Э., Салиниекс Н. Повышение нажености втулочных роликовых цепей методом сбалансированной износостойкости. Вісник Тернопільського державного технічного університету. 1999. Том 4. Число 3. С. 19–23.
10. Томашунс И. А., Узкглингис Г. А., Доминиекс Л. А. Исследование изменения передаточного числа роликовой цепной передачи. Тр. Латв. СХА. Елгава. 1985. Вып. 230. С. 52–61.
11. Кривий П. Д., Сенік А. А. Конструкторське-технологічне забезпечення підвищення якості згортних втулок: монографія. Тернопіль: Тернопільський національний технічний університет ім. І. Пулюя. 2019. 232 с.
12. Назаревич О. Розмірні характеристики приводних ланцюгів. Вісник Тернопільського державного технічного університету. 1999. Том 4. Число 3. С. 126–135.
13. Дубиняк С. А. О неравномерности вращения ведомой системы цепной передачи. Вестник Львовского политехнического института. Роликовые цепные передачи. 1966. № 12. С. 68–78.
14. Дубиняк С. А., Зубченко И. И. О влиянии разноразмерности шагов звеньев на движение ведущей ветви цепной передачи. Вестник Львовского политехнического института. Роликовые цепные передачи. 1966. № 12. С. 61–67.
15. Зубченко І. І., Дубиняк С. А., Рибак В. Є. Кінематика і динаміка ланцюгових передач. Львів: Вид-во Львівського ун-ту. 1972. 124 с.
16. Глушенко И. П. О разноразмерности шагов звеньев роликовых цепей с прямыми пластинами. Вестник Львовского политехнического института. Роликовые цепные передачи. 1966. № 12. С. 5–16.

УДК 621.855

## МАТЕМАТИЧНЕ ПРЕДСТАВЛЕННЯ КІНЕМАТИКИ ВІТКИ ПЕРЕДАЧІ З ДИСКРЕТНО-ГНУЧКИМ ЗВ'ЯЗКОМ

Ігор Луців<sup>1</sup>; Тарас Дубиняк<sup>1</sup>; Олександра Манзій<sup>2</sup>;  
Станіслав Андрейчук<sup>2</sup>

<sup>1</sup>Тернопільський національний технічний університет імені Івана Пулюя,  
Тернопіль, Україна

<sup>2</sup>Національний університет «Львівська політехніка»,  
Львів, Україна

*Резюме.* Конструкції передач гнучкою ланкою, дослідження та підвищення їх надійності й довговічності є однією із нагальних задач у галузі прикладного машинознавства. Розглянуто рух приводних елементів ланцюга, що є одним із характерних прикладів передачі з дискретно-гнучким зв'язком. Досліджено особливість кінематики зачеплення передачі з дискретно-гнучким зв'язком та

встановлено, що в цьому випадку неможливо забезпечити рівномірний хід передачі протягом визначеного періоду часу. Розроблено математичну модель руху, яка дає представлення про реальну інтерпретацію кінематики ланцюгової передачі з урахуванням нерівномірності розмірів окремих ланок. Показано, що складові відхилень руху передачі від заданого можна поділити на дві групи: відхилення, що створюються накопиченою похибкою довжини ділянки ланцюга, який набігає на зірочку, і відхилення, що формуються протягом усього часу зачеплення окремого шарніра, що спричинені нерівномірністю його руху. Можна зробити висновок, що різнорозмірність ланок ланцюга, зважаючи на збільшення розсіювання розмірів ланок через зношування, призводить до збільшення величини коефіцієнта нерівномірності руху ланцюга. Проведені дослідження на прикладі втулково-роликового ланцюга дозволили сформулювати реальну інтерпретацію кінематики ланцюгової передачі з урахуванням нерівномірності розмірів окремих ланок. На підставі проведених досліджень зроблено висновок, що різнорозмірність ланок ланцюга, зважаючи на збільшення розсіювання розмірів ланок через зношування, призводить до збільшення величини коефіцієнта нерівномірності руху ланцюга. Експериментально показано, що при зношенні шарнірів втулково-роликового ланцюга на 2,5% величина коефіцієнта нерівномірності ходу передачі зростає втричі.

**Ключові слова:** передача, ланцюг, ролик, вітка, кінематика, крок.

[https://doi.org/10.33108/visnyk\\_tntu2022.02.005](https://doi.org/10.33108/visnyk_tntu2022.02.005)

Отримано 11.01.2022