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INVESTIGATION OF THE SPATIAL MOTION OF A BODY WITH DISTRIBUTED MASS CONNECTED BY AN INEXTENSIBLE CABLE TO A MOVING TROLLEY

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Summary. *The article considers the spatial motion of a mechanical system where a heavy beam of a given mass and dimensions is suspended at one end by a weightless inextensible cable to a trolley, which can move along horizontal guides without resistance. The system has five degrees of freedom. Based on the apparatus of analytical mechanics and Lagrange equations, a mathematical model of the considered mechanical system in the form of a system of five nonlinear differential equations of the second order is obtained. The mathematical model is implemented in the form of a computer program that allows you to determine the coordinates (positions) of the beam at any time, build the trajectory of the center of mass, determine the kinematic characteristics of the movement, calculate the cable tension and determine its extreme value. Based on the numerical experiment, graphs and phase trajectories of these parameters are constructed, including the 3D trajectory of the center of mass of the beam. The system can show quite complex dynamics depending on the initial conditions, as evidenced by the results of numerical calculations. Under certain conditions, chaotic behavior of the system is possible. Having a mathematical model and a calculation program, it is possible to conduct further studies of the system under consideration, revealing the positions of stable and unstable equilibrium, modes of self-oscillations, revealing areas of periodic and chaotic modes, bifurcations, and so on.*

Key words: *nonlinear dynamics, oscillations, chaos, spatial problem, double spherical pendulum, Lagrange equation of the 2nd kind, mathematical model, numerical experiment.*

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Problem statement. The study of the spatial motion of bodies with distributed mass is quite urgent due to the wide use of lifting mechanisms and machines, such as tower, traveling gantry, cantilever, bridge, gantry, and other types of cranes. One of the types of multilink oscillating pendulum systems is taken into consideration in this study. This is a nonlinear system whose motion can be disordered under certain conditions. While taking into consideration the system motion it is important to be able to determine the body position at certain moments of time and to calculate the cable maximum tension to forecast its possible break.

Analysis of the well-known research results. An elliptical pendulum (a flat problem) consisting of a slider that is sliding along the smooth horizontal planed of the specified mass and a heavy ball connected with the slider by a weightless inextensible bar is taken into consideration in almost all textbooks on classical mechanics as well as in a number of publications [1]. The impact of external factors on the elliptical pendulum motion and oscillation reduction control are studied [2–5].

Different options of spherical pendulum motion like the motion with a movable or vibrating suspension point, the motion under the influence of variable external load, and the spherical pendulum motion on elastic suspension are studied in the papers [6–13].

Besides, multilink pendulum systems can be quite often seen in different fields in practice [14]. A flat problem is more often taken into consideration for double pendulums [15–17]. A model of the flat double pendulum motion has been described in the paper [15]. A flat model of a double mathematical pendulum suspended to a trolley that is incrementally moving

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along the guides is considered in the paper [16]. V. Loveikin and P. Lymar have analyzed the motion of a system consisting of a trolley, a pickup, and a load with the shifted center of mass relative to the pickup in the process of run-up [17].

A model of a double pendulum analyzed in the paper [18] is very similar to the mechanical system in the paper under discussion. The law of the trolley motion is specified in that paper while in the paper under consideration a trolley with a connected double spherical 3D pendulum is moving along the horizontal guides without any resistance forces action. Moreover, another system of generalized coordinates has been used (the same system was used in the paper [7]), as well as another approach to the inertia moment determination of a body with distributed mass was applied.

The purpose of the paper is to build a 3D model and to study the motion of a double spherical pendulum with both a movable suspension point and a load with distributed mass as well.

Task setting. The mechanical system under consideration includes a trolley m_1 moving along the horizontal guides without resistance force. A load in the form of a beam AB of the length $2a$ and of the mass m_2 is suspended to the trolley on the non-extensible cable of constant length ℓ (fig. 1). The cable is considered to be weightless.

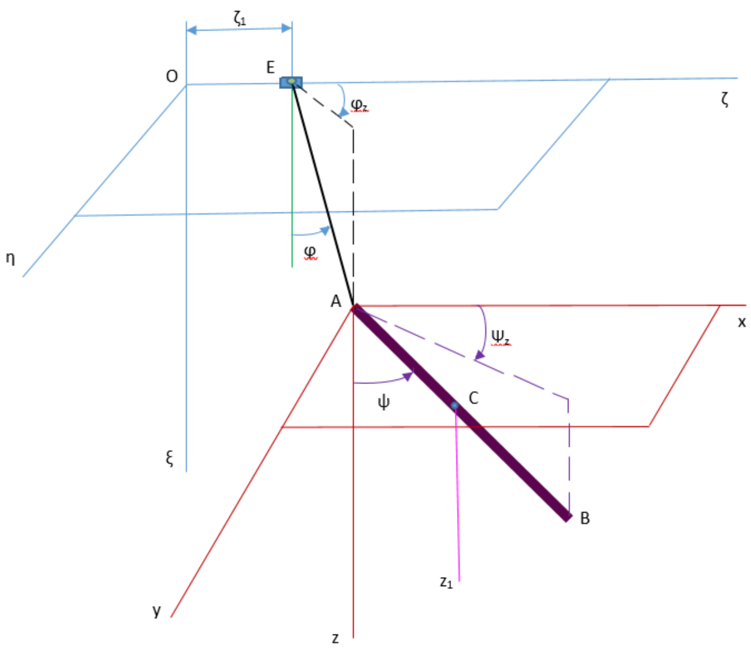

Figure 1. Calculation scheme of a double spherical pendulum with a load in the form of a beam AB and a movable suspension point (slider E)

Let's introduce a fixed (absolute or reference) coordinate system $O\zeta\eta\xi$. We have connected the coordinate system $Axyz$ with the beam. The system has 5 degrees of freedom. As generalized coordinates, we have assumed: $q_1=\zeta_1$, $q_2=\varphi_z$, $q_3=\varphi$, $q_4=\psi_z$, $q_5=\psi$.

The coordinates of point A relative to the fixed system of coordinates can be found by the formulae:

$$\zeta_A=\zeta_1+\ell\cdot\sin(\varphi)\cdot\cos(\varphi_z),$$
$$\eta_A=\ell\cdot\sin(\varphi)\cdot\sin(\varphi_z), \quad (1)$$
$$\xi_A=\ell\cdot\cos(\varphi).$$

The coordinates of the beam mass center of point *C* relative to the fixed system of coordinates:

$$\zeta_C=\zeta_A+a\cdot sin(\psi)\cdot cos(\psi_z)=\zeta_1+\ell\cdot sin(\varphi)\cdot cos(\varphi_z)+a\cdot sin(\psi)\cdot cos(\psi_z),$$
$$\eta_C=\eta_A+a\cdot sin(\psi)\cdot sin(\psi_z)=\ell\cdot sin(\varphi)\cdot sin(\varphi_z)+a\cdot sin(\psi)\cdot sin(\psi_z), \quad (2)$$
$$\xi_C=\xi_A+a\cdot cos(\psi)=\ell\cdot cos(\varphi)+a\cdot cos(\psi).$$

The projections of the velocity t. *C* on the axis of coordinates are equal to the derivative over time formulae (2):

$$V_{\zeta_C}=\dot{\zeta}_C=\zeta_1'+\ell\cdot cos\varphi\cdot cos\varphi_z\cdot\varphi'-\ell\cdot sin\varphi\cdot sin\varphi_z\cdot\varphi_z'+$$
$$+a\cdot cos\psi\cdot cos\psi_z\cdot\psi'-a\cdot sin\psi\cdot sin\psi_z\cdot\psi_z';$$
$$V_{\eta_C}=\dot{\eta}_C=\ell\cdot cos\varphi\cdot sin\varphi_z\cdot\varphi'+\ell\cdot sin\varphi\cdot cos\varphi_z\cdot\varphi_z'+ \quad (3)$$
$$+a\cdot cos\psi\cdot sin\psi_z\cdot\psi'+a\cdot sin\psi\cdot cos\psi_z\cdot\psi_z';$$
$$V_{\xi_C}=\dot{\xi}_C=-\ell\cdot sin\varphi\cdot\varphi'-a\cdot sin\psi\cdot\psi'.$$

The absolute velocity t. C can be presented as three constituents, taken from (3):

$$V_C=\sqrt{(\dot{\zeta}_C)^2+(\dot{\eta}_C)^2+(\dot{\xi}_C)^2}. \quad (4)$$

Kinetic energy of the system is equal to:

$$T=T_1+T_2, \quad (5)$$

where $T_1=m_1\cdot\frac{(\dot{\zeta}_1)^2}{2}$ – kinetic energy of the trolley translational motion;
T_2 – kinetic energy of the beam AB complex motion.

Kinetic energy of the beam AB is defined as the sum of kinetic energy of the translational motion together with the mass center and the relative motion relative to the mass center:

$$T_2=m_2\cdot\frac{(V_C)^2}{2}+T_C^{(r)}. \quad (6)$$

Kinetic energy of the relative motion:

$$T_C^{(r)}=\frac{m_2\cdot a^2}{3}\cdot\frac{(\dot{\psi}_z)^2}{2}+J_{Cz_1}\cdot\frac{(\dot{\psi})^2}{2}. \quad (7)$$

The beam inertia moment relative to the axis Cz_1:

$$J_{Cz_1}=\frac{m_2\cdot a^2}{3}\cdot sin^2(\psi). \quad (8)$$

By substituting successively (8) in (7), then (7) in (6) and (6) in (5) we have obtained the formula for calculation of total kinetic energy of the mechanical system under consideration:

$$T=m_1\cdot\frac{(\dot{\zeta}_1)^2}{2}+m_2\cdot\frac{(V_C)^2}{2}+\frac{m_2\cdot a^2}{6}\cdot((\dot{\psi}_z)^2+(\dot{\psi})^2\cdot sin^2(\psi)). \quad (9)$$

Potential energy of the system:

$$P = m_2 g \cdot \xi_C. \tag{10}$$

Lagrange

$$L = T - P$$

Or in expanded form using (3), (4), (9), (10) after transformations we have obtained:

$$\begin{gathered}L = \frac{1}{6}(6g\mathrm{m}_2(-a - \ell + \ell \cdot \cos\varphi + a \cdot \cos\psi) + 3m_1(\zeta_1{}')^2 + \\ +a^2\mathrm{m}_2((\sin\psi)^2 \cdot (\psi')^2 + (\psi_z{}')^2) + 3\mathrm{m}_2((\ell \cdot \sin\varphi \cdot \varphi' + a \cdot \sin\psi \cdot \psi')^2 + \\ +(\ell \cdot \cos\varphi \cdot \sin\varphi_z \cdot \varphi' + \ell \cdot \cos\varphi_z \cdot \sin\varphi \cdot \varphi_z{}' + a \cdot \cos\psi \cdot \sin\psi_z \cdot \psi' + \\ +a \cdot \cos\psi_z \cdot \sin\psi \cdot \psi_z{}')^2 + (\zeta_1{}' + \ell \cdot \cos\varphi \cdot \cos\varphi_z \cdot \varphi' - \\ -\ell \cdot \sin\varphi \cdot \sin\varphi_z \cdot \varphi_z{}' + a \cdot \cos\psi \cdot \cos\psi_z \cdot \psi' - a \cdot \sin\psi \cdot \sin\psi_z \cdot \psi_z{}')^2)).\end{gathered} \tag{11}$$

The system motion is described by five Lagrange equations of the 2nd kind:

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{L}}{\partial \dot{\mathrm{q}}_{\mathrm{i}}}\right) - \frac{\partial \mathrm{L}}{\partial \mathrm{q}_{\mathrm{i}}} = 0, \text{ where } i=1, 2, \ldots, 5. \tag{12}$$

By substituting (11) in (12), we have obtained the system of five nonlinear differential equations of the second order in expanded form:

$$\begin{gathered}m_1\zeta_1{}'' = m_2(-\zeta_1{}'' + \ell \cdot \cos\varphi \cdot (2 \cdot \sin\varphi_z \cdot \varphi' \cdot \varphi_z{}' - \cos\varphi_z \cdot \varphi'') + \\ \ell \cdot \sin\varphi(\cos\varphi_z \cdot ({\varphi'}^2 + {\varphi_z{}'}^2) + \sin\varphi_z \cdot \varphi_z{}'') + a \cdot \cos\psi \cdot (2 \cdot \sin\psi_z \cdot \psi' \cdot \psi_z{}' - \\ \cos\psi_z \cdot \psi'') + a \cdot \sin\psi \cdot (\cos\psi_z \cdot ({\psi'}^2 + {\psi_z{}'}^2) + \sin\psi_z \cdot \psi_z{}''));\end{gathered} \tag{13}$$

$$\begin{gathered}\ell \cdot m_2 \cdot \sin\varphi \cdot (2 \cdot \ell \cdot \cos\varphi \cdot \varphi' \cdot \varphi_z{}' - \sin\varphi_z \cdot \zeta_1{}'' + \ell \cdot \sin\varphi \cdot \varphi_z{}'' + \\ a \cdot (\sin\psi \cdot \sin(\varphi_z - \psi_z) \cdot {\psi'}^2 + 2 \cdot \cos\psi \cdot \cos(\varphi_z - \psi_z) \cdot \psi' \cdot \psi_z{}' + \\ \sin(\varphi_z - \psi_z) \cdot (\sin\psi \cdot {\psi_z{}'}^2 - \cos\psi \cdot \psi'') + \cos(\varphi_z - \psi_z) \cdot \sin\psi \cdot \psi_z{}'')) = 0;\end{gathered} \tag{14}$$

$$\begin{gathered}\ell \cdot m_2 \cdot (g \cdot \sin\varphi - \ell \cdot \cos\varphi \cdot \sin\varphi \cdot {\varphi_z{}'}^2 + a \cdot (\cos\psi \cdot \sin\varphi - \\ \cos\varphi \cdot \cos(\varphi_z - \psi_z) \cdot \sin\psi) \cdot {\psi'}^2 + 2a \cdot \cos\varphi \cdot \cos\psi \cdot \sin(\varphi_z - \psi_z) \cdot \psi' \cdot \psi_z{}' + \\ \cos\varphi \cdot \cos\varphi_z \cdot \zeta_1{}'' + \ell \cdot \varphi'' + a \cdot \cos\varphi \cdot \cos\psi \cdot \cos(\varphi_z - \psi_z) \cdot \psi'' + a \cdot \sin\psi \cdot \\ (\sin\varphi \cdot \psi'' + \cos\varphi \cdot (-\cos(\varphi_z - \psi_z) \cdot {\psi_z{}'}^2 + \sin(\varphi_z - \psi_z) \cdot \psi_z{}''))) = 0;\end{gathered} \tag{15}$$

$$\begin{gathered}a \cdot m_2 \cdot (6a \cdot \sin(2\psi) \cdot \psi' \cdot \psi_z{}' + 6 \cdot \sin\psi \cdot (-\sin\psi \cdot \zeta_1{}'' + \\ \ell \cdot (-\sin\varphi \cdot \sin(\varphi_z - \psi_z) \cdot {\varphi'}^2 + 2 \cdot \cos\varphi \cdot \cos(\varphi_z - \psi_z) \cdot \varphi' \cdot \varphi_z{}' + \\ +\sin(\varphi_z - \psi_z) \cdot \left(-\sin\varphi \cdot {\varphi_z{}'}^2 + \cos\varphi \cdot \varphi''\right) + \cos(\varphi_z - \psi_z) \cdot \\ \sin\varphi \cdot \varphi_z{}'')) + a \cdot (5 - 3 \cdot \cos(2\psi)) \cdot \psi_z{}'') = 0;\end{gathered} \tag{16}$$

$$\begin{gathered}a \cdot m_2 \cdot (6\ell \cdot (-\cos\psi \cdot \cos(\varphi_z - \psi_z) \cdot \sin\varphi + \cos\varphi \cdot \sin\psi) \cdot {\varphi'}^2 - \\ 12\ell \cdot \cos\varphi \cdot \cos\psi \cdot \sin(\varphi_z - \psi_z) \cdot \varphi' \cdot \psi_z{}' - 6\ell \cdot \cos\psi \cdot \cos(\varphi_z - \psi_z) \cdot \\ \sin\varphi \cdot {\varphi_z{}'}^2 + a \cdot \sin(2\psi) \cdot ({\psi'}^2 - 3{\psi_z{}'}^2) + 6(g \cdot \sin\psi + \cos\psi \cdot \cos\psi_z \cdot \zeta_1{}'' + \\ \ell \cdot (\cos\varphi \cdot \cos\psi \cdot \cos(\varphi_z - \psi_z) + \sin\varphi \cdot \sin\psi) \cdot \varphi'' - \ell \cdot \cos\psi \cdot \sin\varphi \cdot \\ \sin(\varphi_z - \psi_z) \cdot \varphi_z{}'') - a \cdot (-7 + \cos(2\psi)) \cdot \psi'') = 0.\end{gathered} \tag{17}$$

Let's determine the cable tension during the beam AB motion.

The differential equation of the beam motion in the projection in the projection on the vertical axis of a fixed coordinate system looks like:

$$m_2 \cdot \ddot{\xi}_C = m_2 \cdot g - N \cdot \cos\varphi.$$

Hence,

$$N = m_2 \cdot \frac{g - \ddot{\xi}_C}{\cos\varphi}. \qquad (18)$$

On the basis of the obtained system of equations (13)-(17) the numerical modeling of the system motion was done at the following initial data and initial conditions: $m_1=400\ kg$, $m_2=3000\ kg$, $\ell=8\ m$, $a=1,8\ m$, $\zeta_{10}=-0,4\ m$, $\varphi_{z0}=0,4$, $\varphi_0=0,2$, $\psi_{z0}=0,2$, $\psi_0=0,3$, $\zeta'_{10}=0,1\ m/s$, $\varphi'_{z0}=-0,1\ rad/s$, $\varphi'_0=-0,2\ rad/s$, $\psi'_{z0}=-0,1\ rad/s$, $\psi'_0=-0,2\ rad/s$.

The results of the modeling are shown on the figures 2-10. The law of motion (the law of change of generalized coordinates over time) is given on figures 2-3. Represented on figures 4-8 the phase trajectories have shown the evolution of the given physical system. A spatial motion trajectory of the beam mass center is shown on fig. 9 (t. C).

The conducted studies have proved, that the system motion displays various dynamic behavior. A great number of different modes can be observed which depend significantly on the specified initial conditions. The various regular and chaotic movements of a double 3D pendulum are realized.

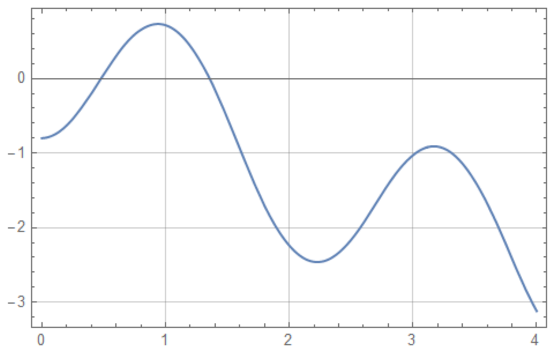

Figure 2. Coordinate ζ1-time dependence graph.

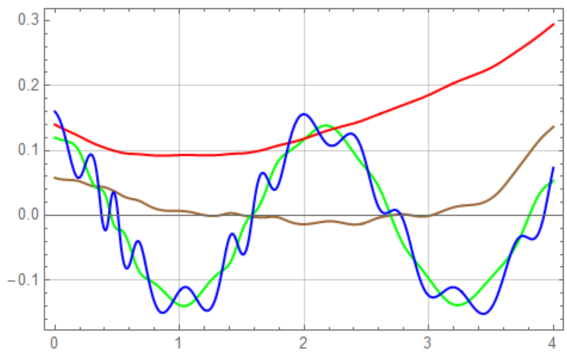

Figure 3. Dependence graph of angular generalized coordinates change over time:
– φ_z, – φ, – ψ_z, – ψ.

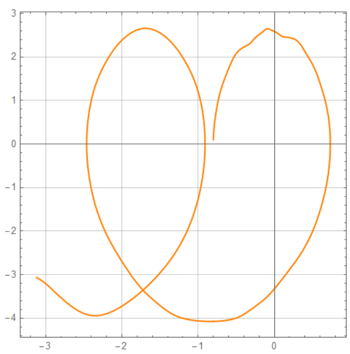

Figure 4. Phase trajectory ζ_1-ζ'_1

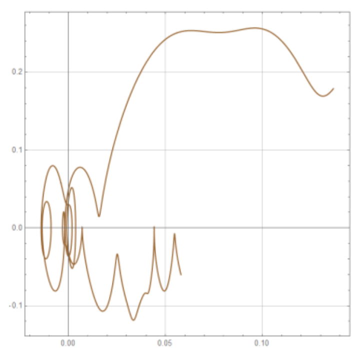

Figure 5. Phase trajectory φ_z-φ_z'

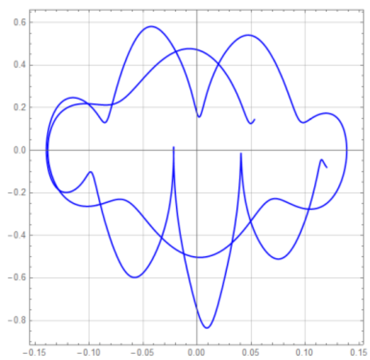

Figure 6. Phase trajectory φ-φ'

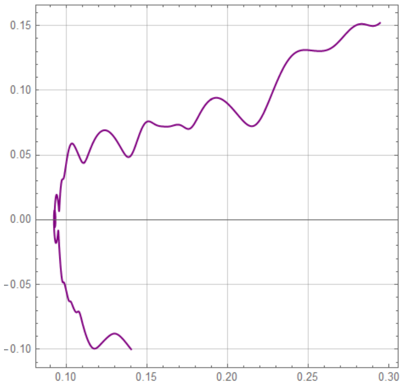

Figure 7. Phase trajectory ψ_z-ψ_z'.

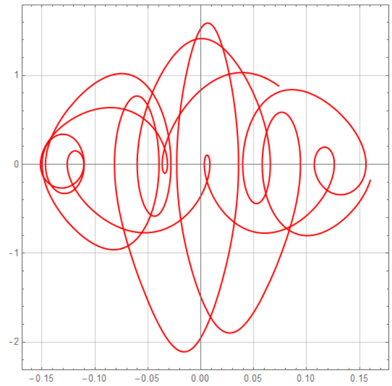

Figure 8. Phase trajectory ψ-ψ'.

Using the formula (18) one can calculate a cable tension in the process of the beam motion AB (fig. 10), and its maximum value can also be calculated by means of software. Here, under specified parameters the maximum tension is equal to 34211 H and it is observed at the moment of time 0,216 s.

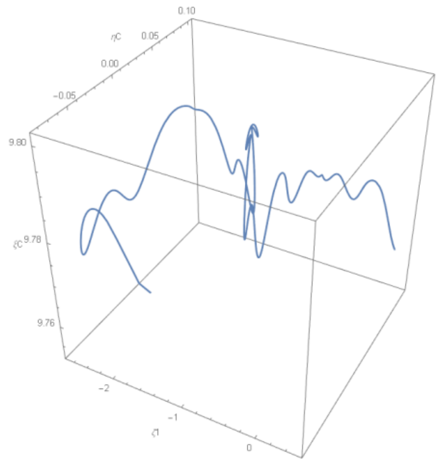

Figure 9. 3D trajectory of the center of mass of the beam (p. C)

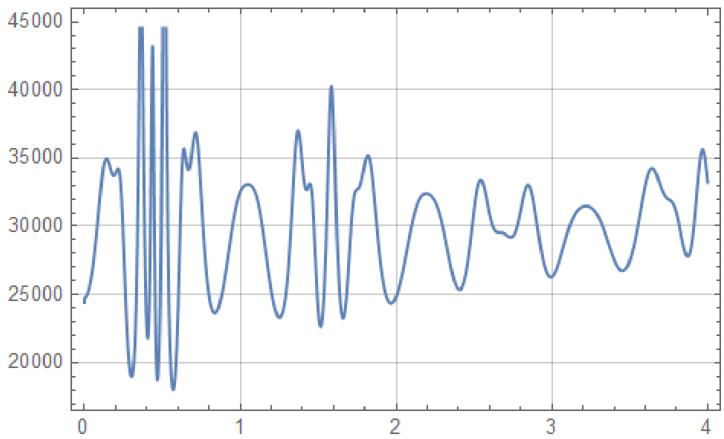

Figure 10. **C**able tension change during the beam motion

Conclusions. A spatial mechanical system of a double spherical pendulum with the load with distributed mass and a movable suspension point. 3D motion of the model is described by the system of five nonlinear differential equations of the second order. A program has been developed and a computer simulation of the system motion has been done. The laws have been determined and the curves, as well as phase trajectories describing the changes over time of linear and angular coordinates and velocities have been constructed. A cable tension has been calculated, and its maximum value has been found. The system can demonstrate quite complex (including chaotic one) dynamics depending on the initial conditions, that was proved by the results of numerical calculations in the form of graphs and phase trajectories.

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ДОСЛІДЖЕННЯ ПРОСТОРОВОГО РУХУ ТІЛА З РОЗПОДІЛЕНОЮ МАСОЮ, ПРИЄДНАНОГО НЕРОЗТЯЖНИМ ТРОСОМ ДО РУХОМОГО ВІЗКА

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Резюме. *Розглянуто просторовий рух механічної системи, де важка балка заданої маси і розмірів підвішена одним кінцем невагомим нерозтяжним тросом до візка, який без опору може рухатися по горизонтальних напрямних. Система має п'ять ступенів вільності. На основі апарату* *аналітичної механіки і рівнянь Лагранжа отримано математичну модель розглядуваної механічної системи у вигляді системи п'яти нелінійних диференціальних рівнянь другого порядку. Математична модель реалізована у вигляді комп'ютерної програми, яка дозволяє визначити координати (положення) балки в будь-який момент часу, побудувати траєкторію руху центра мас, визначити кінематичні характеристики руху, розрахувати натяг троса й визначити його екстремальне значення. На підставі проведеного числового експерименту побудовано графіки й фазові траєкторії цих параметрів, у тому числі 3D траєкторія руху центра мас балки. Система може демонструвати досить складну динаміку залежно від початкових умов, про що свідчать результати чисельних розрахунків. За певних умов можлива хаотична поведінка системи. Маючи математичну модель і програму розрахунку, можна проводити подальші дослідження розглянутої системи, виявляючи положення стійкої та нестійкої рівноваги, режими автоколивань, виявляючи області різних за характером періодичних і хаотичних режимів, біфуркації та ін. Дослідження проведено за нелінійною моделлю без використання асимптотичних методів, що дозволило виключити методологічну похибку рішення. Отримані результати можуть бути використані при моделюванні керованих маятникових рухів різних механічних систем. Методика і програма рекомендуються для вирішення прикладних завдань проектування та експлуатації різних вантажопідйомних систем і технічних пристроїв, здатних демонструвати складну поведінку. У методичному плані пропонований матеріал цікавий для студентів і аспірантів у плані навчання принципам побудови та аналізу складних нелінійних просторових динамічних систем.*

Ключові слова: *нелінійна динаміка, коливання, хаос, просторова задача, подвійний сферичний маятник, рівняння Лагранжа 2-го роду, математична модель, числовий експеримент.*

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