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USE OF p-ADIC NUMBERS IN URBAN RESOURCE NETWORKS DATA ANALYSIS

Oleksandr Matsiuk

Ternopil Ivan Puluj National Technical University, Ternopil, Ukraine

***Summary**. The implementation of the concept «smart city» has required, in particular, the development and efficient use of new mathematical methods and approaches enabling the data hypercube analysis to be made efficiently and constructively, data transfer processes to be initiated, etc.*
***Key words**: smart city, data hypercube, p-adic numbers, hierarchy tree.*

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Problem statement. One of the key components of the processes maintenance in «smart city» resource networks is the data warehousing and the use of information technologies of data multidimensional analysis – OLAP. The use of this technology within the information-technological maintenance of the processes in urban resource and socio-communicational networks has allowed the current state of the city material resources to be analyzed in detail, the tendencies of their change and state to be forecasted due to data processing which belongs to different periods of time.

The processes of generalization of detailed data collections gathered in the resource networks are used in the «smart city» OLAP systems that enable us to obtain new knowledge from interrelated information sets consolidated according to different aspects of analysis [1]. Initial data sets for analytical processing by OLAP information about urban resource networks is the consumption of the resources used (water, gas, electricity and heat).

The authors of the paper [6] have presented the study on improvement of the processes of data hypercubes formation which are the information model of the processes of urban resource networks functioning based on the data obtained from the systems of the IoT devices, and multidimensional presentation of information dealing with COVID-19 based on data hypercubes has been considered in detail in the paper [5]. A mathematical apparatus of p-adic numbers was proposed to be used to store and process big data collections obtained from different information sources in the above-mentioned paper.

p-adic numbers were introduced by K. Hensel in 1897 [2]. As a rule, they were used in the theory of numbers and algebraic geometry. The use of p-adic numbers apparatuses was exclusively mathematical. Even mathematicians considered p-adic analysis a very specific theory that could be applied only in the theory of numbers. Since the 1980s when the scientists (mathematicians) have been actively involved in the study of complex systems evolution where the classical concept of continuity was difficult to apply, it was evident, that p-adic analysis has perfect possibilities for the study of phenomena which can't be studied by known and traditional methods.

The more detailed material on the mathematical aspect of p-adic numbers has been given in the papers [3, 4].

 Corresponding author: Oleksandr Matsiuk; e-mail: oleksandr.matsiuk@gmail.com

We have distinguished two main characteristics of p-adic analysis, which have no comparable counterparts in the classical mathematical analysis based on the basic concept, i.e. the concept of continuity. We speak about the concept of discreteness and hierarchy, in particular. The essence of the discreteness concept can be described by the following fact:

If the norm $\|a\|$ of p-adic number a is higher than the norm $\|b\|$ p-adic number b, then the norm of the sum of these numbers is invariable, i.e. $\|a+b\| = \|a\|$.

These properties have allowed us to develop a comprehensive p-adic quantum mechanics – a branch of quantum mechanics which studies the phenomena under ultrasmall distances conditions, smaller than Planck's constant $e_p \approx 1,616 \cdot 10^{-35}$ m. It results from the quantum mechanics, that lengths smaller than Planck's constant cannot be measured, i.e. for them there is no classic concept of length $|x-y|$ between points. In this case, the measurement of p-adic distances between points is a natural one.

The second fundamentally distinctive feature p-adic numbers is their hierarchical pattern. A set of p-adic numbers has a simple geometric structure. p-adic numbers can be written as trees with branching into p-parts (p-random fixed prime number). A certain p-adic number can be obtained if one travels over the specified distance recording the figures in the treetops step- by-step. In this case, every piece of travel corresponds to one certain p-adic number and vice versa.

This fundamental specific feature has opened a wide range of possibilities of p-adic numbers use in the study of hierarchy systems of various nature and areas of usage which can solve tasks and problems of artificial intelligence systems creation, new genetic sequences, problems of astrophysics. In particular, this approach can be used efficiently in the processes of data analysis in a case when each process is presented as a hierarchy structure.

Paper aim. Development and efficient use of new mathematical methods and approaches enabling the data hypercube analysis to be made efficiently and constructively, data transfer processes to be initiated etc.

Problem setting. We write $\mathbb{Q}$ as a set of rational numbers. We claim that the function $\|\bullet\|$ on $\mathbb{Q}$ is a norm when it satisfies the following characteristics

1) For each $x \in \mathbb{Q}$ $\|x\| \geq 0$ and $\|x\| = 0 \Leftrightarrow x = 0$

2) $\|x \cdot y\| = \|x\| \cdot \|y\|$ for each $x, y \in \mathbb{Q}$ (1)

3) $\|x+y\| \leq \|x\| + \|y\|$ for each $x, y \in \mathbb{Q}$

An example of the norm on a set of rational numbers $\mathbb{Q}$ is a standard module $\|\bullet\| = |\bullet|$.

Other examples of norm can be built in the following way: we will fix a random prime number p. Then, a random nonzero rational number x can be written as:

$$x = p^{\gamma} \frac{m}{n}, \tag{2}$$

where $\gamma, m, n \in \mathbb{Z}$ (a set of whole numbers) and numbers m and n are not divided by p.

In the schedule (2) it is very important, under the above-mentioned restrictions, that the number γ is determined as a single-valued one. It makes possible to specify the function on the set of rational numbers $\mathbb{Q}$ correctly

$$\|x\|_p = \begin{cases} \frac{1}{p^{\gamma}} & x \neq 0 \\ 0 & x = 0 \end{cases}. \tag{3}$$

We can show, that the function $\|\bullet\|_p$ is a norm on $\mathbb{Q}$ (i.e. the conditions are satisfied for it (1). Further we will call this norm a p -adic norm. From the definition of a p -adic norm it is resulted that p -adic norm accepts only countable number of values $\frac{1}{p^{\gamma}}$, when it γ runs through a set of whole numbers.

We must admit, that p -adic norm, unlike a standard module, $|\bullet|$ satisfies the enhanced condition (3) in (1)

$$\|x+y\|_p \leq \max\left\{\|x\|_p, \|y\|_p\right\}. \tag{4}$$

It is this enhanced condition of the triangle that causes the unusual features of a p -adic norm.

The norms satisfying (4) are called nonArchimedes. It is caused by the fact that they don't satisfy the Archimedes' lemma: if there is a small interval I_1 of the length x, and also a big interval I_2, including I_1 and its length is much bigger than x (i.e. $y \gg x$), then the big interval I_2 can be presented as subset of length intervals sum x, i.e. there is $n \in \mathbb{N}$, then $n \cdot x > y$. (fig. 1).

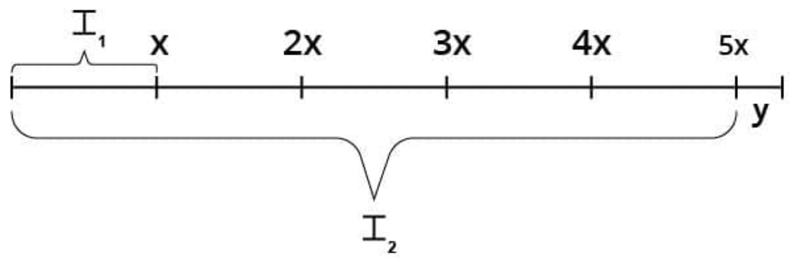

Figure 1. Picture of Archimedes' Lemma

This Lemma is valid if the lengths of intervals are measured by means of a standard module.

If the lengths are measured by means of p -adic norm, then the lemma is not valid, as

$$\|2x\|_p = \|x+x\|_p \le \max\left\{\|x\|_p, \|x\|_p\right\} = \|x\|_p . \tag{5}$$

From the mathematical induction it has resulted, that $\|nx\|_p \le \|x\|_p \quad \forall n \in \mathbb{N}$.

We must admit, that p -adic norms is not a unique phenomenon at all. We can show that, if a nontrivial function $\|\bullet\|$ on a set of rational numbers is a norm (i.e. the properties are satisfied (1–3)), then there is such $\alpha > 0$, that $\|x\| = \|x\|_p^{\alpha}$ $\forall x \in \mathbb{Q}$ at a certain prime p or $\|x\| = |x|^{\alpha}$.

It means that the specified length concept on the set $\mathbb{Q}$ by means of Archimedes' norm, i.e. by means of the module: $\|\bullet\| = |\bullet|$ is rather the exception. The rest norms are nonArchimedes ones.

Definition. The completion of a set of rational numbers $\mathbb{Q}$ relative to the p -adic norm $\|\bullet\|_p$ is called a set of p -adic numbers and is written as $\mathbb{Q}_p$.

It is clear that $\mathbb{Q} \subset \mathbb{Q}_p$ and each p -adic number $a \in \mathbb{Q}_p$ can be approximated by means of a rational numbers sequence $\{a_n\}$ from$\mathbb{Q}$ i.e.

$$\lim_{n\to\infty} \|a - a_n\|_p = 0 . \tag{6}$$

If we consider a standard module as a norm $|\bullet|$, the completion $\mathbb{Q}$ gives a set of real numbers $\mathbb{R}$. Besides, we must admit that in this case irrational numbers have no sense in the set of p -adic numbers $\mathbb{Q}_p$.

On the set of p -adic numbers $\mathbb{Q}_p$ we can define the concepts of a ball and a sphere which will be widely used in the next subsection:

Let $a \in \mathbb{Q}_p$. A set of p -adic numbers will be written as a ball with the center in point a and radius $\frac{1}{p^{\gamma}}$:

$$B_{\frac{1}{p^{\gamma}}}(a) = \left\{b \in \mathbb{Q}_p:\ \|a-b\|_p \le \frac{1}{p^{\gamma}}\right\}. \tag{7}$$

Similarly, an open ball $B^0_{1/p^{\gamma}}(a)$ is

$$B^0_{\frac{1}{p^{\gamma}}}(a) = \left\{b \in \mathbb{Q}_p:\ \|a-b\|_p < \frac{1}{p^{\gamma}}\right\}. \tag{8}$$

A sphere $S_{1/p^{\gamma}}(a)$ is

$$S_{\frac{1}{p^{\gamma}}}(a) = \left\{b \in \mathbb{Q}_p:\ \|a-b\|_p = \frac{1}{p^{\gamma}}\right\}. \tag{9}$$

It is clear that $B_{1/p^{\gamma}}(a) = B^0_{1/p^{\gamma}}(a) \bigcup S_{1/p^{\gamma}}(a)$.

Besides, we must admit, that choice of radius in the form $\frac{1}{p^{\gamma}}$ is specified by the fact, that the values of p -adic norm can be taken only from the set of nonnegative numbers $\left\{0, \frac{1}{p^{\gamma}},\ \gamma \in \mathbb{Z}\right\}$ (see the definition of p -adic norm)

Unlike the balls on the set $\mathbb{R}$, p -adic balls $\mathrm{B}_{1/p^{\gamma}}(a)$ have an unusual property: the point $b \in \mathrm{B}_{1/p^{\gamma}}(a)$ will be also its center, i.e.

$$\mathrm{B}_{1/p^{\gamma}}(a) = \mathrm{B}_{1/p^{\gamma}}(b)\,. \tag{10}$$

The similar phenomenon will be true for an open ball as well.

From the perspectives of use, we will be interested in p -adic whole numbers $\mathbb{Z}_p \subset \mathbb{Q}_p$, which can be determined in the following way

$$\mathbb{Z}_p = \left\{a \in \mathbb{Q}_p:\ \|a\|_p \leq 1\right\}. \tag{11}$$

We can show, that

$$\mathbb{Z}_p = S_1(0) \cup S_{\frac{1}{p}}(0) \cup S_{\frac{1}{p^2}}(0) \cup S_{\frac{1}{p^3}}(0) \cup \ldots \tag{12}$$

It is known, that every p -adic whole number $a \in \mathbb{Z}_p$ can be recorded as a series (canonical representation) which is convergent relative to p -adic norm

$$a = \sum_{n=0}^{\infty} \alpha_n p^n\,, \tag{13}$$

where $\alpha_n \in \{0,1,\ldots p-1\}$

The picture (13) is exclusively useful. In particular, it helps to describe the operations of addition and multiplication on a set of p -adic whole numbers as standard operations of addition and multiplication for polynomials of degree p .

We must also admit, that the norm of the number a , is

$$\|a\|_p = \frac{1}{p^{n_0}}\,, \tag{14}$$

where $n_0 = min\{n \in \mathbb{N} \cup \{0\}, \alpha_n \neq 0\}$, and the series convergence (13) is thought of as a convergence in p -adic norm.

From the canonical representation (13) it follows, that every $a \in \mathbb{Z}_p$ can be identified with an infinite set of correspondent coefficients α_n

$$a \leftrightarrow (\alpha_o, \alpha_1, \alpha_2, \ldots, \alpha_n, \ldots)\,, \qquad \alpha_n \in \{0,1,\ldots, p-1\}\,. \tag{15}$$

Let $b \leftrightarrow (\beta_0, \beta_1, \beta_2, \ldots, \beta_{n,\ldots})$ is another whole p -adic number. Then we can see easily

$$\|a-b\|_p \le \frac{1}{p^m} \Leftrightarrow \alpha_j = \beta_j,\ j = 0,1,\dots,m-1 \tag{16}$$

Similarly

$$\|a-b\|_p < \frac{1}{p^m} \Leftrightarrow \alpha_j = \beta_j,\ j = 0,1,\dots,m, \tag{17}$$

$$\|a-b\|_p = \frac{1}{p^m} \Leftrightarrow \alpha_j = \beta_j,\ j = 0,1,\dots,m-1, \alpha_m \ne \beta_m,\ , \tag{18}$$

***p*-adic analysis of data hypercube.** Each cell of data hypercube is determined by a unique value of measuring attributes. Mathematically it can be interpreted as a certain hypercube in the Euclidean space $\mathbb{R}^n$. In this case, each cell of data hypercube will be determined by a unique set of natural numbers.

A cell $\leftrightarrow (m_1, m_2, m_3, \dots, m_n),\ m_i \in \mathbb{N}$.

To have access to the data in the hypercube a user should choose one or several cells by fixing the required coordinates.

Multidimensional analysis of the data hypercube is made not by all dependent measurements simultaneously, but the data are selected for the specified values of the fixed set of parameters.

Making such data analysis enables us to find some insights of the smart city networks functioning, that can facilitate its further development.

In this regard, a natural problem dealing with mathematical methods development is to be solved which will accelerate the multidimensional analysis making and provide the clear interpretation of the obtained data slices. One of the possible directions is the use of Boolean algebra formalism which was discussed in the thesis subsection.

Let's discuss the perspectives of p-adic numbers theory use for data analysis.

We start from the following idea: making the required multidimensional analysis is impossible without the hypercube designing aimed at creation of a certain hierarchy to simplify and accelerate the analysis under discussion. In the case of a data hypercube for a «smart city» the following hierarchical dependence showing the resource consumption is possible to be formed, as a rule:

City districts	(α_0)
Microdistricts	(α_1)
Streets	(α_2)
Blocks of flats	(α_3)
Riser blocks of flats	(α_4)
Flats	(α_5)
Resource daily consumption per month of a certain flat	(α_6)

We must admit, that the thorough analysis of the above-described consumption hierarchy is an especially urgent and incredibly important task nowadays due to the rapid development of green energy that requires scientifically substantiated forecast of the resource consumption.

An attempt pf implementation of the given hierarchy by means of coordinates $(m_{1,...}, m_n)$ vectors of space $\mathbb{R}^n$ requires some imposition of additional function dependencies between the coordinates m_i, which take into consideration the hierarchy specific properties. A set of dependencies should be quite big to take onto account all possible slices of data on the incomplete number of parameters. All this has complicated the data analysis considerably. Taking into consideration that p -adic numbers have natural hierarchy, there is a natural idea of p-adic numbers use to describe the hierarchical structure of the hypercube data.

Let's illustrate this approach using the example of the above-mentioned hierarchy.

Let's consider a case of gas consumption. To begin with, we'll take into consideration a prime number p . We assume, that it is big enough but we won't say its exact value.

We will put a subset of p -adic numbers $a \in \mathbb{Z}_p$to correspond the hierarchy.

$$a = \sum_{n=0}^{6} \alpha_n p^n , \qquad (18)$$

where $\alpha_n \in \{0,1,2,....,5\}$.

We assume, that

$\alpha_0 = 0$ when there are no data of the city district;

$\alpha_0 = 1$ when we have the first district;

$\alpha_0 = k_0$ when the last city district is taken into consideration.

Now let's pass to the definition of the parameter α_1. As before, we will accept, that $\alpha_1 = 0$ if there are no data of the city microdistrict. Furthermore, we will enumerate the list of microdistricts in each district. Let k_1 is the biggest number among the microdistricts indexing. We assume that, apart from 0, α_1 can take values $\alpha_1 = 1, 2, ..., k_1$.

In the same way we will consider all the next parameters:

α_2 streets in the specified microdistrict (the biggest number is k_2);

α_3 blocks of flats on the street (the biggest number is k_3);

α_4 riser blocks of flats for the specified block of flats (the biggest number is k_4);

α_5 flats in the specified riser blocks of flats (the biggest number is k_5);

α_6 daily gas consumption of the certain flat during a month (the biggest number is $k_6 = 31$).

Now we can choose a prime number p , as the closest prime number, which exceeds $\max\{k_0, k_1, k_2, ..., k_6\}$

The sequence $(\alpha_0, \alpha_1, ..., \alpha_6)$ specified above can be identified as a p -adic number $a = \sum_{n=0}^{6} \alpha_n p^n$.

We write Ψ as a set of all p-adic numbers which are determined by means of the above-mentioned algorithm. It is evident, that $\Psi \subset \mathbb{Z}_p$.

Further, we won't differentiate p-adic number $a = \sum_{n=0}^{6} \alpha_n p^n$ and the correspondent set of numbers $(\alpha_0, \alpha_1, ..., \alpha_6)$.

We must admit, that the set Ψ is a subset of the set of whole p-adic numbers $\mathbb{Z}_p$. If $a = (\alpha_0, \alpha_1, \dots, \alpha_n) \in \mathbb{Z}_p \backslash \Psi$ then the data cell does not correspond to this p-adic number. We can say, that in this case the cell has a symbol NULL. Similarly, if at least one of the parameters α_i is zero, then we consider, that the cell contains incomplete information, i.e. the information which must be checked (get more accurate information).

p-adic number $a = (\alpha_0, \alpha_1, ..., \alpha_6) \in \Psi$ is the cell identifier with the specified gas consumption of the specified flat per certain day. This presentation is quite useful. In particular, if one is interested in gas consumption per first week of the month in the flats of the certain block-of-flats of the certain microdistrict, it will be enough to consider the cells corresponding to p-adic numbers.

We should admit, that parameters α_i can't be zero.

Let's illustrate this fact using the interpretation of p-adic numbers as a hierarchical tree.

Data slices in the suggested approach can be interpreted by means of an open ball and a ball on the set of p-adic numbers.

Let's consider some examples of data slices. We assume that we would like to assess a set of data regarding gas consumption of a certain flat (we consider it as a standard consumption). Hence, a cell is specified as p-adic number $a = (\alpha_0, \alpha_1, ..., \alpha_6)$.

Let's consider the cells which will attract attention when we consider the sphere cross-section with the center in this point with the set Ψ

$$S_{1/p^m}(a) \cap \Psi = \left\{ b \in \Psi_p : \|a - b\|_p = \frac{1}{p^m} \right\} \quad m \in \{1, 2, ..., 6\}. \tag{19}$$

We assume that $m = 6$. Then, as it was mentioned above

$$b = (\beta_0, \beta_1, ..., \beta_6) \in S_{\frac{1}{p^6}}(a) \cap \Psi \Leftrightarrow \alpha_i = \beta_i \ i = 0,5 \ \alpha_6 \neq \beta_6. \tag{20}$$

Thus, on the surface of the sphere $S_{1/p^6}(a) \cap \Psi$ there are some cells containing some information of the daily gas consumption by the specified flat, except the above-mentioned day α_6.

Another example. Let's consider the cross-section of an open ball with Ψ

$$B_{1/p^m}(a) \cap \Psi = \left\{ b \in \Psi_p : \|a - b\|_p < \frac{1}{p^m} \right\}. \tag{21}$$

If $m = 5$, then from (17) it follows, that $\alpha_i = \beta_i \ i = 0, ..., 5$, that an open ball $B_{1/p^5}(a) \bigcap \Psi$ contains some information of gas consumption in a certain flat per each day of the month. We must admit that each p -adic number $b \in B_{1/p^5}(a) \bigcap \Psi$ will also be the center of this ball. Thus, when we deal with ball we don't have «standard consumption» and each cell with $B_{1/p^5}(a)$ is of the same value.

Conclusions. The development and efficient use of some new mathematical methods and approaches enabling us to make effective and constructive analysis of the data hypercube has been taken into consideration in the paper under discussion. The obtained results have been used in construction of an information-technological platform of analytical processing of urban data collections.

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ВИКОРИСТАННЯ p -АДИЧНИХ ЧИСЕЛ ДЛЯ АНАЛІЗУ ДАНИХ МІСЬКИХ РЕСУРСНИХ МЕРЕЖ

Олександр Мацюк

Тернопільський національний технічний університет імені Івана Пулюя, Тернопіль, Україна

Резюме. *Імплементація концепту «розумного міста» вимагає, зокрема, розроблення та ефективного використання нових математичних методів і підходів, які б дозволяли ефективно та конструктивно проводити аналіз гіперкубів даних, ініціювати процеси передавання даних і т.ін.*

Одним із ключових компонентів супроводу процесів у ресурсних мережах «розумного міста» є організації сховищ даних та інформаційних технологій багатовимірного аналізу даних – OLAP. Використання цієї технології в рамках інформаційно-технологічного супроводу процесів у міських ресурсних та соціокомунікаційних мережах дозволяє детально аналізувати поточний стан матеріальних ресурсів міста, виявляти тенденції їх зміни та станів шляхом опрацювання даних, що належать до різних часових періодів.

В OLAP системах «розумного міста» використовуються процеси узагальнення деталізованих колекцій даних, зібраних у ресурсних мережах, що дозволяє отримувати нові знання з консолідованих за різними аспектами аналізу взаємопов'язаних інформаційних наборів. Вихідними наборами даних для аналітичного опрацювання за допомогою OLAP відомостей щодо міських ресурсних мереж є показники витрат спожитих ресурсів (вода, газ, електроенергія і тепло). Багатовимірний аналіз гіперкуба даних здійснюється не за всіма залежними вимірами одночасно, а вибірка даних виконується для конкретних значень фіксованого набору параметрів. Проведення такого аналізу даних дозволяє знайти (приховані) закономірності функціонування мереж розумного міста, що може сприяти його подальшому розвитку. В зв'язку з цим виникає природна задача розвитку математичних методів, які дозволяють пришвидшити проведення багатовимірного аналізу й забезпечать прозору інтерпретацію отриманих зрізів даних.

Наведено основні результати досліджень щодо застосування р-адичного аналізу до аналізу гіперкубів даних, представлено основні поняття так званого р-адичного аналізу для інших цілей. проаналізовані підходи щодо застосування р-адичних чисел до аналізу даних у гіперкубах. Запропонований підхід проілюстровано на прикладі газоспоживання міста (квартира, будинок, мікрорайон і т.д.). Зрізи даних у запропонованому підході інтерпретуються за допомогою сфери, відкритої кулі і кулі на множині р-адичних чисел.

Ключові слова: *розумне місто, гіперкуб даних, p-адичні числа, ієрархічне дерево.*

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