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MATHEMATICAL MODEL OF FUNCTIONING OF THE SCREW CONVEYOR LOADING HOPPER

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Summary. *Increasing productivity and reducing energy costs of the process of operation of screw devices and technological lines of processing complexes, where they are installed in agricultural sector, in the first stage depends mainly on the throughput capacity of loading hoppers. At the same time, the need for rational management of the flows, that have quite significant random deviations from the measure of average of the root crops flow, is quite an urgent task. The objective of investigation is to increase the efficiency of screw conveyor-crushers by developing a mathematical model which functionally describes the technological process of movement of the material in the loading hopper. The article provides an analysis of the process root crops movement in the screw conveyor loading hopper through the development of deterministic mathematical models that describe and determine the quantitative indicators of per second feeding of root crops to the auger depending on the hopper loading.*

Key words*: root crops, discharge outlet, per second feeding, root crops stock, consumption of root crops, flow velocity, acceleration of consumption of roots crops.*

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Statement of the problem. The development of effective technological processes of operation of screw devices, that provide simultaneous crushing and movement of large root crops should be based on integrated methods of solving technical tasks, the implementation of which will improve the technological performance and expand the functionality of screw conveyors-crushers of root crops [1].

These technological and constructive tasks are realized by developing new or improved methods of substantiation of technological process and calculation of structural and kinematic parameters and operating modes of screw conveyor-crushers of root crops.

Increasing productivity and reducing energy costs of the process of operation of screw devices and technological lines of processing complexes, where they are installed in agricultural sector, in the first stage depends mainly on the throughput capacity of loading hoppers [2, 3].

At the same time, the need for rational management of the flows, that have quite significant random deviations from the measure of average of the root crops flow, is quite an urgent task [4, 5].

Analysis of available research. It is found that the way of movement of a body in a loading hopper depends on the physical and mechanical properties of the product, the frictional properties of the walls of the main feeding part (throat) and the aperture (discharge outlet) of the hopper bottom, as well as their structural and geometric shape, and parameters [6, 7].

In works [8–11] it is proved, that the geometrical form of aperture (discharge outlet) of the bottom and its dimensional parameters have a predominant influence on the discharge characteristics of hoppers. Predominantly the bottom part of production hoppers for bulk materials is made in the form of a truncated cone, pyramid, or volumetric trapezium.

In addition, it is generalized, that this effect is due because such forms of a hopper bottom of a hopper provide a uniform distribution of the product body particles over the cross-sectional area of the aperture of the hopper [12].

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Such movement of the product reduces the pulsating character of their movement, caused by the presence of dynamic consolidated formation of discrete product particles in the hopper [13, 14].

In addition, the bottom of such hoppers must comply with the condition according to which the given angle of external friction of the body φ_m should be less than the angle of inclination of the hopper bottom wall α_z to the horizontal plane [15].

However, compliance with these requirements for the design of the hopper does not mean that the hopper is a hopper with a rational throughput capacity. This requires another condition, that is the condition of rational shape, that forms the bottom discharge outlet of the hopper.

Objective of investigation. To increase the efficiency of screw conveyor-crushers by developing a mathematical model which functionally describes the technological process of movement of the material in the loading hopper.

Statement of the task. In general, technological efficiency of any loading hopper, which is a component of a conveying device, depends on the coordination of its structural geometric shape (conical, pyramidal, cylindrical, combined), method of movement (hydraulic, normal, mixed) and transportation of loaded materials, size and mass parameters, and physical and mechanical properties of conveyed materials with general design of screw conveyor or technological line in which it is installed, as well as the actual needs of the production necessity.

Slot hoppers are widely used in agricultural machinery. However analytical questions on definition of the shape of a longitudinal cross-section of a hopper, that provides required throughput capacity of root crops, are not solved sufficiently. The need for such solution is due to different characteristics of root crops compared to bulk material and discrete processes of their evenly flowing output.

In the first step of developing a mathematical model that describes the process of functioning of a hopper (Fig. 1) of a screw conveyor-crusher of a product transportation, we take the starting points, assumptions, and simplifications:

- feeding part 2 of a hopper (Fig. 1) is taken as a truncated rectangular pyramid; we indicate its height by $h_з$, and dimensional parameters of its top – base by $a_з \times b_з$;

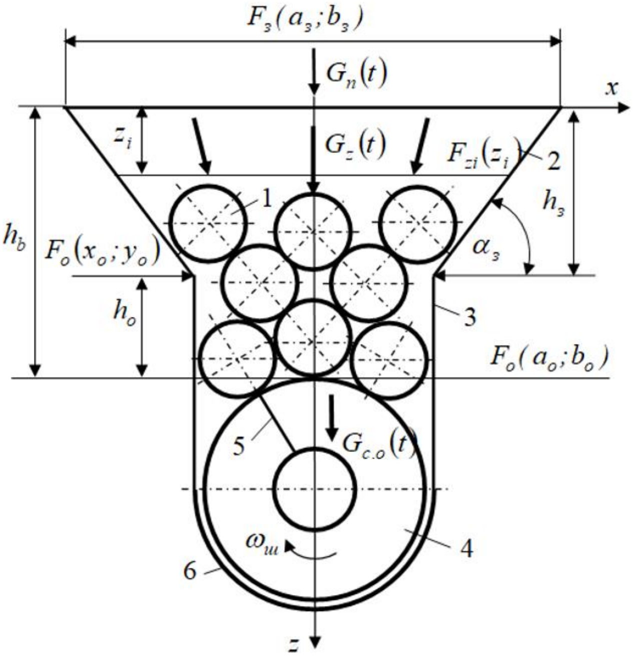

Figure 1. Functional diagram of the hopper: 1 – root crops; 2 – feeding part; 3 – discharge outlet; 4 – auger; 5 – spiral coil of the auger; 6 – cover

- the discharge outlet 3 of the hopper is taken as a rectangular parallelepiped; we indicate the height of the pyramid by h_o, and dimensional parameters of the bottom – base by $a_o \times b_o$;
- in the working space of the hopper most of the root crop 1 is in horizontal position, that is, the longitudinal axis of the root crop is parallel to the horizontal plane;
- the movement of the root crop in the hopper to the screw obeys the law of normal motion.

There is a need to develop a mathematical model that describes the process of movement root crops in the loading hopper to the screw conveyor, and coordinates or ensures the required throughput of the root crops and the productivity of the auger.

Presentation of the material. Suppose the quantity of root crops that flows into the main body of the hopper (feeding part) in relative time t is $G_n(t)$, the number of root crops accumulated in the hopper in the intermediate stage of the production cycle (or the stock of roots crops) is $G_z(t)$, and the number of root crops that leave the discharge outlet (or the consumption from the stock) is $G_{c.o}(t)$. In this case, the root crops stock $G_z(t)$ and the retention capacity of this stock characterize the accumulation of material during the entry of the root crops to the feeding throat and the consumption of root crops from the stock, while the stock allows to compensate for the effect of eddy in the flows.

Hence a condition for increase in the root crops stock is $G_n(t) > G_{c.o}(t)$, and a condition for decrease in the root crops stock is $G_n(t) < G_{c.o}(t)$.

In this context, the equation describing the resultant mass flow rate of the root crops looks like

$$\frac{dG_z(t)}{dt} = \sum G_z = G_n(t) - G_{c.o}(t) = \varDelta G_z(t), \tag{1}$$

where $\varDelta G_z(t)$ – is a remaining stock of root crops in the intermediate stage of the production cycle.

For further analysis, we express the current imaginary technological flows as the actual product, or the appropriate quantitative mass flow of root crops, thus:

- the bulk quantity of the root crops $G_n(t)$ that enter the feeding part of a hopper at a relative time t is

$$\begin{aligned} G_n(t) &= K_{1G_n}(t)m_{k1} + K_{2G_n}(t)m_{k2} + \ldots + K_{iG_n}(t)m_{ki} = \\ &= \sum_{i=1}^{n}\left[K_{1G_n}(t)m_{k1} + K_{2G_n}(t)m_{k2} + \ldots + K_{iG_n}(t)m_{ki}\right] \end{aligned}; \; i = 1, 2, \ldots, n, \tag{2}$$

where $K_{1G_n}, K_{2G_n}, \ldots, K_{iG_n}$ – respectively is the number of root crops of the 1st, 2nd, ..., i-size mass group, that enters the feeding part of a hopper, pcs.; $m_{k1}, m_{k2}, \ldots, m_{ki}$ respectively is the weight of a root crop of 1st, 2nd,..., i-size group, that enters the feeding part of a hopper, kg;

- the bulk quantity of root crops stored in the hopper or root crops stock $G_z(t)$ in relative time t is

$$\begin{aligned} G_z(t) &= K_{1G_z}(t)m_{k1} + K_{2G_z}(t)m_{k2} + \ldots + K_{iG_z}(t)m_{ki} = \\ &= \sum_{i=1}^{n}\left[K_{1G_z}(t)m_{k1} + K_{2G_z}(t)m_{k2} + \ldots + K_{iG_z}(t)m_{ki}\right] \end{aligned}; \; i = 1, 2, \ldots, n, \tag{3}$$

where $K_{1G_z}, K_{2G_z}, \ldots, K_{iG_z}$ – respectively is the number of root crops of the 1st, 2nd, ..., i-size mass group, which is stored in the hopper, pcs.;

- the mass number of root crops that leave the discharge outlet of the hopper, or consumption from the stock $G_z(t)$ in relative time t is

$$G_{c.o}(t) = K_{1G_c}(t)m_{k1} + K_{2G_c}(t)m_{k2} + \ldots + K_{iG_c}(t)m_{ki} = \\ = \sum_{i=1}^{n}\left[K_{1G_c}(t)m_{k1} + K_{2G_c}(t)m_{k2} + \ldots + K_{iG_c}(t)m_{ki}\right]; \; i = 1,\, 2,\, \ldots,\, n, \tag{4}$$

where $K_{1G_c}, K_{2G_c}, \ldots, K_{iG_c}$ – respectively is the number of root crops of the 1st, 2nd, ..., i -size mass group, which leave the discharge outlet of the hopper, pcs.

Substituting (2)–(4) in equation (1) we obtain

$$\frac{d\sum\limits_{i=1}^{n}\left[\begin{array}{l}\left[K_{1G_z}(t) - K_{1G_n}(t) + K_{1G_c}(t)\right]m_{k1} + \left[K_{2G_z}(t) - K_{2G_n}(t) + K_{2G_c}(t)\right]m_{k2} + \ldots + \\ + \left[K_{iG_z}(t) - K_{iG_n}(t) + K_{iG_c}(t)\right]m_{ki}\end{array}\right]}{dt} = \varDelta G_z(t) \cdot \tag{5}$$

If we assume that the shape of the root crops is conical, which corresponds to the shape of most large root crops, then we can state that

$$m_{k1} = \frac{\pi D_{k1}^2 L_{k1}\gamma_k}{12} = \frac{\pi D_{k1}^3\gamma_k}{24tg\left(0{,}5\alpha_{k1}\right)}; \; m_{k2} = \frac{\pi D_{k2}^3\gamma_k}{24tg\left(0{,}5\alpha_{k2}\right)}; \; m_{ki} = \frac{\pi D_{ki}^3\gamma_k}{24tg\left(0{,}5\alpha_{ki}\right)}, \tag{6}$$

where V_{k1} – is the volume of root crops of the 1st size mass group, m^3;
γ_k – is specific weight of root crops, kg / m^3;
D_{k1} – is diameter of root crops of the 1st size mass group, m; L_{k1} is length of root crops of the 1st size mass group, m;
α_{k1} – is a cone angle of growth of root crops of the 1st size mass group, deg.;
$D_{k2}, \ldots, D_{ki}$ – is diameter of root crops of the 2nd, ..., i -size mass group, m;
α_{ki} – is a cone angle of growth of root crops of 2-nd, ..., i -size mass group, deg.

According to (6) the dependence (5) looks like

$$d\sum_{i=1}^{3}\frac{\pi\gamma_k}{24}\left\{\begin{array}{l}\left[K_{1G_z}(t) - K_{1G_n}(t) + K_{1G_c}(t)\right]\dfrac{D_{k1}^3}{tg\left(0{,}5\alpha_{k1}\right)} + \\ + \left[K_{2G_z}(t) - K_{2G_n}(t) + K_{2G_c}(t)\right]\dfrac{\pi D_{k2}^3}{tg\left(0{,}5\alpha_{k1}\right)} + \\ + \left[K_{3G_z}(t) - K_{3G_n}(t) + K_{3G_c}(t)\right]\dfrac{D_{k3}^3}{tg\left(0{,}5\alpha_{k3}\right)}\end{array}\right\} = \varDelta G_z(t)dt \cdot \tag{6}$$

The dependence between the current stock (remaining root crops stock in the intermediate stage of the production cycle) and the resulting mass flow rate of the material stock change is obtained by integrating the equation (1).

In this case

$$G_z(t) = \int\left[G_n(t) - G_{c.o}(t)\right]dt = \int \varDelta G_z dt\,. \tag{7}$$

Taking into account (2)–(4) and (6), the integral equation (7) will look like:

$$\int \varDelta G_z dt = \int \left[\sum_{i=1}^{3} \frac{\pi\gamma_k}{24} \left\{ \begin{array}{l} \left[K_{1G_n}(t) - K_{1G_c}(t)\right] \dfrac{D_{k1}^3}{tg(0{,}5\alpha_{k1})} + \left[K_{2G_n}(t) - K_{2G_c}(t)\right] \dfrac{D_{k2}^3}{tg(0{,}5\alpha_{k2})} + \\ + \left[K_{3G_n}(t) - K_{3G_c}(t)\right] \dfrac{D_{k3}^3}{tg(0{,}5\alpha_{k3})} \end{array} \right\} \right] dt \, . \tag{8}$$

The obtained dependence (8) is a mathematical model written in a general integral form and that characterizes functioning of a hopper of screw conveyor-crushers of root crops or a process of movement of root crops at a relative time t, or the nature of the change between the current remaining stock of root crops and the resulting mass flow rate of the material stock change in the hopper, depending on the mass and size composition of root crops under condition of their normal movement.

According to the provisions of [17] is known that in any cross-section of a hopper under condition of normal movement of root crops the flow velocity of the product ϑ (m/s) is governed by the law of dry friction (Coulomb’s law) and is defined by the well-known formula

$$\vartheta = \frac{G_{n.n}}{F_{zi}(z_i)} = \frac{G_{n.n}}{F_{zi}(x_{zi}\,;y_{zi})} = \frac{G_{n.n}}{F_{zi}(a_{zi}\,;b_{zi})}, \tag{9}$$

where $G_{n.n}$ – is the volume flow rate of the root crops, or the resulting volume flow rate of the material stock change, m^3/s;

$F_{zi}(a_{zi}\,;b_{zi})$ is area of the cross-section dimension of the feeding part of the hopper at a height z_i, m^2.

Then the single-piece volume flow rate of root crops $G_{n.n}$ (pc. m^3/s), or the resulting volume flow rate of the material stock change in the feeding part of the hopper of a screw conveyor-crusher of root crops can be found by the formula (4)

According to (8) we have:

$$G_{n.n} = \frac{\sum\limits_{i=1}^{3} \dfrac{\pi}{24} \left[K_{1G_c} \dfrac{D_{k1}^3}{tg(0{,}5\alpha_{k1})} + K_{2G_c} \dfrac{D_{k2}^3}{tg(0{,}5\alpha_{k2})} + K_{3G_c} \dfrac{D_{k}^3}{tg(0{,}5\alpha_{k3})} \right]}{t}. \tag{10}$$

Then, according to (8), (10) and under accepted conditions, the root crops flow velocity, or $\vartheta_{c.z_i}$ (pc. m/s) and acceleration $a_{c.z}$ (pc. m/s^2) of the consumption of root crops from the hopper stock according to (9) of the feeding part is determined by the formula:

$$\vartheta_{c.zi} = \frac{\pi}{24} \frac{\sum\limits_{i=1}^{3} \left[K_{1G_c} \dfrac{D_{k1}^3}{tg(0{,}5\alpha_{k1})} + K_{2G_c} \dfrac{D_{k2}^3}{tg(0{,}5\alpha_{k2})} + K_{3G_c} \dfrac{D_{k3}^3}{tg(0{,}5\alpha_{k3})} \right]}{\left[F_{zi}(a_{zi}\,;b_{zi})\right]t}. \tag{11}$$

$$a_{c.zi} = \frac{d\vartheta_{c.k}}{dt} = \frac{d}{dt} \left(\frac{\pi}{24} \frac{\sum\limits_{i=1}^{3} \left[K_{1G_c} \dfrac{D_{k1}^3}{tg(0{,}5\alpha_{k1})} + K_{2G_c} \dfrac{D_{k2}^3}{tg(0{,}5\alpha_{k2})} + K_{3G_c} \dfrac{D_{k3}^3}{tg(0{,}5\alpha_{k3})} \right]}{\left[F_{zi}(a_{zi}\,;b_{zi})\right]t} \right). \tag{12}$$

According to Fig. 2, the root crops flow velocity, or the rate of consumption of root crops $\vartheta_{c.zi}$ (pc. m/s) from the hopper stock is within the average range of 0.6 to 1.06 m/s, the acceleration $a_{c.zi}$ (pc. m/s^2) of consumption of root crops from the stock according to Fig. 2.3 is within the average range of 0.5 to 2.0 m/s^2.

Putting in dependence (12) the acceleration of consumption of root crops from the stock as $a_{c.zi} = g$, where g – is gravitational acceleration (m/s^2) and differentiating the function (12), we find the ratio between actual remaining quantity of mass of root crops and acceleration of consumption of root crops from the hopper stock or we obtain a differential equation for consumption of root crops from the hopper at a relative time t, if the cross-sectional fraction is $F_{zi}(x_{zi};y_{zi}) = F_{zi}(a_{zi};b_{zi})$

$$d\left(\frac{\pi}{24\gamma_k}\,\frac{\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0{,}5\alpha_{k1})}+K_{2G_c}\frac{D_{k2}^3}{tg(0{,}5\alpha_{k2})}+K_{3G_c}\frac{D_{k3}^3}{tg(0{,}5\alpha_{k3})}\right]}{\left[F_{zi}(a_{zi};b_{zi})\right]t}\right)=gdt\,\cdot \tag{13}$$

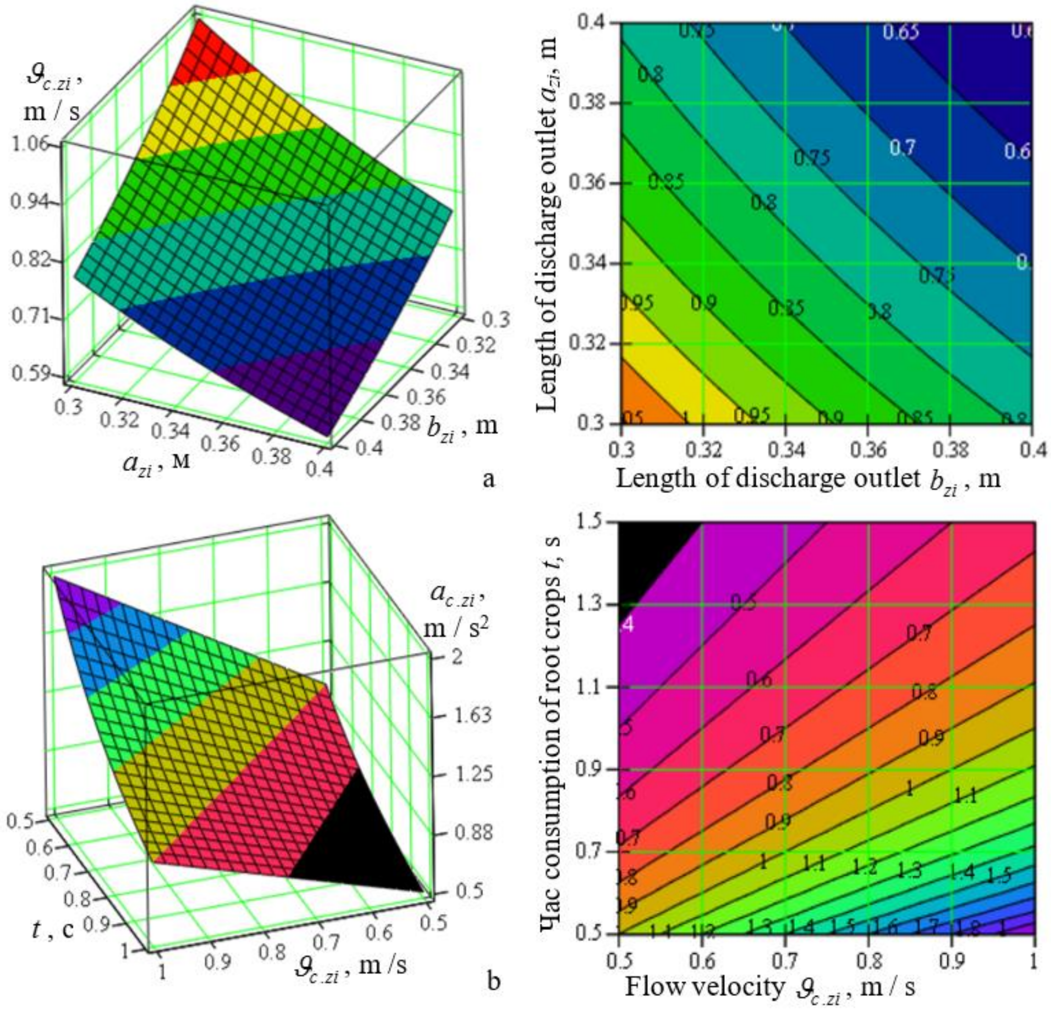

Figure 2. Dependence of change: a – root crops flow velocity as a function $\vartheta_{c.zi} = f_{\vartheta}(a_{zi};b_{zi})$;
b – acceleration of root crops flow as a function $a_{c.zi} = f_a(\vartheta_{c.zi};t)$

Having differentiated the equation (13) we obtain

$$\frac{d}{dt}\frac{\left\{\frac{\pi}{24}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg\left(0,5\alpha_{k1}\right)}+K_{2G_c}\frac{D_{k2}^3}{tg\left(0,5\alpha_{k2}\right)}+K_{3G_c}\frac{D_{k3}^3}{tg\left(0,5\alpha_{k3}\right)}\right]\right\}}{\left[F_{zi}\left(a_{zi};b_{zi}\right)\right]t}=gF_{zi}\left(a_{zi};b_{zi}\right)+$$

$$+\frac{\frac{dF_{iz}\left(a_{zi};b_{zi}\right)}{dt}}{\left[F_{zi}\left(a_{zi};b_{zi}\right)\right]^2}\left\{\frac{\pi}{24t^2}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg\left(0,5\alpha_{k1}\right)}+K_{2G_c}\frac{D_{k2}^3}{tg\left(0,5\alpha_{k2}\right)}+K_{3G_c}\frac{D_{k3}^3}{tg\left(0,5\alpha_{k3}\right)}\right]\right\}^2. \qquad (14)$$

Let’s integrate the differential equation (14) by separation of variables

$$\int\frac{\left\{\frac{\pi}{24}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0,5\alpha_{k1})}+K_{2G_c}\frac{D_{k2}^3}{tg(0,5\alpha_{k2})}+K_{3G_c}\frac{D_{k3}^3}{tg(0,5\alpha_{k3})}\right]\right\}}{F_{zi}(a_{zi};b_{zi})t}dt= .$$

$$=\int\left(\begin{array}{l} gF_{zi}(a_{zi};b_{zi})+\frac{\frac{dF_{zi}(a_{zi};b_{zi})}{dt}}{[F_{zi}(a_{zi};b_{zi})]^2}\times \\ \times\left\{\frac{\pi}{24t^2}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0,5\alpha_{k1})}+K_{2G_c}\frac{D_{k2}^3}{tg(0,5\alpha_{k2})}+K_{3G_c}\frac{D_{k3}^3}{tg(0,5\alpha_{k3})}\right]\right\}^2\end{array}\right)dt \qquad (15)$$

The solution of the differential equation (15) looks like:

$$\frac{\pi}{24t}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0,5\alpha_{k1})}+K_{2G_c}\frac{D_{k2}^3}{tg(0,5\alpha_{k2})}+K_{3G_c}\frac{D_{k3}^3}{tg(0,5\alpha_{k3})}\right]=$$

$$=F_{zi}(z_i)\sqrt{\frac{\frac{\pi}{24t}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0,5\alpha_{k1})}+K_{2G_c}\frac{D_{k2}^3}{tg(0,5\alpha_{k2})}+K_{3G_c}\frac{D_{k3}^3}{tg(0,5\alpha_{k3})}\right]F_{zi}(a_{zi};b_{zi})}{F'_{zi}(a_{zi};b_{zi})}}\times, \qquad (16)$$

$$\times th^2\left(\sqrt{-\frac{gF'_{zi}(a_{zi};b_{zi})}{F_{zi}(a_{zi};b_{zi})}}\right)\cdot t$$

where th is a hyperbolic tangent;

$$\frac{d\left\{\frac{\pi}{24t}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0,5\alpha_{k1})}+K_{2G_c}\frac{D_{k2}^3}{tg(0,5\alpha_{k2})}+K_{3G_c}\frac{D_{k3}^3}{tg(0,5\alpha_{k3})}\right]\right\}}{dt}= .$$

$$=gF_{zi}(a_{zi};b_{zi})\times$$

$$\times\left\{1-th^2\left[\sqrt{-\frac{\frac{\pi}{24t}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0,5\alpha_{k1})}+K_{2G_c}\frac{D_{k2}^3}{tg(0,5\alpha_{k2})}+K_{3G_c}\frac{D_{k3}^3}{tg(0,5\alpha_{k3})}\right]F'_z(a_z;b_z)}{F_{zi}(a_{zi};b_{zi})}}\cdot t\right]\right\} \qquad (17)$$

If in formula (17), (18) time $t \to \infty$, or if in formula (14) we assume that $d\left\{\frac{\pi}{24t}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0{,}5\alpha_{k1})} + K_{2G_c}\frac{D_{k2}^3}{tg(0{,}5\alpha_{k2})} + K_{3G_c}\frac{D_{k3}^3}{tg(0{,}5\alpha_{k3})}\right]\right\} / dt = 0$ and multiply the obtained formula by total bulk weight of root crops γ_k, we get maximum permissible mass consumption of crops from the hopper stock at time $t = 1$ second, or respectively, the required maximum entry of root crops $G_{c.z.max}$ (kg/s) to the discharge outlet of the hopper, or the maximum possible consumption of the root crops if the cross-section $F_{zi}(x_{zi};y_{zi}) = F_{zi}(a_{zi};b_{zi})$ is a conditional instant aperture, i.e.

$$0 = g\gamma_k F_{zi}(a_{zi};b_{zi})\sqrt{-\frac{d\left(\frac{1}{F_{zi}(a_{zi};b_{zi})}\right)t}{dt}} + \frac{\frac{dF'_{zi}(a_{zi};b_{zi})}{dt}}{\left[F'_{zi}(a_{zi};b_{zi})\right]^2}\cdot 0 \cdot \sqrt{-\frac{d\left(\frac{1}{F'_{zi}(a_{zi};b_{zi})}\right)t}{dt}}, \tag{18}$$

or

$$G_{c.z.max} = \gamma_k F_{zi}(a_{zi};b_{zi})\sqrt{-\frac{gF_{zi}(a_{zi};b_{zi})}{F'_{zi}(a_{zi};b_{zi})}}. \tag{19}$$

The area of the cross-section $F_{zi\backslash}(a_{zi};b_{zi})$ of the instant aperture of discharge outlet of a hopper is determined by the formula $F_{zi\backslash}(a_{zi};b_{zi}) = b_{zi}2r_{zi}$, where b_i, r_{zi} – is the length and hydraulic radius of the aperture, m. Still $2r_{zi} = a_{zii} \le b_z$. The coefficient of filling of the hopper is $k_з$.

Then the function (19) will look like

$$G_{c.z.max} \le \gamma_k a_{zi} b_{zi} k_з \left(\sqrt{-\frac{ga_{zi}b_{zi}}{F'_{zi}(a_{zi};b_{zi})}}\right), \tag{20}$$

and the flow velocity of the root crops or the rate of consumption of root crops $\vartheta_{c.o}$ (pc. m/s) from the stock of the discharge outlet when $F_{zi}(x_{zi};y_{zi}) = F_o(a_o;b_o)$ is

$$\vartheta_{c.o} = \frac{\sum_{i=1}^{3}\left[K_{1G_c}m_{k1} + K_{2G_c}m_{k2} + K_{3G_c}m_{k3}\right]}{\left[F_o(a_o;b_o)\right]\gamma_k t}, \tag{21}$$

or

$$\vartheta_{c.o} = \frac{\pi}{24}\frac{\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0{,}5\alpha_{k1})} + K_{2G_c}\frac{D_{k2}^3}{tg(0{,}5\alpha_{k2})} + K_{3G_c}\frac{D_{k3}^3}{tg(0{,}5\alpha_{k3})}\right]}{\left[F_o(a_o;b_o)\right]t}. \tag{22}$$

The rate of acceleration $a_{c.o}$ (pc. m/s^2) of consumption of root crops from the stock, or the maximum possible consumption of root crops if the cross-section $F_{zi}(x_{zi};y_{zi}) = F_o(a_o;b_o)$ is the aperture of the hopper is determined by the formula

$$a_{c.o} = \frac{d\vartheta_{c.k}}{dt} = \frac{d}{dt}\left(\frac{\pi}{24} \frac{\sum\limits_{i=1}^{3}\left[K_{1G_c}\dfrac{D_{k1}^3}{tg(0{,}5\alpha_{k1})} + K_{2G_c}\dfrac{D_{k2}^3}{tg(0{,}5\alpha_{k2})} + K_{3G_c}\dfrac{D_{k3}^3}{tg(0{,}5\alpha_{k3})} \right]}{[F_o(a_o;b_o)]t} \right). \tag{23}$$

Based on a graphical analysis, it was found that the root crops flow velocity, or the rate of consumption of root crops $\vartheta_{c.o}$ (pc. m/s) from the stock is within the average range of 2.0 to 3.0 m/s, the acceleration $a_{c.o}$ (pc. m/s^2) of the consumption of root crops from the stock according is within the average range of 0.6 to 6.0 m/s^2.

Having compared (14) and (23), we can write that

$$\begin{gathered} \frac{\pi}{24t}\sum_{i=1}^{3}\left[K_{1G_c}\frac{D_{k1}^3}{tg(0{,}5\alpha_{k1})} + K_{2G_c}\frac{D_{k2}^3}{tg(0{,}5\alpha_{k2})} + K_{3G_c}\frac{D_{k3}^3}{tg(0{,}5\alpha_{k3})} \right] = F_o(z)\times \\ \times\sqrt{\frac{\dfrac{\pi}{24t}\sum\limits_{i=1}^{3}\left[K_{1G_c}\dfrac{D_{k1}^3}{tg(0{,}5\alpha_{k1})} + K_{2G_c}\dfrac{D_{k2}^3}{tg(0{,}5\alpha_{k2})} + K_{3G_c}\dfrac{D_{k3}^3}{tg(0{,}5\alpha_{k3})} \right]F_o(a_o;b_o)}{F_z'(a_z;b_z)} th^2\left(\sqrt{-\frac{gF_o'(a_o;b_o)}{F_o(a_o;b_o)}} \right)\cdot tk_{\text{з}}} \end{gathered}, \tag{24}$$

Then the required maximum permissible per-second bulk entry of root crops to the auger $G_{c.o.max}$ (kg / s), or the maximum possible consumption of root crops if the cross-section $F_{zi}(x_{zi};y_{zi}) = F_o(a_o;b_o)$ is an aperture of a discharge outlet of hopper by analogy with (2.27) is determined by the formula

$$G_{c.o.max} \le \gamma_k F_o(a_o;b_o)k_{\text{з}}\sqrt{-\frac{gF_o(a_o;b_o)}{F_o'(a_o;b_0)}} = \gamma_k a_o b_o k_{\text{з}}\sqrt{-\frac{ga_o b_o}{F_o'(a_o;b_0)}}. \tag{25}$$

In this case, a rational technological operation of the screw conveyor-crusher is possible under the condition that the per-second entry of the root crops $G_{c.o.max}$ to the auger is less than or equal to the productivity of the screw conveyor-crusher. Thus, to determine the actual number of discrete root crops that come out of the discharge outlet to the auger, we must find the lowest minimum of the function (20) or the lowest value of the function $G_{c.o} = f(z)$ in the interval $0 < z \le h_b$. As a rule, during the research the function $u = (G_{c.o})^2 = \varphi(z)$ is considered.

Setting the derivative of the function $u = (G_{c.o})^2 = \varphi(z)$ to zero, or $u' = (G_{c.o}')^2 = \varphi'(z) = 0$, we get a differential equation that describes the surface of a hopper, which ensures maximum entry of the root crops to the auger.

$$F_{z.u}(a_{z.u};b_{z.u})F_{z.u}''(a_{zu};b_{z.u}) = 3[F_{z.u}'(a_{z.u};b_{z.u})]^2. \tag{26}$$

After integrating into equation (26) under initial conditions $F_{z.u}(0)=F_u(a_u;b_u)$, $F'_{z.u}(0)=F'_u(a_u;b_u)$, we obtain the ratio between areas and abscissa of cross-section of a hopper, which characterize the maximum flow rate of root crops.

$$F_{z.u}(a_{z.u};b_{z.u})=F_u(a_u;b_u)\left(\sqrt{\frac{F_u(a_u;b_u)}{F_u(a_u;b_u)-2F'_u(a_u;b_u)z}}\right);\ G_{c.u.max}=\gamma_k F_u(a_u;b_u)k_{\text{з}}\sqrt{-\frac{gF_u(a_u;b_u)}{F'_u(a_u;b_u)}}. \qquad (27)$$

Expressing formula (28) as regard to $F'_u(a_u;b_u)$ and substituting values $F'_u(a_u;b_u)$ into function (27), we obtain a functional dependence between area $F_u(a_u;b_u)$ and ordinate z for the maximum possible bulk quantity of root crops, coming out of the discharge outlet or consumption from the stock, and the area of the cross-section of the discharge outlet of the hopper

$$F_{z.u}(a_{z.u};b_{z.u})=\gamma_k F_u(a_u;b_u)k_{\text{з}}\,/\sqrt{b_u-\frac{2g\left[F_u(a_u;b_u)\right]^2 z}{G^2_{c.o.max}}}, \qquad (29)$$

where де $F_u(a_u;b_u)=2a_ub_u$, a_u is half the width of the aperture of the discharge outlet, m; b_u – length of the aperture of a discharge outlet, m.

If we assume that $F_u(a_u;b_u)=F_o(a_o;b_o)_o$, then we have:

$$G_{c.o.max}=\gamma_k F_o(a_o;b_o)k_{\text{з}}\left(\sqrt{-\frac{gF_o(a_0;b_o)}{F'_o(a_o;b_0)}}\right);\ F_0(a_o;b_o)=\frac{\gamma_k F_o(a_0;b_0)k_{\text{з}}}{\sqrt{b_0-\frac{2g\left[F_0(a_0;b_0)\right]^2 x_o}{G^2_{c.o.max}}}}, \qquad (30)$$

or

$$G_{c.o.max}=2\gamma_k b_0 x_o k_{\text{з}}\left(\sqrt{-gx_o/x'_o}\right). \qquad (31)$$

Assuming that $x_o=0{,}5a_o$, and $x'_o=0{,}5(a_o)'=\left(0{,}5h_{\text{з}}ctg\alpha_{\text{з}}\right)'=-0{,}5tg\alpha_{\text{з}}$, then according to formula (31) the maximum per-second entry of the root-crops of the discharge outlet of the hopper to the auger, taking into account the coefficient of the hopper filling $k_{\text{з}}$, or the maximum consumption of root crops $G_{c.o.max}$ from the stock is determined by the formula

$$G_{c.o.max}=\gamma_k b_0 a_o k_{\text{з}}\left(\sqrt{ga_o/tg\alpha_{\text{з}}}\right). \qquad (32)$$

Fig. 3a shows the dependence of changes in the maximum consumption of root-crops $G_{c.o.max}$ from the stock as a function $G_{c.o.max}=f(a_o;b_o)$, Fig. 3b shows it as a function $G_{c.o.max}=f(a_o)$.

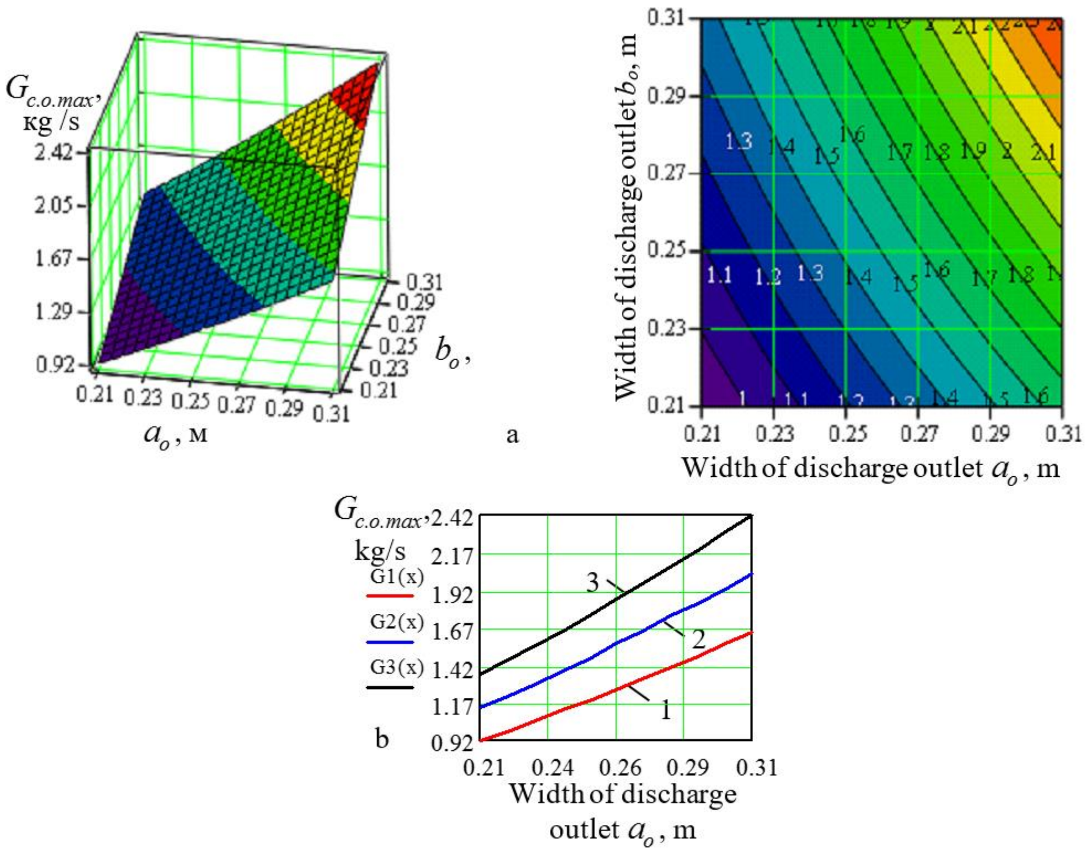

Figure 3. Dependence of changes in the maximum consumption of root-crops $G_{c.o.max}$ from the stock as a function: a – $G_{c.o.max} = f(a_o;b_o)$; б – $G_{c.o.max} = f(a_o)$; 1, 2, 3 – respectively, b_o = 0,21; 0,26; 0,31 m

The obtained dependence (32) can be used for theoretical analysis of estimated productivity of a screw conveyor-crusher of root crops and for substitution of structural and kinematic parameters of its parts.

According to Fig. 3, the value of maximum per-second entry of root crops through the discharge outlet of the hopper to the auger or the maximum consumption of root crops $G_{c.o.max}$ from the hopper stock is in the range from 0.9 to 2.4 kg/s.

Conclusions. The smooth process of product transportation by parts of a screw conveyor-crusher is described by mathematical models (30)–(32).

Thus, the maximum per-second entry of root crops through the discharge outlet of the hopper to the auger or the maximum consumption of root crops $G_{c.o.max}$ from the hopper stock is in the range from 0.9 to 2.4 kg/s, depending on the dimensions of the discharge outlet.

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МАТЕМАТИЧНА МОДЕЛЬ ПРОЦЕСУ ФУНКЦІОНУВАННЯ ЗАВАНТАЖУВАЛЬНОГО БУНКЕРА ШНЕКОВОГО КОНВЕЄРА

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Резюме. *Підвищення продуктивності та зменшення енергетичних затрат процесу роботи шнекових пристроїв і технологічних ліній переробних комплексів, де вони встановлені в секторі аграрного виробництва загалом, на першому етапі в основному залежить від пропускної здатності завантажувальних бункерів. При цьому необхідність раціонального керування такими потоками, які мають доволі значні випадкові відхилення від середньої величини потоку коренеплодів, – досить актуальна задача. Тому метою дослідження є підвищення ефективності процесу роботи шнекових конвеєрів-подрібнювачів шляхом розроблення математичної моделі, яка функціонально описує* *технологічний процес переміщення матеріалу в завантажувальному бункері та узгоджує або забезпечує необхідну пропускну здатність коренеплодів і продуктивність шнека. Наведено теоретичний аналіз процесу переміщення коренеплодів у завантажувальному бункері шнекового конвеєра шляхом розроблення детермінованих математичних моделей, які описують та визначають кількісні показники секундної подачі коренеплодів до шнека залежно від завантаження бункера. Отримано математичну модель, яку записано в загальному інтегральному вигляді та яка характеризує процес функціонування бункера шнекового транспортера-подрібнювача коренеплодів або процес переміщення коренеплодів у відносному часі* t*, або характер зміни між поточним залишковим запасом коренеплодів і результуючою масовою витратою зміни запасу матеріалу в бункері залежно від масового розмірно-фракційного складу коренеплодів за умови їх нормального руху. Отримана модель може бути застосована для проведення теоретичного аналізу розрахункової продуктивності роботи шнекового транспортера-подрібнювача коренеплодів і обґрунтування конструктивно-кінематичних параметрів його робочих органів. Встановлено, що швидкість витікання коренеплодів, або швидкість споживання коренеплодів із запасу знаходиться у середніх межах від 2,0 до 3,0 шт. м/с, прискорення споживання коренеплодів із запасу – у середніх межах від 0,6 до 6,0 шт. м/с*^{2}*. Значення максимального секундного надходження коренеплодів через вихідну горловину бункера до шнека, або максимальне споживання коренеплодів із запасу бункера знаходиться у діапазоні від 0,9 до 2,4 кг/с.*

Key words*: коренеплоди, вихідна горловина, секундна подача, запас коренеплодів, витрата коренеплодів, швидкість витікання, прискорення витрат коренеплодів.*

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