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ANALYSIS OF ELECTRICALLY PERMEABLE CRACKS ON THE INTERFACE BETWEEN TWO ONE-DIMENSIONAL PIEZOELECTRIC QUASICRYSTALS

Abstract. A set of finite number collinear cracks along the interface of two 1D piezoelectric hexagonal quasicrystals is considered. The cracks can have arbitrary lengths and distances between their tips. The problem of linear relationship is formulated and solved in an analytical form. The analytical formula for the ERR have been obtained. The variations of the phonon and phason crack faces displacement jumps, stresses along the interface and the ERR are presented in graph and table forms.

Consider the plane problem in $x_1 - x_3$ plane for a set of n electrically permeable cracks $a_1 \le x_1 \le b_1$, $a_2 \le x_1 \le b_2$, …, $a_n \le x_1 \le b_n$ in the interface between two semi-infinite 1D piezoelectric hexagonal quasi-crystalline spaces with point group 6mm. The plane $(x_1, 0, x_2)$ is periodic and x_3-axis is identical to the quasi-periodic direction. It is assumed that the cracks are electrically permeable, and uniformly distributed phonon $(\sigma^\infty, \tau^\infty)$ and phason H^∞ stresses are prescribed at infinity. The conditions along the material interface can be written in the following form

$$\sigma_{13}^{(m)} = 0,\ \sigma_{33}^{(m)} = 0,\ H_{33}^{(m)} = 0,\ \langle \varphi \rangle = 0,\ \langle D_3 \rangle = 0 \text{ for } x_1 \in L_I, \tag{1}$$
$$\langle \sigma_{13} \rangle = 0,\ \langle \sigma_{33} \rangle = 0,\ \langle H_{33} \rangle = 0,\ \langle u_1 \rangle = 0,\ \langle u_3 \rangle = 0,$$
$$\langle W_3 \rangle = 0,\ \langle \varphi \rangle = 0,\ \langle D_3 \rangle = 0 \text{ for } x_1 \in L_{II},$$

where $L_I = \bigcup_{k=1}^{n} (a_k, b_k)$, $L_{II} = (-\infty, \infty) \setminus L_I$, $m = 1, 2$; the upper index (1) corresponds to the upper material and (2) is related to the lower one; the designation $\langle f \rangle$ means the jump of the function f through the material interface $x_3 = 0$.

The following presentations were obtained in paper [1]

$$\sigma_{33}^{(1)}(x_1, 0) + m_{j5} H_{33}^{(1)}(x_1, 0) + i m_{j1} \sigma_{13}^{(1)}(x_1, 0) = \Phi_j^+(x_1) + \lambda_j \Phi_j^-(x_1), \tag{2}$$
$$n_{j1} \langle u_1'(x_1) \rangle + i n_{j3} \langle u_3'(x_1) \rangle + i n_{j5} \langle W_3'(x_1) \rangle = \Phi_j^+(x_1) - \Phi_j^-(x_1), \tag{3}$$

where $j = 1, 3, 5$; $\Phi_j(z)$ are the functions analytic in the whole complex plane except the cracks region; (…)' means the differentiation on x_1; m_{ji}, n_{ji}, λ_j are the constants defined by materials properties.

Satisfying the interface conditions (1) by using Eqs. (2), one arrives at the following problem of linear relationship:

$$\Phi_j^+(x_1) + \lambda_j \Phi_j^-(x_1) = 0 \text{ for } x_1 \in L_I \tag{4}$$

The solution of Eq. (4) can be presented in the form

$$\Phi_j(z) = F_j(z) \Psi_{nj}(z), \tag{5}$$

where

$$F_j(z)=\prod_{k=1}^{n}(z-a_k)^{-\frac{1}{2}+i\kappa_j}\cdot(z-b_k)^{-\frac{1}{2}-i\kappa_j}\ ,\ \kappa_j=\frac{\ln\lambda_j}{2\pi}, \quad (6)$$

$$\Psi_{nj}(z)=c_{0j}z^n+c_{1j}z^{n-1}+\ldots+c_{nj}. \quad (7)$$

c_{kj} $(k=0,1,...n)$ are arbitrary complex coefficients, which are found from the conditions at infinity.

$$\Phi_j(z)|_{z\to\infty}=\sigma_j^*-i\tau_j^*, \quad (8)$$

the single-valued ness of the displacements and zero net charge for each crack.

All required quantities at the interface are found on the base of the relations (2), (3) and the solution (5).

According to the crack closure integral the energy release rate (ERR) at a crack tip b_k can be defined in the form:

$$G_{b_k}=\lim_{\Delta l\to 0}\frac{1}{2\Delta l}\left\{\int_{b_k}^{b_k+\Delta l}\left[\sigma_{33}^{(1)}(x_1,0)\langle u_3(x_1-\Delta l)\rangle+\sigma_{13}^{(1)}(x_1,0)\langle u_1(x_1-\Delta l)\rangle+\right.\right.$$
$$\left.\left.H_{33}^{(1)}(x_1,0)\langle W_3(x_1-\Delta l)\rangle\right]dx_1\right\}.$$

Substituting the phonon and phason asymptotic expressions for the required quantities into the last formula and performing the integration, we arrive to the following expression for the ERR:

$$G_{b_k}=\frac{\pi(1+4\kappa_1^2)}{8\cosh(\pi\kappa_1)}\eta_1\eta_{2k}+\frac{\pi}{4}\eta_3\bar{J}_{5k}Q_{5k}, \quad (9)$$

where all coefficients in this formula are expressed via elementary functions.

For the certain quasicristalline bimaterial , the set of three cracks of length $20\ mm$ is considered. The positions of two cracks with $a_1=-200\ mm$ and $a_2=-10\ mm$ are fixed and the right crack (a_3,b_3) changes its location. The values of ERR of the middle crack, obtained for $\sigma^\infty=10^7\,Pa$, $\tau^\infty=H_{33}^\infty=0$ are presented in Table 1.

Table 1. The ERRs of the middle crack for the fixed position of the left crack and variable position of the right one

Locations of the left crack tip a_3, mm	180	80	40	20	12	10.2
ERR $G_{b_2}, N/m$	23.512	23.778	24.734	29.012	52.142	211.59
ERR $G_{a_2}, N/m$	23.512	23.717	24.296	25.936	29.521	34.094

It is clearly seen from these results that the position of right crack influences the ERRs of both crack tips of the middle crack, but the growth of the ERR for the neighboring tip b_2 is much more intensive than for the remote crack tip a_2.

The variation of the phonon and phason crack openings for the middle crack and approaching to the right crack are shown in Figures 1 and 2, respectively. The same position of the left and right cracks and loading as for Table 1 are considered. The solid lines correspond to $a_3=40mm$, dashed – to $a_3=20mm$, dash-dotted – to $a_3=12mm$ and dotted– to $a_3=10.2mm$.

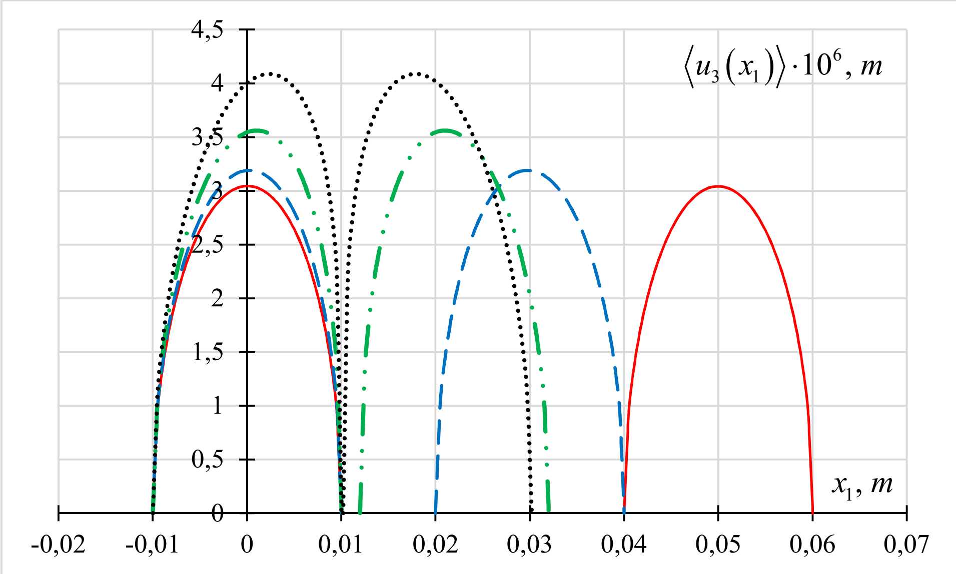

Fig. 1 The influence of the cracks convergence on their phonon opening

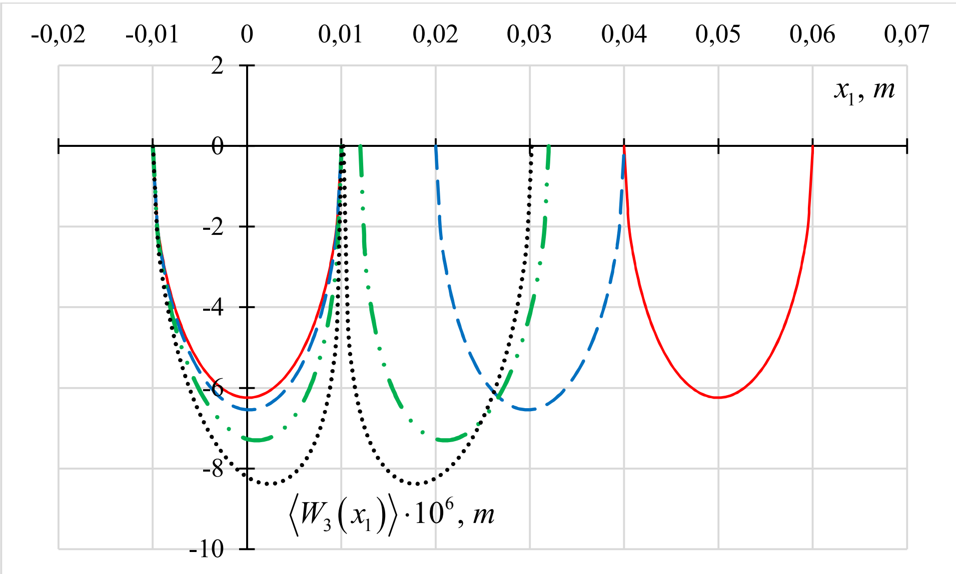

Fig. 2 The influence of the cracks convergence on their phason opening

References

1. Loboda V, Komarov O, Bilyi D, Lapusta Y. An analytical approach to the analysis of an electrically permeable interface crack in a 1D piezoelectric quasicrystal. Acta Mech. 2020; 231: 3419–3433.