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## DISCRETE–TIME CONTROLLER DESIGN FOR PITCH CHANNEL

This research proposes a discrete time controller design for the pitch channel for a two degree of freedom helicopter using the root locus method. The proposed lead-lag controller uses zero and pole placement to increase the stability and controllability of the system. Simulation is provided to insure the validity of the proposed controller.

*Keywords—Two-degree-of-freedom, Helicopter, discrete time control, root locus, lead lag compensator*

The two degree of freedom (2 DOF) helicopter consists of a fixed base with two propellers that are driven by DC motors. One propeller controls the elevation of the helicopter nose about the pitch axis and the other propeller controls the side to side motion about the yaw axis. These non-linearities and model uncertainties make designing a controller for helicopters an open research problem [1]. The interest in this research problem has increased recently due to their potential military and civil applications [2]. Various approaches for stabilization and tracking control of helicopters have been reported in the literature. A fuzzy control technique was presented in [3], a State Dependent Riccati Equation (SDRE) methodology in [4], back-stepping based approach in [5], and linear and non linear feedback control was presented among others [6].

### SYSTEM DYNAMICS & PROBLEM STATEMENT

The two degrees of freedom (2DOF) helicopter system is a popular modeling tool due to its highly non-linear nature. The modeling and control tools of this system can be used in multiple areas such as aerospace. The system used in the model is a twin rotor single input single output system. The twin rotors are the yaw rotor and the pitch rotor which control the yaw and pitch of the system respectively. The system can be seen in figure 1.

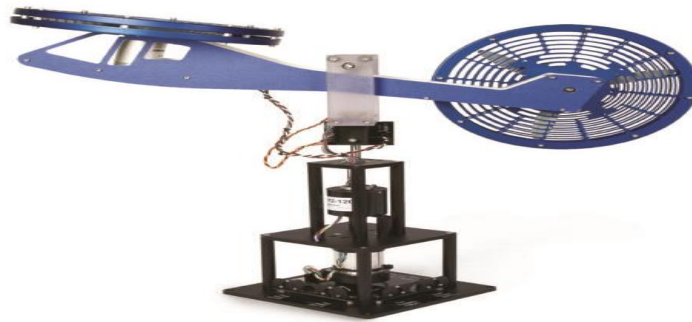


Figure. 1. 2DOF helicopter system

The free body diagram of the 2-DOF helicopter is illustrated in figure 2. The diagram illustrates the degrees of freedom for the helicopter using the two rotors. In this system the two degrees of freedom are around the yaw axis and pitch axis. The pitch angle increases positively,  $\theta(t) > 0$ , when the nose is moved upwards, and the body rotates in the counter-clockwise (CCW) direction. The yaw angle increases positively,  $\psi(t) > 0$  when the body rotates in the clockwise (CW) direction. When the pitch thrust force is positive the pitch increases, and when the yaw thrust force is positive the yaw increases.

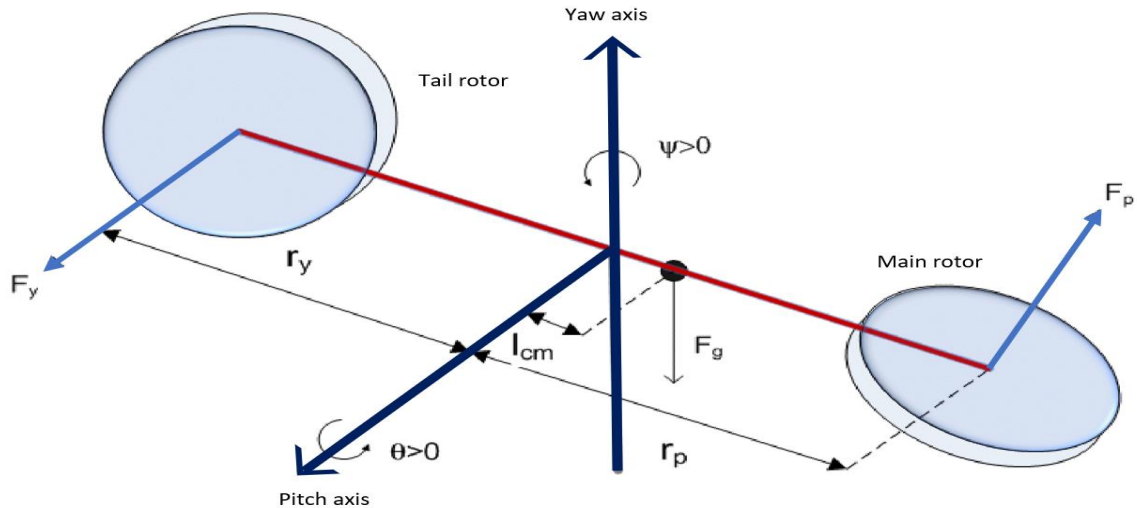


Figure. 2. Simple free-body diagram of 2-DOF Helicopter

The thrust forces acting on the pitch and yaw axes from the front and back motors are then defined. The non-linear equations of motion for the system are derived. Linearization can be used to simplify the non-linear dynamics of the system about a set of preselected equilibrium conditions and presented in the form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

The (linearized) state-space equations describing the system are:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.7451 & -0.2829 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.2701 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 37.2021 & 3.5306 \\ 0 & 0 \\ 2.3892 & 7.461 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t)$$

Where:

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix}, \quad u = \begin{bmatrix} v_p \\ v_y \end{bmatrix}, \quad y = \begin{bmatrix} \theta \\ \psi \end{bmatrix}$$

The closed loop system presentation for the pitch channel is shown in figure 3.

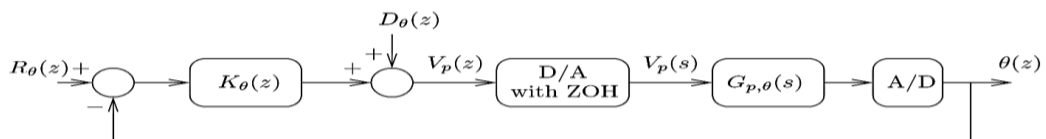


Figure. 3. Closed loop system (pitch channel).

The response of the closed-loop system ( $\theta[n]$ ) to unit step disturbance



$$\frac{\theta(z)}{D(z)} = \frac{G_\theta(z)}{1 + K(z)G_\theta(z)}$$

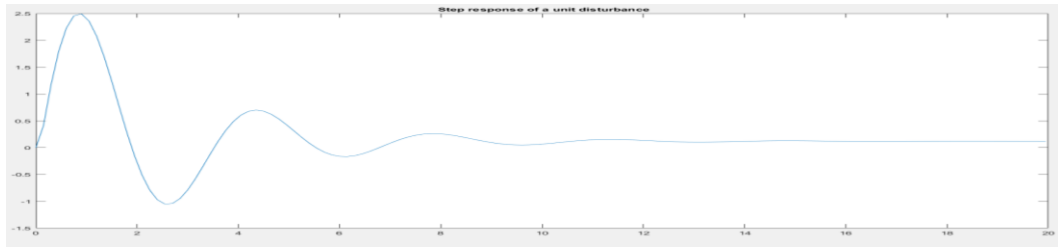


Figure 4. The response of the system to a unit step disturbance

The steady state error of the system is 0.0 to a step input disturbance.

$$\theta_{ss} = \lim_{z \rightarrow 1} \frac{G(z)}{1 + K(z)G(z)}$$

$$\theta_{ss} = \lim_{z \rightarrow 1} (1 + K(z)G(z)) = \infty$$

$$\theta_{ss} = 0$$

Table 1: closed loop system specifications (with step response to the disturbance)

Overshoot	2.0032e+03
Rise Time	0
Settling Time	10.0500
Steady state error	0

From the table it can be seen that the settling time of systems reaction to a step disturbance is less than 16 seconds which means that the system specifications has been met.

Obtain motor voltage  $v_p[n]$  in response to step reference input in the pitch channel.

From the step response the peak voltage can be derived. From this the maximum size of step input that does not result in motor saturation is calculated as follows.

$$\frac{V_{max}}{V_p} = \frac{8}{1.0720} = 7.46$$

## CONCLUSION.

In this research a controller for a 2-DOF helicopter was designed using the root locus method. The poles and zeros of the lead lag controller were strategically placed to allow the maximum controllability and stability of the system. Simulation results were presented to show the result of the proposed controller under various conditions.

## REFERENCES

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