Міністерство освіти і науки України
Тернопільський національний технічний університет імені Івана Пулюя

Кафедра електричної інженерії

Electrical circuits.

LECTURES

Ternopil
2019

УДК 621.3
М54

Укладач:
Куземко Н. А., канд. техн. наук, доцент.

Рецензент:
Г. М. Осухівська, канд. техн. наук, доцент.

Затверджено на засіданні кафедри електричної інженерії
Тернопільського національного технічного університету імені Івана Пулюя.
Протокол № 6 від 1 березня 2019 року.

Схвалено й рекомендовано до друку методичною радою факультету
прикладних інформаційних технологій та електроінженерії
Тернопільського національного технічного університету імені Івана Пулюя.
Протокол № 6 від 26 березня 2019 року.

М 54 Методичні вказівки з курсу «Електротехніка» для студентів-іноземців «Electrical circuits. Lectures» / Укладач : Кузeмко Н.А. – Тернопіль : Тернопільський національний технічний університет імені Івана Пулюя, 2019. – 64 с.

УДК 621.3

Посібник складено відповідно до робочих програм курсів “Теорія електричних та магнітних кіл”, “Електротехніка, електроніка та основи МПТ“, “Електротехніка в будівництві”.

Комп’ютерний набір: Куземко Н. А.

PART 1. DIRECT CURRENT CIRCUITS

Sources and consumers

Electrical circuit is the multitude of devices designed for transforming, distribution and conversion of electrical energy, while the processes which are taking places in these devices can be described by the concepts of current, voltage and electromotive force (e.m.f.).

The simplest electrical circuit contains three main elements: electrical source (active element), consumer (passive element) and the wires. Besides, the circuit can have also additional elements: measuring devices, switches, fuses, contactors, etc.

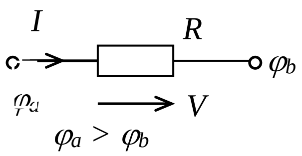


Fig. 1.1

Electrical power is transformed into heating, mechanical energy, etc. at the *consumers*. The measure of this transformation is resistance *R* (fig. 1.1). You can see the directions of the electrical values at fig. 1.1.

Ohm's law for this element is as follow $V = RI$ or $I = GV$, where R - is resistance, $G = 1/R$ - is conductivity. The power on resistive element is $P = RI^2 = GV^2$.

Heating, mechanical energy, etc. is transformed into electrical power at the *electrical sources*. The measure of this transformation is electromotive force (e.m.f.) *E* (fig. 1.2). You can see the directions of the electrical values at fig. 1.2.

The *ideal electrical source* (without losses) is characterized only by *E*. The power on the electrical source is $P = EI$.

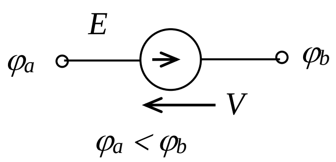


Fig. 1.2

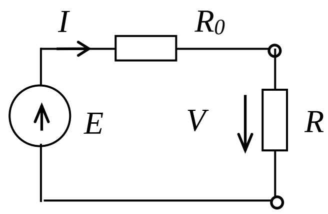


Fig. 1.3

The *real electrical source* has losses and is characterized by *E* and R_0 (internal resistance), which reflects the losses. The simplest electrical circuit is shown at fig. 1.3. For this circuit:

$$I = E/(R_0 + R), \text{ then } \quad V = E - R_0 I, \quad V = RI, \quad R_0 I + RI = E.$$

We can represent real electrical circuit by two substitution schemes: serial (fig. 1.4) and parallel (fig. 1.6). The external volt-ampere characteristic (fig. 1.5) *V(I)* is the main characteristic of the source. Its analytical expression is $V = E - R_0 I$. At fig. 1.5 solid line indicates the characteristic of real source, dashed line - the characteristic of ideal source. Boundary points of this characteristic correspond to the

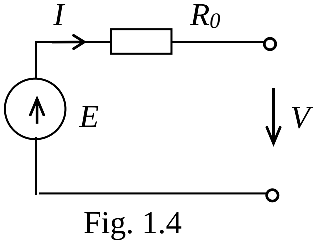


Fig. 1.4

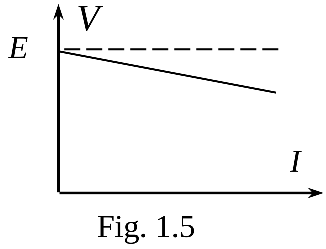


Fig. 1.5

source boundary modes – open circuit (idle) mode (without loading), when $I = 0$, $V = E = V_{OC}$ and short circuit mode, when $V = 0$, $I = I_{SC}$. The external characteristic of ideal source $V = E$ is represented by dashed line at fig. 1.5.

Parallel substitution scheme (fig. 1.6) consists of ideal current source J and internal conductivity G_0, which characterizes the losses. The external characteristic of real source (fig. 1.7) is described by the equation $I = J - G_0 U$. The external characteristic of ideal source $I = J$ is represented by dashed line at fig. 1.7.

Serial and parallel schemes are equivalent, it means you can transform one into

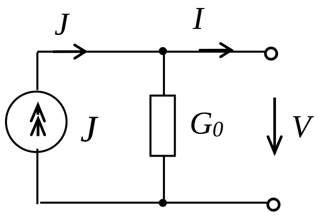


Fig. 1.6

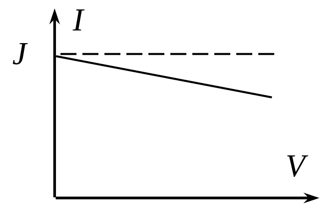


Fig. 1.7

another using such formulas:

$$E = G_0 J,\ R_0 = 1/G_0,\ J = E/R_0,\ G_0 = 1/R_0.$$

The efficiency factor of the source characterizes the efficiency of energy transforming from the source to consumer:

$$\eta = \frac{P_R}{P_E} = \frac{VI}{EI} = \frac{V}{E}, \quad V = \eta E,$$

where P_R - is a consumer power, P_E - is a source power.

We can also write down the efficiency factor using the elements parameters:

$$\eta = \frac{P_R}{P_R + \Delta P} = \frac{RI^2}{RI^2 + R_0 I^2} = \frac{R}{R_0 + R} = \frac{1}{1 + R_0/R},$$

where ΔP - are power losses.

There are three main *electrical circuit modes:* nominal, operating and boundary.

The nominal mode is the best mode for the working device, the device nominal parameters are shown in its technical passport ($I_{NOM}, V_{NOM}, P_{NOM}$).

Operating mode is a mode, where the deviation from the nominal parameters is not big.

Boundary modes are: open circuit or idle (non-working) and a short circuit (emergency) modes. For the open circuit (o.c.) mode $R=\infty$, then using the scheme at fig.1.3, we can write down:

$$I = \frac{E}{R_0 + R} = \frac{E}{R_0 + \infty} = 0,\ V = E - R_0 I = E,\ \eta = 1.$$

For short circuit mode (s.c.) *R=0,* then using the scheme at fig.1.3, we can write down:

$$V = RI,\ V=0,\ I = E/R_0 = I_{SC},\ \eta = 0.$$

The methods of open circuit and short circuit experiments can be used for defining the parameters of the source (E, R_0): $V_{OC} = E$, $R_0 = E/I_{SC}$. The experiment of s.c. mode is provided at low voltage.

Electrical source operating modes:

- voltage generator, when the voltage at the clamps of the source practically does not depend on the current, thus $V \approx E$, and this mode is close to o.c. In this mode $R_0 I << RI$ (fig.1.4), that's why the condition of it is $R_0 << R$ and $\eta \approx 1$. This is the main operating mode of electrical engineering devices.
- current generator, when the current at the clamps of the source practically does not depend on the voltage, thus $I \approx J$, and this mode is close to to s.c. In this mode $G_0 U << GU$ (fig.1.6), that's why the condition of it is $G_0 << G$ ($R_0 >> R$).

-balanced mode – the maximum power $P = R I^2$ is transferred from the source to the consumer at this mode? $I = E/(R_0 + R)$, and $P = RE^2/(R_0 + R)^2$ at this mode.

The condition of this mode comes out from the expression $dP/dR = 0$, that means $R_0 = R$ and $\eta = 0.5$. This mode is used in electronics.

Connections of elements

There are two types of elements connections in electrical circuits, they are simple and complex. The major difference between those two types is that we know the directions of currents before we calculate the circuit with simple connection and don't know the directions of currents at the circuits with complex connections, that's why we choose them arbitrarily.

There are three types of *simple* connection: serial, parallel and mixed.

When the elements are connected in *serial* (fig.1.8), the same current I is flowing through them. The total resistance of serial connection is $R = \Sigma\, R_n$.

The input voltage (fig.1.8)

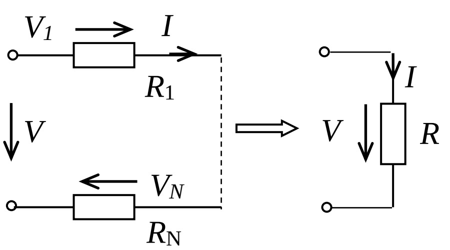


Fig. 1.8

$$V = V_1 + ... + V_N = \Sigma V_N = \Sigma R_N I\,.$$

The power of this circuit

$$P = VI = \Sigma R_N II = \Sigma R_N I^2 = \Sigma P_N\,,$$

where $P = VI$ – the power of the source, ΣP_N – the power of the consumers.

When the elements are connected in *parallel* (fig. 1.9), the same voltage V is applied to them. The total conductivity of parallel connection is $G = \Sigma\, G_n$.

The total current of the circuit (fig.1.9):

$$I = I_1 + ... + I_N = \Sigma I_N = \Sigma G_N V\,.$$

The power of this circuit:

$$P = VI = \Sigma G_N VV = \Sigma G_N V^2 = \Sigma P_N\,.$$

For two elements connected in parallel:

$$G = G_1 + G_2 = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}\,,$$

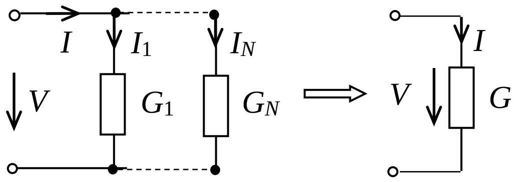


Fig. 1.9

$$R = \frac{1}{G} = \frac{R_1 R_2}{R_1 + R_2}.$$

The circuit with two elements connected in serial (fig.1.10) can be used as *voltage divider*.

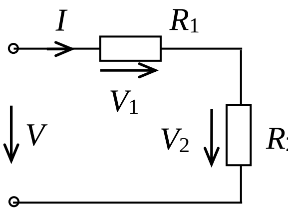


Fig. 1.10

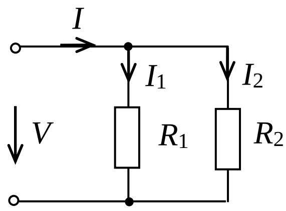


Fig. 1.11

$$I = \frac{V}{R_1 + R_2}, \qquad V_1 = R_1 I = V\frac{R_1}{R_1 + R_2}, \qquad V_2 = R_2 I = V\frac{R_2}{R_1 + R_2}.$$

The circuit with two parallel connected elements (fig.1.11) can be used as *current divider*.

$$V = I\frac{R_1 R_2}{R_1 + R_2},\ I_1 = V/R_1 = I\frac{R_2}{R_1 + R_2},\ I_2 = V/R_2 = I\frac{R_1}{R_1 + R_2}.$$

We can replace the *mixed* (serial-parallel) connection (fig. 1.12) by one equivalent (total) resistance R:

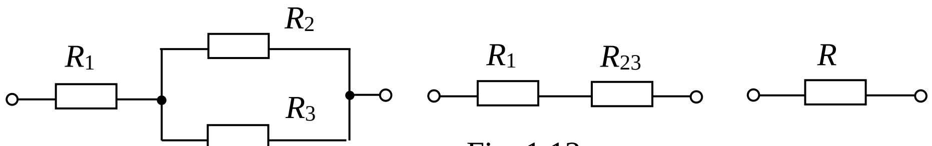


Fig. 1.12

$$R_{23} = R_2 R_3 /(R_2 + R_3), \qquad R = R_1 + R_{23}.$$

We can also replace the *mixed* (parallel-serial) connection (fig. 1.13) by one equivalent (total) resistance R:

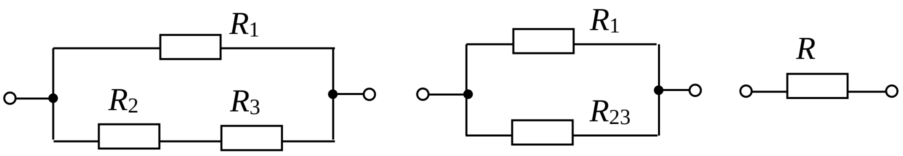


Fig. 1.13

$$R_{23} = R_2 + R_3, \qquad R = R_1 R_{23} /\left(R_1 + R_{23}\right)$$

The *complex* connections are DELTA (fig. 1.14) and WYE (fig. 1.15).

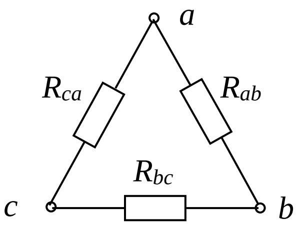


Fig. 1.14

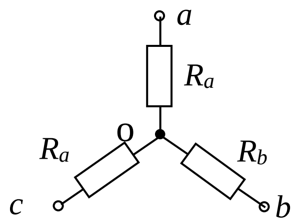


Fig. 1.15

We can know real directions of the currents only after calculation. We can also transform DELTA into WYE using such expressions:

$$R_a = \frac{R_{ab} R_{ca}}{R_{ab} + R_{bc} + R_{ca}}, \qquad R_b = \frac{R_{ab} R_{bc}}{R_{ab} + R_{bc} + R_{ca}}, \qquad R_c = \frac{R_{ca} R_{bc}}{R_{ab} + R_{bc} + R_{ca}}.$$

We use the *simplification method* to calculate the circuits with one source. To use this method we must:

- simplify the circuit to one equivalent resistance;
- calculate the total current by using Ohm's law;
- revert back the circuit and calculate the branch currents and voltages across the elements;
- verify the calculation by using the power balance equation.

The calculation of electrical circuits with several sources

We can use several methods, which are based on Kirchhoff's laws.

Current law (Kirchhoff's first law) states that the sum of the currents entering the node is equal to the sum of the currents leaving the node $\sum I_{\text{n}} = 0$ (the algebraic sum of the currents in the node is equal to zero).

Voltage law (Kirchhoff's second law) states that the algebraic sum of all voltages across passive elements around a loop is equal the algebraic sum of electro-motive forces around the same loop $\Sigma\, R_n I_n = \Sigma E_n$.

Branch of the circuit is the part of the circuit with the same current, it may be consisted from one or several elements connected in serial.

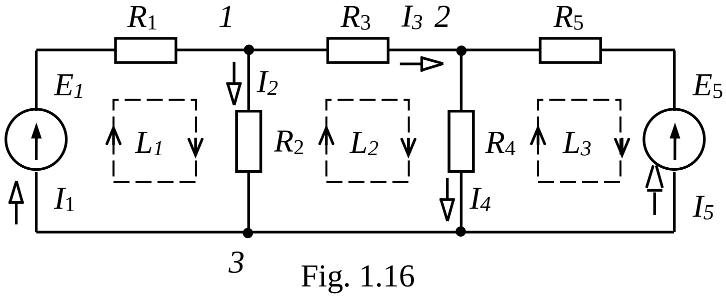


Fig. 1.16

Node is the place where three or more branches are connected.

Loop is any closed path around the circuit.

Kirchhoff's laws method

Let`s suppose the circuit has p branches and q nodes. There'll be p unknown currents. We must solve the system of p equations to find them.

First, you have to choose the directions of branch currents arbitrarily and mark them at the scheme, then mark the nodes and the loops. After this, it is necessary to write down $q-1$ nodes equations according to Current lawand $p-q-1$ loop equations according to the Kirchhoff's second law.

After the system of equations is solved, some currents may have sign "-", it means that the real directions of that current is opposite to the one we have chosen at the beginning.

Let`s write down the system of equations for the scheme at fig.1.16. There are 5 branches $p=5$ and 3 nodes $q=3$ here.

The equations according to current law($q-1=2$) for the nodes *1* and *2:*

$$“1“\ \ I_1 = I_2 + I_3$$

$$“2“\ \ I_3 + I_5 = I_4 \qquad (1)$$

The equations according to voltage law ($p - q - 1 = 3$) for the loops L_1, L_2, L_3 (we choose the directions along the loops clockwise, if the directions of our bypass and the voltage or e.m.f. are the same, we denominate it with “+“, if opposite with “-“).

$$“\mathrm{L}_1“\ \ +R_1 I_1 + R_2 I_2 = +E_1$$

$$“\mathrm{L}_2“ - R_2 I_2 + R_3 I_3 + R_4 I_4 = 0 \qquad (2)$$

$$“\mathrm{L}_3“ - R_4 I_4 - R_5 I_5 = -E_5$$

So, the equation system according this method will be:

$$\begin{cases} +I_1 - I_2 - I_3 = 0 \\ +I_3 - I_4 + I_5 = 0 \\ +R_1 I_1 + R_2 I_2 = +E_1 \\ -R_2 I_2 + R_3 I_3 + R_4 I_4 = 0 \\ -R_4 I_4 - R_5 I_5 = -E_5 \end{cases} \qquad (3)$$

After solving this system we get the unknown branch currents.

We apply the equation of power balance to verify our calculations: the total power of the sources should be equal to the total power of the consumers $\Sigma P_R = \Sigma P_E$. The total power of the sources $\Sigma\, P_E = \Sigma\, E_n I_n = E_1 I_1 + E_5 I_5$. The total power of the consumers $\sum P_R = \sum R_n I_n^2 = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 + R_4 I_4^2 + R_5 I_5^2$.

Loop currents method

This method has less equations than previous and is based on the voltage law.

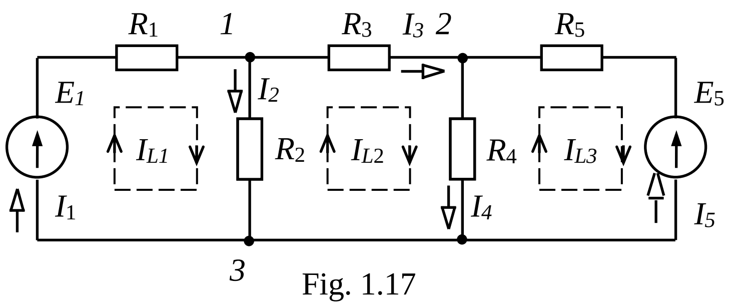


Fig. 1.17

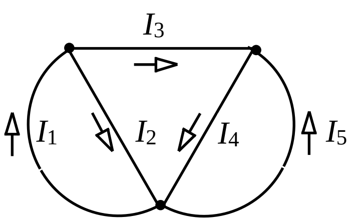


Fig. 1.18

Let’s suppose that we have three loop currents I_{L1}, I_{L2}, I_{L3} at circuit (fig. 1.17), the directions of these currents we choose arbitrarily. Then we can write down branch current by using loop currents: $I_1 = I_{L1}$, $I_2 = I_{L1} - I_{L2}$, $I_3 = I_{L2}$, $I_5 = -I_{L3}$, $I_4 = I_{L2} - I_{L3}$.

We have to substitute these expressions in the equations of voltage law:

$$“\mathrm{L}_1“\ \ +R_1 I_1 + R_2 I_2 = +E_1$$

$$“\mathrm{L}_2“ - R_2 I_2 + R_3 I_3 + R_4 I_4 = 0$$

$$“\mathrm{L}_3“ - R_4 I_4 - R_5 I_5 = -E_5$$

We get the following:

$$\begin{cases}(R_1+R_2)I_{L1}-R_2I_{L2}=E_1\\ -R_2I_{L1}+(R_2+R_3+R_4)I_{L2}-R_4I_{L3}=0\\ -R_4I_{L2}+(R_4+R_5)I_{L3}=-E_5\end{cases} \qquad (4)$$

Let's mark:

$R_{11}=R_1+R_2$, $R_{22}=R_2+R_3+R_4$, $R_{33}=R_4+R_5$ - it'll be individual resistances of the loops, which are equal to the sum of all the resistances of the loop;

$R_{12}=R_{21}=R_2$, $R_{13}=R_{31}=0$, $R_{23}=R_{32}=R_4$ - mutual resistances of the loops, the resistances of the branches which are mutual for the respective loops;

$E_{L1}=E_1$, $E_{L2}=0$, $E_{L3}=-E_5$ - loops e.m.f., is equal to the algebraic sum of the electromotive forces of the loops.

Using these markings, system (4) looks like (5), that can be used for any circuit with three independent loops:

$$\begin{cases}+R_{11}I_{L1}-R_{12}I_{L2}-R_{13}I_{L3}=E_{L1}\\ -R_{21}I_{L1}+R_{22}I_{L2}-R_{23}I_{L3}=E_{L2}\\ -R_{31}I_{L1}-R_{32}I_{L2}+R_{33}I_{L3}=E_{L3}\end{cases}. \qquad (5)$$

Nodal potential method

This method has less equations than previous one and is based on current law. Let's analyze the circuit on fig.1.19. There are two independent nodes a,b. Try to suppose that the potential of the basic (dependent) node is equal to zero, so the potentials of other nodes are marked at the scheme as φ_a,φ_b (fig. 1.19).

We can also write down the branch currents using node potentials:

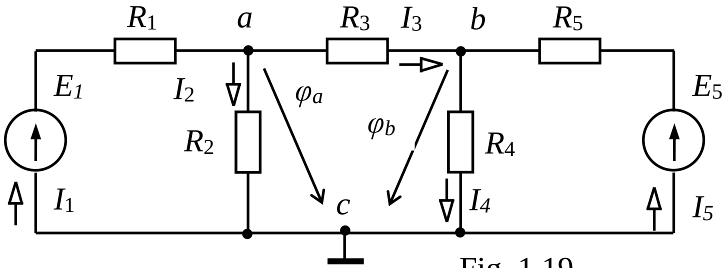


Fig. 1.19

$$\varphi_a=E_1-R_1I_1,\qquad I_1=\frac{(E_1-\varphi_a)}{R_1}=(E_1-\varphi_a)G_1,$$

$$\varphi_a=R_2I_2,\ I_2=\frac{\varphi_a}{R_2}=\varphi_aG_2,\qquad \varphi_b=R_4I_4,\ I_4=\frac{\varphi_b}{R_4}=\varphi_bG_4,$$

$$\varphi_a-\varphi_b=R_3I_3,\ I_3=\frac{(\varphi_a-\varphi_b)}{R_3}=(\varphi_a-\varphi_b)G_3,$$

$$\varphi_b=E_5-R_5I_5,\ I_5=\frac{(E_5-\varphi_b)}{R_5}=(E_5-\varphi_b)G_5.$$

Let's substitute these expressions into the equations for the nodes a,b

$$+I_1 - I_2 - I_3 = 0$$
$$+I_3 - I_4 + I_5 = 0 \text{ ,}$$

we get

$$\begin{cases}(G_1 + G_2 + G_3)\varphi_a - G_3\varphi_b = G_1 E_1 \\ -G_3\varphi_a + (G_3 + G_4 + G_5)\varphi_b = G_5 E_5\end{cases} . \qquad (6)$$

Let's mark:

$G_{11} = G_1 + G_2 + G_3$, $G_{22} = G_3 + G_4 + G_5$ - the individual conductivities of the nodes, it's the sum of the branch conductivities which coming in the node;

$G_{12} = G_{21} = G_3$ - the mutual conductivities of the nodes, the conductivity of the branch, which connects respective nodes;

$J_a = G_1 E_1$, $J_b = G_5 E_5$ - the algebraic sum of the currents of current sources, which are flowing in the respective nodes. If the current J of the source flows in the node, we mark it by the sign "+", when it flows out – with sign "-".

Using these markings, system (6) looks like (7), that can be used for any circuit with two independent nodes:

$$\begin{cases}G_{11}\varphi_a - G_{12}\varphi_b = J_a \\ -G_{12}\varphi_a + G_{22}\varphi_b = J_b\end{cases} . \qquad (7)$$

Two nodes method

This method is used for calculating the circuits with only two nodes and several parallel branches. The example of such circuit is on fig.1.20. This method is also based on the Current law and is partly the method of nodal potentials. First of all, we calculate the inter-node voltage $V = \Sigma G_n E_n / \Sigma G_n$, where G_n - conductivity of *n* branch, E_n - e.m.f.of *n* branch. For the circuit on fig. 1.20 it'll be

$$V_{ab} = \frac{G_1 E_1 + G_2 E_2}{G_1 + G_2 + G_3} .$$

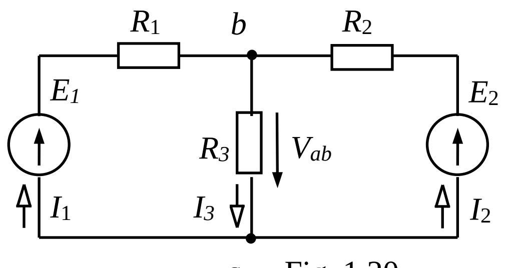


Fig. 1.20

Then we calculate the branch currents using such expressions:

$$V_{ab} = R_3 I_3, \; I_3 = \frac{V_{ab}}{R_3} = V_{ab} G_3 ,$$

$$V_{ab} = E_1 - R_1 I_1, \quad I_1 = \frac{(E_1 - V_{ab})}{R_1} = (E_1 - V_{ab})G_1 ,$$

$$V_{ab} = E_2 - R_2 I_2, \quad I_2 = \frac{(E_2 - V_{ab})}{R_2} = (E_2 - V_{ab})G_2 .$$

The superposition method

We can use this method when the e.m.f. of one source is changed. The method based on the superposition principle, means that every e.m.f. acts in the circuit independently. So, the calculation of one circuit (fig. 1.20) with two sources, for example, can be reduced to the calculation of two circuits with one source (fig. 1.21, 1.22).

According to this method, we must calculate two partial circuits with partial currents. We have only e.m.f. E_1 in the first partial circuit (fig. 1.21).

The total resistance of this circuit: $R' = R_1 + \dfrac{R_2 R_3}{R_2 + R_3}$.

The partial branches currents: $I_1' = E_1 / R'$, $I_2' = I_1' \dfrac{R_3}{R_2 + R_3}$, $I_3' = I_1' \dfrac{R_2}{R_2 + R_3}$.

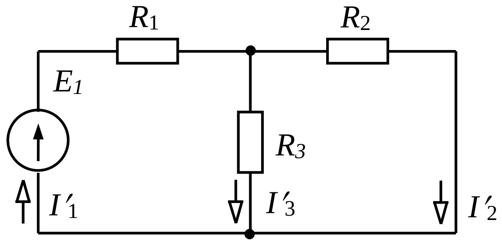


Fig. 1.21

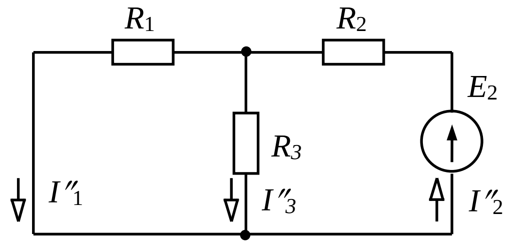


Fig. 1.22

We have only e.m.f. E_2 in the second partial circuit (fig. 1.22).

The total resistance of this circuit: $R'' = R_2 + \dfrac{R_1 \cdot R_3}{R_1 + R_3}$.

The partial branches currents: $I_2'' = E_2 / R''$, $I_1'' = I_2'' \dfrac{R_3}{R_2 + R_3}$, $I_3'' = I_2'' \dfrac{R_1}{R_1 + R_3}$.

Then we have the real branch currents as an algebraic sum of the respective partial currents (fig. 1.20):

$$I_1 = I_1' - I_1'',\ I_2 = I_2'' - I_2',\ I_3 = I_3'' + I_3'.$$

Equivalent generator method

The method is used when it is necessary to calculate the current of only one branch of the circuit (for example it is varying resistor or non-linear element in this branch). We select the branch with unknown current (e.g. I_3) from the circuit on fig. 1.20 and the rest of the circuit is replaced by the equivalent generator (fig. 1.23) with parameters E_{eqv} - equivalent e.m.f., which is equal to the open circuit voltage on

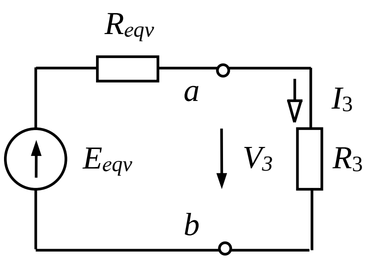


Fig. 1.23

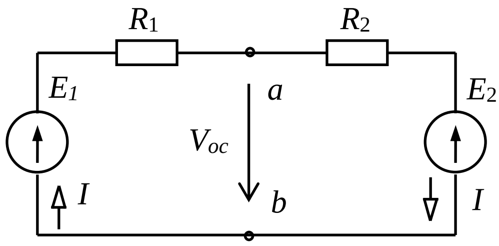


Fig. 1.24

the clamps of an open branch *ab* and R_{eqv} – equivalent resistance, which is equal to the input resistance of the circuit in respect to the open branch *ab*. The problem is to calculate the parameters of equivalent generator E_{eqv} and R_{eqv}. For the circuit at fig. 1.20 $R_{екв} = \dfrac{R_1 R_3}{R_1 + R_3}$. Then we can calculate E_{eqv} using fig. 1.24 $V_{OC} = E_{eqv} = E_1 - R_1 I$, where $I = \dfrac{E_1 - E_2}{R_1 + R_3}$.

According to the fig. 1.23 we calculate unknown current $I_3 = E_{eqv} / (R_{eqv} + R_2)$.

Direct currents non-linear circuits

Non-linear circuits consist of one or more non-linear elements. We call an element non-linear when its resistance is not constant and depends on voltage, current, temperature, light, etc. The volt-ampere characteristic (VAC) $V(I)$ is the main characteristic of non-linear element and it's non-linear (fig. 1.25).

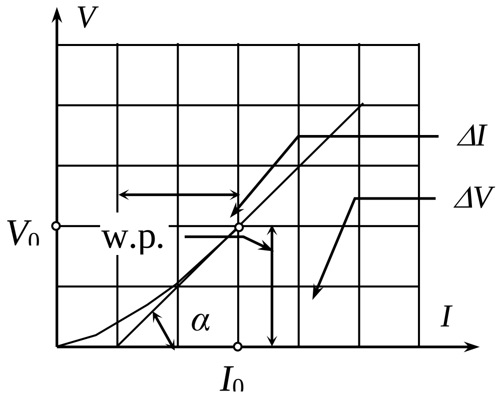


Fig. 1.25

There are non-controlled and controlled non-linear elements. Non-controlled elements have two clamps (lamps, diodes), controlled elements have three or more clamps (transistors, thyristors). VAC of non-linear elements may be symmetrical or non-symmetrical. If the resistance of the element doesn't depend on the direction of the current and the polarity of voltage then the characteristic is symmetrical. We can present VAC by graphs, tables or formulas $V(I)$.

Non-linear circuits can be calculated by analytical or graph methods. If we use graph method we define the voltage and current of the circuit using VACs of the elements. We can use Ohm's and Kirchhoff's laws as well. Analytical methods (two nodes method and equivalent generator method) can be used when the VAC is presented by a formula.

Non-linear element is characterized by static and dynamic resistance. We can calculate them for every point of VAC (at fig.1.25 for work point – w.p.):

$$R_S = V_0 / I_0, \ R_{ä} = \Delta V / \Delta I = dV / dI = tg\alpha,$$

α – the angle between axe X and tangent to working point (w.p.). $R_S > 0$, $R_{ä} > 0$ when VAC rise and $R_{ä} < 0$ when VAC drops.

PART 2. ALTERNATING CURRENT (AC)

Instantaneous value of AC is a value at every time moment, so it depends on the time: $i(t) = I_m \sin(\omega t + \psi_I)$. Instantaneous value of alternating voltage is $v(t) = V_m \sin(\omega t + \psi_V)$ (fig. 2.1).

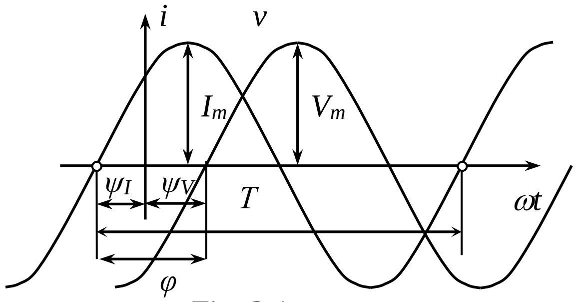


Fig. 2.1

AC is characterized by such *parameters*: I_m - amplitude, maximum value during the period, period T, cyclic frequency $f = 1/T$ (quantity of periods per second) (Hz), angular frequency $\omega = 2\pi f$ (rad/s), phase $\beta = (\omega t + \psi_I)$, initial phase ψ_I (phase shift from zero).

Phase shift angle is: $\varphi = \psi_V - \psi_I$ (fig. 2.1).

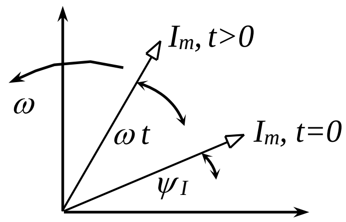


Fig. 2.2

Average current value per half of period is*:*

$$I_{AV} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\, dt\,,\ I_{AV} = 2I_m / \pi = 0.637\, I_m\,.$$

Effectiv e value of AC $i(t)$ (RMS – root-mean-square) is equal to such a value of DC I, which generates the same amount of energy per period $T = 2\pi$, as AC $i(t)$.

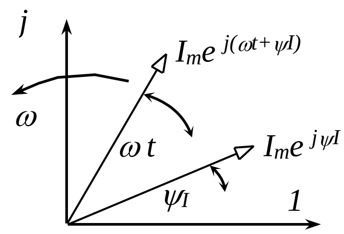


Fig. 2.3

Amount of energy per period of AC:

$$Q_{\approx} = \int_0^T Ri^2 dt = RI_m^2 T \cdot \int_0^T Ri^2 dt$$

Amount of energy per half a period of DC: $Q_{-} = RI^2 T$.

$Q_{\approx} = Q_{-}$, so $Q_{\approx} = \int_0^T Ri^2 dt = RI^2 T$ and RMS value will be equal: $I = \sqrt{\frac{1}{T}\int_0^T Ri^2 dt}$.

$Q_{\approx} = RI_m^2 T = Q_{-} = RI^2 T$, thus AC effective value is $I = I_m / \sqrt{2} = 0.707\, I_m$.

AC can be represented by the *time diagram* (fig. 2.1), vector (fig. 2.2) and complex number.

When AC $i = I_m \sin(\omega t + \psi_I)$ is represented by *vector*, the length of this vector is proportional to the amplitude I_m, and angle between this vector and axis X is $\beta = \omega t + \psi_I$. The positive rotation direction will be counterclockwise. In that case, the vectors of current and voltage will be rotating with the same angular frequency ω counterclockwise. It is convenient to fix them at the time moment $t = 0$ (fig. 2.2), in that case the angle $\beta = \psi_I$ (initial phase).

Vector diagram consists of several vectors of currents and voltages, which represent real sinusoidal currents and voltages starting from the same point. It's better to build a vector diagram for the effective values of the currents and voltages $I = I_m / \sqrt{2}$, $V = V_m / \sqrt{2}$. One of the vectors is chosen as a basic one, it is the vector of current when the connection is in serial one and the vector of voltage when the connection is in parallel.

The AC can also be designated by the *complex number* $i(t) = I_m \sin(\omega t + \psi_I) \div I_m e^{j(\omega t + \psi i)} = I_m e^{j\psi i} e^{j\omega t} = \underline{I}_m e^{j\omega t}$ (it's an exponential form of complex number). AC on the complex surface is shown on fig. 2.3, where "+1" is a real axis and "j" is an imaginary axis. $\underline{I}_m = I_m e^{j\psi i}$ is then called an amplitude complex, $\underline{I} = (I_m / \sqrt{2}) e^{j\psi i}$ is accordingly an effective complex that corresponds to the instantaneous current (at the moment $t = 0$) and doesn't depend on the time (fig. 2.3). Amplitude complex doesn't contain the frequency but it is not so important because circuit's voltage and current have the same frequency.

Complex numbers

Complex number $\underline{c}$ has two presentation forms: algebraic $\underline{c} = a + jb$ (where a is a real part and b is an imaginary part) and exponential $\underline{c} = ce^{j\alpha}$ (where c is a module and α is an argument) (fig. 2.3a). One form can be converted into another by using the following expressions: $c = \sqrt{a^2 + b^2}$, $\alpha = arctg(b/a)$, $a = c\cos\alpha$, $b = c\sin\alpha$, j is a symbol for the imaginary part (also known as rotating operator – see below why). Thus $\underline{c} = ce^{j\alpha} = a + jb$. It's more convenient to use the algebraic form when adding complex numbers $(a_1 + jb_1) + (a_2 + jb_2) = (a_1 + a_2) + j(b_1 + b_2) = a + jb$, and exponential form for multiplication and division of the complex numbers: $c_1 e^{j\alpha_1} \cdot c_2 e^{j\alpha_2} = c_1 \cdot c_2 e^{j(\alpha_1 + \alpha_2)} = c e^{j\alpha}$, $\dfrac{c_1 e^{j\alpha_1}}{c_2 e^{j\alpha_2}} = \dfrac{c_1}{c_2} e^{j(\alpha_1 - \alpha_2)} = c e^{j\alpha}$. The number $\underline{c}^* = a - jb = c e^{-j\alpha}$ is called a *complex conjugate* to number $\underline{c} = a + jb = c e^{j\alpha}$.

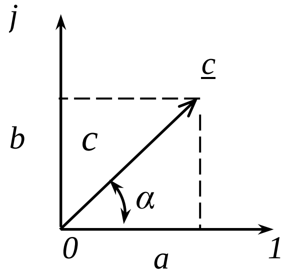


Fig. 2.3a

Consumers at AC circuit

Expressions for instantaneous current and voltage are correspondingly:

$$i = I_m \sin(\omega t + \psi_I),\ v = V_m \sin(\omega t + \psi_U).$$

The voltage for the *resistive element* (fig.2.4) (active resistance) is

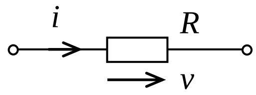


Fig. 2.4

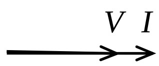


Fig. 2.5

$$v = V_m \sin(\omega t + \psi_v) = Ri = RI_m \sin(\omega t + \psi_i)$$

according to Ohm`s law, where $V_m = RI_m$, $V = RI$, phase expression $\psi_v = \psi_i$ and phase shift angle makes $\varphi = \psi_v - \psi_i = 0$. Resistance of this element is R (Ω) and conductance is thus $G = 1/R$ (Sm). Vector diagram for this element is shown on fig. 2.5. Active power of resistive element is accordingly $P = RI^2 = GV^2$ (W).

Inductance L (H) is correspondingly the main parameter for the ideal *inductive element* (fig. 2.6). The differential form of Ohm`s law is thus applied accordingly:

$$v_L = V_m \sin(\omega t + \psi_V) = L\frac{di_L}{dt} = L\frac{d}{dt} I_m \sin(\omega t + \psi_i) =$$
$$= \omega L I_m \cos(\omega t + \psi_i) = \omega L I_m \sin(\omega t + \psi_i + \pi/2),$$

where $V_m = \omega L I_m$, $V = X_L I$, reactance $X_L = \omega L$ (Ω), susceptance $B_L = 1/\omega L$ (Sm), phase expression $\psi_v = \psi_i + \pi/2$, phase shift angle makes $\varphi = \psi_v - \psi_i = \pi/2$, it means voltage leads current. In case of DC: $\omega = 0$, $X_L = 0$, $B_L = \infty$. Vector diagram for this element is shown on fig. 2.7. Reactive power for L element makes $Q_L = X_L I^2 = B_L V^2$ (VAr).

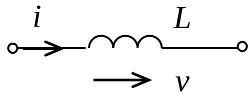


Fig. 2.6

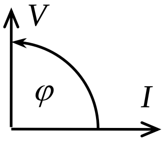


Fig. 2.7

Capacitance C is the main parameter for the ideal *capacitive element* (fig. 2.8). Integral form of Ohm`s law is applied in this case:

$$v_C = V_m \sin(\omega t + \psi_V) = \frac{1}{C}\int i\, dt = \frac{1}{C}\int I_m \sin(\omega t + \psi_i) = -\frac{1}{\omega C} I_m \cos(\omega t + \psi_i) =$$
$$= -\frac{1}{\omega C} I_m \cos(\omega t + \psi_i) = \frac{1}{\omega C} I_m \sin(\omega t + \psi_i - \pi/2),$$

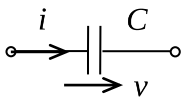


Fig. 2.8

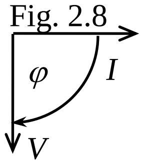


Fig. 2.9

where $V_m = \frac{1}{\omega C} I_m$, $V = X_C I$, reactance $X_C = 1/(\omega C)$ (Ω), susceptance $B_C = \omega C$ (Sm), phase expression $\psi_v = \psi_i - \pi/2$, phase shift angle makes $\varphi = \psi_v - \psi_i = -\pi/2$, it means voltage lags current. In case of DC: $\omega = 0$, $X_C = \infty$, $B_C = 0$. Vector diagram for this element is shown on fig. 2.9. Reactive power for this element makes thus $Q_C = X_C I^2 = B_C V^2$ (VAr).

The complex designation for current, voltage, derivative and integral functions are accordingly:

$$i \div \underline{I} = \underline{I}_m/\sqrt{2} = I\, e^{j\psi_i},\ v \div \underline{V} = \underline{V}_m/\sqrt{2} = V\, e^{j\psi_V},$$
$$d/dt \div j\omega,\ \int dt \div 1/(j\omega) = -j/\omega\ .$$

Complex form of Ohm`s law equation for R-element is thus:

$$v = Ri \div \underline{V} = R\underline{I},\ Ve^{j\psi_V} = RI\, e^{j\psi_i},$$
$$R = V\, e^{j\psi_V} / I\, e^{j\psi_i} = (V/I)\, e^{j(\psi_V - \psi_i)} = \mathrm{Re}^{j\varphi}, \varphi = 0.$$

Complex form of Ohm law equation for L-element is accordingly:

$$v = Ldi/dt \div \underline{V} = j\omega L\underline{I},\ V\, e^{j\psi_V} = j\omega LI\, e^{j\psi_i},$$

complex reactance is

$$j\omega L = Ve^{j\psi_V} / I\, e^{j\psi_i} = (V/I)e^{j(\psi_V - \psi_i)} = X_L e^{j\varphi},\ \varphi = 90^\circ,$$

complex susceptance is $B_L e^{-j\varphi} = -j/\omega L$. Multiplication by j means counterclockwise rotation for $\varphi = 90^\circ$. That's why j is called a rotation operator.

Complex form of Ohm`s law equation for *C*-element is accordingly:

$$v = \int idt/C \div \underline{V} = -j(1/\omega C)\underline{I},\ V e^{j\psi_V} = -j(1/\omega C) I e^{j\psi_i},$$

complex reactance is

$$-j(1/\omega C) = V e^{j\psi_V} / I e^{j\psi_i} = (V/I) e^{j(\psi_V - \psi_i)} = X_C e^{j\varphi},\ \varphi = -90^\circ,$$

complex susceptance is $B_C e^{-j\varphi} = j\omega C$. Multiplication by $-j$ means clockwise rotation for $\varphi = -90^\circ$.

Serial connection of consumers at AC circuit

Electrical status equations for the circuit (fig. 2.10) for voltage instantaneous values and voltage vectors are accordingly:

$$v_R + v_L + v_C = v,\ \overline{V}_R + \overline{V}_L + \overline{V}_C = \overline{V}.$$

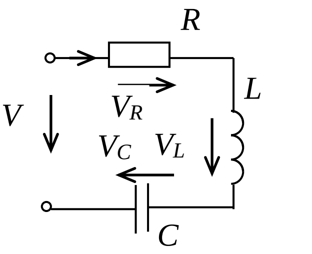


Fig. 2.10

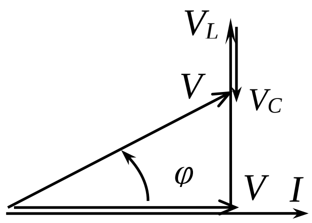


Fig. 2.11

Vector diagram is shown on fig 2.11. The calculated triangles for voltages, resistances and powers (fig. 2.12) are obtained from this diagram. Out of those triangles:

$$V = \sqrt{V_R^2 + (V_L - V_C)^2},\quad \varphi = arctg\,(V_L - V_C)/V_R,$$

$$V_R = V\cos\varphi = V_a,\ V_L - V_C = V\sin\varphi = V_r,$$

-these are active and reactive constituents of the applied voltage V. Therefore circuit impedance makes:

$$Z = \sqrt{R^2 + (X_L - X_C)^2},\quad \varphi = arctg\,((X_L - X_C)/R),$$

$$R = Z\cos\varphi,\ X = X_L - X_C = Z\sin\varphi\ -$$

these are resistance and reactance of the circuit.

Total power makes thus:

$$S = \sqrt{P^2 + (Q_L - Q_C)^2}\ \text{(VA)},\ \varphi = arctg\,(Q_L - Q_C)/P,$$

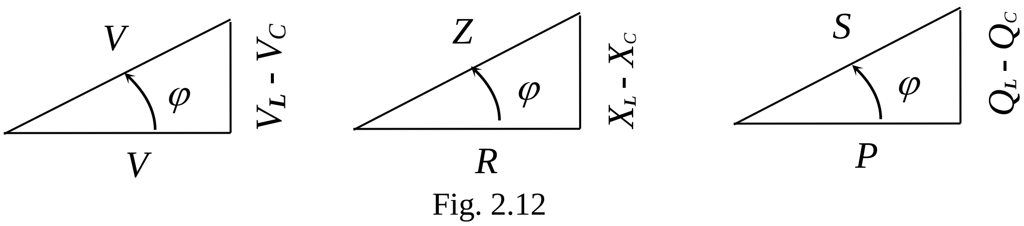


Fig. 2.12

$$P = S\cos\varphi = VI\cos\varphi,\ Q = Q_L - Q_C = S\sin\varphi = VI\sin\varphi$$

- these are active and reactive powers of the circuit.

Circuit complex form electrical status equation is:

$$R\underline{I} + j\omega L\underline{I} - j/(\omega C)\underline{I} = \underline{V}.$$

Complex impedance makes thus: $\underline{Z} = R + j\omega L - j/(\omega C) = R + j(X_L - X_C)$.

Expression for Ohm`s law is accordingly:

$$\underline{Z} = \underline{V}/\underline{I} = \frac{Ve^{j\psi_V}}{I e^{j\psi_I}} = \frac{V}{I} e^{j(\psi_V - \psi_I)} = Ze^{j\varphi} = Z\cos\varphi - jZ\sin\varphi = R + jX.$$

Parallel connection of consumers at AC circuit

Circuit electrical status equations (fig. 2.13) for current instantaneous values and current vectors are accordingly:

$$i_R + i_L + i_C = i,\ \bar{I}_R + \bar{I}_C + \bar{I}_L = \bar{I}.$$

Vector diagram is shown on fig. 2.14. The calculated triangles of currents and conductivities are obtained from this diagram (fig. 2.15). From those triangles we get subsequently:

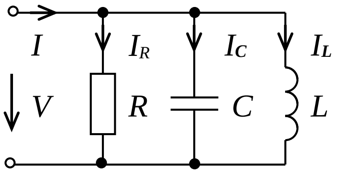


Fig. 2.13

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}, \qquad \varphi = arctg((I_C - I_L)/I_R),$$

$I_R = I\cos\varphi = I_a$, $I_C - I_L = I\sin\varphi = I_r$ - these are active and reactive constituents of the current.

Circuit admittance makes:

$$Y = \sqrt{G^2 + (B_C - B_L)^2}, \qquad \varphi = arctg\,\varphi(B_C - B_L)/G,$$

$$G = Y\cos\varphi,\ B = B_C - B_L = Y\sin\varphi$$

- these are conductance and susceptance of the circuit.

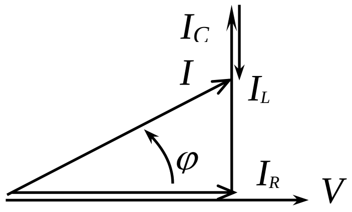


Fig. 2.14

Electrical status equation for the circuit in complex form is accordingly: $G\underline{V} + j\omega C\underline{V} - j/(\omega L)\underline{V} = \underline{I}$.

Complex admittance makes: $\underline{Y} = G + j\omega C - j/(\omega L) = G + j(B_C - B_L)$.

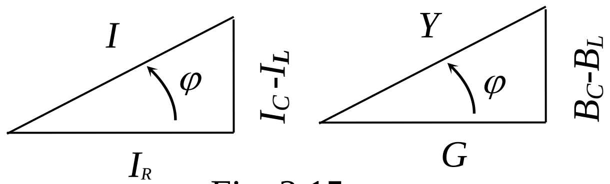


Fig. 2.15

Expression for Ohm`s law is thus: $\underline{Y} = \underline{I}/\underline{V} = \dfrac{Ie^{j\psi_V}}{Ve^{j\psi_I}} = \dfrac{I}{V}e^{j(\psi_I - \psi_V)} =$

$Ye^{-j\varphi} = Y\cos\varphi - jY\sin\varphi = G + jB$.

Total complex power makes accordingly:

$$\underline{S} = \underline{V}\,\underline{I}^* = Ve^{j\psi_V} \cdot Ie^{-j\psi_I} = VIe^{j(\psi_V - \psi_i)} = Se^{j\varphi} =$$
$$= S\cos\varphi + jS\sin\varphi = P + jQ,$$

where real part of complex number $P = S\cos\varphi$ – is an active power, imaginary part of complex number $Q = S\sin\varphi$ – is a reactive power.

To check the calculation of the circuit you may use *power balance equations*: the active power of the source must be equal to the active powers of the consumers:

$$P_{ps} = \sum P_{cons},$$

$$P_{ps} = VI\cos\varphi,\ \sum P_{cons} = R_1 I_1^2 + R_2 I_2^2 + ... = \sum R_n I_n^2,$$

the reactive power of the source must be equal to the reactive powers of the consumers: $Q_{ps} = \sum Q_{cons}$,

$$Q_{ps} = VI\sin\varphi,\ \sum Q_{cons} = X_1 I_1^2 + X_2 I_2^2 + ... = \sum X_n I_n^2,$$

where I_n – is an effective value of the branch n-th current, R_n– resistance of the n-th branch, $X_n = X_{Ln} - X_{Cn}$ – reactance of the n-th branch.

The transformation formulas must be used to calculate the alternating current

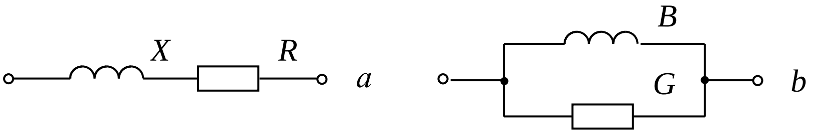


Fig. 2.16

circuits. The admittance is inversely to impedance:

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = G - jB\,.$$

So, the following formulas must be used to transform serial connection into parallel (fig. 2.16):

$$G = \frac{R}{(R^2 + X^2)},\ \ B = -\frac{X}{(R^2 + X^2)}.$$

It is obtained from the calculation triangles of resistances (Fig. 2.17a) and conductivities (Fig. 2.17b):

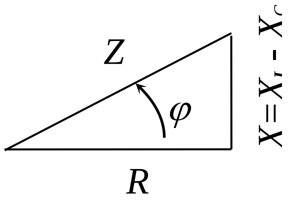


Fig.2.17a

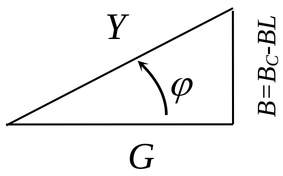


Fig.2.17b

$$\cos\varphi = \frac{R}{Z} = \frac{G}{Y}\,,\ \sin\varphi = \frac{X}{Z} = \frac{B}{Y}\,,$$

Thus the following formulas must be used to transform parallel connection into serial (fig. 2.18):

$$R = \frac{GZ}{Y} = \frac{G}{Y^2} = \frac{G}{G^2 + B^2}\ \ ,\ X = \frac{BZ}{Y} = \frac{B}{Y^2} = \frac{B}{G^2 + B^2}\,.$$

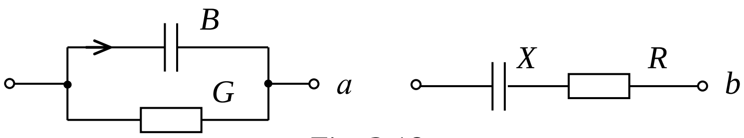


Fig. 2.18

The real coil can be represented by serial and parallel substitution schemes (fig. 2.16). The elements of this scheme: L - is an ideal inductance $X = \omega L,\ B = 1/(\omega L)$, R (G) – represents power losses. The coil quality can be estimated by Q - *factor*:

$$d = Q / P = X_L / R = tg\,\varphi\,.$$

The real capacitor can be represented by serial and parallel substitution schemes (fig. 2.18). The elements of this scheme: C - is an ideal capacitance $B = \omega C,\ X = 1/(\omega C)$ R (G) – represents power losses.

The capacitor quality can be estimated by *loss tangent*:

$$tg\delta = P / Q = R / X_C,\ \delta = 90^\circ - \varphi\,.$$

Power factor determines the efficiency of using electrical energy:

$$\cos\varphi = P/S = P/\sqrt{P^2+Q^2}\ ,$$

P - is an active useful power, Q - reactive, non-useful power (for electromagnetic field creation).

In ideal case power factor depends on the loading character

$$\cos\varphi = R/Z = R/\sqrt{R^2+X^2}\ .$$

$\cos\varphi = 1$ must be provided to avoid the work of electrical devices at idle mode. Most of the devices consume the active-inductive power (P and Q_L). Capacitors C must be connected in parallel to such devices to enhance $\cos\varphi$, thus $Q_L = Q_C$. Reactive power, which is non-useful power $Q_L - Q_C = Q = 0$, so $\cos\varphi = 1$ is maximum.

Voltage resonance

Voltage resonance take place at the circuit with serial connection of *L, C* elements (fig. 2.10). $V_L = V_C$ at resonance mode, so the condition of voltage resonance is $X_L = X_C$, it means

$$\omega_0 L = 1/(\omega_0 C)\,,$$

Thus $\omega_0^2 LC = 1$ and, resonance frequency

$$\omega_0 = 1/\sqrt{LC}\ .$$

Resonance can be reached by changing C, L or ω_0.

$\rho = \omega_0 L = 1/(\omega_0 C)$, $\rho = \sqrt{L/C}$ is called wave resistance.

At resonance mode

$$X = X_L - X_C = 0\,,\ Z = \sqrt{R^2+X^2} = R\,,$$

$$Q_L - Q_C = Q = 0\,,\ S = \sqrt{P^2+Q^2} = P\,,\ \varphi = 0\,.$$

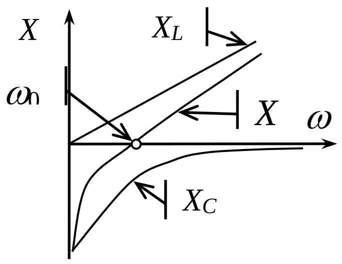


Fig. 2.19

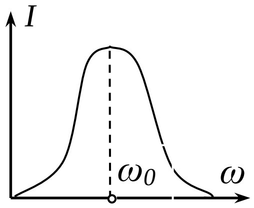


Fig. 2.20

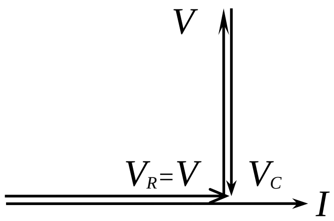


Fig. 2.21

Total current $I = V/Z = V/R$ is at maximum, what is an indication of the voltage resonance. Frequency characteristics of the circuit $X_L(\omega) = \omega L$, $X_C(\omega) = 1/(\omega C)$, $X(\omega) = X_L(\omega) - X_C(\omega)$ are shown at fig. 2.19. When $\omega < \omega_0$, $X < 0$, $\varphi < 0$, reactance has inductive character. When $\omega > \omega_0$ $X > 0$, $\varphi > 0$ reactance has capacitive character.

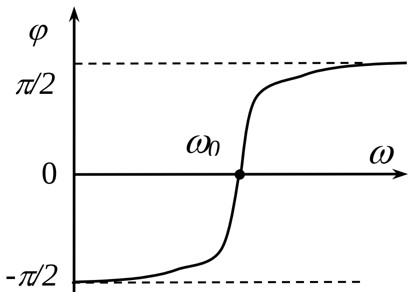


Fig. 2.22

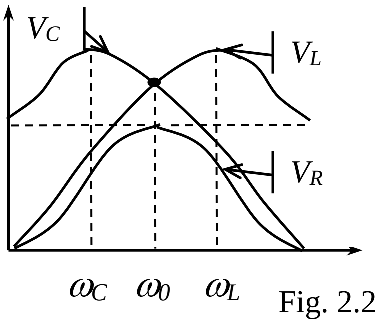


Fig. 2.23

At fig. 2.20 resonance curve $I(\omega)$ and at fig. 2.21 vector diagram for resonance mode are shown.

Phase-frequency characteristic $\varphi(\omega) = arctg\,\dfrac{\omega L - 1/(\omega C)}{R}$ is shown at fig. 2.22 and the resonance curves of voltages $V_R(\omega)$, $V_L(\omega)$, $V_C(\omega)$ at fig. 2.23 accordingly.

Voltage resonance should be avoided, because the voltage across the elements may several times exceed the nominal value.

Current resonance

Current resonance takes place at the circuit with parallel connection of *L*, *C* elements (fig. 2.24). $I_L = I_C$ at resonance mode, so the condition of voltage resonance for real circuit is $B_L = B_C$, that means

$$\omega_0 L/(R^2 + (\omega_0 L)^2) = 1/(\omega_0 C).$$

For ideal circuit ($R = 0$) the condition is $\omega_0 L = 1/(\omega_0 C)$.

Thus $\omega_0^2 LC = 1$, resonance frequency $\omega_0 = 1/\sqrt{LC}$.

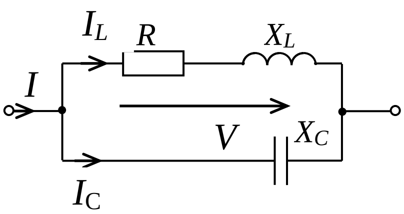


Fig. 2.24

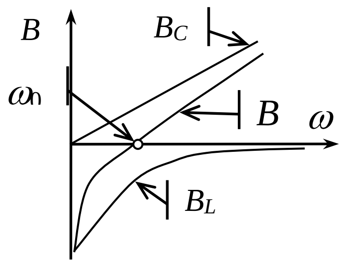


Fig. 2.25

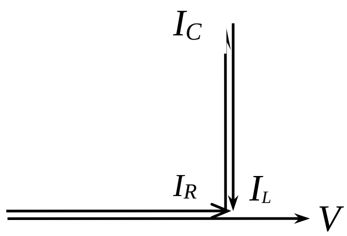


Fig. 2.26

The resonance can be reached by changing C , L or ω_0.

At resonance mode

$$B = B_C - B_L = 0,\ Y = \sqrt{G^2 + B^2} = G\ ,$$

$$Q_L - Q_C = Q = 0\ ,\ S = \sqrt{P^2 + Q^2} = P\ ,\ \varphi = 0\ .$$

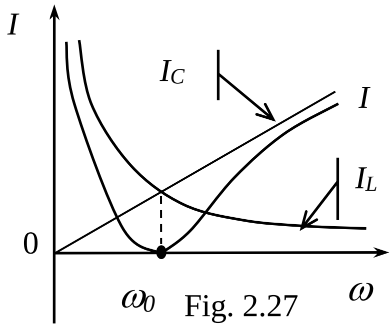


Fig. 2.27

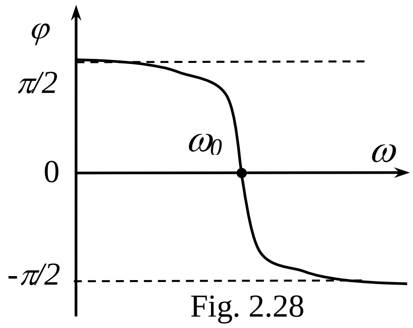


Fig. 2.28

Total current $I = VY = VG$ is at minimum, what is the indication of the current resonance.

Frequency characteristics of the ideal (*R=0*) circuit $B_L(\omega) = 1/(\omega L)$, $B_C(\omega) = \omega C$, $B(\omega) = B_C(\omega) - B_L(\omega)$ are shown at fig. 2.25. Susceptance has an inductive character when $\omega < \omega_0$, $B < 0$, $\varphi < 0$. Susceptance has a capacitive character, when $\omega > \omega_0$ $B > 0$, $\varphi > 0$.

Vector diagram for resonance mode is shown at fig. 2.26. Resonance curves $I(\omega)$, $I_L(\omega) = B_L V$, $I_C(\omega) = B_C V$ and phase-frequency characteristic $\varphi(\omega)$ are shown at fig. 2.27 and 2.28.

Voltage resonance on one hand should be avoided, because the current across the elements may several times exceed the nominal current, but on the other hand the resonance can be applied for rising power factor and as the working mode of some electronic devices.

PART 3. THREE-PHASE CIRCUITS

Three-phase electro-motive-force circuit system is the set of three sinusoidal e.m.f. with the same frequency ω and out of phase with each other by $2\pi/3$ (120°). *Phase* is the part of the circuit with the same current. The amplitudes of e.m.f. are marked accordingly: E_{Am}, E_{Bm}, E_{Cm}, if they are equal, such system is called *balanced*.

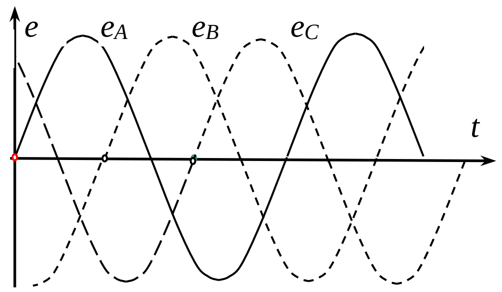


Fig. 3.1

The instantaneous values of e.m.f. (fig. 3.1) are:

$$e_A = E_{Am} \sin \omega t,\ e_B = E_{Bm} \sin(\omega t - 120^\circ),\ e_C = E_{Cm} \sin(\omega t + 120^\circ).$$

Phase sequence is the time order in which the e.m.f. pass through their respective maximum values (or through zero value). Phase sequence *ABC* is called positive (fig. 3.1), the reverse phase sequence *ACB* be called negative.

The following requirements are met for three-phase balanced electro-motive force system:

$$E_A = E_B = E_C = E_{ph}.$$

The following expressions are true having disregarded losses at power sources:

$$E_A = V_A, \qquad E_B = V_B, \qquad E_C = V_C,$$

where V_A, V_B, V_C – are *source phase voltages* (between the lines and neutral point N (fig. 3.3). These voltages in complex form are presented as:

$$\underline{V}_A = V_A e^{j0}, \qquad \underline{V}_B = V_B\, e^{-j120^\circ}, \qquad \underline{V}_C = V_C\, e^{j120^\circ}.$$

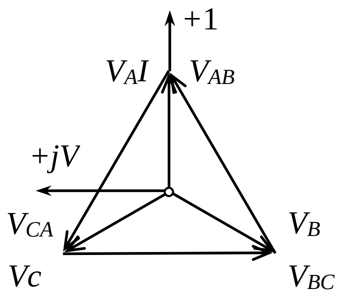


Fig. 3.2

Linear voltages (between lines, which connect the sources and the consumers) (fig. 3.4) in complex form are:

$$\underline{V}_{AB} = \underline{V}_A - \underline{V}_B = V_{AB} e^{j30^\circ},$$

$$\underline{V}_{BC} = \underline{V}_B - \underline{V}_C = V_{BC}\, e^{-j90^\circ}$$

$$\underline{V}_{CA} = \underline{V}_C - \underline{V}_A = V_{CA}\, e^{j150^\circ}.$$

Linear voltage is equal to the difference between corresponding phase voltages and lead the phase of the first one for 30° (fig. 3.2). Vector diagram (fig. 3.2) illustrates relationship between phase and linear voltages.

Three-phase circuit consists of three-phase electro-motive force system, three-phase loads and connection wires.

The most common types of connection the three-phase sources and consumers are WYE (Y) (fig. 3.3) and DELTA (Δ) (fig. 3.7).

At WYE connection the ends of source phases windings (fig. 3.3) are connected in common neutral point N, and the beginnings of phases A, B, C are connected to the linear wires. The ends of consumer phase windings (fig. 3.3) are connected in common neutral point n, and the beginnings of phases a, b, c are connected to the linear wires.

The source phase voltages are called the voltages between phase and neutral

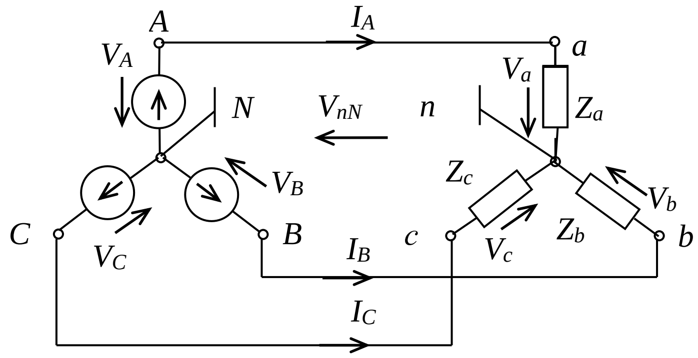


Fig. 3.3

points V_A, V_B, V_C, for consumer V_a, V_b, V_c. The source linear voltages are called the voltages between phase points (fig. 3.4) V_{AB}, V_{BC}, V_{CA}, for consumer V_{ab}, V_{bc}, V_{ca}. The directions of these voltages are shown at fig. 3.4. The effective values of phase and linear voltages are related according to the expression $V_L = \sqrt{3} V_{ph}$.

For WYE connection (fig. 3.4) *phase currents* (flowing through the phase) I_{ph}

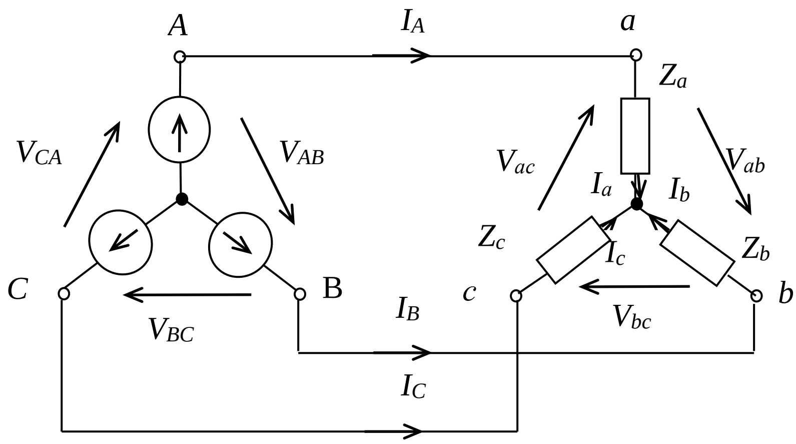


Fig. 3.4

(I_a, I_b, I_c), are equal to the *linear currents* (flowing through the lines connecting the source and the consumer) I_L (I_A, I_B, I_C), $I_{ph} = I_L$. The directions of these currents are shown at fig. 3.4. Balanced load is one in which the phase impedances are equal in magnitude and in phase:

$$\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = \underline{Z}_{ph}.$$

In this case:

$$V_a = V_A, \qquad V_b = V_B, \qquad V_c = V_C.$$

$$\underline{I}_A = \underline{V}_a / \underline{Z}_a, \qquad \underline{I}_B = \underline{V}_b / \underline{Z}_b, \qquad \underline{I}_C = \underline{V}_c / \underline{Z}_c.$$

The effective values of the currents are also equal: $I_A = I_B = I_C = I_{ph} = I_L$.

If the load is unbalanced ($\underline{Z}_a \neq \underline{Z}_b \neq \underline{Z}_c$) the voltage between the neutral points of source and consumer appears – $\underline{V}_{nN}$ (fig. 3.3). This voltage is called the *bias*

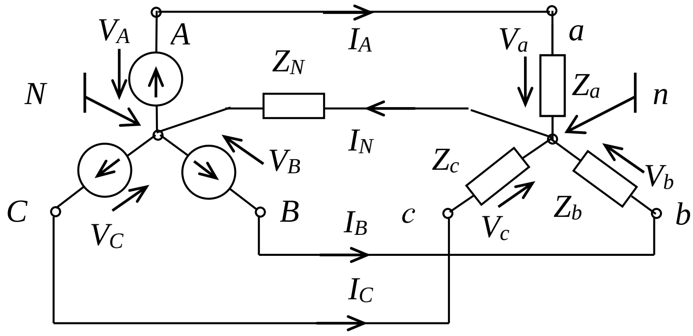


Fig. 3.5

neutral and can be calculated by using the method of two nodes:

$$\underline{V}_{nN} = \frac{\underline{V}_A \underline{Y}_a + \underline{V}_B \underline{Y}_b + \underline{V}_C \underline{Y}_c}{\underline{Y}_a + \underline{Y}_b + \underline{Y}_C},$$

where $\underline{Y}_a = 1/\underline{Z}_a = I_a / V_a$, $\quad \underline{Y}_b = 1/\underline{Z}_b = I_b / V_b$, $\quad \underline{Y}_c = 1/\underline{Z}_c = I_c / V_c$.

In that case the consumer phase voltages are calculated according to the following expressions:

$$\underline{V}_a = \underline{V}_A - \underline{V}_{nN}, \quad \underline{V}_b = \underline{V}_B - \underline{V}_{nN}, \quad \underline{V}_c = \underline{V}_C - \underline{V}_{nN},$$

Phase currents complexes are:

$$\underline{I}_a = \underline{V}_a / \underline{Z}_a, \ \underline{I}_b = \underline{V}_b / \underline{Z}_b, \ \underline{I}_c = \underline{V}_c / \underline{Z}_c.$$

There is also a *neutral wire* at three-phase four-wires circuits, which connects neutral points of source N and consumer n (fig. 3.5). In this case $\underline{V}_{nN} = 0$.

The following is true according to the Kirchhoff`s first law for node n:

$$\underline{I}_A + \underline{I}_B + \underline{I}_C = \underline{I}_N.$$

When the load is balanced ($\underline{Z}_a = \underline{Z}_b = \underline{Z}_c$): $\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$, $I_N = 0$, $\underline{V}_{nN} = 0$.
The vector diagram of currents for unbalanced load is shown in fig.3.6.

At DELTA connection the end of one source (consumer) winding is connected to the beginning to the second source (consumer) winding (fig. 3.7). For this

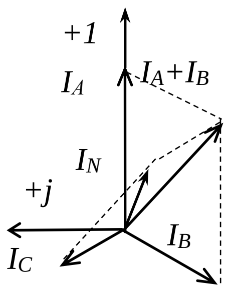


Fig. 3.6

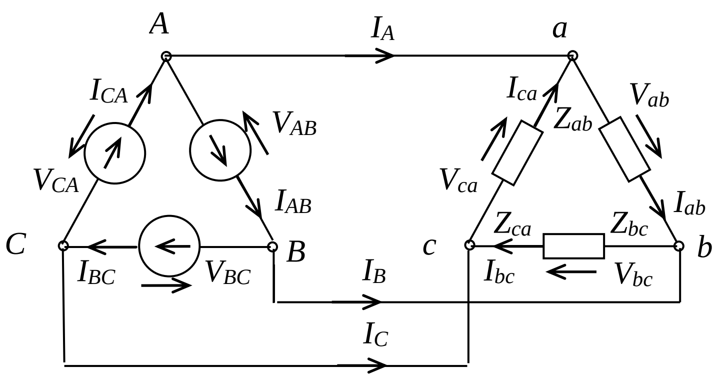


Fig. 3.7

connection the following is true: $V_{ph} = V_L$, $V_{AB} = V_{BC} = V_{CA} = V_L$.

The phase (linear) complex voltages can be represented:

$$\underline{V}_{AB} = V_{AB} e^{j0}, \quad \underline{V}_{BC} = V_{BC} e^{-j120^\circ}, \quad \underline{V}_{CA} = V_{CA} e^{j120^\circ}.$$

The consumer linear (phase) voltages are equal to the source linear voltages:

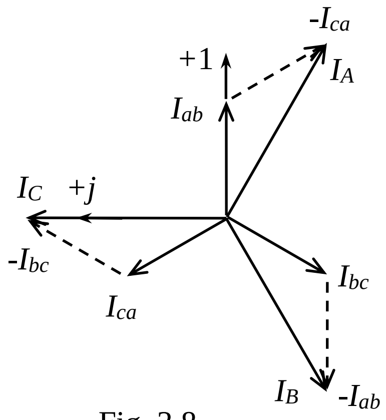


Fig. 3.8

$\underline{V}_{ab} = \underline{V}_{AB}$, $\underline{V}_{bc} = \underline{V}_{BC}$, $\underline{V}_{ca} = \underline{V}_{CA}$.

If the phase load is active ($\varphi = 0$), the vectors of phase currents I_{ab}, I_{bc}, I_{ca} have the same directions as the vectors of corresponding phase voltages V_{AB}, V_{BC}, V_{CA}.

If the phase load is an active-inductive one ($\varphi > 0$), the phase current lags behind the corresponding phase voltage by an angle of $\varphi = arctg\,(X_{ph} / R_{ph})$.

If the phase load is an active-capacitive one ($\varphi < 0$), the phase current leads the corresponding phase voltage by an angle of $\varphi = arctg\,(X_{ph} / R_{ph})$.

The load is balanced when $\underline{Z}_{ab} = \underline{Z}_{bc} = \underline{Z}_{ca} = \underline{Z}_{ph}$ and unbalanced when $\underline{Z}_{ab} \neq \underline{Z}_{bc} \neq \underline{Z}_{ca}$.

The following is true for the nodes a,b,c (fig. 3.7) according to the first Kirchhoff law:

$$\underline{I}_A + \underline{I}_{ca} - \underline{I}_{ab} = 0, \qquad \underline{I}_B + \underline{I}_{ab} - \underline{I}_{bc} = 0, \qquad \underline{I}_C + \underline{I}_{bc} - \underline{I}_{ca} = 0,$$

Then:

$$\underline{I}_A = \underline{I}_{ab} - \underline{I}_{ca}, \qquad \underline{I}_B = \underline{I}_{bc} - \underline{I}_{ab}, \qquad \underline{I}_C = \underline{I}_{ca} - \underline{I}_{bc}.$$

The linear current is equal to the difference between corresponding phase currents and lags the first one for 30° (fig. 3.8). Vector diagram (fig. 3.8) illustrates relationship between phase and linear currents.

The effective values of the phase and the linear currents are connected by expression: $I_L = \sqrt{3}\, I_{ph}$.

Complex phase currents can be defined according to Ohm's law:

$$\underline{I}_{ab} = \underline{V}_{ab} / \underline{Z}_{ab}, \ \underline{I}_{bc} = \underline{V}_{bc} / \underline{Z}_{bc}, \ \underline{I}_{ca} = \underline{V}_{ca} / \underline{Z}_{ca}.$$

For balanced load: $I_A = I_B = I_C$, $I_{ab} = I_{bc} = I_{ca}$.

Complex total power of three-phase unbalanced circuit is:

$$\underline{S} = \underline{V}_A \underline{I}_A^* + \underline{V}_B \underline{I}_B^* + \underline{V}_C \underline{I}_C^* = P + jQ.$$

Active power of three-phase unbalanced circuit is:

$$P = V_A I_A \cos\varphi_A + V_B I_B \cos\varphi_B + V_C I_C \cos\varphi_C = P_A + P_B + P_C.$$

Reactive power of three-phase unbalanced circuit is:

$$Q = V_A I_A \sin\varphi_A + V_B I_B \sin\varphi_B + V_C I_C \sin\varphi_C = Q_A + Q_B + Q_C.$$

These formulas can be used for WYE or DELTA connections.

Active P, reactive Q and total S powers of the consumer can be calculated by using phase or linear voltages for balanced load:

$$P = P_a + P_b + P_c = 3P_{ph} = 3V_{ph} I_{ph} \cos\varphi_{ph} = \sqrt{3} V_L I_L \cos\varphi_L,$$

$$Q = Q_a + Q_b + Q_c = 3Q_{ph} = 3V_{ph} I_{ph} \sin\varphi_{ph} = \sqrt{3} V_L I_L \sin\varphi_L,$$

$$\cos\varphi_{ph} = R_{ph} / Z_{ph}, \qquad \sin\varphi_{ph} = X_{ph} / Z_{ph},.$$

$$S = S_a + S_b + S_c = 3S_{ph} = 3V_{ph} I_{ph}, \qquad S = \sqrt{P^2 + Q^2}.$$

The same formulas can be used for WYE and DELTA connection.

PART 4. THE NON-SINUSOIDAL CURRENT CIRCUITS

Non-sinusoidal voltages or currents are the ones which are changed with the time according to periodical non-sinusoidal law. The cause of non-sine currents (voltages) is the source of non- sinusoidal voltage or the non-linear element of the circuit.

Such circuits may be represented by the Fourier series as the sum of sinusoidal functions in order to get calculated:

$$v = V_0 + V_{m1}\sin(\omega t + \psi_{V1}) + V_{m2}\sin(2\omega t + \psi_{V2}) + ... + V_{mk}\sin(k\omega t + \psi_{Vk}) =$$

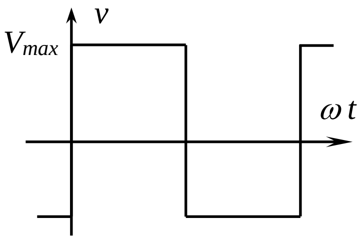


Fig. 4.1

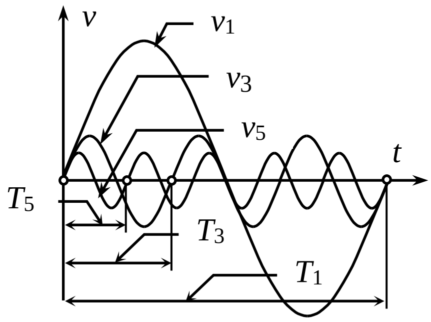


Fig. 4.2

$$= V_0 + \sum_{k=1}^{\infty} V_{mk}\sin(k\omega t + \psi_{Vk}),$$

where V_0 is the steady component; $v_1 = V_{m1}\sin(\omega t + \psi_{V1})$ is the first (basic) harmonic component, (ω - the frequency of first harmonic), $v_k = V_{mk}\sin(k\omega t + \psi_{Vk})$ - k harmonic component (called also as *harmonic*), V_{mk} - amplitude, ω - fundamental frequency, $k\omega$ - frequency of k harmonic, ψ_{Vk} - initial phase of k harmonic. The harmonics with the frequencies 2, 3,…k times larger than ω, are called higher harmonics.

We can represent the value $V_{mk}\sin(k\omega t + \psi_{Vk}) \div A_{mk}\sin(k\omega t + \varphi_k)$ by the sum of two constituents:

$$A_{mk}\sin(k\omega t + \varphi_k) = B_{mk}\sin k\omega t + C_{mk}\cos k\omega t,$$

where $B_{mk} = A_{mk}\cos\varphi_k$, $C_{mk} = A_{mk}\sin\varphi_k$, $A_{mk} = \sqrt{B_{mk}^2 + C_{mk}^2}$, $\varphi_k = arctg(C_{mk}/B_{mk})$.

So, the Fourier series we can write down ($v \div f(\omega t)$):

$$f(\omega t) = A_0 + \sum_{k=1}^{\infty} B_{mk}\sin k\omega t + \sum_{k=1}^{\infty} C_{mk}\cos k\omega t.$$

If the function is symmetrical across the X axis $f(\omega t) = -f(\omega t \pm \pi)$ then Fourier series have only odd harmonics:

$$f(\omega t) = A_{m1}\sin(\omega t + \varphi_1) + A_{m3}\sin(3\omega t + \varphi_3) + A_{m5}\sin(5\omega t + \varphi_5) + ... =$$

$$= B_{m1}\sin\omega t + C_{m1}\cos\omega t + B_{m3}\sin 3\omega t + C_{m3}\cos 3\omega t + B_{m5}\sin 5\omega t + C_{m5}\cos 5\omega t + ...$$

If the function is symmetrical across the origin $f(\omega t) = -f(-\omega t)$ then Fourier series have only sin constituents:

$$f(\omega t) = B_{m1}\sin\omega t + B_{m2}\sin 2\omega t + B_{m3}\sin 3\omega t + ...$$

If the function is symmetrical across Y axis $f(\omega t) = f(-\omega t)$ then Fourier series have only steady component and cos constituents:

$$f(\omega t) = A_0 + C_{m1}\cos\omega t + C_{m2}\cos 2\omega t + C_{m3}\cos 3\omega t + ...$$

Fourier series has only steady component and cos constituents:

$$f(\omega t) = A_0 + C_{m1}\cos\omega t + C_{m2}\cos 2\omega t + C_{m3}\cos 3\omega t + ...$$

For example, the square shape of voltage (fig.4.1) can be represented in such a way (fig. 4.2):

$$v = \frac{4V_{\max}}{\pi}(\sin\,\omega t + \frac{1}{3}\sin\,3\omega t + \frac{1}{5}\sin\,5\omega t),$$

Non-sinusoidal current $i = I_0 + \sum_{k=1}^{\infty} I_{mk}\sin(\,k\omega t + \psi_{Ik})$ (i.e. the sum of the sinusoidal currents) is present in the circuit with non-sinusoidal voltage $v = V_0 + \sum_{k=1}^{\infty} V_{mk}\sin(\,k\omega t + \psi_{Vk})$ (the sum of the sine voltages). The calculation of the circuit is based on the principle of superposition. The steady component of the current I_0 can be calculated by using the methods of DC circuits' calculation and harmonic of current i_k by using the methods of AC circuits' calculation.

As known reactance of the coil for *k*-harmonic is equal $X_{Lk} = k\,\omega L = kX_L$ and susceptance $B_{Lk} = 1/(k\,\omega L) = B_L / k$. Reactance of the coil for DC (as effect of the steady voltage component V_0) is $X_L(0) = 0 \cdot L = 0$. The susceptance of the capacitor for *k*-harmonic is $B_{Ck} = k\,\omega C = kB_C$ and reactance is $X_{Ck} = 1/(k\,\omega C) = X_C / k$. Reactance of the capacitor for DC (as effect of the steady voltage component V_0) is $X_C(0) = 1/(0 \cdot C) = \infty$, $I_0 = 0$. The resistance of the circuit doesn't actually depend on the frequency and is the same for every harmonic.

The non-sinusoidal circuit calculation order is:

– the source voltage is expressed by Fourier series as an infinite sum of harmonic (sinusoidal) components (functions);

– the circuit for every harmonic component is calculated separately using DC and AC circuits' calculation methods. Also it should be taken into consideration that the reactances depend on the frequency;

– according to the superposition principle, the current instantaneous value is equal to the sum of currents instantaneous values of all harmonics, that's why the calculation results are considered at each particular moment. The effective values of voltage and current are equal correspondingly:

$$V = \sqrt{V_0^2 + V_1^2 + ... + V_k^2}\,,\; I = \sqrt{I_0^2 + I_1^2 + ... + I_k^2}\,,$$

where V_k, I_k are harmonic voltages and currents effective values.

The *average value* of non-sinusoidal function $A_0\ (V_0,\ I_0)$ for the period:

$$A_0 = A_{AV} = \frac{1}{T}\int_0^T a\,dt\,,$$

The *effective value* of non-sinusoidal function - *A (V, I)* is the mean-square value for the period T :

$$A = \sqrt{\frac{1}{T}\int_0^T a^2 dt} = \sqrt{\sum_{k=o}^{n} A_k^2} = \sqrt{A_0^2 + ... + A_k^2}\,.$$

Shape factor is equal to the relation of function effective value to its average value: $K_{sh} = A / A_{AV}$. ($K_{sh} = 2/\pi = 1.11$ for sinusoidal curve).

Amplitude factor is equal to the relation of function amplitude value to its effective value: $K_a = A_m / A$. ($K_a = \sqrt{2} = 1.41$ for sinusoidal curve).

Distortion factor is equal the relation of first harmonic effective value to the function effective value:

$K_d = A_1 / A$ ($K_d = 1$ for sinusoidal curve).

Harmonic factor is equal the relation of high harmonics effective values to the first harmonic effective value: $K_g = A_g / A_1$, where $A_g = \sqrt{A_2^2 + ... + A_k^2} = \sum_{k=2}^{\infty} A_k^2$ is the mean-square value of high harmonics effective values ($K_g = 0$ for sinusoidal curve).

Active power of non- sinusoidal current is equal to the sum of harmonics active powers: $P = \sum_{k=o}^{n} V_k I_k = P_0 + P_1 + .. + P_k = P_0 + \Sigma P_k$, where $P_0 = V_0 I_0$ is the power of steady voltage component, $P_k = V_k I_k \cos\varphi_k$ is the active power of k harmonic, $\varphi_k = \psi_{Vk} - \psi_{Ik}$ - phase shift angle between k harmonic component of voltage and current. Reactive power of non- sinusoidal current is equal to the sum of harmonics reactive powers: $Q = \Sigma V_k I_k \sin\varphi_k = \Sigma Q_k$.

Total power of non- sinusoidal current is: $S = \sqrt{P^2 + Q^2}$.

PART 5. TRANSIENT PROCESSES

The transient processes occur when devices and circuits change their working regime. Transient processes may have negative effect in electrical engineering, but they can be useful in electronics.

The transient processes start at turning on/off the sources, changing the configuration of the scheme, circuit parameters, changing the current/voltage amplitude, phase, frequency or shape. Still the transient processes are typically caused by commutation (turning on/off the circuit).

The transient process is the process of transition from one energetic state of the circuit into another. This process cannot proceed stepwise, because the stock of energy can´t change abruptly. That’s because the elements' values upon which the energy storage depends (*L,C*) don't allow to change current and voltage stepwise (i_L, v_C). Two main laws of transient processes come out from this point.

The first law states that the current through inductance just after the commutation $i_L(0+)$ is equal to the current through inductance just before the commutation $i_L(0-)$: $i_L(0+) = i_L(0-) = i_L(0)$.

The second law states that the voltage at capacity just after the commutation $v_C(0+)$ is equal to the voltage at capacity just before the commutation $v_C(0-)$: $v_C(0+) = v_C(0-) = v_C(0)$.

Initial conditions (voltage or current values at the commutation moment $t = 0$) are defined by these laws. The steady-state mode before the commutation is at $t < 0$. The steady-state mode after the commutation is after the transient process is over.

The transient process duration depends on the elements parameters. It is estimated as $t_{tr} = 5 \div 6\tau$, where τ is the time constant. It is time during which voltage or current changes *e*=2.7 times of its initial value.

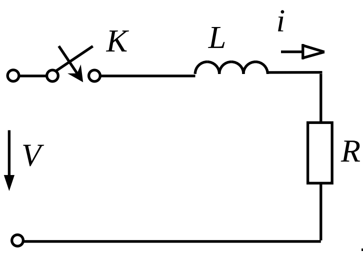


Fig. 5.1

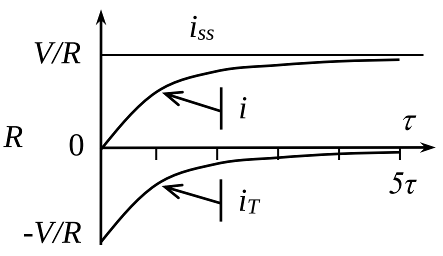


Fig. 5.2

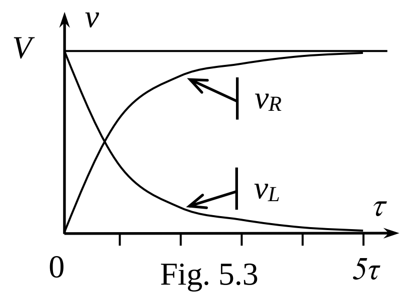


Fig. 5.3

The transient process can be described by linear differential equation, which is formed with the help of Kirchhoff’s laws. Commutation laws should be used to solve this equation.

The partial solution of inhomogeneous differential equation is the steady-state component i_{SS} or v_{SS}. The general solution of homogeneous differential equation is the transient component i_T or u_T, which dies out with time. The solution of linear differential equation is current (voltage), which is equal to the sum of transient and steady-state components $i(t) = i_T + i_{SS}$ ($v(t) = v_T + v_{SS}$). Therefore, to calculate transient process means to find the current or voltage changing rule.

Let's analyse the transient process when *RL* link is connected to DC source (fig. 5.1). According the differential equation to the Voltage lawfor after commutation steady-state mode is: $Ldi/dt + Ri = V$. Its solution is $i(t) = i_T + i_{SS}$. The partial solution i_{SS} of inhomogeneous differential equation $Ldi_{SS}/dt + Ri_{SS} = V$ is equal to the current value when transient process is over $i_{SS} = V/R$ (because $X_L = 0$ for DC).

i_T is the general solution of homogeneous differential equation $Ldi_T/dt + Ri_T = 0$. The characteristic equation corresponding to this differential one is $pL + R = 0$ with its root $p = -R/L$. The time constant is $\tau = 1/p = L/R$. Since the characteristic equation has one real root, the transient component is $i_T = Ae^{pt}$. Constant of integration can be found from initial conditions: $i(0) = i_T(0) + i_{SS}(0) = A + V/R$. According to the first commutation law $i(0) = i(0-) = 0$, so $A = -V/R$, $i_T = -V/R\,e^{-(R/L)t}$.

The solution of differential equation is (fig.5.2):

$$i = i_T + i_{SS} = -V/R\,e^{-(R/L)t} + V/R = V/R(1 - e^{-(R/L)t}).$$

The voltage on resistive element is (fig.5.3):

$$v_R = Ri = R(V/R)(1 - e^{-(R/L)t}) = V(1 - e^{-(R/L)t}).$$

The voltage on inductive element is (fig.5.3):

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left(\frac{V}{R}(1 - e^{-(R/L)t})\right) = L\frac{V}{R}(-\frac{R}{L})\cdot(-e^{-(R/L)t}) = Ve^{-(R/L)t}.$$

Let's analyse the transient process when *RL* link is disconnected from DC source and shortened (fig. 5.4). The differential equation according to the Voltage lawfor after commutation steady-state mode is: $Ldi/dt + Ri = 0$. Its solution is $i(t) = i_T + i_{SS}$. The partial solution i_{SS} of inhomogeneous differential equation $Ldi_{SS}/dt + Ri_{SS} = 0$ is equal to the current value when transient process is over $i_{SS} = 0$

i_T is the general solution of homogeneous differential equation $Ldi_T/dt + Ri_T = 0$. The characteristic equation corresponding to this differential one is $pL + R = 0$ with its root $p = -R/L$. The time constant is $\tau = 1/p = L/R$. Since the characteristic equation has one real root, the transient component is $i_T = Ae^{pt}$. Constant of integration can be found from initial conditions: $i(0) = i_T(0) + i_{SS}(0) = A$. According to the first commutation law $i(0) = i(0-) = V/R$, so $A = V/R$, $i_T = V/R\,e^{-(R/L)t}$.

The solution of differential equation is (fig. 5.6):

$$i = i_T = V/R\,e^{-(R/L)t}.$$

The voltage on resistive element is (fig. 5.5):

$$v_R = Ri = R(V / R)e^{-(R/L)t} = Ve^{-(R/L)t}.$$

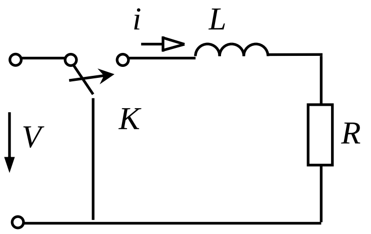


Fig. 5.4

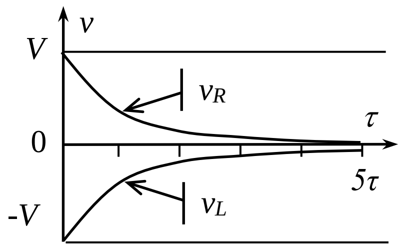


Fig. 5.5

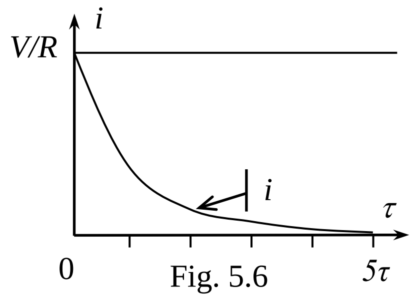


Fig. 5.6

The voltage on inductive element is (fig. 5.5):

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left(\frac{V}{R}e^{-(R/L)t}\right) = L\frac{V}{R}(-\frac{R}{L})\cdot(e^{-(R/L)t}) = -Ve^{-(R/L)t}.$$

Let's analyse the transient process when *RC* link is connected to DC source (fig. 5.7). The differential equation according to the Voltage lawfor after commutation steady-state mode is $Ri + v_C = V$, $i = C(dv_C / dt)$, then $RC(dv_C / dt) + v_C = V$. Its solution is $v_C = v_{CT} + u_{CSS}$. The partial solution v_{CSS} of inhomogeneous differential equation $RC(dv_C / dt) + v_C = V$ equals to the voltage value on *C* when the transient process is over. The circuit current equals zero in this case, because the input voltage is applied directly to capacitance $v_{CSS} = V$.

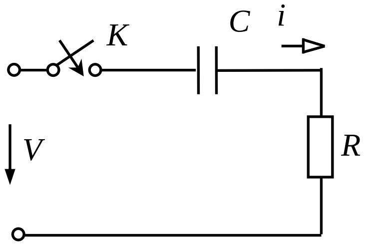


Fig. 5.7

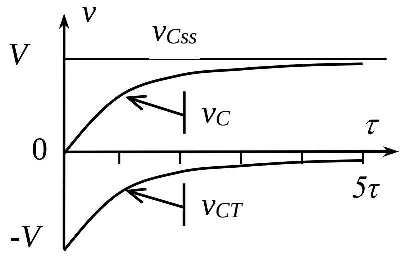


Fig. 5.8

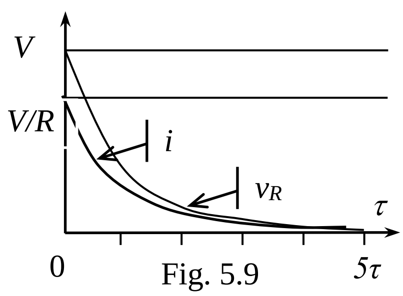


Fig. 5.9

v_{CT} is the general solution of homogeneous differential equation $RC(dv_C / dt) + v_C = 0$. The characteristic equation corresponding to this differential one is $RCp + 1 = 0$ with its root $p = -1/(RC)$. The time constant is $\tau = 1/p = RC$. Since the characteristic equation has one real root, the transient component is $v_{CT} = Ae^{pt}$. Constant of integration can be found from initial conditions: $v_C(0) = v_{CT}(0) + v_{CSS}(0) = A + V$. According to the first commutation law $v_C(0) = v_C(0-) = 0$, so $A = -V$, $v_{CT} = -Ve^{-t/RC}$.

The solution of differential equation is (fig. 5.8):

$$v_C(t) = V - Ve^{-t/RC} = V(1 - e^{-t/RC}) = V(1 - e^{-t/\tau}).$$

The current is (fig. 5.9):

$$i = C(dv_C / dt) = C\frac{d}{dt}\left(V - Ve^{-t/RC}\right) = -CV/(RC)(-e^{-t/RC}) = (V/R)e^{-t/RC}.$$

The voltage on resistive element is (fig. 5.9):

$$v_R = Ri = R(V/R)(e^{-t/RC}) = Ve^{-t/RC}.$$

Let's analyse the transient process when RC link is disconnected from DC source and shortened (fig. 5.10). The differential equation according to the Voltage lawfor after commutation steady-state mode is: $RC(dv_C / dt) + v_C = 0$. Its solution is $v_C = v_{CT} + v_{CSS}$. The partial solution v_{CSS} equals zero, because this equation is homogeneous.

v_{CT} is the general solution of homogeneous differential equation $RC(dv_C / dt) + v_C = 0$. The characteristic equation corresponding to this differential one is $RCp + 1 = 0$ with its root $p = -1/(RC)$. The time constant is $\tau = 1/p = RC$. Since the characteristic equation has one real root, the transient component is $v_{CT} = Ae^{pt}$. Constant of integration can be found from initial conditions: $v_C(0) = v_{CT}(0) + v_{CSS}(0) = A$. According to the first commutation law $v_C(0) = v_C(0-) = V$ (the circuit current is equal to zero in this case, because the input voltage is applied directly to the capacitance), $v_{CT} = Ve^{-t/RC}$.

The solution of differential equation is (fig. 5.11):

$$v(t) = v_{CT}(t) = Ve^{-t/RC}.$$

The current is (fig. 5.12):

$$i = C(dv_C / dt) = C\frac{d}{dt}\left(Ve^{-t/RC}\right) = CV/(RC)(-e^{-t/RC}) = -(V/R)e^{-t/RC}.$$

The voltage on resistive element is (fig. 5.12):

$$v_R = Ri = R(-V/R)(e^{-t/RC}) = -Ve^{-t/RC}.$$

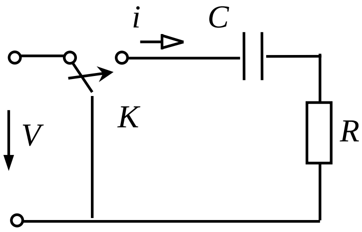


Fig. 5.10

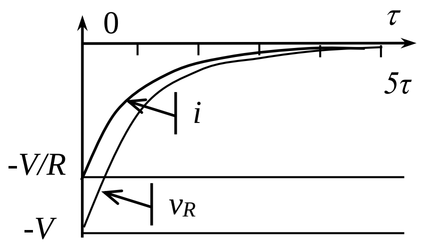


Fig. 5.11

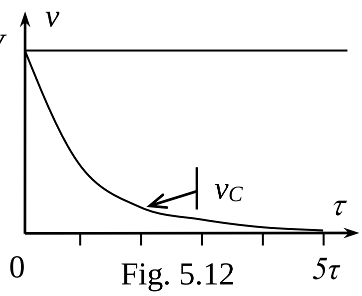


Fig. 5.12

PART 6. MAGNETIC CIRCUITS

The electromagnetic device consists of electric and magnetic circuits.

The electric circuit consists of winding with a current that excites the magnetic field with the tension H. The magnetic circuit in electrical devices is the way in which magnetic field lines are closed. The magnetic circuit has a desired configuration and is characterized by induction of the magnetic field *B.* Magnetic circuit is a combination of magnetic and non-magnetic areas, which close the magnetic flux.

The magnetic field in devices with constant magneto-motive force (m.m.f.) is created by a permanent magnet or a DC powered electromagnet.

Magnetic circuits don’t have air gaps in electrical converters like transformers, magnetic amplifiers, etc. However, air gap is required for electromechanical power converters like relays, contactors, solenoids, starter, electric cars, some measuring devices, etc.

The magnetic field is represented by magnetic field lines, which look like concentric circles. The direction of the lines is determined by the rule of the right screw.

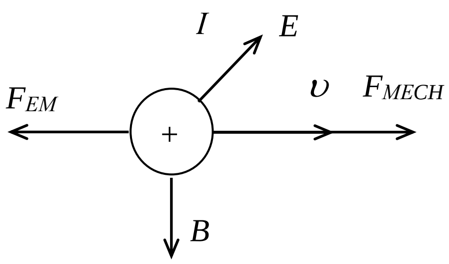


Fig. 6.1

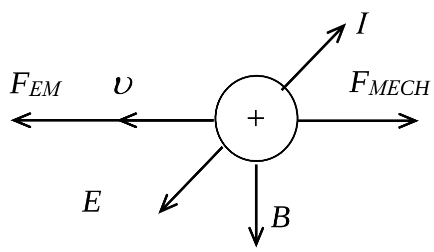


Fig. 6.2

According to the law of electromagnetic induction, a moving in a magnetic field conductor induces electromotive force $E = Bl\upsilon$, where l is the length of the conductor; v - speed of its movement and B - the magnetic field induction. The direction of the electromotive force is determined by the right hand rule (fig. 6.1).

By the electromagnetic force law (Ampere's law), the force acting on the current-carrying conductor, which moves in a magnetic field, makes $F = BlI$, where l is the conductor length. The direction of the electromagnetic force is determined by the left-hand rule (fig. 6.2). Induced in the conductor electromotive force E is directed to suppress the current I.

Magnetic induction *B (T)* describes the intensity of the magnetic field and the magnetic flux *Φ (Wb)* is an integral characteristic of the magnetic field. The magnetic field is considered to be uniform if the magnetic induction at all points of the field is the same ($B = const$). Magnetic flux of uniform magnetic field passing through the surface *S,* placed perpendicular to the lines of magnetic induction is $\Phi = BS$.

Total current law in integral form is: $\oint \overline{H}\, d\,\overline{l} = \int H \cos\alpha\, d\,l = \sum I$, which means that the circulation of magnetic field tension vector *H* along the closed path *l* is equal to the algebraic sum of currents encircled with this path. The magnetic circuit is homogeneous if the magnetic induction is the same along the magnetic circuit. Total current law for homogeneous and heterogeneous (with air gap) magnetic circuits is: $H\,l = w\,I$, $\quad H\,l + H_\delta \delta = w\,I$, where H is the magnetic field tension in the magnetic circuit with length *l*; H_δ - the magnetic field tension in the air gap δ; w - the number

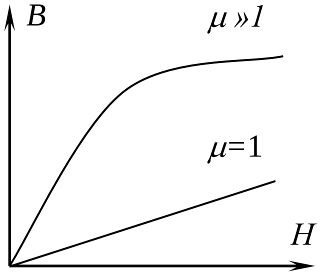


Fig. 6.3

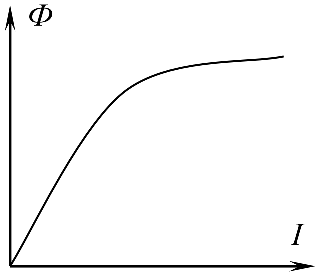


Fig. 6.4

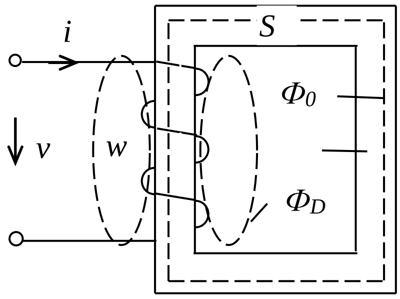


Fig. 6.5

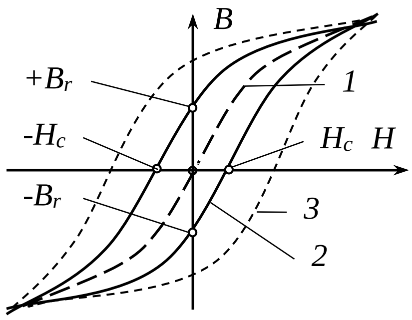
of winding turns and $wI = F$ - a magneto-motive force. The coil linkage is $\Psi = LI = w\Phi$.

Magnetic permeability μ describes the properties of the conducting medium, which shows how many times the medium increases the magnetic field of the coil. Diamagnetics and paramagnetics belong to nonmagnetic materials, where $\mu_r = 1$ and ferromagnetics belong to magnetic materials, where $\mu_r >> 1$. Magnetic induction *B* is connected with magnetic field tension H by the following equation: $B = \mu_0 \mu_r H$, where relative magnetic permeability μ_r, vacuum magnetic permeability $\mu_0 = 4\pi \cdot 10^{-7}$. So, the magnetic resistance is $R_m = l/(\mu_0 \mu_r S)$.

Magnetic flux will be much larger in a coil with the ferromagnetic core than in a coil without it, as the flux is created not only by the current but also by the ferromagnetic substance of magnetic circuit.

The magnetic characteristics $B(H)$ for magnetic ($\mu >> 1$) and nonmagnetic ($\mu = 1$) materials are shown in fig. 6.3. Given that the magnetic flux is proportional to the magnetic induction $\Phi \equiv B$, and the current proportional to the magnetic field tension $I \equiv H$, a dependence $\Phi(I)$- or so called weber-ampere characteristic can be obtained (fig. 6.4). As can be seen from it, an additional core of magnetic (ferromagnetic) material is required to reduce the current needed to generate a given magnetic flux of the coil.

The coil electromagnetic circuit is shown in fig. 6.5. Applied to the coil voltage *v* initiates current *i*, which results in a magnetic flux. Magnetic flux Φ is the vector sum of the main magnetic flux Φ_o, which is closed through the core, and the magnetic flux of dissipation Φ_D, which closes in the air around the coils $\underline{\Phi} = \underline{\Phi}_o + \underline{\Phi}_D$. Dissipation magnetic flux is not involved in energy transmission (in transformers). The core permeability ($\mu >> 1$) is much higher than air



Fig, 6.6

permeability ($\mu = 1$), that means $\Phi_o >> \Phi_D$, but the magnetic fluxes can shift the phase.

The basic magnetization curve (curve 1 in big dashed line on fig. 6.6) passes through the center of coordinate grid. When the coil is powered by alternating voltage the magnetic flux changes in time and core is cyclically re-magnetized. After several AC periods a closed symmetrical across the origin hysteresis loop is set (curve 2 in solid lines fig.6.6), which is called the static hysteresis loop. A residual induction in the core B_r is stored at $H = 0$. $H = H_C$ at $B = 0$ and it is called a coercive (holding) force.

The magnetic losses in the core ΔP_M consist of magnetic hysteresis losses ΔP_H and eddy current losses ΔP_E: $\Delta P_M = \Delta P_H + \Delta P_E$. Hysteresis losses are the losses due to the cyclical magnetization of the core and they are proportional to the area embraced by static hysteresis loop. These are determined by the formula: $\Delta P_H = \gamma_H \, \sigma \, f \, B_m^2$, where γ_H are specific power losses for hysteresis; σ - magnetic circuit mass; f - current frequency; B_m - magnetic induction amplitude. To reduce the magnetic hysteresis losses the magnetic circuits are made of soft magnetic (ferromagnetic) materials, which have a narrow hysteresis loop.

Eddy currents occur in solid metal parts as a result of the magnetic field.

Eddy currents losses occur when the coil is fed with AC, which demagnetize

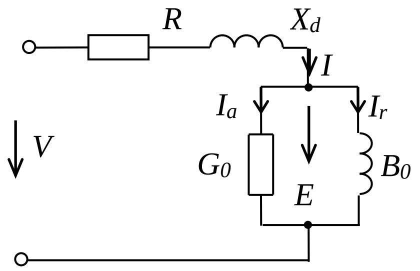


Fig. 6.7

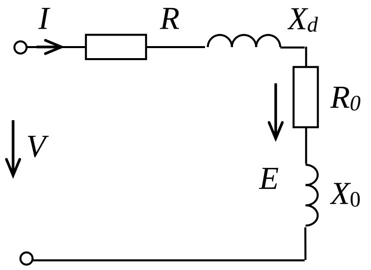


Fig. 6.8

the magnetic circuit. Eddy currents losses are proportional to the difference between the areas of dynamic (curve 3 in small dash line) and static (curve 2 in solid line) hysteresis loops. They are determined by the formula:

$$\Delta P_E = \gamma_E \, \sigma \, f^2 \, B_m^2,$$

where γ_E are specific power losses for eddy currents; σ - magnetic circuit mass; f - current frequency; B_m - magnetic induction amplitude. To reduce eddy currents losses the magnetic cores are collected from thin electrical steel plates (or tape) with a thickness of 0.2 - 0.5 mm, which are isolated from each other by dielectric layers.

Fig. 6.7 shows a series-parallel equivalent circuit of the coil with core, where $\Delta P_E = R\, I^2$ are electrical losses in the coil winding; $X_D = \omega L_D$ - coil reactance caused by dissipation; L_D - inductance, equivalent to the dissipation magnetic flux Φ_D; G_0 - the conductance, equivalent to core magnetic losses $\Delta P_M = G_0 \, E^2$; B_0 - susceptance, equivalent to the main magnetic flux Φ_0.

Fig. 6.8 shows a serial equivalent circuit of the coil, where $\underline{V} = -\underline{E} + R_0 \underline{I} + jX_0 \underline{I}$.

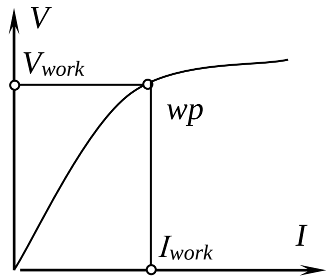


Fig. 6.9

Electromotive forces induced by the main magnetic flux are:

$$e = -w\, d\Phi / dt, \qquad E = \omega w\, \Phi_m / \sqrt{2} = 4.44 fw\, \Phi_m \ .$$

Electromotive forces induced by the dissipation magnetic flux are: $e_D = -L_D di / dt$, . $\underline{E}_D = -j\omega L_D I = -X_D \underline{I}$.

The equation of electric state of the coil is written by the second Kirchhoff 's law in complex form: $\underline{V} = -\underline{E} + R\underline{I} + j X_D \underline{I}$.

Current-voltage characteristic *V(I)* (fig. 6.9) of the coil is derived from weber-voltage characteristic $\Phi(I)$. By zooming magnetic flux $\Phi \equiv E \approx U$ curve *V(I)* is obtained, which coincides with the curve $\Phi(I)$.

Working point (wp) for the coil is chosen at the bent-point of current-voltage characteristic. When chosen below that point the magnetic circuit is irrationally used – it is increased in size. When chosen above that point, the electrical losses are increased due to the increased current.

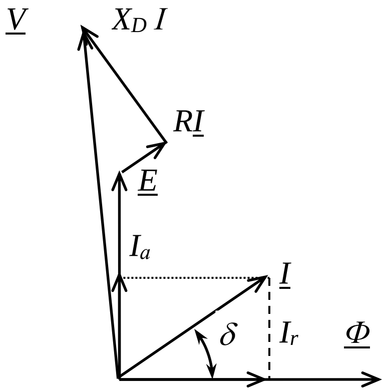


Fig. 6.10

Neglecting the relatively small resistances R, X_D (fig. 6.7), the following can be taken approximately $\underline{V} \approx \underline{E}$ and if $E \equiv \Phi$, so $U \approx E \equiv \Phi$. When the voltage is sinusoidal, electromotive force and magnetic flux are also sinusoidal. It follows from weber-current non-linear characteristic $\Phi(I)$ that the coil current is non-sinusoidal at sinusoidal magnetic flux. When analyzing non-sinusoidal current the first and third harmonics are taken into consideration: $i = I_{1m} \sin \omega_1 t + I_{3m} \sin \omega_3 t$.

To simplify the analysis of the coil with non-sinusoidal current it is substituted by the equivalent sinusoidal current $i = I_m \sin(\omega t + \delta)$ with amplitude $I_m = \sqrt{I_{1m}^2 + I_{3m}^2}$ and frequency $\omega = \omega_1$. Magnetic flux lags behind the phase of current at an angle δ (angle of the magnetic delay or magnetic losses) due to hysteresis effects.

The vector diagram of the coil (fig. 6.10) corresponds to the electrical state equation $\underline{V} = \underline{E} + R\underline{I} + j X_D \underline{I}$. The current at vector diagram is represented by active I_a and reactive I_r components according to the equivalent scheme (fig. 6.7).

If the magnetic circuit has the air gap (magnetic circuit is non-homogenous) the magnetic resistance increases significantly. Therefore, it leads to a reduction of magnetic flux according to the full current law. However, this does not happen, because the magnetic flux is constant at constant voltage. The amplitude of the magnetic flux in electromagnetic devices does not depend on the size of the air gap, but the current effective value in the coil depends on it. Thus, the coil starting current for the core with an air gap is much greater than nominal current.

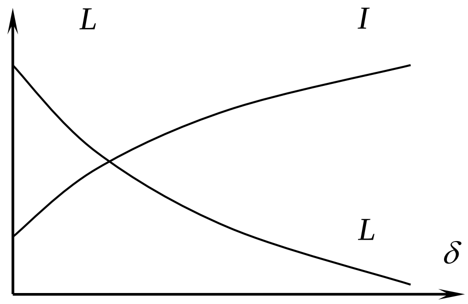


Fig. 6.11

The dependence of coil inductance and current from size of air gap is shown at fig. 6.11. Air gap δ can change its value by changing resistances at AC circuits. The air gap is unavoidable in brake solenoids, relays, contactors, etc.

The variable inductance coil (by change of the air gap) is used to adjust the AC at welding machines and electric ovens. Electromagnetic system of variable inductance coil includes a rod (stationary part) on which a coil is placed (inductor), armature (moving part) and a yoke for connection the rod and the armature in a closed magnetic system. Electromagnets are used in cranes, drive brakes, clutches, electrical switching equipment, measuring devices, machines, relays, etc. The air gap is undesirable in some cases (like transformers, AC engines), since it leads to the current increase, the winding dimensions, reactive power consumption and electromagnetic devices $cos\,\varphi$ reduction.

PART 7. TRANSFORMER

Transformer is a static electromagnetic device that has two (or more) windings inductively connected to each other and is designed to convert primary parameters

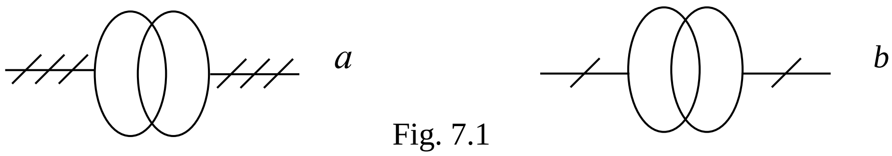


Fig. 7.1

AC into secondary ones. A three-phase transformer is shown on fig. 7.1a and a single-phase transformer is shown on fig. 7.1b. Transformers are used in automatic devices and electrically powered household appliances. There are as well special transformers for converting, welding, measuring and so-called peak transformers.

The transformer's core (fig. 7.2) determines its design. Laminated core is made of insulated electrical steel sheets. Core types like U-and-I (aka core-type) (a), E-and-I (aka shell-type) (b) or toroidal (c) are used in low-power transformers.

Transformer winding is made of insulated copper (aluminum) wire of round or rectangular cross-section. The winding is wound on the frame, which is slid on one of the rods. The principle of the transformer is based on the law of electromagnetic induction. Electromagnetic single-phase transformer circuit is shown on fig. 7.3, and it's schematic diagram on fig. 7.3b. The primary winding w_1 converts electrical

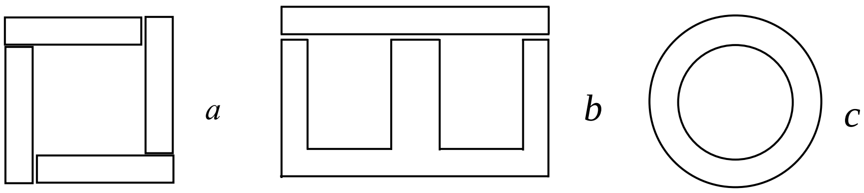


Fig. 7.2

energy into magnetic one, whereas the secondary winding w_2 converts the magnetic energy into the electrical one. Magnetic core strengthens the link between the windings. The current i_1 generates an alternating magnetic flux Φ_1, which closes itself on the core and dissipation magnetic flux Φ_{1d}, which closes itself in the air around the primary winding w_1.. Magnetic flux Φ_1 induces an electromotive force of self-induction in the transformer primary winding and a mutual electromotive force in the secondary winding. If the secondary winding is connected to the load, the current will flow. The current i_2 generates an alternating magnetic flux Φ_2, which closes itself on the core and dissipation magnetic flux Φ_{2d}, which closes itself in the air around the secondary winding w_2.

Transformer electromotive forces e_1 and e_2 create the resulting magnetic flux in the core.

$$e_1 = -w_1\, d\Phi / dt, \qquad E_1 = \omega w_1\, \Phi_m / \sqrt{2},$$

$$e_2 = -w_2\, d\Phi / dt, \qquad E_2 = \omega\, w_2\, \Phi_m / \sqrt{2},$$

where $\omega/\sqrt{2} = 2\pi f/\sqrt{2} = 4.44\,f$; f – is an AC frequency, Φ_m – is the resulting flux amplitude, w_1 - number of primary winding turns, w_2 - number of secondary winding turns.. Induced electromotive force phase lags behind the magnetic flux phase by an angle of $\pi/2$. The e.m.f. effective values are as follows:

$$E_1 = 4.44\,w_1\,f\,\Phi_m, \qquad E_2 = 4.44\,w_2\,f\,\Phi_m.$$

If $w_1 > w_2$ the transformer is a down transformer and its transformation ratio is

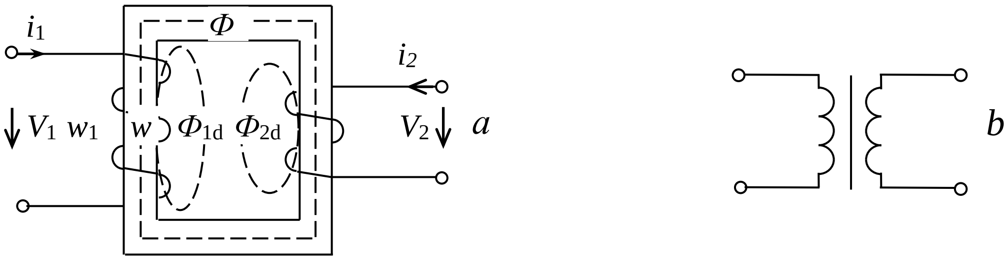


Fig. 7.3

$k_{12} = w_1/w_2 = E_1/E_2 > 1$. If $w_1 < w_2$ the transformer is an up transformer and its transformation ratio is $k_{21} = w_2/w_1 = E_2/E_1 > 1$ (i.e. higher than one).

Transformer electromagnetic substitutional scheme (fig. 7.4) represents an idealized real transformer, which includes active R_1 R_2 (winding) resistances and (dissipation) reactances X_1, X_2.

The transformer electrical state equations are (fig. 7.4):

$$\begin{cases} \underline{V}_1 = -\underline{E}_1 + (R_1 + jX_1)\underline{I}_1 = -\underline{E}_1 + \Delta\underline{V}_1 \\ \underline{V}_2 = \underline{E}_2 - (R_2 + jX_2)\underline{I}_2 = \underline{E}_2 - \Delta\underline{V}_2 \end{cases},$$

where $\Delta\underline{V}_1$, $\Delta\underline{V}_2$ – are accordingly voltage drops across the primary and secondary windings of the transformer. Since $\Delta\underline{V}_1 \approx (0{,}02 \div 0{,}05)V_1$, it can be considered $V_1 \approx E_1$. Thus, $V_1 \approx E_1 = 4.44\,w_1\,f_1\,\Phi_m$. When $V_1 = const$, the magnetic flux is practically independent of the transformer load.

Electrical state equations corresponds to the scheme on fig. 7.5, where E_1 is a receiver of electrical energy (primary winding) and E_2 is an electric energy source (secondary winding).

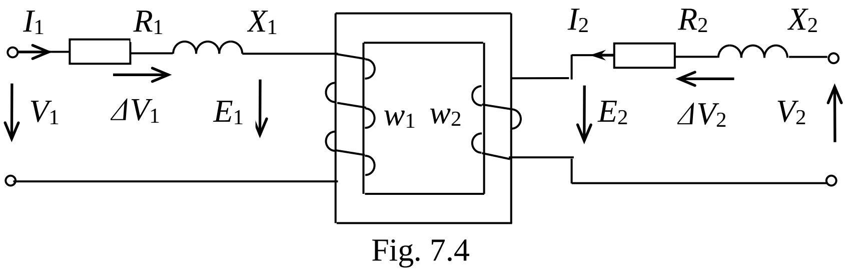


Fig. 7.4

The magnetomotive force of primary winding in idle mode is $w_1 I_{10} = Hl$. It induces the magnetic flux $\Phi = \mu SH$. And magnetic flux of loaded transformer is created by an e.m.f. of primary and secondary windings $w_1 I_1 + w_2 I_2 = Hl$. Taking into consideration that $\Phi \approx const$ and $H \approx const$, transformer magnetic state equation makes: $$w_1 I_{10} = w_1 I_1 + w_2 I_2.$$

Therefore, the expression for current in the primary winding of the transformer makes: $\underline{I}_1 = \underline{I}_{10} - \underline{I}_2 w_2 / w_1 = \underline{I}_1 = \underline{I}_{10} + \underline{I}'_2$,

where $\underline{I}'_2 = -\underline{I}_2\, w_2 / w_1 = -\underline{I}_2 / k$ I'_2 - is a specific secondary winding current.

Since numbers of turns $w_1 \neq w_2$, then $E_1 \neq E_2$. To simplify the analysis of the transformer, the secondary winding number of turns is reduced to the number of turns

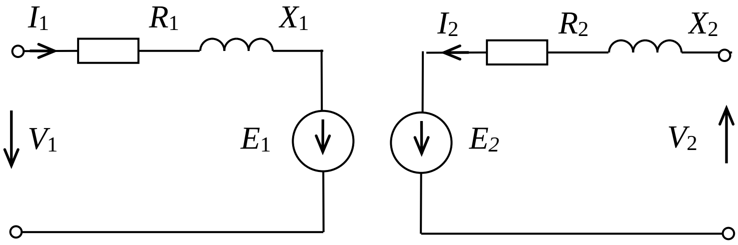


Fig. 7.5

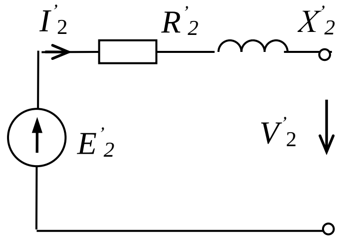


Fig. 7.6

in the primary winding.

The real secondary winding w_2 is replaced by brought-in values $w'_2 = w_1$, $w'_2 = kw_2 = w_1$. Specific electromotive force makes then $E'_2 = kE_2 = E_1$ and specific voltage makes $V'_2 = kV_2 = V_1$ accordingly. Specific resistance is defined from the condition $I'^2_2 R'_2 = R_2 I_2^2$ as $R'_2 = k^2 R_2$ and specific reactance is defined from the condition $\varphi = arctg\,(X'_2 / R'_2) = arctg\,(X_2 / R_2)$ as $X'_2 = k^2\, X_2$. The brought-in secondary winding elements is applied in fig.7.6.

T-shaped substitutional scheme is shown on fig. 7.7, where $\underline{E}_1 = R_0 \underline{I}_{10} + jX_0 \underline{I}_{10}$. The scheme elements bear some physical meaning, as such R_0 - are magnetic losses ΔP_M and X_0 – is the main magnetic flux.

Idle mode scheme $(I'_2 = 0)$ shown on fig.7.8 is obtained from the scheme 7.7 by neglecting resistances R_1 and X_1, that are considerably less than R_0 and X_0. The quality of core steel defines the idle current. Resistance R_0 and impedance $Z_0 = U_{10} / I_{10}$ determine the idle losses: $P_0 = R_0 I_{10}^2$. The transformation ratio makes:

$$k = w_1 / w_2 = E_1 / E_2 \approx V_{10} / V_{20} .$$

The following expressions determines magnetic induction and magnetic flux:

$$V_{10} \cong E_1 = 4.44\, w_1\, f_1\, \Phi_m\,,\ \Phi_m = B_m S\,.$$

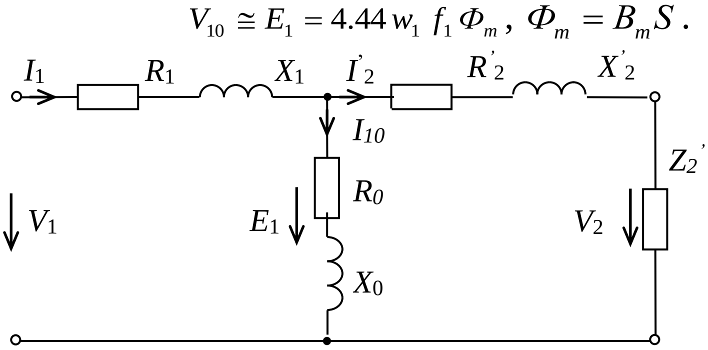


Fig. 7.7

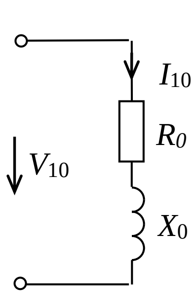


Fig. 7.8

Transformer magnetic losses are constant and independent from load. Any power consumption in idle mode goes over into transformer magnetic core losses ΔP_M and electrical losses in the primary winding ΔP_{1E}: $P_0 = \Delta P_M + \Delta P_{E1}$. Electrical losses in the transformer primary winding are $\Delta P_{E1} = R_1 I_{10}^2$. They can be neglected because idle state current is smaller than the nominal $I_{1N} >> I_{10}$ ($I_{10} = (0.04 \div 0.1) I_{1N}$). Thus, $\Delta P_{E1} << \Delta P_M$. Consequently, the power consumed by the transformer in the idle mode is almost equal to the magnetic losses $P_0 \approx \Delta P_M$.

The short circuit transformer mode is a wrecking one at nominal voltage V_{1N}, since current makes then $I_{1SC} \approx (10 \div 25)\ I_{1N}$. Therefore, short circuit mode calculations

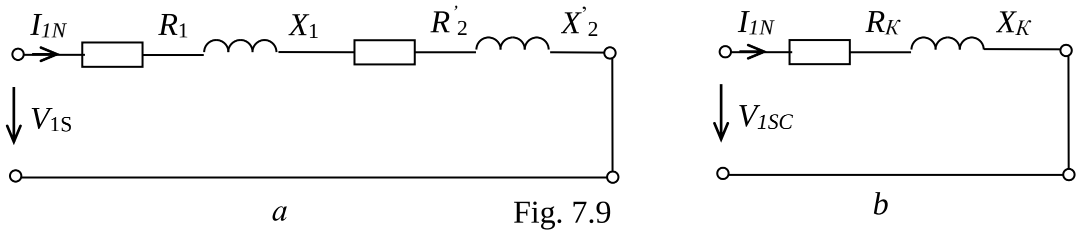


Fig. 7.9

are done out at nominal current $I_{1SC} = I_{1N}$ and considerably less voltage.

Short circuit transformer mode scheme ($V_2' = 0$) is shown on fig. 7.9. It is obtained from the circuit on fig. 7.7 by neglecting R_0, X_0, which are much less than R_2', X_2' and R_1', X_1'. The following elements are used in resulting short circuit scheme (fig. 7.9b): $R_1 + R_2' = R_{SC}$, a $X_1 + X_2' = X_{SC}$. Voltage, current, R_{SC} and impedance $Z_{SC} = V_{1SC} / I_{1N}$ can be defined, when having short circuit losses $P_{SC} = R_{SC} I_{1N}^2$.

The consumed by transformer power goes over into electrical losses in the windings and magnetic losses in the core. However, short circuit voltage is much less than the nominal $V_{1SC} = (0.05 \div 0.1) V_{1N}$, so the magnetic flux is also much less than at nominal mode. Magnetic losses are proportional to the square of the magnetic induction and are insignificant. Allowing that $\Delta P_M << \Delta P_E$, the consumed by transformer power in the short circuit mode goes over into electrical losses in the windings $P_{SC} \approx \Delta P_E$. These are variable and depend on transformer's load.

External characteristics of the transformer (fig. 7.10) is a depending of the voltage from the current at variable load. Active load (curve 1) is at $\cos\varphi_2 < 1$ ($\varphi_2 > 0$). The active-inductive load (curve 2) is at $\cos\varphi_2 < 1$ ($\varphi_2 > 0$). And active-capacitive load (curve 3) is at $\cos\varphi_2 < 1$ ($\varphi_2 > 0$).

Transformer power diagram is shown on fig. 7.11, where input power makes $P_1 = V_1 I_1 \cos\varphi_1$, ΔP_{E1} are electrical losses in the primary winding; $P_{EM} = EI\cos\psi$ - is an electromagnetic power; ΔP_M are magnetic losses in the core; ΔP_{E2} are electrical losses in the secondary winding and $P_2 = V_2 I_2 \cos\varphi_2$ is an output power.

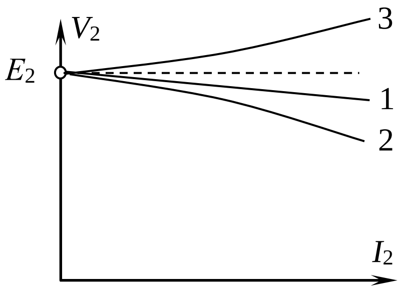


Fig. 7.10

Transformer total power makes $S_1 = S_2 = V_1 I_1 = V_2 I_2$, so $V_1 / V_2 = I_2 / I_1 \approx k$.

The transformer efficiency factor makes:

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_2 + \Delta P} = \frac{\beta\ S_2 \cos\varphi_2}{\beta S_2 \cos\varphi_2 + P_0 + \beta^2 P_{SC}} \quad ,$$

Where $\Delta P = P_0 + \beta^2 P_{SC}$ are the total losses; P_0 are magnetic losses (idle); P_{SC} are short circuit electrical losses; $\Delta P_E = \beta^2 P_{SC}$ are electrical losses, $\beta = I_2 / I_{2N}$ is the transformer load factor.

The dependence of the efficiency factor from the transformer load $\eta(\beta)$ is shown on fig. 7.12. Maximum efficiency factor is at equal electric and magnetic

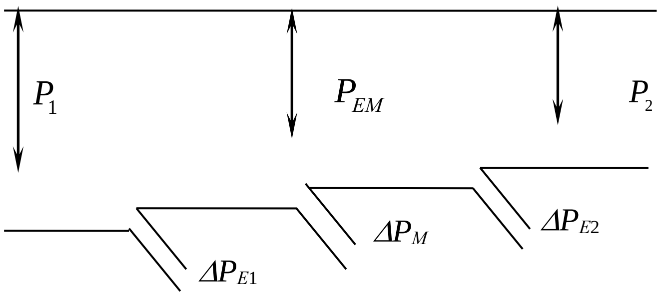


Fig. 7.11

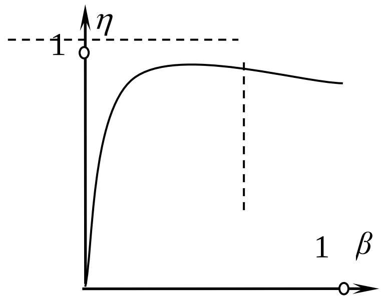


Fig. 7.12

losses, that corresponds to the optimum load. $\eta = (70 \div 95)\%$ is typical for modern transformers.

PART 8. DIRECT CURRENT MACHINES

DC machines can be used either as a generator or as a motor. And they are reversible. Motors are used to drive machine tools, rolling mills, lifting and transport machines, excavators and others. The main advantage of these motors is a wide range of power - from fractions of Watt to several thousand kilowatts, good start and control properties (the ability of smooth frequency regulation over a wide range) However, DC motors are more expensive and less reliable comparing to contactless AC motors. This is because of a collector-brush unit. Special equipment - DC generator or rectifier (when powered by AC) – is needed to feed these motors.

The structure of the DC machine is shown in figure 8.1. Stationary part of it called stator, movable - rotor.

Stator consists of steel bed 8, through which main magnetic flux closes and main stator poles 6. Stator is an inductor, because it contains the main poles for excitation of the main magnetic field. Pole consists of a core 6 and a coil 7 (excitation winding).

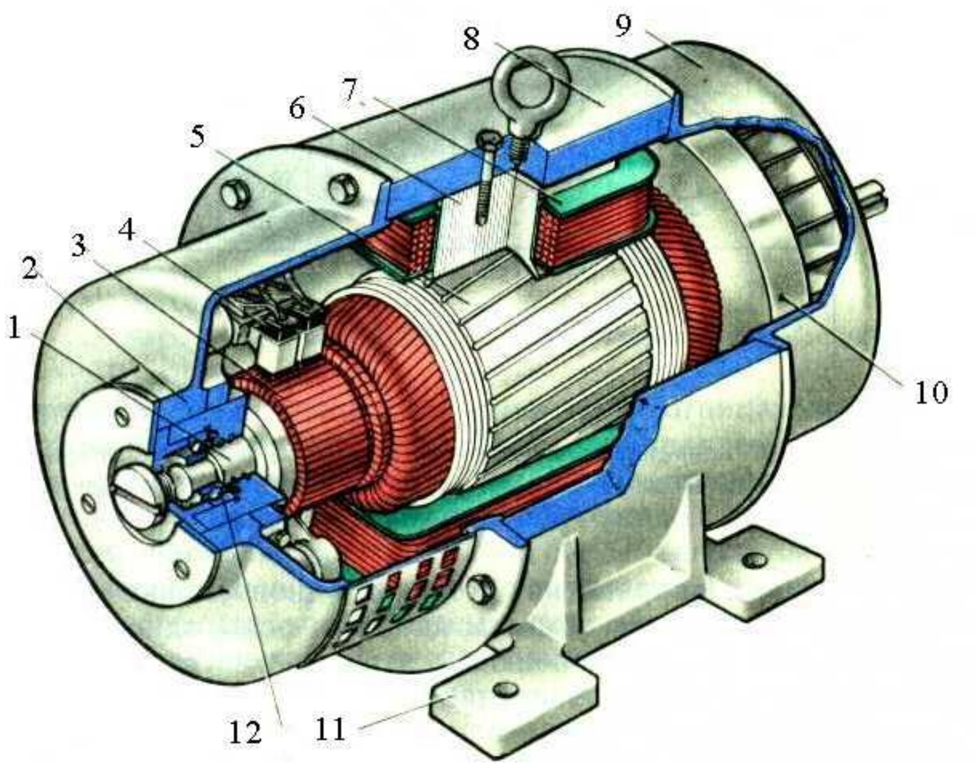


Fig. 8.1. DC motor construction:
1– shaft; 2– front shild; 3– collector; 4– brushes; 5– core with coil; 6–core;
7– coil; 8– steel bad; 9– back shield; 10– fan; 11– lugs; 12– bearings

Rotor is an armature, to which the load is connected. On the shaft of the rotor 1 is a laminated core 5 with a winding and a collector-brush unit 3. Armature winding consists of individual sections 1 (figure 8.2). Each section is placed into the grooves of the armature magnetic circuit and sections are connected with collector lamellas (fig. 8.2). The number of sections is equal to the number of collector plates. Armature winding can be of two types: loop (fig. 8.2a) and wave (fig. 8.2b). Loop winding is used in high power machines and wave one is used in machines of medium and low power.

Next element of DC motor is the brush-collector unit that acts as a mechanical DC to AC adapter to feed armature winding. The main elements of the brush-

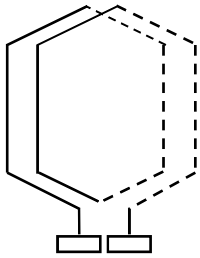


Fig. 3.2a

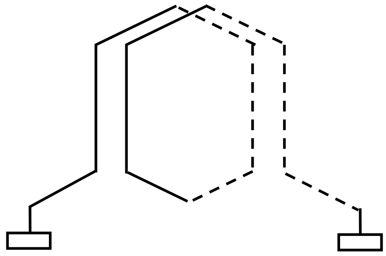


Fig. 3.2b

collector unit 3 (fig. 8.1) are made of copper lamellas.

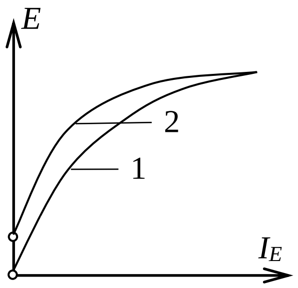


Fig. 8.3

The principle of generator is the following. The excitation winding current creates a magnetic field, which is constant in time and in space. E.m.f. (field induction action) is induced in the rotating rotor winding under the influence of this magnetic field, the direction of e.m.f. is determined by the right hand rule. If the armature winding is connected via brushes to the outer circle, there appears AC I_A. However, at the outer circuit the current direction does not change, because the collector serves as a mechanical current rectifier. When the armature rotates by 180 deg., e.m.f. at the wires changes its direction and there it also changes the collector plates under the brushes at the same time. As a result, the polarity of the brushes and the direction of current in the outer circuit does not change. Armature winding is made of many connected in series sections to get e.m.f. nearly constant in its external-circuit.

Idling characteristic of DC generator (fig. 8.3) - is a dependence of armature winding e.m.f. from excitation current without load and at a constant frequency of armature rotation. The presence of hysteresis phenomena in magnetic circuit provides a parallel excitement due to self-excitation of the generator. When generator rotor rotates, there is a small electromotive force in the winding, which leads to small excitation current and small magnetic flux, respectively. When this flux has the same direction as the flux of residual induction, the resulting flux increases, therefore the armature electromotive force and excitation current increase (curve 1, fig. 8.3) as well. Electromotive force increases to the value $1.25\ E_{nom}$ with increasing excitation current, following the saturation of magnetic circuit. As the excitation current decreases, the electromotive force also decreases to the value $0.05\ E_{nom}$, (curve 2, fig. 8.3). The external generator characteristics (fig. 8.4) is a dependence between voltage at the clamps of the armature winding and current $V(I_A)$ at a constant rotation frequency and excitation current (for generators with independent excitation) or constant rotation frequency and constant resistance of excitation winding (for generators with parallel excitation). Curve 1 is for independent excitation, 2 is for parallel excitation, 3 is for serial

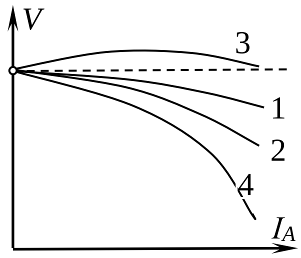


Fig. 8.4

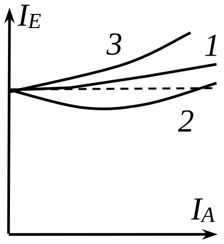


Fig. 8.5

excitation and 4 is for mixed excitation. Adjusting generator characteristic is a dependence between the excitation current and armature current $I_E(I_A)$ at constant voltage at generator ($V=const$) and constant rotation frequency ($n=const$). The generator adjusting characteristics for independent and parallel excitation is shown on figure 8.5 (curve 1), for mixed agreed excitation winding (curve 2) and for mixed counter excitation winding (curve 3).

The generator electrical state equation is: $V = E_A - R_A I_A$, where V- is a power voltage, R_A – is an armature winding resistance. Output current is $I = I_A - I_E$, where I_A – is an armature current, I_E – is an excitation current.

The principle of the DC motor based on power performance of electromagnetic field. Connecting the motor to the DC voltage will result in currents in the windings of the inductor and the armature. The interaction of the armature current with the magnetic field (of inductor) results in growth of electromagnetic torque $M = c_M \Phi I_A$, what makes the rotor rotation (here c_M - is a coefficient that depends of winding construction and poles number; Φ - magnetic flux of one pair of main poles ; I_A- armature current. Induced electromotive force in the armature winding during its rotation is directed opposite to the current: $E_A = c_E \Phi n$, where c_E- is a structural coefficient, n – is a rotor rotation frequency.

The influence of the armature magnetic field (fig. 8.6b) on the main magnetic field (fig. 8.6a) is called an armature reaction. It distorts the main magnetic field (fig. 8.6c) because slope of magnetic field physical neutral (line q-q' at fig. 8.6c) is shifted in the opposite direction of rotor rotation at some angle relatively to its geometrical neutral. Physical neutral is an imaginary line that runs through the points of magnetic circuit, where magnetic induction is zero. Because the armature reaction worsens the commutation process, an arcing occurs between the collector and the brushes. Commutation is a process of switching armature winding sections when rotor rotates.

To improve the commutation between the main poles the additional poles are

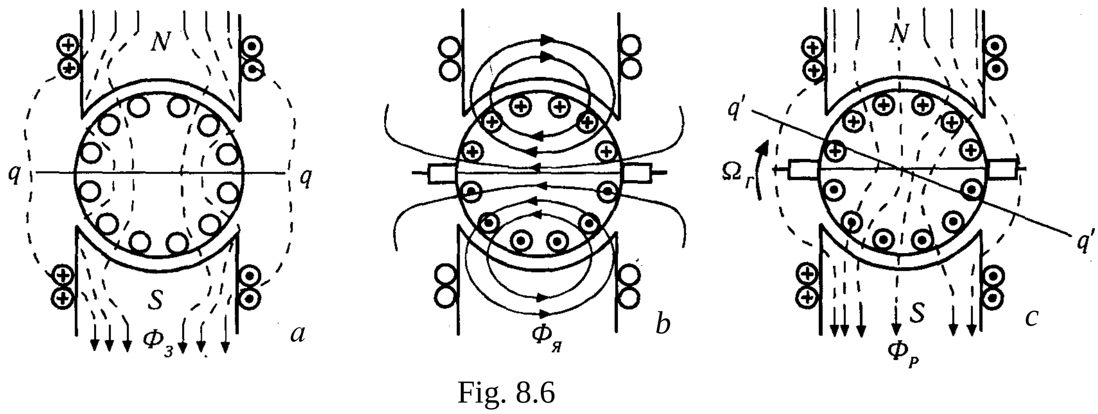


Fig. 8.6

added. The winding of additional poles connects in series to the armature winding and is wound in a way that the magnetic flux generated by it, opposes the magnetic flux of the armature winding, thus it compensates the armature reaction. Brushes position is also shifted from geometrical to physical neutral to improve the commutation. The physical neutral position depends on motor loads and shifts against the direction of rotor rotation.

By way of excitation winding connection - the motors are split in independent

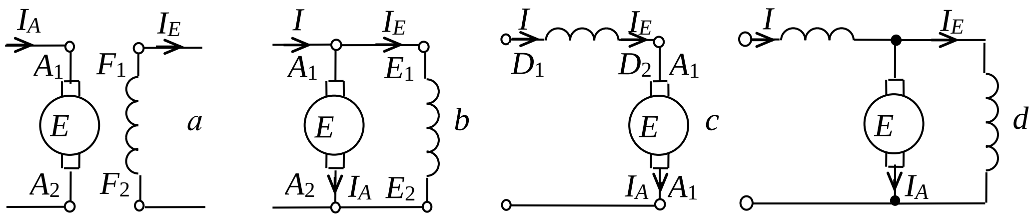


Fig. 8.7

excitation ones and self-excitation ones (parallel, serial and mixed). The machines with independent excitation (fig. 8.7a) feed their excitation winding with a single DC source. The machines with parallel excitation (fig. 8.7b) have the excitation winding connected in parallel to the armature winding. The machines with serial excitation (fig. 8.7c) have the excitation winding connected in serial to the armature winding. The machines of mixed excitation (fig. 8.7d) have two excitation windings connected both in serial and in parallel to the armature winding.

The motor electrical state equation is: $V = E_A + R_A I_A$, where V - is a feeding voltage, R_A – is an armature winding resistance. Input current makes $I = I_A + I_E$, where I_A – is an armature current, I_E – is an excitation current. Therefore:

$$I_A = (V - E_A)/R_A = (V - c_E n\Phi)/R_A .$$

When starting the motor, rotation frequency is $n = 0$, hence E_A=0. Because the armature winding resistance is small, the starting current is large: $I_{AST} = V/R_A$.

To limit the starting current the rheostat is connected in armature circuit, the resistance of which R_{ST} is step by step reduced to zero. The value of armature starting current with the starting rheostat is determined as: $I_{AST} = V/(R_A + R_{ST})$.

Starting rheostat resistance is calculated so that the armature current does not exceed the permissible value of $I_{AST} \le (2.0...2.5) I_{Anom}$.

Direct switching to the electric network, using the starting rheostat in the armature circuit or reduced supply voltage, can start DC motors. When starting the motor directly there are significant starting currents that could break down the motor and reduce the network voltage. So motors mainly are started with starting rheostats.

If we substitute the equation $E_A = V - R_A I_A$ into the expression of armature e.m.f. $E_A = c_E \Phi n$, we'll have the expression of motor frequency characteristic:

$$n = (V - R_A I_A)/c_E \Phi_A .$$

It follows that without load motor frequency is:

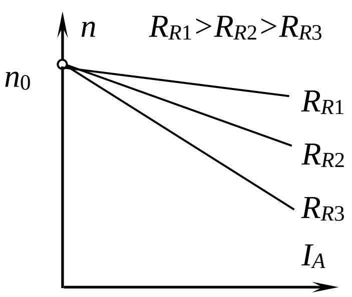


Fig. 8.8

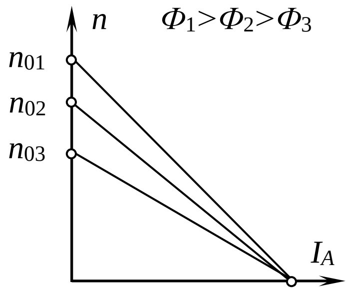


Fig. 8.9

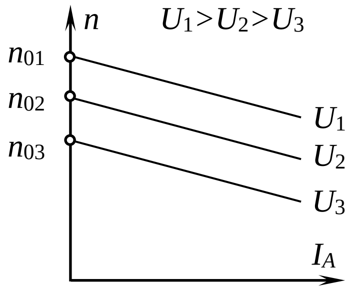


Fig. 8.10

$$n = n_0 = V/(c_E \Phi),$$

where n_0 – is an idling frequency.

The rotation frequency can be adjusted by connecting a circuit of the additional resistance R_R to armature winding. These are characteristics for different values of resistance R_R on fig. 8.8. The advantages of this regulation method is its simplicity, the possibility of smooth adjustment and wide range of frequency adjustment ($0 \le n \le n_{nom}$). Disadvantages are major losses and the inability to adjust the frequency above nominal.

We can also adjust rotation frequency with changing magnetic flux by connecting the rheostat R_E to the excitation circuit. There are characteristics for different values of magnetic flux shown on fig.8.9. The advantages of this adjustment method is its simplicity and efficiency. Disadvantage are the inability to adjust the frequency above nominal and narrow adjustment range of frequencies.

We can adjust the rotation frequency by changing input voltage V in electric drives. There are characteristics for different values of voltage shown on fig. 8.10. The advantage of this method is the wide range of frequency adjustment. Disadvantage is the need for a separate adjustable energy source.

Mechanical characteristic of the motor is the dependence between the rotation frequency and torque $n(M)$ at $V = const$ and $I_E = const$. The expression for frequency characteristics is: $n = (V - R_A I_A) / c_E \Phi_A = V / c_E \Phi_A - R_A I_A / c_E \Phi_A$.

Taking into account the expression for torque as $M = c_M \Phi I_A$, the armature current is defined as $I_A = M / (c_M \Phi)$ and the equation of mechanical characteristic is the following: $n = V / c_E \Phi - R_A M / (c_M c_E \Phi^2) = n_0 - \Delta n$,

where - n_0 is a motor rotation frequency in idling mode, Δn - is a rotation frequency change caused by the change of motor load. Mechanical characteristic without additional resistance at armature circuit is called natural one, and with the additional resistance at armature circuit is called an artificial one. Artificial characteristic slopes rapidly. Type of motor mechanical characteristic depends on the type of excitation.

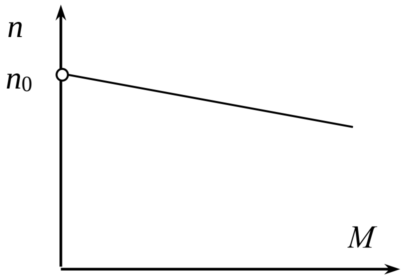


Fig. 8.11

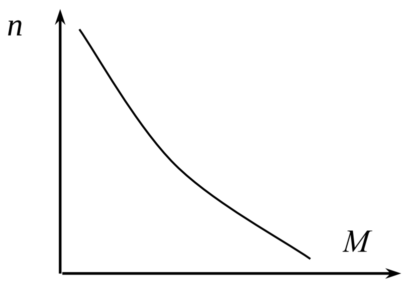


Fig. 8.12

Motors with independent and parallel excitation have rigid characteristics – rotation frequency does not depend from the motor torque (fig. 8.11). They are used to drive machine tools, etc. Motors with serial and mixed excitation have soft characteristic - rotation frequency depends from the motor torque (fig. 8.12). These motors are used in electric transport because they have large torque, which is proportional to the square of the current, what is important in difficult starting conditions. The rotation frequency increases rapidly at idling mode, what can cause the motor collapse. The work of the motor with torque less than $0.25M_{NOM}$ is unacceptable.

The energy balance of the DC motor illustrating its energy diagram is shown

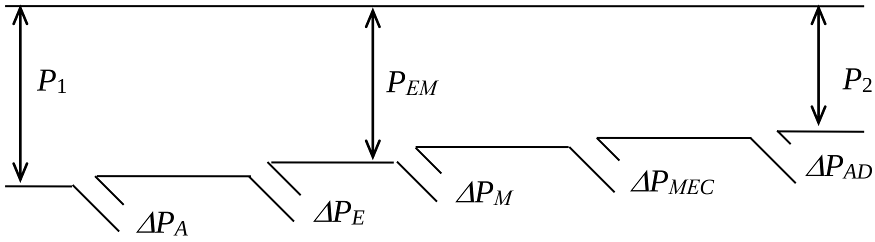


Fig. 8.13

on fig. 8.13. Motor electric power makes $P_1 = VI$. Electromagnetic power is $P_{EM} = E_A I_A$. Mechanical power is - $P_2 = \Omega \cdot M$. Total losses in the motor make:

$$\Delta P = \Delta P_A + \Delta P_E + \Delta P_M + \Delta P_{MECH} + \Delta P_{ADD},$$

where $\Delta P_E = R_E I_E^2$ - are excitation winding losses; $\Delta P_A == R_A I_A^2$- are armature winding losses; - ΔP_M - are magnetic losses; ΔP_{MECH} - are mechanical losses; ΔP_{ADD} - are collector-brush losses. Efficiency factor is defined as the ratio of useful power to the power consumed by the motor: $\eta = P_2 / P_1 = P_2 /(P_2 + \Delta P)$.

Properties of motor as part of the circuit are estimated by working characteristics $I_A(P_2)$, $\eta(P_2)$, (fig. 8.14). Properties of motor as the electric drive element are estimated by working characteristics $n(P_2)$, $M(P_2)$, (fig.8.15).

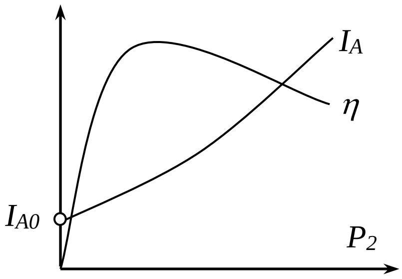


Fig. 8.14

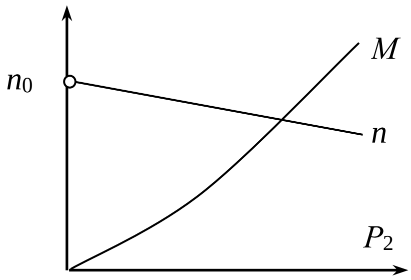


Fig. 8.15

PART 9. ALTERNATIVE CURRENT (INDUCTION) MOTORS

AC induction machines can operate either as generators or as motors. However, when used as generators the characteristics of the machines are not good enough, so they are used mostly as motors. For induction motors, the rotor rotation frequency depends on the load at a constant circuit frequency.

Single-phase induction motors are used in automatic control systems, electric hand tools, low-power machines and various household appliances and machines (refrigerators, washing and sewing machines, fans), where there is no need to adjust the frequency. The use of single-phase power supply for these motors (means two wires instead of three or four in three-phase power supply) provides economic benefits and ease of use.

Three-phase induction motors are more popular, because they have better characteristics comparing to single-phase ones.

Induction motors are widely used because of simpler construction, lower cost, higher operational reliability and less maintenance. This goes alongside with a simpler design and higher reliability.

The drawbacks include greater sensitivity to voltage changes (motor moment is proportional to the square of the applied voltage). The starting moment is small at low voltages that the motor may not start at all. Another drawback of induction motors is significant consumption of reactive power, which reduces the network power factor.

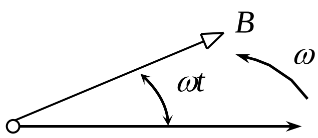


Fig. 9.1

Structural elements of induction motors ensure reliability, rigidity, strength, rotor rotation and cooling. Motor electromagnetic system provides mutual energy conversion, it is a heterogeneous branched symmetrical magnetic circuit, which consists of magnetic circuits of stator and rotor and their windings.

Magnetic circuit increases magnetic connection between stator and rotor windings. The magnetic core consists of thin insulated electrical steel lists. There are

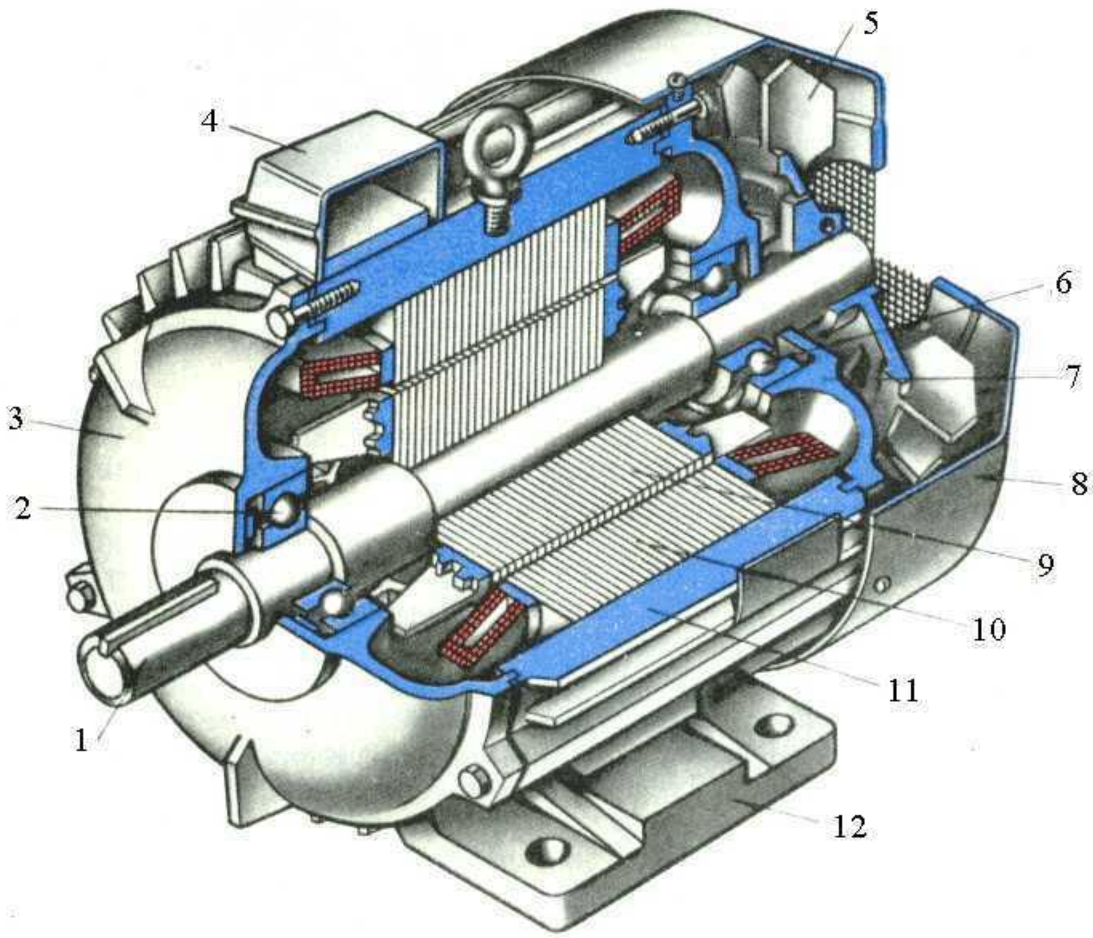


Fig. 9.2

grooves at the inner surface of the stator, where the winding is placed. The windings are made of copper (aluminum) wire around round or rectangular inserts covered with insulating materials.

The condition for creating a rotating magnetic field by the fixed stator winding is spatial symmetry of the phase windings and temporal symmetry of phase currents in the windings. Stator phase windings are placed in the grooves and shifted relative to each other in space at the angle of 120 degrees. The symmetrical three-phase currents flow through these windings, causing a rotating in space $\beta = \omega t$ (fig.9.1) and constant in time $B = 3/2B_m$ magnetic field. The phase windings ends connect to the terminal box 4, which makes it possible to connect them WYE or DELTA.

Magnetic field synchronous frequency is $n_1 = 60 f / p$, where f - the frequency of the power network, p - the number of pole pairs.

The induction motors are designed either with a short circuit rotor or with a phase rotor (with slip rings for winding connection to the outside circuit).

Three-phase induction motor with a short circuit rotor winding is shown at fig. 9.2. Stator consists of frame 11, core 10 and a three-phase winding. There are ribs on the stator surface to increase the cooling effect. The core 10 is placed on the stator frame, it consists of 0,5 mm thick steel lists insulated from each other by oil varnish and packed together. This design enables to significantly reduce eddy currents in the core.

The rotor shaft is positioned in bearings 2 and 6, located in the bearing nests 3 and 7. Motor is cooled by a fan 5, which is covered with a casing 8.

Rotor consists of a shaft 1 and core 9 with a short circuit winding. Short circuit winding is made of copper wire (fig. 9.3) rods, embedded in isolation without grooves and close-circuited with the rings on the ends. An aluminum coil is made by filling in the grooves of a molten aluminum alloy. Short circuit rings 2 and ventilation scoops 3 (fig. 9.3b) are cast the same way. Phase rotor winding is made the same way. It is also a three-phase one (for a three-phase motor) - three coils are placed in the space and their endings connect to the three contact rings placed on the shaft. Rings are isolated from each other and from the shaft of the rotor. The additional rheostats are connected to the windings by brushes. The rheostat is used to improve the motor start-off (increased starting moment) or to adjust the rotor rotation frequency.

The principle of the motor is based on the induction law and power results from the magnetic field. Rotating magnetic field of the stator winding, while crossing rotor winding wires, induces electromotive force, causing currents in the short circuited rotor winding. The interaction of the rotor winding currents and the rotating

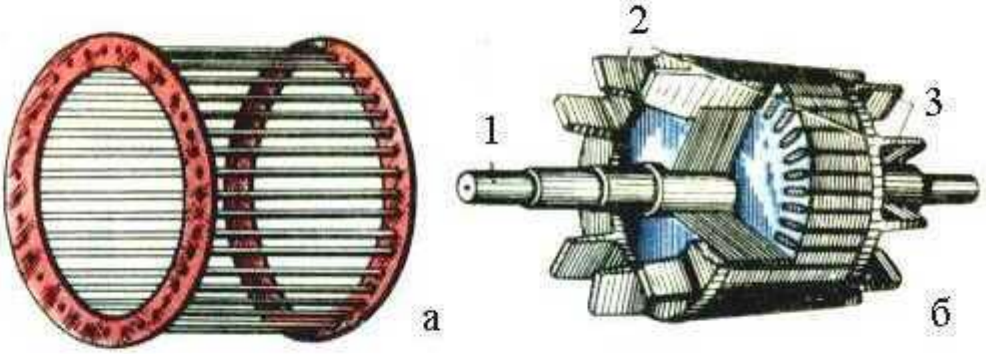


Fig. 9.3

magnetic field create electromagnetic force that rotates the rotor in the direction of the rotating magnetic field. The phases sequence determines the direction of the rotating magnetic field.

A characteristic feature of the induction motor is lag of the rotor rotation frequency from the stator magnetic field rotation frequency.

Slipping s connects rotor mechanical frequency n_2 with the frequency of the rotating magnetic field of the stator n_1:

$$s = (n_1 - n_2)/n_1 .$$

Thus the rotor rotation frequency is $n_2 = n_1(1-s)$.

The expression of rotor rotation frequency for multipolar motor can be written as: $$n_2 = n_1(1-s) = 60\ f_1(1-s)/p .$$

The rotation frequency can be controlled by changing the frequency of network current f by special converters. This method is the most common, because it allows smooth rotation frequency adjusting. Besides, the converters are cheap.

The second method is to change the sliding s by rheostats, which are connected with every phase winding of phase rotor (for motors with phase rotor).

The third way is by changing the number of pole pairs p.

Rotating magnetic field induces the electromotive forces in the phase windings of the stator $E_1 = \omega_1 w_1 \Phi$ and the rotor $E_2 = \omega_2 w_2 \Phi$ whose frequencies are (for short circuit rotor):

$$\omega_1 = 2\pi n_1 / 60,\ (f_1),\quad \omega_2 = s2\pi n_1 / 60 = s\,\omega_1,\ (f_2 = s\, f_1).$$

The electrical state equations of the stator and rotor phases are:

$$\underline{V}_1 = \underline{-E}_1 + \underline{R}_1 \underline{I}_1 + j\underline{X}_1 \underline{I}_1, \qquad \underline{V}_2 = \underline{E}_2 - \underline{R}_2 \underline{I}_2 - j\underline{X}_2 \underline{I}_2 = 0.$$

Motor phase substitutional scheme is shown at fig. 9.4, phase reduced substitutional scheme is shown at fig. 9.5, where $R_{EQV} = R_2(1-s)/s$ is the equivalent of mechanical load.

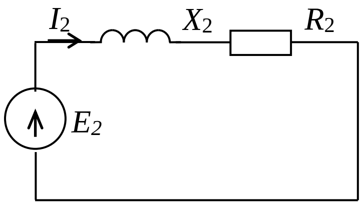


Fig. 9.4

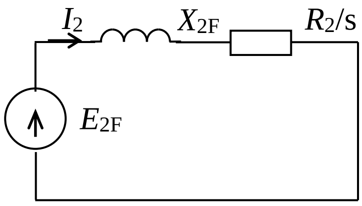


Fig. 9.5

Rotor winding electromotive force E_2 can be expressed as fixed rotor winding electromotive force $E_{2F} = const$:

$$E_2 = \omega_2 w_2 \Phi = s\,\omega_1 w_2 \Phi = sE_{2F},\ \text{where}\ E_{2F} = \omega_1 w_2 \Phi.$$

Rotor winding reactance X_2 can be expressed as fixed rotor winding reactance $X_{2F} = const$

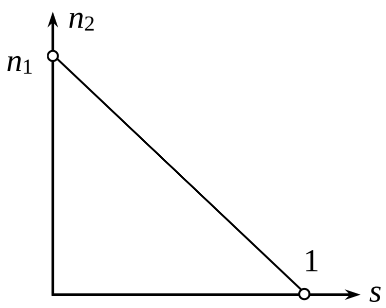


Fig. 9.6

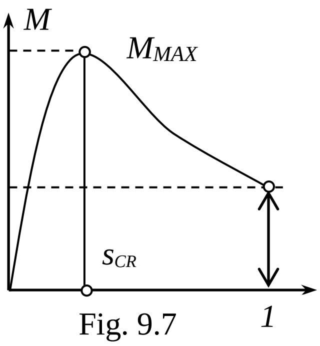


Fig. 9.7

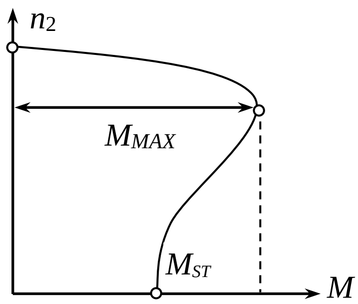


Fig. 9.8

$$X_2 = \omega_2 L_2 = s\omega_1 L_2 = sX_{2F}, \qquad \text{where}\ X_{2F} = \omega_1 L_2.$$

$$I_2 = \frac{sE_{2F}}{R_2 + jsX_{2F}} = \frac{E_{2F}}{R_2/s + jX_{2F}}.$$

For motor idle mode $s = 0$, $n_2 = n_1$, $R_{EQV} = \infty$ and for motor short circuit mode $s = 1$, $n_2 = 0$, $R_{EQV} = 0$.

Motor electromagnetic moment is:

$$M = 2M_{MAX} /(s/s_{CR} + s_{CR}/s), \qquad M = c\Phi I_2 \cos\varphi_2,$$

where s_{CR} is a critical slip, the moment is maximum for this slip.

Motor frequency characteristic $n_2(s)$ at sliding interval $0 < s < 1$ is shown at fig. 9.6, according to motor operation mode.

Motor moment characteristic $M(s)$ at sliding interval $0 < s < 1$ is shown at fig. 9.7. The characteristic is unstable at sliding interval $s_{KP} < s < 1$.

Motor mechanical characteristic $n_2(M)$ is the main characteristic of the motor, it is shown in fig. 9.8 and determines its operational capabilities. Mechanical characteristic is based on the frequency $n_2(s)$ and moment $M(s)$ characteristics.

The value of nominal moment M_{NOM} characterizes motor for slip ranging within $0 < s < s_{CR}$. Motor overload capacity is $\lambda = M_{MAX} / M_{NOM}$. Starting properties are evaluated as starting moment repetition factor $\gamma = M_{ST} / M_{NOM}$ and starting current repetition factor $\beta = I_{ST} / I_{NOM} = 5 - 7$.

Motor energy balance is illustrated in its energy diagram (fig. 9.9), where $P_1 = \sqrt{3} V I \cos\varphi_1$ - electric power, $P_{EM} = 2\pi n_1 M / 60 = \omega_1 M$ - electromagnetic

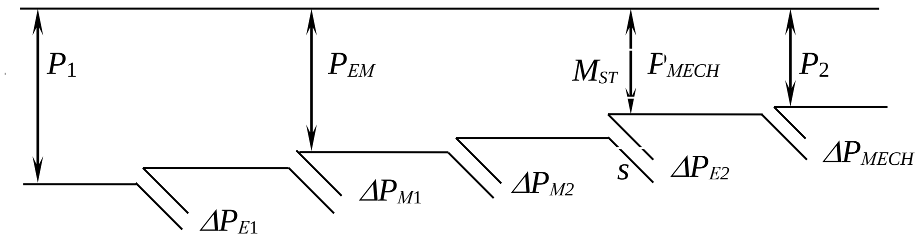


Fig. 9.9

power, $P_2 = 2\pi n_2 M / 60 = \omega_2 M$ - motor mechanical power.

The conversion of electrical energy into mechanical energy are balanced with losses: $\Delta P = \Delta P_E + \Delta P_M + \Delta P_{MECH}$, where $\Delta P_E = \Delta P_{E1} + \Delta P_{E2}$ are electrical losses in the windings, $\Delta P_M = \Delta P_{M1} + \Delta P_{M2}$ - magnetic losses in the core and ΔP_{MECH} - mechanical losses.

The motor efficiency factor is determined by the formula:

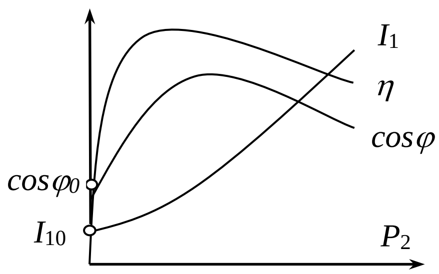


Fig. 9.10

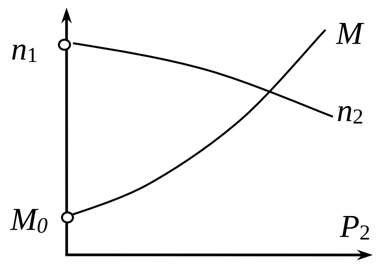


Fig. 9.11

$$\eta = P_2 / P_1 \text{ for } \eta < 0.85 \quad \text{or} \quad \eta = 1 - \Delta P / P_1 \text{ for } \eta > 0.85.$$

Motor properties as the electrical circuit part are estimated according to the working characteristics (fig.9.10), the dependencies of current I_1, efficiency factor η and power factor $\cos\varphi$ on mechanical power P_2.

Power factor of induction motors is less than 1, because the motor uses reactive power (which is necessary to create a magnetic field) together with active power, so

$$\cos\varphi = P_2 / \sqrt{(P_2^2 + Q^2)}.$$

It is important that motor always work with loading close to nominal, for its highest power factor.

Motor properties as the electric drive part are estimated according to the working characteristics (fig. 9.11) and dependencies $n_2(P_2)$, $M(P_2)$.

PART 10. SYNCHRONOUS MACHINES

Synchronous AC machine is the one, which rotor rotates with the same frequency as the rotating magnetic field of the stator. Synchronous machines are widely used as generators, motors and synchronous compensators.

However, the most important role of synchronous machine is a three-phase synchronous generator. In modern power stations, regardless of their type and power, the source of electric current is exclusively a three-phase synchronous generator. In thermal power plants generators are driven with steam or gas turbines. These generators are called turbo generators. Their speed makes *3000 rpm* and *1500 rpm*, their generated power makes up to *800-1000 MW*. Water turbines drive hydrogenerators. Their speed is between *50 to 600 rpm* and their generated power makes - *200-600 MW*.

Synchronous motors operate with power ranging from *10 kW* to couple of thousands of *kW*. They are used mainly in drives that do not need to regulate the frequency of rotation, such as powerful pumps, fans, compressors, ball and rod mills, rolling mills, steel mills, as well as powerful units converting AC to DC. Advantages of synchronous motors make them more welcome at large powers comparing to the asynchronous ones. Therefore, asynchronous motors are widly spread in industry. Synchronous micromotors with power ranging from tenth of watts to hundreds of watts are used in automation, tools making and computer technologies. They also are used in electrical appliances and control systems. These comprise inductor/ reactive/ hysteresis/ stepping motors and motors with permanent magnets. Synchronous compensator - is a synchronous motor working in idle mode that serves to improve the power factor of the network.

Synchronous machines have the following advantages: high efficiency and power factors, motor has the rigid mechanical characteristic, generator e.m.f. is independent from the load. However, they have complex structure, - motors require two voltage sources (three-phase AC and DC). Besides, synchronous motor’s starting is more complicated than the induction motor one.

Electromagnetic system of synchronous machine is a branched symmetrical magnetic circuit. The main parts of it are an unmovable stator with a three-phase winding and a moving rotor with an excitation winding, powered by direct current. Stator has non-poles and rotor can have both designs with clear poles (fig. 10.1) and with non salient ones. Radial gap of machine with clear poles is uneven and it is even for the non-poles machine. For the clear poles rotor each pole is made as a separate unit, which consists of a core 1, pole tip 2 and pole coil 3.

All poles are mounted on the rim 4, which is a yoke of magnetic system, which closes its magnetic fluxes.

In the synchronous motor, as well as in the asynchronous one, the current that flows in the three-phase stator winding creates a rotating magnetic field. Excitation DC I_E flows in the rotor’s winding and it is fed by DC power - opposite to the induction motors, where the current that flows in the rotor winding is induced by rotating magnetic field of the stator. Rotation speed of synchronous motors is constant and equal to the frequency of the rotating magnetic field of the stator. The interaction of stator rotating magnetic field and the rotor current creates the rotating moment (due to the magnetic field force). Synchronous motors are made only with clear poles.

The rotor of the generator acts as an inductor, it is a system of magnets, which windings are powered by direct current through slip rings. Pole coils are fixed by means of pole pieces to the rotor. They are fed by DC power. The low-speed synchronous generator rotor (speed under 300 rpm) is performed with clear poles, and high-speed generators rotor (speed 3000 and 1500 prm) is performed with non-poles.

The rotor excitation DC creates constant in time magnetic field. Rotor's rotating creates a variable in space magnetic field, which induces in three phase stator winding a variable electromotive force (due to the induction magnetic field). If the load is connected to the stator winding, the armature current I_A will be flowing through it because the stator acts as an armature.

In synchronous machines of low power the principle of excitation by permanent magnets is used. They are placed on the rotor. This excitation method does not need the excitation winding and contact rings. As a result, the machine construction is simpler.

Since the generator is designed to generate sinusoidal electromotive force, the magnetic flux (magnetic induction) in the air gap between the stator and rotor should

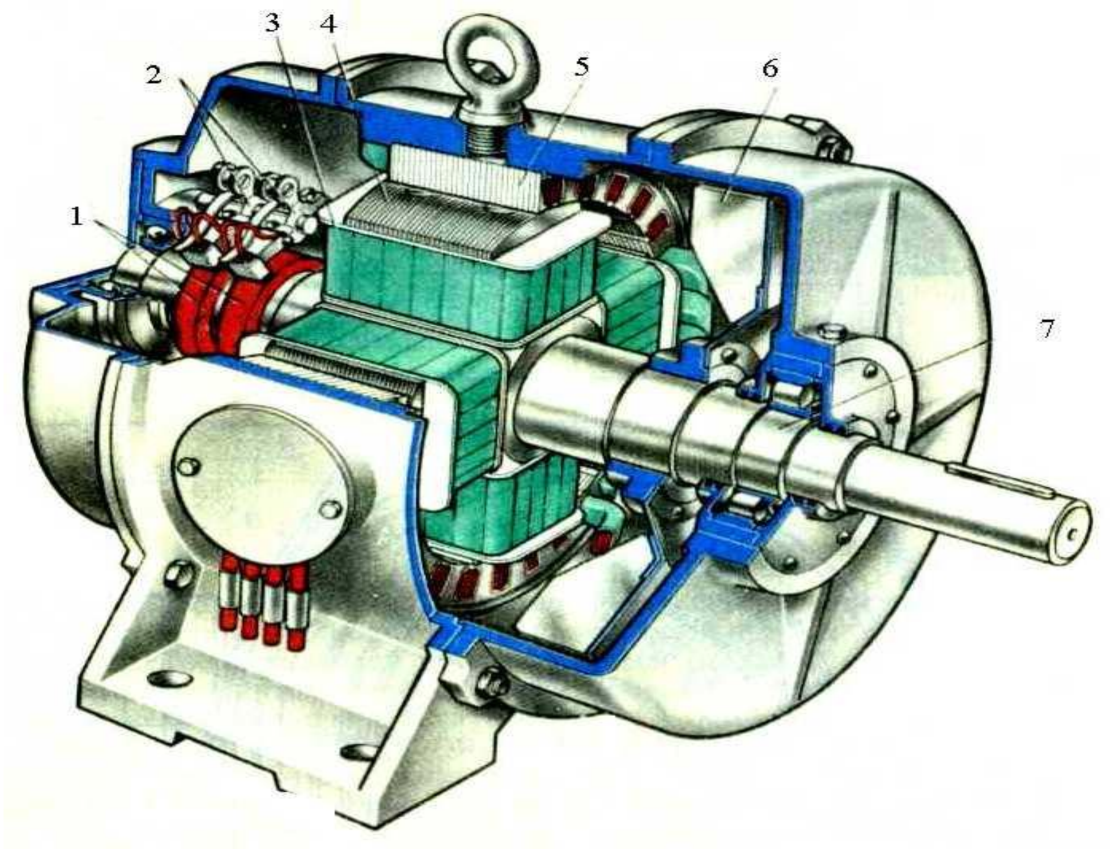


Fig. 10.1. Synchronous generator
1– slip rings; 2 – brush-holder; 3 – rotor pole winding; 4 – pole tip; 5 – stator core; 6 – ventilator; 7– shaft

also change in sinusoidal way. Clear poles machines achieve this with uneven radial gap, providing a form of pole tip (like a mushroom head). The nonsalient rotor machines achieve this effect with uneven distribution of the excitation (rotor) winding.

Synchronous machines, as well as DC machines, have an armature reaction effect, when the armature (stator) rotating magnetic field affects the basic magnetic field of the inductor (rotor winding). The resulting magnetic flux creates the combined effect of the magnetic fluxes of armature and inductor windings. Armature reaction in synchronous machines, unlike DC machines, depends from the load. At generator’s active load armature winding magnetic flux is behind the main magnetic flux at an angle 90^0. This effect is called a transverse armature reaction. At the active-inductive load, the armature winding magnetic flux is opposite to the main magnetic flux, so the resulting magnetic field weakens. At the active-capacitive load, the armature and main magnetic fluxes have the same direction, so the resulting magnetic field increases.

The frequency of the induced in armature winding electromotive force depends on the number of pairs of poles and rotor speed: $n_0 = 60 f / p$.

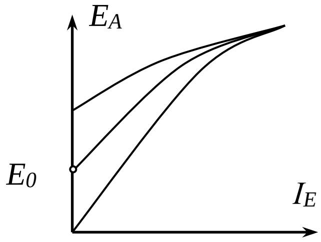


Fig. 10.2

The magnitude of generator-induced electromotive force is: $E_1 = \omega_1 w_1 k_w \Phi_m$,
where w_1- is a number of coils in the armature (stator) winding, k_w- winding ratio, Φ - the magnetic flux amplitude of inductor (rotor) poles.

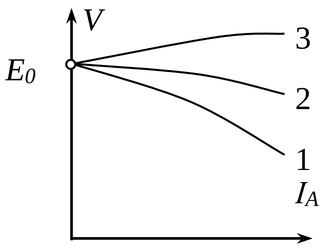


Fig. 10.3

One of the important characteristics of synchronous generator is a characteristic of idling (magnetic characteristic) – that is the dependence of electromotive force in the armature winding E_A from excitation current I_E at a constant speed of rotation, without load. Generator idle characteristics (fig. 10.2) is similar to the spread of branches. This is due to the influence of hysteresis phenomena of machine magnetic system. With increasing hysteresis losses in magnetic circuit, the spread of branches becomes wider.

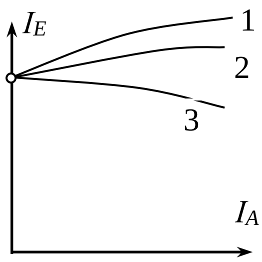


Fig. 10.4

As seen from the idling characteristic, electromotive force increases almost linearly with increasing generator excitation current. The slope characteristic is determined by the value of the air gap, (the bigger gap the bigger slope). With further growth of excitation current generator magnetic system saturates. The generator external characteristic is a dependence of voltage at the stator winding clamps from the load current (armature current) $V(I_A)$ at constant excitation current and speed. Figure 10.3 shows the synchronous generator external characteristics at different nature of the load. The current growth at the active load (curve 2) is accompanied by a voltage dropping because of growing voltage drop in the stator winding. The current growth at inductive load (curve 1), is accompanied by the voltage dropping because of the demagnetizer effect of armature reaction. The current growth at capacitive load (curve 3) is accompanied by voltage growth due to the magnetizer effect of armature magnetic field.

Generator adjusting characteristic is a dependence of excitation current from armature current $I_E(I_A)$ at a constant rotation frequency and constant output voltage. It shows how the excitation current changes in response to the load (armature) current attempts to maintain the constant voltage at generator clamps. Figure 10.4 shows the adjusting characteristics corresponding to different kind of load. The increasing armature current at active load (curve 2) needs to have reduced excitation current to maintain constant voltage at the clamps. The increasing armature current at inductive load (curve 1) needs to have reduced excitation current as well. The increasing armature current at capacitive load (curve 3) needs on the contrary to have an increased excitation current.

Synchronous machines are not reversible as DC machines. That means the same machine cannot work both as a generator and a motor. This is because the motor does not have its own starting moment (unlike induction motor). The currents of three-phase stator winding, which is powered by a three-phase voltage source creates a rotating magnetic field. Winding that is located on the rotor is powered by a DC voltage and DC runs through it. Power performance of the stator rotating magnetic field creates the motor rotation moment on the rotor winding wires and thus rotor, "involved in synchronism", begins to rotate with the frequency n_2 equal to the frequency of the stator rotating magnetic field n_1. They are called synchronous motors because the rotor speed and rotating magnetic field speed are the same $n_2 = n_1 = n_0$. This is a necessary condition of the motor working. However, the motor does not have it`s own starting moment. It is not run simply by connecting to the network.

Consequently, additional motor is needed to start synchronous motor. This start is called a synchronous one. Alternative start is an asynchronous one. Additional short circuit winding is put into the rotor grooves (as at induction motor). It turns on when motor starts, so the motor runs as an induction, and after start (after the motor is "involved in synchronism") the relay turns off this winding, turning on the excitation

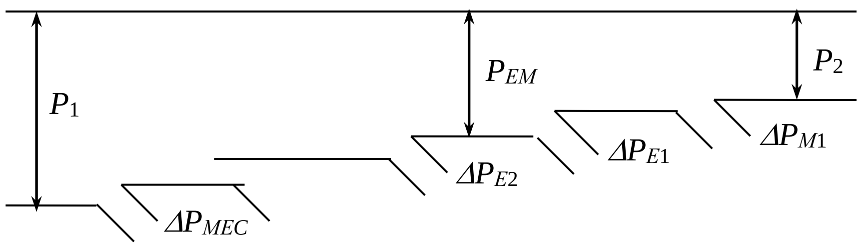


Fig. 10.5

winding for the motor to run as a synchronous one. So, synchronous motors, unlike generators, have additional starting winding. To start high power synchronous motors lower voltage is applied to limit starting currents.

Synchronous generator energy conversion is illustrated in diagram at fig. 10.5. Generator total power losses consist of the sum of electrical ΔP_E, magnetic ΔP_M and mechanical ΔP_{MECH} losses $\Delta P = \Delta P_E + \Delta P_M + \Delta P_{MECH}$. Electrical losses are the losses in the stator $\Delta P_{E1} = 3R_1 I_1^2$ and rotor $\Delta P_{E2} = R_2 I_2^2$ winding, where R_1, R_2 are stator and rotor phase winding resistances, I_1, I_2 - stator and rotor phase currents. General electrical losses are $\Delta P_E = \Delta P_{E1} + \Delta P_{E2}$. Generator magnetic losses are losses in the cores of the stator $\Delta P_M = \Delta P_{M1}$.

The generator efficiency factor is the ratio of power, which generator gives to the network P_2, to mechanical power P_1, which it uses: $\eta = P_2 / P_1 = P_2 /(P_2 + \Delta P)$.

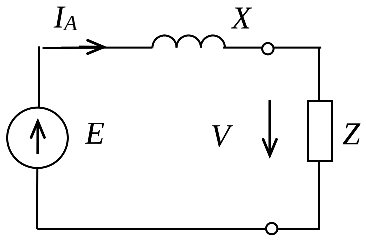


Fig. 10.6

Electromagnetic power transmitted from the rotor to the stator is $P_{EM} = M\omega$,
where M - is an electromagnetic moment and ω - a rotation frequency.

The power losses in the armature winding comparing to an active capacity are minor, so the machine active power is assumed to be equal to electromagnetic power: $P_{EM} = P = 3VI\cos\varphi$. Thus the moment is $M = 3VI\cos\varphi / \omega$.

The generator and motor energy diagrams are similar, but the motor consumes electrical power P_1 and produce mechanical power P_2.

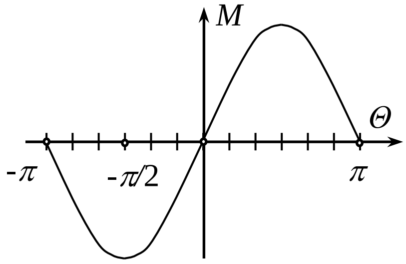


Fig. 10.7

Electromotive forces induced in the stator winding by armature magnetic flux and dissipation magnetic flux are accordingly: $\underline{E}_A = -j\underline{I}_A X_A$, $\underline{E}_D = -j\underline{I}_A X_D$, where X_A- is an armature reactance, X_D - a dissipation reactance. Then the total reactance is $X = X_A + X_D$.

Simplified substitutional scheme of synchronous machine is shown at fig.10.6. Its phase equation according to the second Kirchhoff's law is the following (neglecting the armature active losses): $\underline{E} + \underline{E}_A + \underline{E}_D = \underline{I}_A \underline{Z} = \underline{V}$.

Thus, the equation of a synchronous generator is $\underline{V} = \underline{E} - jX\underline{I}_A$.

Vector of generator electromotive force $\overline{E}$ is ahead of voltage vector $\overline{V}$ at the generator clamps at an angle Θ. This angle increases with the load current, and it is called a synchronous machine load angle. If $\overline{E} > \overline{V}$, synchronous machine operates in generator mode at $\Theta > 0$. If $\overline{E} < \overline{V}$, it operates in the motor mode at $\Theta < 0$. And if $\Theta = 0$ the machine works in an idle mode.

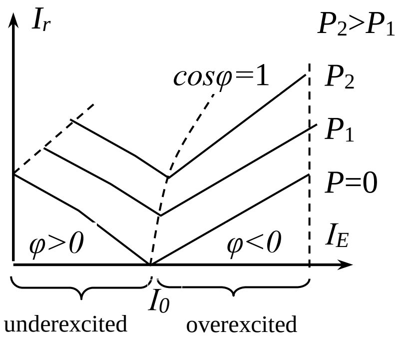


Fig. 10.8

The magnetic field of the armature rotates synchronously with the rotor, but the axis of the armature and inductor magnetic fields is shifted by an angle Θ. The moment on the generator shaft changes according to sinusoidal law (fig.10.7): $M = M_{\max}\sin\Theta$, where $M_{\max}$ - is a maximum moment. The dependence $M(\Theta)$ is the machine angle characteristic. Synchronous machine functions in the motor mode at $M > 0$, when $0 < \Theta < \pi$ and in the generator mode at $M < 0$, when $-\pi < \Theta < 0$.

Generator steady mode corresponds to load angle of $-\pi/2 < \Theta < 0$ and motor steady mode corresponds to load angle $0 < \Theta < \pi/2$.

Mechanical characteristic of synchronous motor $n(M)$ is rigid, the rotation speed does not depend on the moment and it is not regulated. That's why the synchronous motors are not so popular.

If motor moment $M = 3VI\cos\varphi/\omega$ is not changed, it keeps steady active power $P = VI\cos\varphi = const$ and the active component of consumption current I_a. However, if the excitation current I_E changes (for the constant power motor feed current), it changes actually its reactive component I_r (consumption reactive power Q and power factor $cos\,\varphi$). Thus, power factor can be changed with the excitation current. The dependence of the reactive component of the feed current of motor from the excitation current $I_A(I_E)$ at constant active power is called a V-like motor characteristics (fig. 10.8).

The graph shows that the minimum stator currents will be when $cos\,\varphi = 1$ for excitation current I_0. The machine at this mode does not give to the network and does not consume from the network any reactive power. The area located to the left of the current I_0 corresponds to the currents with the phase lagging behind the voltage (inductive character of the load), and is called an underexcited operation mode. The area located to the right of the current I_0 corresponds to the currents which phases are ahead of voltage (capacitive character of the load), and is called an overexcited operation mode. Synchronous motors can create an active-inductive load as well as an active-capacitive load depending on the excitation current value. Besides, their $\cos\varphi$ can also be equal to 1. Synchronous motors that work at idle overexcited mode $I_E > I_0$ are called synchronous compensators. They are connected as capacitor banks in parallel to the consumers with big reactive power (induction motors, transformers). Synchronous compensators give to the network capacitive reactive power Q_C that compensate reactive inductive power of the consumers Q_L, resulting in higher $\cos\varphi$.

The following conclusions can be made when comparing synchronous motors with induction motors:

- their stator structures are the same,
- their rotor structures are different: induction motors are manufactured with short circuit or phase rotor, while synchronous motors are made exclusively as explicit-pole ones. Besides, an additional DC source is to be used to feed their excitation winding,
- mechanical characteristic of induction motor is soft (depending on load) as well as their speed regulation, whereas the synchronous motor mechanical characteristic is rigid (independent of load), so its speed can not be regulated,
- asynchronous starting of synchronous motors is more complicated than the starting of induction motors (due to short circuited rotor), i.e. additional starting winding), but starting characteristics of induction motors with phase rotor are better than starting characteristics of synchronous motors,
- the maximum moment of synchronous motor is proportional to the voltage and the maximum moment of induction motor is proportional to the square of the voltage, so induction motors are more sensitive to voltage changes (that fact decreases their starting moment),
- induction motors create only active-inductive load, that’s why their $\cos\varphi$ is not big, whereas synchronous motors can create both active-inductive and active-capacitive loads depending on the excitement current value. Their $\cos\varphi$ can also reach 1.

Attachment

№	***Greek letters***		
1	*Α*	*α*	*alfa*
2	*Β*	*β*	*beta*
3	*Γ*	*γ*	*gamma*
4	*Δ*	*δ*	*delta*
5	*Ε*	*ε*	*epsilon*
6	*Ζ*	*ζ*	*dzeta*
7	*Η*	*η*	*eta*
8	*Θ*	*θ,*	*teta*
9	*Ι*	*ι*	*jota*
10	*Κ*	*κ*	*kapa*
11	*Λ*	*λ*	*lambda*
12	*Μ*	*μ*	*miu*
13	*Ν*	*ν*	*niu*
14	*Ξ*	*ξ*	*ksi*
15	*Ο*	*ο*	*micron*
16	*Π*	*π*	*pi*
17	*Ρ*	*ρ*	*ro*
18	*Σ*	*σ,ς*	*sigma*
19	*Τ*	*τ*	*tau*
20	*Υ*	*υ*	*ipsilon*
21	*Φ*	*φ*	*fi*
22	*Χ*	*χ*	*hi*
23	*Ψ*	*ψ*	*psi*
24	*Ω*	*ω*	*omega*

Physical values designation and units

Value	***Designation***	***Dimension***
Resistance	*R*, Ω	***Om***
Reactance	*X*, Ω	***Om***
Impedance	*Z*, Ω	***Om***
Conductance	*G, Sm*	***Simens***
Susceptance	*B, Sm*	***Simens***
Admittance	*Y, Sm*	***Simens***
Capacity	*C, F*	***Farada***
Inductance	*L, H*	***Henry***
Inductance mutual	*M, H*	***Henry***
Electromotive force	*E, V*	***Volt***
Potential	*φ, V*	***Volt***
Voltage	*V, V*	***Volt***
Current	*I, A*	***Amper***
Active power	*P, W*	***Watt***
Reactive power	*Q, VAr*	***Volt-Amper reactive***
Total power	*S, VA*	***Volt-Amper***
Magnetomotive force	*F, A*	***Amper***
Magnetic induction	*B, T*	***Tesla***
Magnetic field tension	*H, A/m*	***Amper per meter***
Magnetic stream	*Φ, Wb*	***Weber***
Linkage	*ψ, Wb*	***Weber***
Magnetic permeability (absolute)	μ_a,, *Гн/м*	***Henry per meter***
Magnetic permeability (relative)	μ	
Magnetic constant	μ_0, *Гн/м*	$4\pi\cdot10^7$
Frequency	*f, Hz*	***Herz***
Angular frequency	*ω, rad/s*	***radian per second***
Length	*1, m*	***meter***
Hight, depth	*h, m*	***meter***
Layer	*δ, d, m*	***meter***
Arial	*S*, m^2	***square meter***
Magnetic resistance	R_m	
Number of turns	*w*	
Force	*F, N*	***Newton***
Work (energy)	*W, J*	***Joule***
Charge	*Q, C*	***Coulomb***

CONTENT

ДЛЯ НОТАТОК

Навчально-методична література

Куземко Н. А.

Electrical circuits.

LECTURES

Комп'ютерне макетування та верстка *А. П. Катрич*

Формат 60x90/16. Обл. вид. арк. 2,42. Тираж 30 прим. Зам. № 3242.

Тернопільський національний технічний університет імені Івана Пулюя.
46001, м. Тернопіль, вул. Руська, 56.
Свідоцтво суб'єкта видавничої справи ДК № 4226 від 08.12.11.