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The influence of the cinematic parameters of movement and sprung mass vibrations of wheeled vehicles on the move along the curvedlinear sections of the way

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Abstract: The method of study the stability with regard to the skid of wheeled vehicles with the consideration of variable velocity value along the curvilinear sections of the way and longitudinal angular oscillations of the sprung part has been developed. The longitudinal - angular oscillations of the sprung part reduce the critical speed value of steady motion; accelerated movement of the vehicle along the curvilinear section of the way for larger values of acceleration of the front axle skid occurs at lower velocity values (rear for the larger ones), for slow motion on the contrary. As for the impact of the power characteristics of the sprung system, then for the same values of all other parameters (amplitudes of longitudinal - angular oscillations, static deformations, acceleration (deceleration)), the progressive characteristic of the sprinkler system corresponds to the greater value of the critical speed of the stable motion as a regressive one. The obtained results can serve as a base for study the influence of the kinematic parameters of the vehicle's motion on its controllability.

Keywords: steady motion, sprung and unsprung parts, critical speed value, longitudinal - angular oscillations.

1. Problem statement

Steerability, soft riding and steady motion are one of the most important running quality of wheeled vehicles (WV). And they are to some extent dependent on the external factors (inequality and curvature of the way, the state of its coverage). At the same time, they determine the kinematic parameters of WV movement and the relative oscillations of the sprung and unsprung parts. Regarding to the relative oscillations of the indicated parts, they, in addition to external factors (which are the main causes of their occurrence), are additionally determined by the power characteristics of the sprung system (SS) and the elastic tires. The dynamics of the sprung and unsprung parts of WV, and thus, power characteristics PH, greatly affect into the steady motion along the curvilinear sections of the way reducing the critical speed value of steady motion. If for the case of linear and partially for the nonlinear power characteristics PH in terms of vertical or transverse angular oscillations, the problems of the impact of the dynamics of the WV sprung part on the stability motion have been considered, then they did not find proper development for longitudinal - angular oscillations. The last one, to a lesser extent, have no effect on the stability of the movement both vertically and transversely - angular

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oscillations. In addition, the variable speed velocity of WV along the curvilinear sections of the way also affects into its steady motion. The study has been devoted to the partial development of this problem.

In the main publications concerning the steady motion of WV taking into account the dynamics of the SS, cases of linearly elastic characteristics of shock absorbers and simplest power characteristics of damper devices have been considered [1-5]. However, the SS with linear power characteristics does not fully ensure, first and foremost, ergonomic conditions of WV running [6]. In studies [7-12] an attempt was made to analyze the influence of some classes of SP with the nonlinear power characteristics of shock absorbers on SM dynamics for the simplest cases of oscillations of the sprung part (SP). At the same time, it has been noted that the longitudinal - angular oscillations of WV and variable speeds play very important role in the steady motion as transverse angular and vertical one. For the indicated oscillations, there is a need to consider more complex physical, but on the contrary, mathematical models. The research urgency of the steady motion increases primarily with the growth of the WV speed and the use of suspension brackets with a nonlinear (including controlled) characteristic of the change in the restoring force.

2. Task assignment

Three mass mechanical system as a physical model of the study object has been chosen to study the complex influence of variable speed and nonlinear longitudinal - angular fluctuations of the SP, on the WV steady motion along the curvilinear sections of the way. It includes the sprung (front and rear axles) and unsprung parts. They interact with each other through the elements of the suspension system - elastic shock absorbers and dampers.

Basic assumptions about external factors of motion during the KTZ movement along the path with inequalities:

- the inequalities of the road under the right and left wheels have the same vertical section;
- reaction forces of the skid of the front left ($Q_{1l.}$) and right ($Q_{1r.}$) and rear left ($Q_{2l.}$)and right $Q_{2r.}$ the rights of the rear axle are proportional to the normal dynamic forces of pressure $N_{il.}$ or $N_{ir.}$ on the support surface of the road (($i=1$ for the front and $i=2$ - for rear axles);).

As for the power characteristics of the right and left sides of the sprung system it is obvious that they are identical, but, taking into account the above mentioned statements, follows that the SP of the WV, through the onset of the inequality of the road, carries longitudinal-angular oscillations. The values of the forces of the right and left sides of the sprung system are described by the dependencies:

- Elastic forces of shock absorbers of the front ($F_{1r.}$ and $F_{1l.}$) and rear ($F_{2r.}$ and $F_{2l.}$) suspensions $F_{il}=c_i\Delta_{il}^{\nu+1}$, $F_{ir}=c_i\Delta_{ir}^{\nu+1}$ ($c_i,\ \nu$ - stable, $\Delta_{ir.}$ - deformation of the elastic element of the right side, $\Delta_{il.}$ - deformation of the elastic element of the left side
- the resistance forces of damper devices of the right R_{ir} and left $R_{ir}=$ sides as speed functions of their deformation $R_{il}=\alpha_i\dot{\Delta}_{il}^{s}$, $R_{ir}=\alpha_i\dot{\Delta}_{ir}^{s}$ (α_i, s - stable), $\dot{\Delta}_{ir}$ and $\dot{\Delta}_{il}$ - deformation rate corresponding to the left and right dampers.

The task is to find the critical speed velocity V stable motion along the curvilinear sections of the way with a radius of curvature ρ as functions of the amplitude of longitudinal - angular oscillations, the main parameters that describe the power characteristic of SS and acceleration (deceleration) of WV motion.

3. Method of solving

The basis for solving the problem will be the kinetic-statics equation of the system, which is not sprung and unsprung parts of the WV, for the case of its movement along the curvilinear sections of the way [13].

$$\begin{aligned} &\vec{F}^{e}+\vec{\Phi}=0,\\ &\vec{M}_O^{e}+\vec{M}_O^{\Phi.e}+\vec{M}_O^{\Phi.r}=0, \end{aligned} \tag{1}$$

n which respectively $\vec{F}^{e},\vec{\Phi}$ - the main vectors of external forces and forces of inertia considered by the three mass systems, but $\vec{M}_O^{e},\ \vec{M}_O^{\Phi.e},\vec{M}_O^{\Phi.r}$, - the main points of a relatively arbitrary center O external ($\vec{M}_O^e$) forces and forces of inertia in relative ($\vec{M}_O^{\Phi.r}$)and figurative ($\vec{M}_O^{\Phi.e}$) movement. As for the first of them, it is the active forces of weight: front ($\vec{P}_1$), rear ($\vec{P}_2$) axles (unsprung part), sprung part - ($\vec{P}_3$); driven by the driving force" of the rear left $F_{a2l.}$ and the rear right $F_{a2r.}$ wheels.

Passive forces: normal reaction of road surface acting on front tires ($\vec{N}_{1r.}, \vec{N}_{1l.}$) and rear $\vec{N}_{2r}, \vec{N}_{2l.}$ axles; skid reaction forces of the front left ($Q_{1l.}$) and right ($Q_{1r.}$) and rear left ($Q_{2l.}$)and right $Q_{2r.}$ and rear axles; forces of resistance to rolling of front right $F_{1or.}$ and left $F_{1ol.}$ wheels.

The forces of inertia of the portable (curvilinear) motion are reduced to the main vector of forces of inertia of the front $\vec{\Phi}_1^e$, rear $\vec{\Phi}_2^e$ axles and sprung mass - $\vec{\Phi}_3^e$. They are determined by correlation: $\vec{\Phi}_i^e = \vec{\Phi}_i^{e.\tau} + \vec{\Phi}_i^{e.n}$ $\Phi_i^{e.n} = \frac{P_i}{g}\frac{V^2}{\rho}$, $\Phi_i^{e.\tau} = \frac{P_i}{g}w$, w - the value of acceleration (deceleration) of the WV motion. The vectors $\vec{\Phi}_i^{e.n}$ are directed toward the convexity of the trajectory of motion, a $\vec{\Phi}_i^{e.\tau}$ - in the opposite direction to the velocity vector of the center of mass of the WV (for accelerated motion) and to the side of the velocity vector - for the slowed motion.

Notes 1:

1. In the work, it is considered that the WV is moving along the curvilinear sections of the road with the constant acceleration in magnitude, therefore V is the meaning of the speed of the WV at the given time;
2. The change of the radius of the trajectory curvature to the points which coincide with the centers of the masses of the sprung part, as well as the front and rear axles are not taken into account;
3. The influence of the road inequalities on the vertical fluctuations of the WV may be the subject of separate investigations, and fluctuations of the SM are considered to be due to initial disturbances.
4. As for the forces of inertia in relative motion (longitudinally - angular oscillations) then their relative transverse axis, passing through the center of mass C can be reduced to the moment of inertia forces $M_C^{\Phi.r}$, the value of which is equal $M_C^{\Phi.r} = -I_C\varepsilon$, where I_C - moment of inertia of the sprung mass relative to the axis indicated above, but ε - angular acceleration of the SM movement. The basis for determining the angular acceleration of the SM may be longitudinal differential equation - angular oscillations of this part and the relevant initial conditions. The differential equation of the specified part of the WV, more precisely, its right-hand side is determined by the forces acting on it (see Fig. 1).

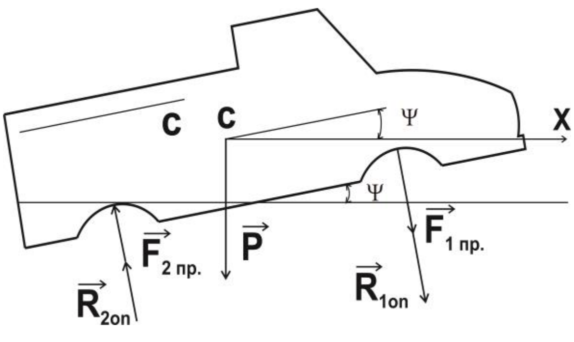

Figure 1. Scheme for studying the dynamics of the SM of the WV

On the given figure $\vec{F}_i = \vec{F}_{ir.} + \vec{F}_{il.}$, $\vec{R}_i = \vec{R}_{ir.} + \vec{R}_{il.}$ are the superficial forces of shock absorbers or damping devices of the right and left side of the front ($i=1$) and ($i=2$) rear suspension, as well as $\psi(t)$ - value of the rotation angle of the SP at an arbitrary time. Thereby, $M_C^{\Phi.r} = -I_C\frac{d^2\psi}{dt^2}$, namely, the differential longitudinal-angular oscillation takes the form

$$I_C\frac{d^2\psi}{dt^2} = -c_1a\left(a\psi(t) - \Delta_{cm.}\right)^{\nu+1} - c_2a\left(b\psi(t) + \Delta_{cm.}\right)^{\nu+1} - \alpha_1a\left(a\frac{d\psi}{dt}\right)^s - \alpha_2b\left(b\frac{d\psi}{dt}\right)^s \quad (1)$$

where a, b - parameters that determine the position of the center of mass of the SM, $\Delta_{cm.}$ - the static deformation of elastic shock absorbers.

In order to determine the critical value of the velocity of steady motion, taking into account the fluctuations of the Sm, it is not necessary to have the law of changing these oscillations of the specified part of the WV, but only their amplitude, for example, the amplitude of the initial perturbation. Then max $M_C^{\Phi.r}$ =max $I_C\frac{d^2\psi}{dt^2}$, but max $I_C\frac{d^2\psi}{dt^2}$ = max $G(\psi,\dot{\psi})$, where $G(\psi,\dot{\psi})$ - right part of the equation (2). In addition it is known that the resistance forces of the damping devices cause the PM to decay the oscillations of the specified part of the WC, so the right-hand side of equation (2) assumes the maximum value at the moment of maximum amplitude, that is, at the "initial" time point for which $a_{\psi}|_{t=0}=\bar{a}_{\psi}$. Thereby, max $M_C^{\Phi.r}$ =max $\left[c_1 a\left(a\psi(t)-\Delta_{cm.}\right)^{\nu+1}+c_2 b\left(b\psi(t)+\Delta_{cm.}\right)^{\nu+1}\right]$. Considering the above, it can be argued that the maximum value of the inertia forces moment of the PM in relative motion relative to the axis $C\zeta$ takes the value

$$\max M_C^{\Phi.r} = \bar{M}_C^{\Phi.r} = \left(c_1 a(a\bar{a}_{\psi}-\Delta_{cm.})^{\nu+1}+c_2 b(b\bar{a}_{\psi}+\Delta_{cm.})^{\nu+1}\right) \qquad (2)$$

The above considerations allowed to avoid time-consuming procedures for integrating the nonlinear differential equation (1), and hence, go directly to the procedure for finding the critical speed of steady motion taking into account longitudinal - angular oscillations of the WV and the variable at the speed of movement. To do this, let's turn from the vector relations (1) to their scalar counterparts

$$\begin{aligned}
&F_{2lppy.}+F_{2rppy.}-F_{1loo.}-F_{1roo.}-\Phi_1^{e.\tau}-\Phi_2^{e.\tau}-\Phi^{e.\tau}=0,\\
&Q_{1r.}+Q_{1l.}+Q_{2r.}+Q_{2l.}-\Phi_1^{e.n}-\Phi_2^{e.n}-\Phi^{e.n}=0,\\
&N_{1r.}+N_{1l.}+N_{2r.}+N_{2l.}-(P_3+P_1+P_2)=0,\\
&\left(\Phi_1^{e.n}+\Phi_2^{e.n}\right)R+\Phi^{e.n}(R+h)+\left(N_{1r.}+N_{2r.}\right)d-(P_3+P_1+P_2)\frac{d}{2}=0,\\
&P_3a+P_2(a+b)-(N_{2r.}+N_{2l.})(a+b)+\left(\Phi_1^{e.\tau}+\Phi_2^{e.\tau}\right)R+\Phi^{e.\tau}(R+h)-\bar{M}_y^{\Phi.r}=0,\\
&(Q_{2r.}+Q_{2l.})(a+b)-\Phi_2^{e.n}(a+b)-\Phi^{e.n}a-(\Phi_1^{e.\tau}+\Phi_2^{e.\tau}+\Phi^{e.\tau})\frac{d}{2}+(F_{2lppy.}-F_{1loo.})d=0.
\end{aligned} \qquad (3)$$

where R, h, d - in accordance: tire radius; the parameter characterizing the height of the masses center placement of the SP over the axels, the distance between the axles of the front (rear) wheels. If it taken into account that the anti-skidding forces $\vec{Q}_1$, $\vec{Q}_2$ are determined by dependencies $Q_1=Q_{1r}+Q_{1l}=k_1\left(N_{1r.}+N_{1l.}\right)$, $Q_2=Q_{2r}+Q_{2r}=k_2\left(N_{2r.}+N_{2l.}\right)$, and the main vectors of inertial forces take the above values, then the system of equations (3) has been transformed into a form

$$\begin{aligned}
&F_{2lpyш.}+F_{2rpyш.}-F_{1lon.}-F_{1ron.}=\frac{P_3+P_1+P_2}{g}w,\\
&k_1(N_{1r.}+N_{1l.})+k_2(N_{2r.}+N_{2l.})=\left(P+P_1+P_2\right)\frac{V^2}{\rho g},\\
&N_{1r.}+N_{1l.}+N_{2r.}+N_{2l.}=P+P_1+P_2,\\
&P_3a+P_2(a+b)-(N_{2r.}+N_{2l.})(a+b)=\bar{M}_y^{\Phi.r}-\left[\frac{P_1+P_2}{g}R+\frac{P_3}{g}(R+h)\right]w,\\
&(P_3+P_1+P_2)\frac{d}{2}-\left(N_{1r.}+N_{2r.}\right)d=\frac{P_1+P_2}{g}\frac{V^2}{\rho}R+\frac{P_3}{g}\frac{V^2}{\rho}(R+h),\\
&k_2\left(N_{2r.}+N_{2l.}\right)(a+b)+(F_{2lpyш.}-F_{1lon.})d=\frac{P_2}{g}\frac{V^2}{\rho}(a+b)+\frac{P_3}{g}\frac{V^2}{\rho}a+\frac{P_3+P_1+P_2}{2g}dw.
\end{aligned} \qquad (4)$$

Notes 2.

1. Equations (4) have been written in projections on a Cartesian system with a start at the point of contact of the right front wheel to the road, and two axles are directed along and across of the WV;
2. The critical speed does not depend on the reference system choice, and the form of the equation (4) changes with the coordinate system.

In the above dependencies $\bar{M}_y^{\Phi.r}$ - the moment of the inertial forces of relative motion relative to the transverse axis and the point of contact of the left front wheel to the road, takes the following value $\bar{M}_y^{\Phi.r}=I_y\frac{\left(c_1a(a\bar{a}_\psi-\Delta_{cm.})^{\nu+1}+c_2b(b\bar{a}_\psi+\Delta_{cm.})^{\nu+1}\right)}{I_C}$, I_y - the inertia moment of the absorbed mass relative to the transverse axis, which passes through the contact point of the front left wheel and the road. The indicated value in accordance with the Huygens-Steiner theorem is determined by the relation $I_y=I_C+\frac{P_3}{g}\left(a^2+(R+h)^2\right)$.

After simple transformations from it critical values in view of the drift of the front V_1 and rear V_2 axles of WV can be done

$$V_1=\sqrt{\left(k_1g-\frac{k_1\left[\left(P_1+P_2\right)wR+P_3w(R+h)+g\bar{M}_y^{\Phi.r}\right]+(P_1+P_2+P_3)\frac{d}{2g}w-(F_{2lpyu.}-F_{1lon.})d}{P_1(a+b)+P_3b}\right)\rho},$$

$$V_2=\sqrt{\left(k_2g+\frac{k_2\left[(P_1+P_2)Rw+P_3(R+h)w-g\bar{M}_y^{\Phi.r}\right]-\left(P_3+P_1+P_2\right)\frac{d}{2g}w+(F_{2rpyu.}-F_{1ron.})d}{P_2(a+b)+P_3a}\right)\rho}. \quad (5)$$

In the specified ratios, the summands $(F_{2lpyu.}-F_{1lon.})$ and $(F_{2rpyu.}-F_{1ron.})$ describe the forces acting in the longitudinal direction of the WV on the tires and force it to accelerated (or slowed) motion. If further consider that $F_{1lon.}=F_{1ron.}$ and $F_{2lpyu.}=F_{2rpyu.}$, then the acceleration of the WV motion is determined by the ratio $w=\frac{F_{2lpyu.}+F_{2rpyu.}-F_{1lon.}-F_{1lon.}}{P_3+P_1+P_2}g$. The above gives the critical values for the introduction of the front and rear axles in the form

$$\tilde{V}_1=\sqrt{\left(k_1g-\frac{k_1\left[\left(P_1+P_2\right)wR+P_3w\left(R+h\right)+g\bar{M}_y^{\Phi.r}\right]}{P_1\left(a+b\right)+P_3b}\right)\rho},$$

$$\tilde{V}_2=\sqrt{\left(k_2g+\frac{k_2\left[\left(P_1+P_2\right)Rw+P_3\left(R+h\right)w-g\bar{M}_y^{\Phi.r}\right]}{P_2\left(a+b\right)+P_3a}\right)\rho}. \quad (6)$$

In this manner, critical $\tilde{V}$ taking into account the fact that the value of the speed of steady motion along the curvilinear section of the road is equal $\tilde{V}=\min\left(\tilde{V}_1,\tilde{V}_2\right)$. It should be noted that in the equations for $\tilde{V}_1$ and $\tilde{V}_2$ before $\bar{M}_y^{\Phi.r}$ is sign "–". This is due to the fact that SP performs relative longitudinal - angular oscillations, and the moment of inertia forces in the specified motion changes its size and direction. The most dangerous because of the steady motion will be the case, when the moment of inertia forces reduces the force of pressure on the road surface, and therefore the critical value of the speed corresponds to the minimum value of the moment of inertia forces.

A special case of the above dependencies for $w=0$ is the critical value of steady motion speed, provided by constant speed value, and for $w<0$ the results refer to the slow motion, i.e.

$$\tilde{\tilde{V}}_1=\sqrt{\left(k_1g+\frac{k_1\left[\left(P_1+P_2\right)wR+P_3w(R+h)-g\bar{M}_y^{\Phi.r}\right]}{P_1(a+b)+P_3b}\right)\rho},$$

$$\tilde{\tilde{V}}_2=\sqrt{\left(k_2g-\frac{k_2\left[(P_1+P_2)Rw+P_3(R+h)w+g\bar{M}_y^{\Phi.r}\right]}{P_2(a+b)+P_3a}\right)\rho}. \quad (7)$$

4. Conclusions

The obtained analytical dependencies show that the critical value of the steady motion along the curvilinear sections of the road:

- without taking into account longitudinal - angular oscillations of the SM is significantly overestimated;

- the greater e value of the oscillations amplitude corresponds the less value of the critical velocity;

- for the progressive power characteristic of the sprung system at small amplitudes of oscillations is greater than for the linear power characteristic (provided that the static strain is equal), and for the "big" ones - on the contrary;

- for higher values of the parameters of the WV base is smaller;

- critical velocity value is greater for the progressive characteristic of the sprung system at higher values of the static deformation of the sprung system, and for regressive - on the contrary.

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