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METHOD OF SEGMENTATION OF DETERMINED CYCLIC SIGNALS FOR THE PROBLEMS RELATED TO THEIR PROCESSING AND MODELING

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Summary: *In this work the method of adaptive segmentation of determined cyclic signals has been developed. The problem of segmentation (breakdown, division of the signal into distinctive segments) of the cyclic determined signal (of the cyclic numeric function and the cyclic interval function) with a segment structure has been solved. Examples of segmentation of simulated and real cyclic signals are provided; an assessment of the accuracy of the developed method of segmentation has been made.*
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Statement of the problem. Cyclic events, characterized by repeatability and in some cases identicality of stages developing in time or space, occur in various fields of science and technology. For example, activity of the heart is characterized by cyclic development of the contraction phases of the heart's compartments after time; process of breathing is characterized by a rhythmic, cyclic filling of lungs with oxygen. Registered during these processes cyclic signals, contain diagnostics information, which is displayed in relevant segments of implementation that characterize stages of development of the cyclic process in time. Therefore, automated detection and analysis of such time segments for a registered cyclic signal – is an important and relevant scientific and technical dilemma, salving of which will allow to create new diagnostic systems; including systems for automated diagnostics of heart diseases.

Analysis of the available investigations. In many works devoted to the study of cyclic signals, deterministic and stochastic approaches are used in order to build mathematical models for processing tasks. In particular, tasks related to the segmentation of the cyclic signals [1-3]. Such task are relevant when conducting a morphological analysis of cardiac signals; an analysis of the heart rhythm based on a cardiointervalogram, which has been obtained both in a state of rest of the patient and in a state of physical activity. Methods of segmentation provide an opportunity to obtain information about the segment structure of the cyclic signal, which on the other hand provides an opportunity to consider a sampling step [1,4], to evaluate the rhythmic structure (define the rhythm function) of the cyclic signal[5], to process and analyze it [6-9].

There are many methods of segmentation [10-16] developed for specific cyclic signals of different nature (electro-cardio signal, reocardial signal etc.). In practice, metric methods of segmentation in the time domain are often used: method of amplitude characteristics of the signal [11], method of the form function [13], method of standards (etalons) [14] and others.

Method of analysis of the amplitude characteristics of the signal suggests the measurement of amplitudes of the cyclic cardio sygnal and their comparison with the previously defined threshold values for the purpose of selecting segments that match the diagnosed segments (R-zones, which are used to be mentioned in medical practice in electrocardio diagnostics). One of the most common methods for selecting QRS segments (QRS - complexes), is the method of analyzing the difference function of first order (first derivative) and the differance function of the second order and the comparison of their extremums with the threshold values [12].

Though there exists a great number of developed methods of segmentation, when put into practice, they don't always correctly process real signals, since methods lack the mechanisms of adaptation to the peculiarities of the cyclic signal. Which are necessary, for instance, with pathologies,when the split of key segments of electrocardio signal (R-zones) appears. Furthermore, a problem appears, the essence of which is in the impossibility of using known methods when working on tasks of coherent processing of cyclic signals of different nature. The result of using these different methods, which are based on different mathematical models is that there is no single approach and methodology for segmentation of different cyclic signals. Such an issue leads to a necessity of creating new methods of processing cyclic signals and methods of segmentation, which would eliminate existing problems.

Purpose of this article. This article is devoted to solving the problem of segmentation of a deterministic cyclic signal with a segment structure.

Main part

Mathematical model of the deterministic cyclic signals. The simplest representation of deterministic cyclic functions is a cyclic numeric function that is a generalization of a periodic numeric function [1,2].

Definition. 1. Cyclic numeric function is the function $f(t) \in \mathbf{R}, t \in \mathbf{W}$, for which exists a numeric $T(t,n)$, that satisfies the conditions of the rhythm function [1] and thus the following relationship exists:

$$f(t) = f(t + T(t,n)), t \in \mathbf{W}, n \in \mathbf{Z}, \tag{1}$$

where, $\mathbf{W}$ – is the area of definition of cyclic numeric function.

Function $T(t,n)$ must satisfy the following properties:

$$\begin{aligned} &\text{a) } T(t,n) > 0, \text{ if } n > 0 \ (T(t,1) < \infty); \\ &\text{b) } T(t,n) = 0, \text{ if } n = 0; \\ &\text{c) } T(t,n) < 0, \text{ if } n < 0, \ t \in \mathbf{W}. \end{aligned} \tag{2}$$

For any $t_1 \in \mathbf{W}$ and $t_2 \in \mathbf{W}$, where $t_1 < t_2$, for function $T(t,n)$ a strict inequality needs to be maintained:

$$T(t_1, n) + t_1 < T(t_2, n) + t_2, \forall n \in \mathbf{Z}. \tag{3}$$

Let's define a cyclic numeric function with a segment structure.

Definition 2. Cyclic numeric function with a segment structure is called a function $f(t) \in \mathbf{R}, t \in \mathbf{W}$ with rhythm function $T(t,n)$, which is presented in a form (through process cycles, segment process cycles):

$$f(t) = \sum_{i \in \mathbf{Z}} f_i(t), t \in \mathbf{W}. \tag{4}$$

In practice, when conducting the processing of the cyclic signal it is required to define the full number of cycles of the studied signal, meaning $i = \overline{1,C}$, where C – is the number of segments cycles of the cyclic function; $f_i(t)$ – relates to the i – st cycle of the cyclic numeric deterministic function with a segment structure which is described as:

$$f_i(t) = f(t) \cdot I_{\mathbf{W}_i}(t), i = \overline{1,C}, t \in \mathbf{W}, \tag{5}$$

where $I_{\mathbf{W}_i}(t)$ – is the indicator function of the i - st cycle, which equals to:

$$I_{\mathbf{W}_i}(t)=\begin{cases}1,\ t\in \mathbf{W}_i,\\ 0,\ t\notin \mathbf{W}_i.\end{cases} \qquad (6)$$

Areas of definition W_i of the indicator function of the i-st process cycle, is defined through a half interwal in the case of the continuous process, meaning $\mathbf{W}=\mathbf{R}$:

$$\mathbf{W}_i=[t_i,t_{i+1}), \qquad (7)$$

where t_i – is the time of the start of the i - st process cycle.

In case of the discrete process $\mathbf{W}=\mathbf{D}$ of the area of definition W_i of the indicator function of the i - st process cycle, the next is determined:

$$\mathbf{W}_i=\{t_{i,l}, l=\overline{1,L}\},\ i=\overline{1,C}\ , \qquad (8)$$

where L – is the number of discrete countdowns per a cycle, $L=const$.

Cyclic deterministic function can also be submitted in such a form (through process zones, segment process zones):

$$f\ (t)=\sum_{i\in\mathbf{Z}}\sum_{j=1}^{Z} f_{i_j}(t),\ t\in\mathbf{W}\ , \qquad (9)$$

where Z – is the number of the zone segments per each cycle of the cyclic process;
$f_{i_j}(t),\ t\in \mathbf{W}_{i_j}$ – j - st zone in the i - st cycle of the cyclic deterministic process, which equals to:

$$f_{i_j}(t)=f\ (t)\cdot I_{\mathbf{W}_{i_j}}(t)=f_i(t)\cdot I_{\mathbf{W}_{i_j}}(t),\ i=\overline{1,C},\ j=\overline{1,Z},\ t\in\mathbf{W}\ . \qquad (10)$$

where $I_{\mathbf{W}_{i_j}}(t)$ – is the indicator function of the j - st zone in i - st cycle, which equals to:

$$I_{\mathbf{W}_{i_j}}(t)=\begin{cases}1,\ t\in \mathbf{W}_{i_j},\\ 0,\ t\notin \mathbf{W}_{i_j}.\end{cases} \qquad (11)$$

Areas of definition $\mathbf{W}_{i_j}$ of the indicator function of the j - st zone in the i - st cycle of the process is defined through a half interval in the case of the continuous process, meaning $\mathbf{W}=\mathbf{R}$:

$$\mathbf{W}_{i_j}=[t_{i_j},t_{i_{j+1}}), \qquad (12)$$

where t_{i_j} – is the time of the start of the j - st zone in the i - st process.

In case of the discrete process $\mathbf{W}=\mathbf{D}$ of the area of definition W_i of the indicator function of the i - st process cycle the next is determined:

$$\mathbf{W}_{i_j}=\{t_{i_{j,l}}, l=\overline{1,L_j}\},\ L=\sum_{j=1}^{Z} L_j,\ i=\overline{1,C},\ j=\overline{1,Z}\ \ , \qquad (13)$$

where L – is the number of the discrete countdowns per cycle, $L=const$;

L_j - is the is the number of the discrete countdowns per j -st zone.

Process (5), which relates to the i - st cycle of the cyclic discrete process is associated with the process (10), which relates to j - st zones of the cyclic discrete process, through such a dependency:

$$f_i(t)=\sum_{j=1}^{Z} f_{i_j}(t),\ t\in \mathbf{W},\ \forall i=\overline{1,C}\ . \qquad (14)$$

Areas of definition of the zone segments and cycle segments of the process with the segment structure are justified with the following:

$$\mathbf{W}_i=\bigcup_{j=1}^{Z}\mathbf{W}_{i_j}\ ,\ \bigcup_{i=1}^{C}\bigcup_{j=1}^{Z}\mathbf{W}_{i_j}=\mathbf{W}\ ,\ \mathbf{W}_{i_j}\neq\varnothing\ ,\ \mathbf{W}_{i_{j_1}}\bigcap\mathbf{W}_{i_{j_2}}=\varnothing,\ j_1\neq j_2. \qquad (15)$$

Cyclic zone structure of the cyclic process is set by the multitude of time points, which equal to the starting points of the zone segments of the cyclic process (segment zone structure):

$$\mathbf{D}_z=\left\{t_{i_j},\ i=\overline{1,C},\ j=\overline{1,Z}\right\},\ t_i=t_{i_1},\ \forall i=\overline{1,Z}\ . \qquad (16)$$

In case when in the studied signal it is possible to allocate only the multitude of the starting time points which equal to the starting points of the cycle segments, then the segment structure is defined through its cycles (cyclic segment structure):

$$\mathbf{D}_c=\left\{t_i,\ i=\overline{1,C}\right\}. \qquad (17)$$

Figure 1 shows a schematic description of the segment structure of the cyclic signal with a denotation of the duration of cycles segments $\left\{T_i,\ i=\overline{1,C}\right\}$, duration of zone segments $\left\{T_{i_j},\ i=\overline{1,C},\ j=\overline{1,Z}\right\}$, and the boundaries of starting points of the j - st segment t_{i_j} and its ending point $t_{i_{j+1}}$ in each cycle.

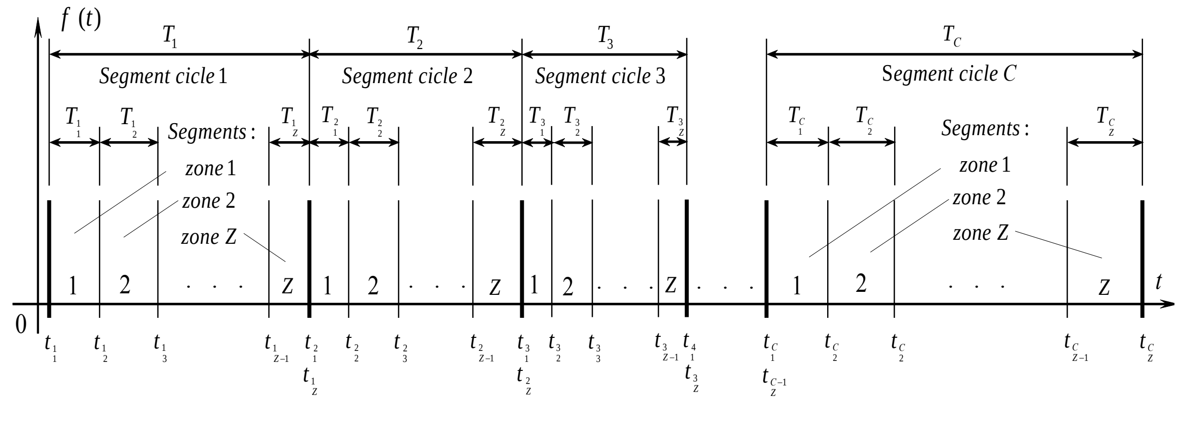


Figure 1. Schematic reflection of the segment structure for the implementation of a cyclic signal

By identifying the segment structure of the cyclic signal it is possible to evaluate it's rhythm structure. Rhythm function of the cyclic signal is described as a continuous rhythm function [1]. Discrete rhythm function is included into a discontinuous rhythm function, the starting points of which equal the starting points of the defined segment structure, and the values of the discrete rhythm function are defined as follows (in case when the segment structure describes the cyclic zone structure and is defined through the starting points of the segment zone structure):

$$T(t_{i_j}, n) = t_{i+n_j} - t_{i_j},\ \forall i = \overline{1,C},\ j = \overline{1,Z},\ n \in \mathbf{Z}. \tag{18}$$

where n– is the number of cycles, through which one-phase values of the investigated signal are set at a distance. Considering the current situation, $n = 1$, was used, meaning one-phase starting points of the first, second, third etc. cycles of the studied signal.

Discrete rhythm function of the cyclic signal, the segment structure of which describes the cyclic structure, will be defined through the countdown points of the process cycles:

$$T(t_i, n) = t_{i+n} - t_i,\ \forall i = \overline{1,C}, n \in \mathbf{Z}. \tag{19}$$

Therefore, the details about the beginning of both zone segments and cycle segments of the cyclic signal provide a possibility to define the signal's discrete rhythm structure, the details about which are contained in the discrete rhythm function $T(t_{i_j}, n)$, Which in itself is contained in a continuous rhythm function $T(t,n)$ of the cyclic process of a continuous argument. Figure 2 provides a schematic description of the rhythm structure, the discrete rhythm function and a continuous piecewise linear rhythm function, a dashed line.

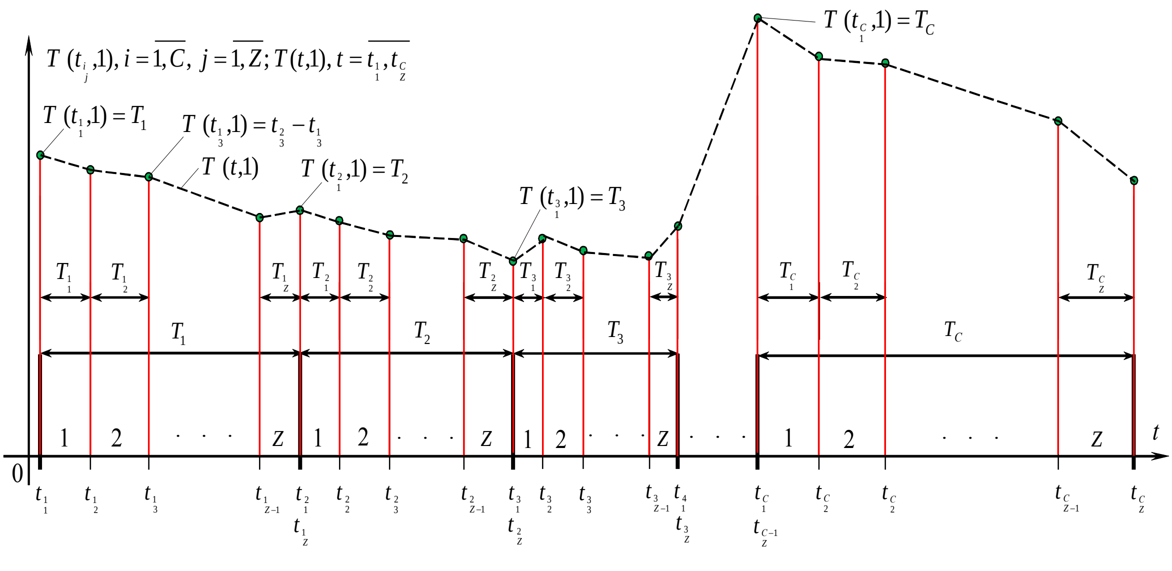


Figure 2. Schematic reflection of the discrete and continuous rhythm functions for the implementation of a cyclic signal. (continuous rhythm function, piecewise linear, marked with a dotted line)

Before determining the problem of segmentation of deterministic cyclic functions it is required to consider the notion of an attribute of a cyclic signal. An attribute or attributes for a cyclic signal is a characteristic that means the equivalence between values of single-phase cycles. An example of an ateribute may be an equation of values in different cycles of a cyclic signal [1]. Meaning that the following the condition is fulfilled $f(t) = f(t + T(t,1)), t \in \mathbf{W}$ the attribute is the $p(f(t)) = f(t)$.

Defining the problem of segmentation of deterministic cyclic functions

In the paper [17] the definition of the problem of segmentation of cyclic functions in a general situation is provided. The essence of the problem is to determine the range of division of the cyclic signal. This means that when solving the multitude of unknown time (spatial) countdowns of j- st zone segments in the related i- st cycle segments $\mathbf{D}_z = \left\{ t_{i_j}, i = \overline{1,C},\ j = \overline{1,Z} \right\}$, or, the multitude of time (spatial) countdowns $\mathbf{D}_c = \{ t_i, i = \overline{1,C} \}$ - related to i- st cycle signals which is analogical in solving the $\mathbf{W} = \left\{ \mathbf{W}_{i_j}, i = \overline{1,C},\ j = \overline{1,Z} \right\}$ range of definition of the cyclic signal or $\mathbf{W} = \{ \mathbf{W}_i, i = \overline{1,C} \}$.

It is also required for the multiple time points $\mathbf{D}_z = \left\{ t_{i_j}, i = \overline{1,C},\ j = \overline{1,Z} \right\}$, or $\mathbf{D}_c = \{ t_i, i = \overline{1,C} \}$ to have fulfilled the requirements of the bijective function of the countdowns of the cycles in a strict ordering of countdowns (20) that refer to the segments and also the attributes countdowns of the segments (21), meaning,

for the multitude (16), segment zone structure:

$$t_{i_j} \leftrightarrow t_{i+1_j}, \ldots;\ t_{i_{j+1}} > t_{i_j},\ t \in \mathbf{W}, i = \overline{1,C},\ j = \overline{1,Z}, \tag{20}$$

$$p(f(t_{i_j})) = p(f(t_{i+1_j})) \to \mathbf{A},\ t \in \mathbf{W}, i = \overline{1,C},\ j = \overline{1,Z}, \tag{21}$$

for the multitude (17), segment cyclic structure:

$$t_i \leftrightarrow t_{i+1}, \ldots;\ t_{i+1} > t_i,\ t \in \mathbf{W}, i = \overline{1,C}, \tag{22}$$

$$p(f(t_i)) = p(f(t_{i+1})) \to \mathbf{A},\ t \in \mathbf{W}, i = \overline{1,C}, \tag{23}$$

where $\mathbf{A}$ – is the set of attributes, for example, an attribute is the equality of the values of all single-phase countdowns.

Taking into account the statement of the segmentation problem for cyclic deterministic functions, let's describe a developed method for the segmentation of deterministic cyclic signals.

Description of the method of segmentation of deterministic cyclic signals

Let's consider the developed method of segmentation step by step. In order to ensure the uniformity of the results of segmentation let's consider that the studied signal has went through pre-processing during which the trend component was eliminated and the beginning point of the signal synchronized with the start of a scale reading (start of cycle's countdown).

1. Evaluation of the segmental structure (preset segmentation)

Initial data is $f(t), t \in \mathbf{W}$ – deterministic cyclic signal. For the initial segmentation the adaptive threshold level needs to be defined with the help of such a formula

$$f_{aver} = \frac{Max\ f\ (t_{\max}) - Min\ f\ (t_{\min})}{2} + Min\ f\ (t_{\min}),\ t \in \mathbf{W}\,, \tag{24}$$

where $Max\ f\ (t_{\max})$ – is the maximum value of the studied cyclic signal; $Min\ f\ (t_{\min})$ – is the minimum value of the studied cyclic signal.

The indicator functions for the segments are defined. They will be set either higher or lower then the adaptive threshold level (24):

$$I_{\tilde{\mathbf{W}}_1}(t) = \begin{cases} 0, & f(t) \le f_{aver} \\ 1, & f(t) > f_{aver} \end{cases},\ \tilde{\mathbf{W}}_1 = \{t_q\},\ q = \overline{1,Q}\,, \tag{25}$$

$$I_{\tilde{\mathbf{W}}_2}(t) = \begin{cases} 0, & f(t) \ge f_{aver} \\ 1, & f(t) < f_{aver} \end{cases},\ \tilde{\mathbf{W}}_2 = \{t_r\},\ r = \overline{1,R}\,, \tag{26}$$

where $\tilde{\mathbf{W}}_1$, $\tilde{\mathbf{W}}_2$ – are the areas of segments' definition (25) and (26); t_q, t_r – are the discrete countdowns for the related areas of the defined segments.

For the areas of segments definition $\tilde{\mathbf{W}}_1$ and $\tilde{\mathbf{W}}_2$ the fair condition would be:

$$\tilde{\mathbf{W}}_1 \bigcup \tilde{\mathbf{W}}_2 = \mathbf{W}\,. \tag{27}$$

Let's define such functions that would reflect areas of segments and indicator functions which have their value equal to one:

$$\tilde{f}_1(t) = f\ (t) \cdot I_{\tilde{\mathbf{W}}_1}(t),\ t \in \mathbf{W}\ ,\ \tilde{f}_2(t) = f\ (t) \cdot I_{\tilde{\mathbf{W}}_2}(t),\ t \in \mathbf{W}\,. \tag{28}$$

At the stage of initial segmentation let's select the first countdown (28) from the following conditions:

$$\text{If } t_q < t_r,\ q = 1, r = 1,\ \text{then } \tilde{t}_1 = t_q,\ q = 1; \tag{29}$$

$$\text{Then also in the case } \tilde{t}_k = t_q,\ k = 2,\ldots \text{ if } f\ (\tilde{t}_1) = \tilde{f}_1(t_q),\ q = \overline{2,Q}\,. \tag{30}$$

$$\text{If } t_q > t_r,\ q = 1, r = 1,\ \text{then } \tilde{t}_1 = t_r,\ r = 1; \tag{31}$$

$$\text{Then also in the case } \tilde{t}_k = t_r,\ k = 2,\ldots \text{ if } f\ (\tilde{t}_1) = \tilde{f}_2(t_r),\ r = \overline{2,R}\,. \tag{32}$$

After this procedure, we would obtain a preliminary breakdown of the cyclic deterministic signal. Considering this the initial data would be $\tilde{\mathbf{D}}_s = \{\tilde{t}_k,\ k = \overline{1,K}\}$ - the multitude of countdowns of time moments of the segments (segment cycles and segment zones). The next step would be the evaluation of the cyclic structure.

2. Evaluation of the cyclic structure (segment cyclic structure)

At the current stage the initial data would be $\tilde{\mathbf{D}}_s = \{\tilde{t}_k,\ k = \overline{1,K}\}$ – multitude of the time countdowns of the segments. Since the start of the countdown of the studied signal is synchronized with the start of the countdown of the first cycle, let's consider $\tilde{t}_1$ – as the countdown of the start of the cycle in the studied signal, meaning $\hat{t}_1 = \tilde{t}_i = \tilde{t}_k, i,k = 1$ – the countdown of the beginning of the first cycle.

The next step would be to evaluate the countdowns of the cycles by the attributes according to table 1. There are 2 possible variants for estimation by the attributes for the deterministic cyclic numeric function: attribute equality of values and for the cyclic interval numeric function for which the values are set in a range $2 \cdot \Delta$, where Δ – the range of possible values that are set.

Table 1. Usage of attributes for specifying the cyclic structure

Attribute	Analytical record
1. Equality of values	$p(f(t)) = f(t)$
2. Values are in the range of values	$p(f(t)) = f(t) - \Delta \le f(t) \le f(t) + \Delta$, duration of the range $2 \cdot \Delta$ of possible values

At this stage the conditions of equality by the attributes and the conditions of isomorphism of the countdowns of cycles need to be executed, meaning

$$\tilde{t}_g = \begin{cases} \tilde{t}_k, if\ p(f(\hat{t}_1)) = p(f(\tilde{t}_k)) \to \mathbf{A}, g = 2..., k = \overline{2,K}, \\ \tilde{t}_k - \text{countdown is not included}, \end{cases} \quad (33)$$

$$\tilde{t}_i = \begin{cases} \tilde{t}_g, if \begin{cases} p(f(\hat{t}_1 + l)) = p(f(\tilde{t}_g + l)), g = \overline{2, G-1}, \tilde{t}_g \le l \le \tilde{t}_{g+1}, \\ \hat{t}_1 + l < \tilde{t}_g + l, g = \overline{2, G-1}, l - \text{countdowns within the limits of the segment - cycle}, \end{cases} \\ \tilde{t}_g - \text{countdown is not included}, \end{cases} \quad (34)$$

Initial data for this stage would be $\tilde{\mathbf{D}}_c = \{\tilde{t}_i, i = \overline{1, \tilde{C}}\}$ – multitude of the time countdowns which relate to the starting points of the cycles; $\tilde{C}$ – the number of estimated cycles at this stage, while the segments of the zone on the cycles are not specified.

3. Specifying and forming of the segmental structure (zonal-cyclic structure or segmental zone structure)

At the stage of specifying the zone structure and accordingly – the cyclic structure, the initial data would be $\tilde{\mathbf{D}}_c = \{\tilde{t}_i, i = \overline{1, \tilde{C}}\}$ and $\tilde{\mathbf{D}}_s = \{\tilde{t}_k, k = \overline{1, K}\}$.

The current stage needs to be separated into two sub-stages. A multitude of time countdowns has to be formed $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1, C}, j \in \mathbf{Z}\}$, which belong between time countdowns of the limits of cycles (this multitude would be specified).

3.1 Specifying the segment structure (segment zone structure)

If between the defined limits of the countdowns of the cycles segments there are no countdowns of the zones segments, meaning that the the condition is fulfilled $p(f(\tilde{t}_1)) = p(f(\tilde{t}_2)) = ... = p(f(\tilde{t}_i)) = ... = p(f(\tilde{t}_{\tilde{C}})), i = \overline{1, \tilde{C}}$, then the studied signal contains only countdowns of the segment cycles and doesn't contain the countdowns of small segment zones. (within the limits of the cycles) then $\hat{\mathbf{D}}_c = \tilde{\mathbf{D}}_c = \{\hat{t}_i = \tilde{t}_i, i = \overline{1, C = \tilde{C}}\}$, thus $\hat{\mathbf{D}}_c = \{\hat{t}_i, i = \overline{1, C}\}$, we receive a segmental cyclic structure and proceed to stage 4.

Otherwise, if between the limits of the countdowns of the segment cycles there is one or more ountdowns of zone segments, in case such a condition is fulfilled, $p(f(\tilde{t}_1)) = p(f(\tilde{t}_3)) = p(f(\tilde{t}_5)) = \ldots = p(f(\tilde{t}_i)) = \ldots = p(f(\tilde{t}_{\tilde{C}})), i = \overline{1, \tilde{C}}$ – countdowns of the segments cycles and accordingly $p(f(\tilde{t}_2)), p(f(\tilde{t}_4))$ – countdowns of the segment zones, we obtain a zone-cyclic structure (segment zone structure).

Let's conduct the evaluation of received countdowns considering the equality by the attribute, and the fulfillment of the condition of isomorphism of the zones countdowns.

Let's consider $\tilde{t}_{i_j} = \tilde{t}_i, i, j = 1$ – as the start of the countdown of the first zone in the first cycle.

$$\tilde{t}_{g_j} = \begin{cases} \tilde{t}_{k_j}, if\ p(f(\hat{t}_1)) = p(f(\tilde{t}_k)) \to \mathbf{A}, g = 2..., k = \overline{2, K}, j = 2... \\ \tilde{t}_{k_j} - \text{countdown is not included}, \end{cases} \qquad (35)$$

$$\tilde{t}_{i_j} = \begin{cases} \tilde{t}_{g_j}, if \begin{cases} p(f(\hat{t}_1 + l)) = p(f(\tilde{t}_g + l)), g = \overline{2, G-1}, \tilde{t}_g \le l \le \tilde{t}_{g+1}, \\ \hat{t}_1 + l < \tilde{t}_g + l, g = \overline{2, G-1}, l - \text{countdowns within the limits of the segment - zone}, \end{cases} \\ \tilde{t}_{g_j} - \text{countdown is not included}, \end{cases} \qquad (36)$$

In a segment structure it is postulated that the number of sample segments of zones on each cycle is the same, thus the number of zones in cycles must also be the same. Considering this for the multitude $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1, C}, j = \overline{1, \tilde{Z} + 1}\}$, the current multitude doesn't include the countdowns of the cycles. Whereas $\tilde{Z}$ - is the number of zone countdowns which would need to be specified.

The following is true:

If the number of specified zone countdowns in each cycle is the same then the number of zones in cycles is same and equal to $Z_i = Z$ then $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1, C}, j = Z_i = Z\}$; where Z – is the number of zones for which there is equality by the attribute between the countdowns of the limits of cycles; Z_i - number of zones in i –st cycle.

Let's combine the multitude of cycles countdowns $\hat{\mathbf{D}}_c = \{\hat{t}_i, i = \overline{1, C}\}$ thus we obtain the multitude of countdowns of the segment zones $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1, C}, j = Z_i = Z\}$ we obtain the multitude of countdowns $\hat{\mathbf{D}}_z = \{\hat{t}_{i_j}, i = \overline{1, C}, j = \overline{1, Z}\}$, which takes into account both the countdowns of cycles and the countdowns of zones (35).

$$\hat{\mathbf{D}}_c \bigcup \tilde{\mathbf{D}}_z = \hat{\mathbf{D}}_z, \qquad (37)$$

Let's continue with step 4.

If the number of zone countdowns in each cycle is not the same (this may be due to the choice of the frequency of the discretization of the investigated signal), as a result it is necessary to clarify the zone countdowns for the the equation of the valus by the attribute, step 3.2/

3.2 Specifying the countdowns of the segment zones

At this stage the initial data would be $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1,C}, j = \overline{1,\tilde{Z}}\}$ and the countdowns that are specified by $\tilde{t}_k$, $\tilde{t}_{k_j}$.

When specifying the countdowns, it is necessary to set the range of specification, which equals $2 \cdot e$, where e - is the number of countdowns for the specification.

Table 2. Specifying the starting points of zones' segments by using the matrics of close values of the starting points of these segments

Metric of proximity (absolute)	Metric of proximity (Euclidean)
$\tilde{t}_k$ – countdown that is being specified, t_e – countdown for specification, $\min \rho = \left\| f(\tilde{t}_k) - f(t_e) \right\|$, The condition for choosing the countdowns for specification: If the countdown that is being specified is in the middle of the signal: $t_e = \begin{cases} t_{k-e}, k \ge 1+e \\ t_{k+e}, k \le K-e \end{cases}$, If the countdown that is being specified is in the end or beginning of the signal: $t_e = \begin{cases} t_{k-e}, k = K \\ t_{k+e}, k = 1 \end{cases}$, $2 \cdot e$ - the number of countdowns within which the specification of the countdown occurs	$\tilde{t}_k$ – countdown that is being specified, t_e – countdown for specification, $\min \rho = \sqrt{(f(\tilde{t}_k) - f(t_e))^2}$, The condition for choosing the countdowns for specification: If the countdown that is being specified is in the middle of the signal: $t_e = \begin{cases} t_{k-e}, k \ge 1+e \\ t_{k+e}, k \le K-e \end{cases}$, If the countdown that is being specified is in the end or beginning of the signal: $t_e = \begin{cases} t_{k-e}, k = K \\ t_{k+e}, k = 1 \end{cases}$, $2 \cdot e$ - the number of countdowns within which the specification of the countdown occurs

After specifying the zone countdowns let's unite the multitudes of cyclic countdowns $\hat{\mathbf{D}}_c = \{\hat{t}_i, i = \overline{1,C}\}$ and the obtained specified zone countdowns $\tilde{\mathbf{D}}_z = \{\tilde{t}_{i_j}, i = \overline{1,C}, j = Z_i = Z\}$ thus we obtain $\hat{\mathbf{D}}_z = \{\hat{t}_{i_j}, i = \overline{1,C}, j = \overline{1,Z}\}$ – the multitude of countdowns which takes into account both the countdowns of cycles and the countdowns of zones.

The initial data at this stage would be $\hat{\mathbf{D}}_z = \{\hat{t}_{i_j}, i = \overline{1,C}, j = \overline{1,Z}\}$, where Z – is the number of specified zones or $\hat{\mathbf{D}}_c = \{\hat{t}_i, i = \overline{1,C}\}$, where C – is the number of specified cycles.

4. Rhythm analysis

Rhythm in cyclic signals can be either stable or variable. In the case when we have stable rhythm then the values of time (spatial) distances, intervals between single-phase values are constant values; and the cycles' duration can be considered as the duration of the period as in the case of periodic deterministic signals (cyclic signals). In the case when the distance between the single-phase values in different cycles is different, meaning different in terms of the length of the cycles, we are dealing with a signal that is characterized by a variable rhythm.

The initial data for at this stage would be the segment zone structure $\hat{\mathbf{D}}_z = \{\hat{t}_{\substack{i\\j}},\ i = \overline{1, C},\ j = \overline{1, Z}\}$ or the segment cyclic structure $\hat{\mathbf{D}}_c = \{\hat{t}_i,\ i = \overline{1, C}\}$.

After having obtained the segment structure, we turn to the analysis of the rhythm that can be carried out by choosing either stage 4.1 or 4.2

4.1 If $t_{\substack{i+1\\j}} - t_{\substack{i\\j}} = t_{i+1} - t_i = const = T - \text{period},\ \forall i = \overline{1, C},\ j = \overline{1, Z}$ - is a stable rhythm, meaning $f(t) = f(t + n \cdot T)$.

Let's evaluate the value of the period by using the next formula:

$$\hat{T} = t_{\substack{i+1\\j}} - t_{\substack{i\\j}} = t_{i+1} - t_i,\ \forall i = \overline{1, C},\ j = \overline{1, Z} \tag{38}$$

It is possible to provide the discrete rhythm function, which is defined through the estimated period:

- for the countdowns of segment zones,

$$\hat{T}(t_{\substack{i\\j}}, n) = n \cdot \hat{T},\ \forall t_{\substack{i\\j}} \in \mathbf{W},\ i = \overline{1, C},\ j = \overline{1, Z},\ n \in \mathbf{Z}\ ; \tag{39}$$

- for the countdowns of segment cycles,

$$\hat{T}(t_i, n) = n \cdot \hat{T},\ \forall t_i \in \mathbf{W},\ i = \overline{1, C},\ n \in \mathbf{Z}\,. \tag{40}$$

4.2 If $t_{\substack{i+1\\j}} - t_{\substack{i\\j}} \neq t_{i+1} - t_i \neq const,\ \forall i = \overline{1, C},\ j = \overline{1, Z}$ – variable rhythm, meaning $f(t) \neq f(t + n \cdot T)$, thus $f(t) = f(t + T(t, n))$.

Let's evaluate the discrete rhythm function:

- for the countdowns of segment zones,

$$\hat{T}(t_{\substack{i\\j}}, n) = t_{\substack{i+n\\j}} - t_{\substack{i\\j}},\ \forall t_{\substack{i\\j}} \in \mathbf{W},\ i = \overline{1, C},\ j = \overline{1, Z},\ n \in \mathbf{Z}\ ; \tag{41}$$

- for the countdowns of segment cycles,

$$\hat{T}(t_i, n) = t_{i+n} - t_i,\ \forall t_i \in \mathbf{W},\ i = \overline{1, C},\ n \in \mathbf{Z}\,. \tag{42}$$

Considering the stated above algorithms, figure 3 reflects the structural scheme of the algorithmic result of the developed method of segmentation. The developed method has been implemented into practice as software program written on the programming language Delphi.

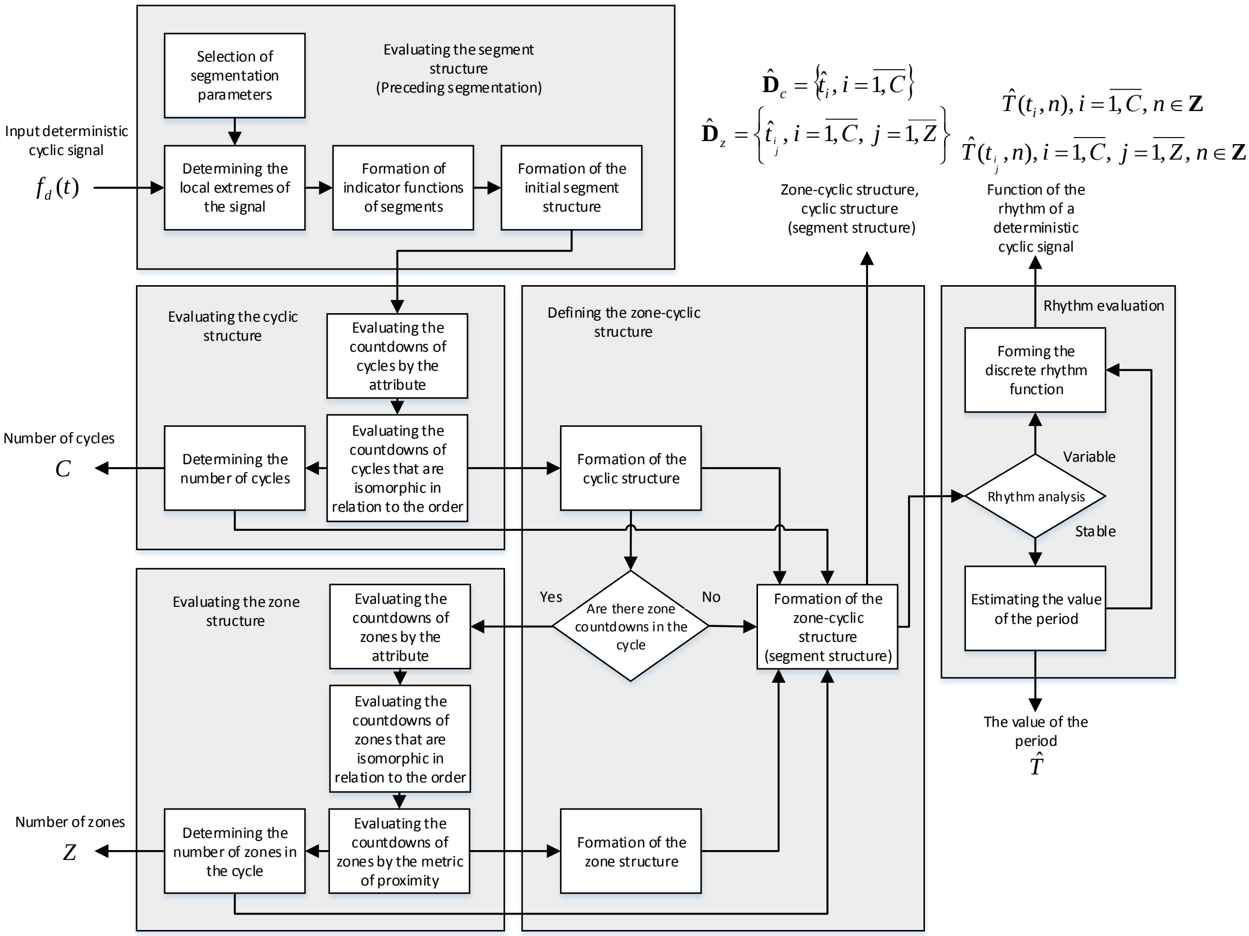
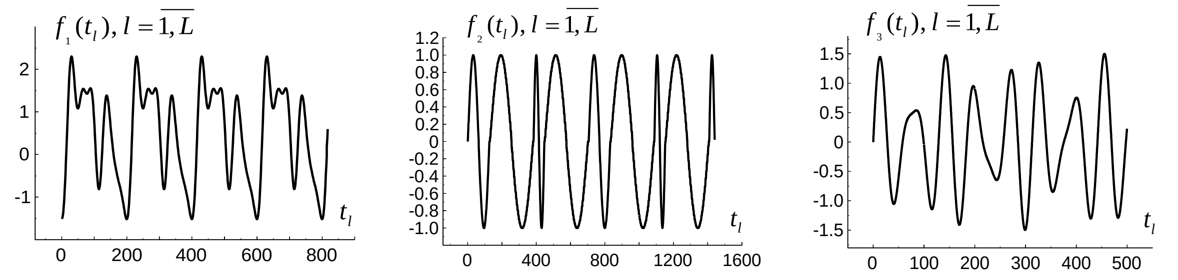


Figure 3. Algorithmic support for the method of segmentation of deterministic cyclic signals

The developed method allows to obtain information about the segment structure: $\hat{\mathbf{D}}_z = \{\hat{t}_{i_j}, i = \overline{1,C}, j = \overline{1,Z}\}$ or $\hat{\mathbf{D}}_c = \{\hat{t}_i, i = \overline{1,C}\}$, to evaluate the rhythm (rhythm structure) in the case of stable rhythm to evaluate the value of the period, and in the case of variable rhythm – to evaluate the discrete rhythm function.

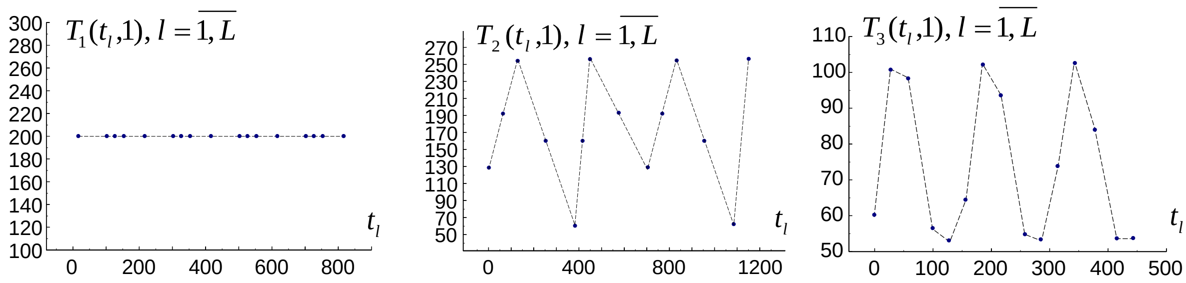
The results of application of the developed method and estimation of the accuracy of the method of segmentation

Figure 4 shows, as an example, the test signals of deterministic cyclic signals, in standard units, to check the developed method of segmentation, they were modeled taking into account the rhythm functions shown in Figure 5.



а) б) в)

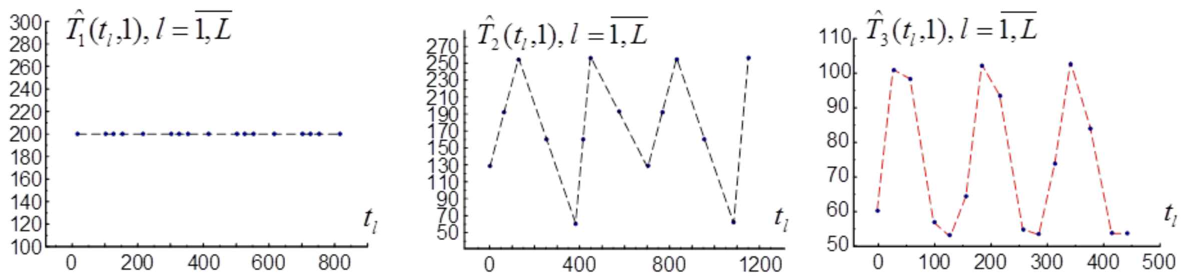
Figure 4. Simulated test cyclic signals for estimating the accuracy of the method of segmentation of deterministic cyclic signals: a) periodic cyclic signal (attribute – equality of values); b) cyclic signal (attribute – equality of values); c) cyclic signal (attribute – values in range $2\cdot\Delta$)



а) б) в)

Figure 5. Discrete rhythm functions of the simulated test cyclic signals: for the periodic cyclic signal (period, attribute – equality of values); b) for the cyclic signal (attribute – equality of values); c) for the cyclic signal (attribute – values in range $2\cdot\Delta$), dashed line – is a continuous rhythm function

After applying the developed method of segmentation to the test signals it was possible to received a segment structure and evaluate the functions of the rhythm. The results are shown in Figure 6.



а) б) в)

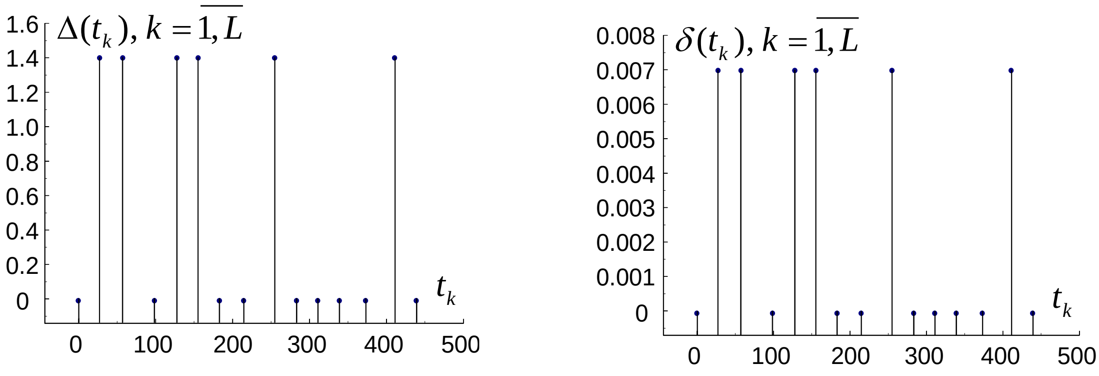
Figure 6. Defined discrete rhythm functions of the simulated test cyclic signals by the method of segmentation of the determined cyclic signals: a) for the periodic cyclic signal (attribute – equality of values); b) for the cyclic signal (attribute – equality of values); c) for the cyclic signal (attribute – values in range $2\cdot\Delta$), dashed line – is a continuous rhythm function

Absolute and relative error of segmentation were determined by the formulas:

$$\Delta(t_k) = \sqrt{\frac{1}{L}\sum_{l=1}^{L}(t_l - \hat{t}_l)^2}\;, \quad k = \overline{1, L}, \tag{43}$$

$$\delta(t_k) = \frac{\Delta(t_k)}{\sqrt{\frac{1}{L}\sum_{l=1}^{L}\hat{t}_l^{\,2}}} \cdot 100\%, \quad k = \overline{1, L}, \tag{44}$$

where t_l – countdown of time of the simulated discrete rhythm function (used to simulate a cyclic signal); $\hat{t}_l$ – countdown of time of the defined through the segmentation method discrete rhythm function; L – number of countdowns of the discrete rhythm function, $l = \overline{1, L}$; t_k – countdown for absolute and relative errors, $k = \overline{1, L}$.



a) б)

Figure 7. Mean square absolute and relative errors defined discrete rhythm function for the cyclic signal (attribute – values in range $2 \cdot \Delta$)

Table 3. Errors of the defined starting points of the discrete rhythm functions of the cyclic signals

Segmentation of deterministic cyclic signals	**Absolute error, (not more)**	**Relative error, (not more), %**
Periodic signals (attribute – the equality of values)	0	0
Cyclic signals (attribute – the equality of values)	1,35	0,2
Cyclic signals (attribute – value in range)	1,4	0,7

Since a cyclic signal, model of which is a periodic deterministic function – can be only a single case, current method can be used both for segmentation and period evaluation. However, sometimes it is enough to determine the period by using one of the well known methods and then it would be possible to find the segment structure for the cyclic periodic signals.

Conclusion. In this paper, the method of segmentation of a deterministic cyclic signal has been developed based on considering information about its segmental structure and attributes.The developed method provides the possibility of segmentation of cyclic signals the mathematical models of which are the deterministic cyclic numeric function and the cyclic interval numeric function with the segment structure.

Current method of segmentation complements the studies for processing cyclic signals and can be used in digital systems of automated processing of real cyclic data; in fields such as medicine, mechanics or economics for the task related to segmentation. The method provides the possibility to evaluate if the rhythm is stable or variable; in case of stable rhythm – it provides the possibility to evaluate the value of the period; in case of variable rhythm – it provides the possibility to evaluate the rhythm function. An estimation of the accuracy of the method of segmentation of deterministic cyclic signals has been carried out; the relative error of segmentation of the studied signals does not exceed 0.7%.

In further research it is planned to expand the approaches of the developed method in order to create methods of segmentation of random cyclic signals considering the segment structure and attributes.

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УДК 004.67

МЕТОД СЕГМЕНТАЦІЇ ДЕТЕРМІНОВАНИХ ЦИКЛІЧНИХ СИГНАЛІВ ДЛЯ ЗАДАЧ ЇХ ОБРОБКИ ТА МОДЕЛЮВАННЯ

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Резюме. *Розроблено метод адаптивної сегментації детермінованих циклічних сигналів. Розв’язано задачу сегментації (розбиття, поділу сигналу на характерні сегменти) циклічного детермінованого сигналу (циклічної числової функції та циклічної інтервальної функції) із сегментною структурою. Наведено приклади сегментації змодельованих та реальних циклічних сигналів,оцінено точності розробленого методу сегментації.*

Ключові слова*: циклічний сигнал, сегментація, сегментна структура, функція ритму.*

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