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THE PROBLEM OF REDUCING THE STRESS CONCENTRATION ON THE BOUNDARY OF THE CAVITIES MEDIUM UNDER THE DYNAMIC LOAD ACTION

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Summary. *The effectiveness of the use of additional barriers in the form of tunnel cavities of the elliptic section located on the wave propagation to reduce the dynamic concentration of stresses is investigated in this paper. The distribution of dynamic hoop stresses on the boundary of cavities and radial stresses in a medium are studied on the basis of the method combining the use of Fourier transformation in time and the method of boundary integral equations. Fixed time snapshots of stress fields are developed.*

Key words*: non-stationary problem, tunnel cavity.*

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Statement of the problem. The problem of reducing the dynamic concentration of stresses by the action of a modified in time load is one of the important tasks in the theoretical aspect as well as in practical applications. It is of great interest while extracting coal and other mineral deposits using the mine method, where fracturing is used. In this case the wave processes that have a significant effect on the existing structures occur.

Analysis of the available investigations. The complexity of the solution of the dynamic non-stationary problems of the deformable solid mechanics is concerned not only with the awkwardness of calculations. When transient processes occur in the environments, the solution of such tasks requires the use of integral transformations to distinguish the time variable.

Therefore, the analytical solutions are obtained by Guz O.M., Zozulia V.V., Kubenko V.D. only for a number of problems on the basis of the combined use of the series method and the Laplace integral transform.

The application of direct numerical methods of finite differences and finite elements in Chen Y.M., Shahani A.R., Zhang J.Y. papers made it possible to investigate the dynamic stressed state of bodies under conditions of variable in time load. However, while discretizating the motion equations for regions with complex defects for fast loading, numerical differentiation requires the thickening of the discretization grid to ensure the calculations accuracy.

The method for investigation of the dynamic stress state of bodies weakened by systems of tunnel cavities of arbitrary cross section, based on the consistent use of Fourier transformation in time, the method of boundary integral equations, the theory of the function of a complex variable and the method of mechanical quadratures are offered in [1]. The advantage of applying the developed approach is the ability to calculate the dynamic stresses in bodies with cavities of almost arbitrary cross section.

The objective of the paper is to investigate the effectiveness of the additional obstacles use to reduce the dynamic stresses concentration in bodies with systems of tunnel cavities under the action of a dynamic axisymmetric impulse applied to the boundary on the basis of the method of the combined use of Fourier transform and boundary integral equations [1].

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Statement of the task. Let us consider the environment under conditions of flat deformation with Young's modulus E and Poisson's ratio ν weakened by the system of two tunnel cavities of the elliptic and arched cross section (Fig. 1,*a*). Let us denote by L_1, L_2 the contours of their cross sections. To reduce the dynamic stresses concentration on the boundary of the arched form by the action of the dynamic axisymmetric load applied to the boundary of the elliptic cavity, we use the properties of the wave diffraction phenomenon, placing the elliptical tunnel cavity in the wave path (Fig. 1,*б*) and denote its contour by L_3.

Statement of the main material. Using the technique developed in [1] for the research of the dynamic stressed state of elastic bodies with tunnel cavities, we refer the body under consideration to the Cartesian coordinate Ox_1x_2 system. Let us investigate the distribution of hoop stresses on the boundary of cavities and radial stresses in the medium with the system of two (Fig. 1,*a*) and three (Fig. 1,*b*) cavities.

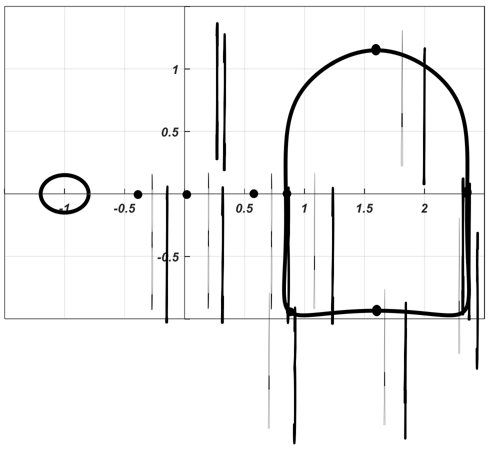


Figure 1*a*. Model of section of biconnected elastic medium

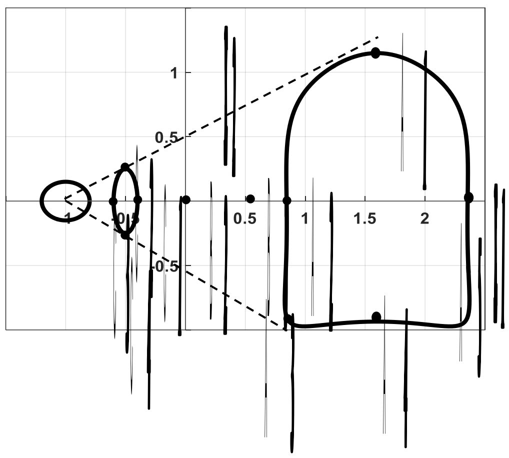


Figure 1*b*. Model of section of triply connected elastic medium

To solve the diffraction problems in the presence of transients the Fourier integral transformation in time is used

$$\tilde{f}(x,\,\omega) = \int_{-\infty}^{\infty} f(x,\,t)e^{-i\omega t}dt,$$

to separate the time variable. Hence, the non-stationary dynamic problem is reduced to a system of problems equivalent to cases of stable oscillations with cyclic frequency ω [2].

In the Fourier-figure area the boundary conditions of the problem for the case of biconnected and triply connected endless domains are presented in the form of conditions for images:

$$\tilde{\sigma}_n\big|_{L_1} = -\tilde{\phi}(\omega),\ \tilde{\sigma}_n\big|_{L_2} = \tilde{\phi}(\omega),\ \tilde{\tau}_{sn}\big|_{L_1,L_2} = 0, \qquad (1')$$

$$\tilde{\sigma}_n\big|_{L_1} = -\tilde{\phi}(\omega),\ \tilde{\sigma}_n\big|_{L_2,L_3} = \tilde{\phi}(\omega),\tilde{\tau}_{sn}\big|_{L_1,\,L_2,L_3} = 0, \qquad (1'')$$

where $\tilde{\phi}(\omega)=\int\limits_{-\infty}^{\infty}\phi(t)e^{-i\omega t}dt,$ – is the function representation describing load change in time presented in the following way:

$$\phi(t)=\begin{cases}0, & \tau<0;\\ p_1\tau^{n_1}e^{-\alpha_1\tau}H(\tau), & 0\le\tau\le\alpha_1;\\ H(\tau), & \alpha_1\le\tau\le\alpha_2;\\ p_2\tau^{n_2}e^{-\alpha_2\tau}H(\tau), & \tau>\alpha_2,\end{cases} \quad (2)$$

where p_1, p_2, α_1, n_1, α_2, n_2 – are numerical constants; $H(t)$ – is Heaviside function; $\tau=t\cdot c_1/a$ – is dimensionless time parameter; $c_1=\sqrt{(\lambda+\mu)/\rho}$ – are wave expansion velocity; λ, μ – are Lame constants; a – is a certain characteristic dimension.

To solve the stated problem, the method of boundary integral equations [3] is used. The potential image of the general solution in displacements in the case of the first main task for the biconnected and triply connected domains is as follows:

$$\tilde{u}_i(\mathbf{x},\omega)=\sum_{k=1}^{2}\sum_{j=1}^{2}\int\limits_{L_k}p_j^k(\mathbf{x}^0\big|_{L_k},\omega)\cdot U_{ji}^*(\mathbf{x},\mathbf{x}^0\big|_{L_k},\omega)\,ds; \quad (3')$$

$$\tilde{u}_i(\mathbf{x},\omega)=\sum_{k=1}^{3}\sum_{j=1}^{2}\int\limits_{L_k}p_j^k(\mathbf{x}^0\big|_{L_k},\omega)\cdot U_{ji}^*(\mathbf{x},\mathbf{x}^0\big|_{L_k},\omega)\,ds, \quad (3'')$$

where p_j^k, $j=1,2$ – being unknown complex potential functions. Integration along the boundary is carried out according to the variables $\mathbf{x}^0=\{x_1^0,\ x_2^0\}$. Expressions for U_{ij}^* image functions are chosen taking into account Sommerfeld conditions [2] in the form of [1].

Meeting the boundary requirements (1) for the case of biconnected infinite domain and (1″) for triply connected one, we have the system of integral equations for determination of unknown quantities on the of functions boundary p_j^k, $j=1,2$, presented in the matrix form in the following way:

$$\mathbf{B}\cdot\mathbf{P}=\mathbf{D}, \quad (4)$$

where $\mathbf{B}=\{F_{jk}\}$, F_{jk} – are known functions containing Bessel functions of the third kind, $\mathbf{P}=\{p_j^k\}$ – is the vector matrix of unknown quantities on the functions boundary; $\mathbf{D}$ – the vector of the given load determined in accordance with the boundary conditions (1′) and (1″).

After defining unknown quantities from the system (4), calculation of the hoop stresses at the boundary of tunnel cavities and radial stresses in the medium is carried out numerically on the basis of dependences [4]:

$$\begin{matrix}\tilde{\sigma}_\theta\\ \tilde{\sigma}_r\end{matrix}=\frac{\tilde{\sigma}_{11}+\tilde{\sigma}_{22}}{2}/2\pm\frac{1}{4}\left(e^{-2i\alpha}\left(\tilde{\sigma}_{11}-\tilde{\sigma}_{22}+2i\tilde{\sigma}_{12}\right)+e^{2i\alpha}\left(\tilde{\sigma}_{11}-\tilde{\sigma}_{22}-2i\tilde{\sigma}_{12}\right)\right).$$

To determine the originals of dynamic stresses, the inverse Fourier transformation is used:

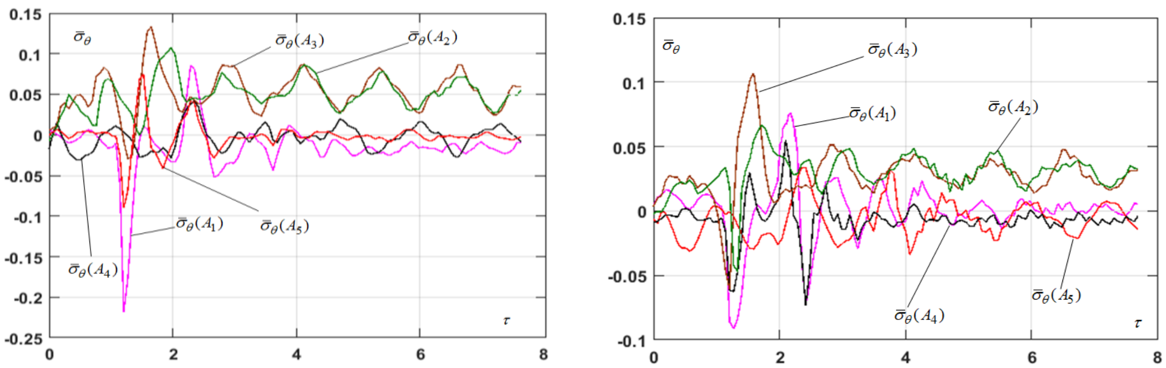
$$\sigma(\mathbf{x}, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\sigma}(\mathbf{x}, \omega) e^{i\omega t} d\omega,$$

which for numerical calculation of the hoop stresses is implemented on the basis of the discrete Fourier transformation [5].

The results of the investigations. On the basis of the developed approach, let us investigate the dynamic distribution of the hoop stresses on the boundary of cavities and radial stresses in the medium for the case where L_1 – is an ellipse with semiaxes $0.2a$ and $0.15a$, L_2 – is the arc with the boundary equation presented on the basis of conformal mapping formulas [2], L_3 – is the ellipse with semiaxes of $0.1a$ and $0.25a$. The distance between the centers of the holes is $2.6a$. The numerical calculations are carried out for soil rocks with the density ρ=1.35·10^3 *kg/m*^{3}, Young modulus E=5.6·10^4 *MPa* and Poisson coefficient ν=0.28.

Load changes in time on the boundary of the elliptic cavity is represented as (2) when $p_1 = 272,\ \alpha_1 = 10,\ n_1 = 2,\ p_2 = 0.272,\ \alpha_2 = 0.1,\ n_2 = 1.$

Fig. 2 shows the results of numerical calculations of relative dynamic hoop stresses $\bar{\sigma}_\theta = \sigma_\theta / \sigma_0$ at five points of the arched cross section boundary when there is no "unloading" tunnel cavity (Fig. 2*a*) and when it is available (Fig. 2*b*). Here $\sigma_0 = 1$.



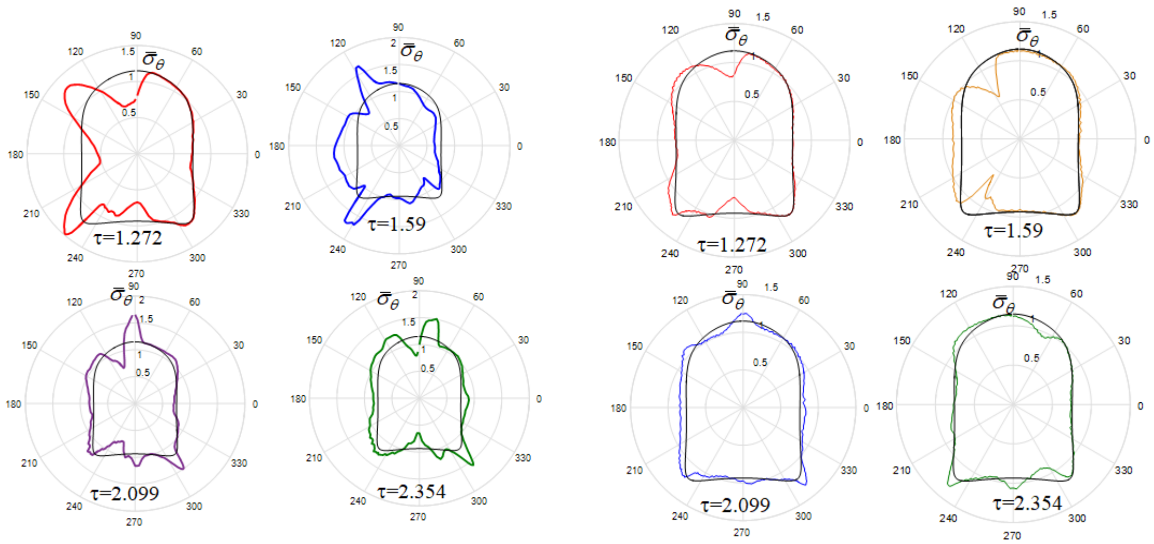
a *b*

Figure 2. Distribution of dynamic hoop stresses on the boundary of the arched cavity in the absence (*a*) and availability (*b*) of the "unloading" tunnel cavities

It is evident from Fig. 2 that if the "unloading" tunnel cavity of the elliptic cross section is available there is a decrease in the dynamic concentration of the hoop stresses on the boundary of the arched tunnel cavity. The maximum values of dynamic stresses are reduced by 2.5 times in the upper part of the cross-section (point A_1), by 2 times in the middle of the left vertical part (point A_2) and by 1.27 times in the left part of the cross section base (point A_3).

This is explained by the basic properties of wave diffraction, arising from the action of dynamic load. When the wave reaches the "unloading" cavity, a part of the waves are reflected from its boundary, and some of them disperse, changing their propagation path. This reduces the dynamic effect on the cavity of the arched cross section.

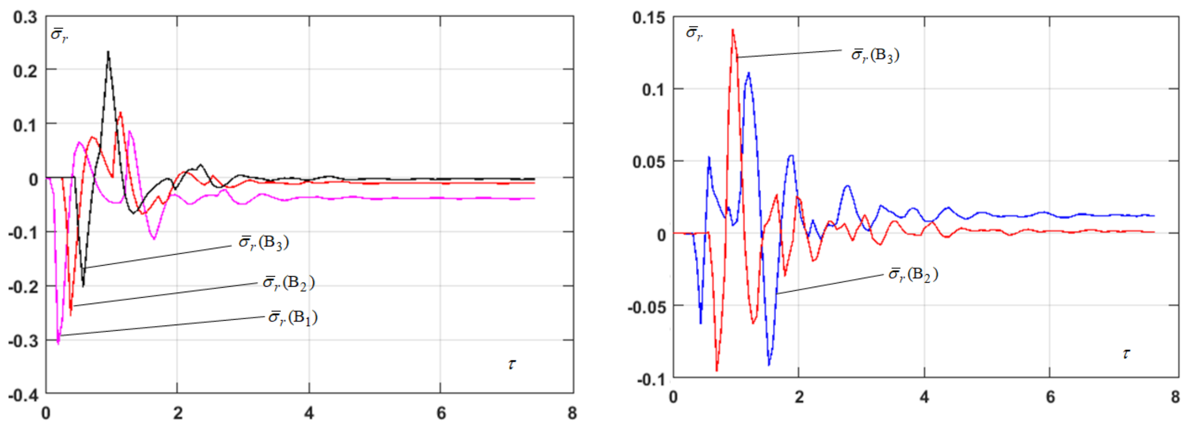
For the qualitative evaluation of the character of the dynamic stresses distribution along the boundary of the cavity, the time snapshots of the relative dynamic stresses by increased four times, for fixed moments of the dimensionless time parameter τ without "unloading" tunnel (Fig. 3*a*) and with it (Fig. 3*b*) are shown in Fig. 3.



a *b*

Figure 3. Fixed time snapshots of dynamic hoop stresses on the boundary of the vaulted cavity in the absence (*a*) and availability (*b*) of the "unloading" tunnel cavities

In order to investigate the wave propagation in the medium under dynamic load, the relative radial stresses $\bar{\sigma}_r = \sigma_r / \sigma_0$ at the points B_1, B_2 and B_3 separated from the center of the cavity to the boundary of theapplied load at a distance $\delta=0.6a$, $\delta=a$ та $\delta=1.6a$ respectively are calculated. The results of the corresponding numerical calculations are shown in Fig. 4 for the case of the absence of the "unloading" tunnel (Fig. 4*a*) and, if available, (Figure 4*б*).



a *b*

Figure 4. Distribution of dynamic radial stresses in medium in the absence (*a*) and availability (*b*) of the "unloading" tunnel cavities

The reliability of the obtained results is proved by the fact that before the wave reaches the corresponding cross section, the values of the dynamic stresses are zero.

Fig. 4 shows that the presence of an additional obstacle in the form of a tunnel elliptical cross section significantly changes the character of the wave propagation and their intensity. The maximum value of dynamic radial stresses in the cross section *1.6a* distant from the center of the cavity with the boundary of applied load with the "unloading" tunnel is 1.786 times less than without it.

Conclusions. The effectiveness of the application of additional obstacles in the form of tunnel cavities of the elliptic cross section to reduce the dynamic concentration of the hoop stresses on the boundary of the vaulted cavity is proved on the basis of numerical calculations

The approach developed on the basis of the combined use of the Fourier transformation over time, the method of boundary integral equations can be used to calculate the dynamic stressed state of the medium weakened by the system of cavities with arbitrary cross section.

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ДО ПИТАННЯ ЗМЕНШЕННЯ КОНЦЕНТРАЦІЇ НАПРУЖЕНЬ НА ГРАНИЦІ ПОРОЖНИН СЕРЕДОВИЩА ЗА ДІЇ ДИНАМІЧНОГО НАВАНТАЖЕННЯ

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Резюме. *Досліджено ефективність використання додаткових перепон у вигляді тунельних порожнин еліптичного перерізу, що розміщені на шляху хвилі, для зменшення динамічної концентрації напружень. На основі методу, що поєднує використання перетворення Фур'є за часом, метод граничних інтегральних рівнянь досліджено розподіл динамічних кільцевих напружень на границі порожнин та радіальних напружень у тілі, побудовано часові зрізи полів напружень.*

Ключові слова: *нестаціонарна задача, тунельна порожнина.*

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