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MATHEMATICAL MODELING. MATHEMATICS

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ. МАТЕМАТИКА

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COMPUTATION OF IMPROPER INTEGRAL ACCORDING TO EIGEN-ELEMENTS OF 1ST-GENUS HANKEL – (KONTOROVYCH-LEBEDEV) HYBRID DIFFERENTIAL OPERATOR – 2ND GENUS LEGENDRE 2ND GENUS – FOURIER ON POLAR AXIS

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Summary. *There were calculated using comparison method of solving the boundary problem on the polar axis segment with three junction points for the separate system consisting of Hankel, Kontorovich-Lebedev, Legendre and Fourier differential equations for the modified functions, which was built, on one side, by Cauchy function method, and on the other side, by the definite hybrid integral transformation, poly-parametric family of the improper integrals according to the eigen-elements of 1st-genus Hankel differential operator – (Kontorovich-Lebedev) 2nd–genus – Legendre 2nd–genus – Fourier.*

Key words: *improper integrals, Eigen-elements, hybrid differential operator, integral transformation, main solutions.*

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Problem setting. Thin-walled construction composite elements usually are in steady mode after series of dramatic alterations of temperature or power stress. The investigation of their physical and technical characteristics leads to thermo-mechanic (mechanic) problems of lump-heterogeneous ambience [1]. The practice shows that even in the simplest cases the values, which characterize the steady conditions of composite material, are marked as multi-parametric integrals that can be conventionally convergent even when they describe analytic function. Hence, it is necessary to convert the improper integral due to its convergence (function) that is especially important during engineering computing. The issues [2, 3] are dedicated to calculation of improper integrals.

Research objective is to elaborate methods of calculation of multi-parametrical medium of improper integrals according to Eigen-elements of 1st-genus Hankel – (Kontorovych-Lebedev) Hybrid Differential Operator – 2nd genus Legendre 2nd genus – Fourier.

Task setting. Let us construct the limited on multiplication $I_3 = \{r : r \in (0; R_1) \bigcup (R_1; R_2) \bigcup (R_2; R_3) \bigcup (R_3; +\infty)\}$ solution of separated system of ordinary differential of 1st-genus Hankel – (Kontorovych-Lebedev) Hybrid Differential Operator – 2nd genus Legendre and Fourier equation for modified functions

$$\left(B_{\nu,\alpha_1} - q_1^2\right)u_1(r) = -g_1(r), \quad r \in (0; R_1),$$

$$\left(B_{\alpha_2} - q_2^2\right)u_2(r) = -g_2(r), \quad r \in (R_1; R_2),$$

$$\left(\Lambda_{(\mu)} - q_3^2\right)u_3(r) = -g_3(r), \quad r \in (R_2; R_3), \tag{1}$$

$$\left(\frac{d^2}{dr^2} - q_4^2\right)u_4(r) = -g_4(r), \quad r \in (R_3; +\infty)$$

under conjugation conditions

$$\left[\left(\alpha_{j1}^k \frac{d}{dr} - \beta_{j1}^k\right)u_k(r) - \left(\alpha_{j2}^k \frac{d}{dr} - \beta_{j2}^k\right)u_{k+1}(r)\right]\Big|_{r=R_k} = \omega_{jk},\ j = 1,2;\ k = 1,2,3. \tag{2}$$

The differential operators of Fourier – $\frac{d^2}{dr^2}$, Hankel – $B_{\nu,\alpha_1} = \frac{d^2}{dr^2} + \frac{2\alpha_1 + 1}{r}\frac{d}{dr} - \frac{\nu^2 - \alpha_1}{r^2}, \nu \ge \alpha_1 \ge -\frac{1}{2}$; Kontorovych-Lebedev – $B_{\alpha_2} = r^2 \frac{d^2}{dr^2} + (2\alpha_2 + 1) r \frac{d}{dr} + \alpha^2 - \lambda^2 r^2, \alpha_2 > -\frac{1}{2}, \lambda \in (0; +\infty)$, Legendre generalized operator – $\Lambda_{(\mu)} = \frac{d^2}{dr^2} + cthr\frac{d}{dr} + \frac{1}{4} + \frac{1}{2}\left(\frac{\mu_1}{1 - chr} + \frac{\mu_2}{1 + chr}\right), \mu_1 > \mu_2 > -\frac{1}{2}$ participate in the system (1) [4].

Let us assume that conditions on coefficients are accomplished $q_s > 0$, $\alpha_{jk}^m \ge 0$, $\beta_{jk}^m \ge 0$, $c_{jk} = \alpha_{2j}^k \beta_{1j}^k - \alpha_{1j}^k \beta_{2j}^k$, $c_{1k}c_{2k} > 0$, $j = 1,2$, $k, m = 1,2,3$, $s = \overline{1,4}$.

Research results. I. Method of Cauchy functions. The fundamental solution system for Hankel differential equation $\left(B_{\nu,\alpha_1} - q_1^2\right)v = 0$ are created by 1st-genus Bessel imaginary argument functions (modified Bessel functions) першого роду $v_1 = I_{\nu,\alpha_1}(q_1 r)$ та другого роду $v_2 = K_{\nu,\alpha_1}(q_1 r)$ [4]. The fundamental solution system for Kontorovych-Lebedev differential equation $\left(B_{\alpha_2} - q_2^2\right)v = 0$ are created by 1st genus Bessel modified cylindrical functions $v_1 = I_{q_2,\alpha_2}(\lambda r)$ and 2nd-genus $v_2 = K_{q_2,\alpha_2}(\lambda r)$ [4]. The fundamental solution system for Lagrange differential equation $\left(\Lambda_\mu - q_3^2\right)v = 0$ are created by 1st-genus Legendre generalized functions $v_1 = P_{\nu_3}^{(\mu)}(chr)$ and 2nd-genus $v_2 = L_{\nu_3}^{(\mu)}(chr)$, where $\nu_3 = -\frac{1}{2} + q_3$ [3]. The fundamental solution system for Fourier differential equation $\left(\frac{d^2}{dr^2} - q_4^2\right)v = 0$ are created with functions $v_1 = e^{q_4 r}$ та $v_2 = e^{-q_4 r}$ [4].

The available fundamental solution systems facilitate building of solution for boundary problems (1), (2) with Cauchy method of functions [4]:

$$u_1(r) = A_1 I_{\nu,\alpha_1}(q_1 r) + \int_0^{R_1} E_1(r,\rho) g_1(\rho) \rho^{2\alpha_1+1} d\rho ,$$

$$u_2(r) = A_2 I_{q_2,\alpha_2}(\lambda r) + B_2 K_{q_2,\alpha_2}(\lambda r) + \int_{R_1}^{R_2} E_2(r,\rho) g_2(\rho) \rho^{2\alpha_2-1} d\rho ,$$

$$u_3(r) = A_3 P_{\nu_3}^{(\mu)}(chr) + B_3 L_{\nu_3}^{(\mu)}(chr) + \int_{R_2}^{R_3} E_3(r,\rho) g_3(\rho) sh\rho d\rho , \tag{3}$$

$$u_4(r) = B_4 e^{-q_4 r} + \int_{R_3}^{+\infty} E_4(r,\rho) g_4(\rho) d\rho ,$$

where Cauchy's functions are represented as:

$$E_1(r,\rho) = \frac{q_1^{2\alpha_1}}{U_{\nu,\alpha_1;11}^{11}(q_1 R_1)} \begin{cases} I_{\nu,\alpha_1}(q_1 r) \Psi_{\nu,\alpha_1;11}^{1}(q_1 R_1, q_1 \rho), \; 0 < r < \rho < R_1, \\ I_{\nu,\alpha_1}(q_1 \rho) \Psi_{\nu,\alpha_1;11}^{1}(q_1 R_1, q_1 r), \; 0 < \rho < r < R_1. \end{cases} \tag{4}$$

$$E_2(r,\rho) = \frac{\lambda^{2\alpha_2}}{\Delta_{q_2,\alpha_2;11}(\lambda R_1, \lambda R_2)} \begin{cases} \Psi_{q_2,\alpha_2;12}^{1}(\lambda R_1, \lambda r) \Psi_{q_2,\alpha_2;11}^{2}(\lambda R_1, \lambda \rho), \; R_1 < r < \rho < R_2, \\ \Psi_{q_2,\alpha_2;12}^{1}(\lambda R_1, \lambda \rho) \Psi_{q_2,\alpha_2;11}^{2}(\lambda R_1, \lambda r), \; R_1 < \rho < r < R_2. \end{cases} \tag{5}$$

$$E_3(r,\rho) = \frac{B_{(\mu)}(q_3)}{\Delta_{\nu_3;11}^{(\mu)}(chR_2, chR_3)} \begin{cases} F_{\nu_3;12}^{(\mu),2}(chR_1, chr) F_{\nu_3;11}^{(\mu),3}(chR_1, ch\rho), \; R_2 < r < \rho < R_3, \\ F_{\nu_3;12}^{(\mu),2}(chR_1, ch\rho) F_{\nu_3;11}^{(\mu),3}(chR_1, chr), \; R_2 < \rho < r < R_3. \end{cases} \tag{6}$$

$$E_4(r,\rho) = \frac{1}{q_4(\alpha_{12}^3 q_4 + \beta_{12}^3)} \begin{cases} e^{-q_4(\rho - R_3)} \Phi_{12}^3(q_4 R_3, q_4 r), \; R_3 < r < \rho < +\infty, \\ e^{-q_4(r - R_3)} \Phi_{12}^3(q_4 R_3, q_4 \rho), \; R_3 < \rho < r < +\infty. \end{cases} \tag{7}$$

The functions participating in equations (4) – (7), are universally accepted [2, 3, 4].

The conjugation conditions (2) to determine the values A_i $(i = 1,2,3)$ and B_j $(j = 2,3,4)$ produce the algebraic system of six equations

$$U_{\nu,\alpha_1;11}^{11}(q_1 R_1) A_1 - U_{q_2,\alpha_2;12}^{11}(\lambda R_1) A_2 - U_{q_2,\alpha_2;12}^{12}(\lambda R_1) B_2 = \omega_{11} ,$$

$$U_{\nu,\alpha_1;21}^{11}(q_1 R_1) A_1 - U_{q_2,\alpha_2;22}^{11}(\lambda R_1) A_2 - U_{q_2,\alpha_2;22}^{12}(\lambda R_1) B_2 = \omega_{21} + G_{12} ,$$

$$U_{q_2,\alpha_2;11}^{21}(\lambda R_2) A_2 + U_{q_2,\alpha_2;11}^{22}(\lambda R_1) B_2 - Z_{\nu_3;12}^{(\mu),21}(chR_2) A_3 - Z_{\nu_3;12}^{(\mu),22}(chR_2) B_3 = \omega_{12} , \tag{8}$$

$$U_{q_2,\alpha_2;21}^{21}(\lambda R_2) A_2 + U_{q_2,\alpha_2;21}^{22}(\lambda R_1) B_2 - Z_{\nu_3;22}^{(\mu),21}(chR_2) A_3 - Z_{\nu_3;22}^{(\mu),22}(chR_2) B_3 = \omega_{22} + G_{23} ,$$

$$Z^{(\mu),31}_{\nu_3;11}(chR_3)A_3 + Z^{(\mu),32}_{\nu_3;11}(chR_3)B_3 - V^{32}_{12}(q_4R_3)B_4 = \omega_{13},$$

$$Z^{(\mu),31}_{\nu_3;21}(chR_3)A_3 + Z^{(\mu),32}_{\nu_3;21}(chR_3)B_3 - V^{32}_{22}(q_4R_3)B_4 = \omega_{13} + G_{34}.$$

The following functions participate in the system (8)

$$G_{12} = \frac{c_{11}}{R_1^{2\alpha_1+1}}\int_0^{R_1}\frac{I_{\nu,\alpha_1}(q_1\rho)}{U^{11}_{\nu,\alpha_1;11}(q_1R_1)}g_1(\rho)\rho^{2\alpha_1+1}d\rho - \frac{c_{21}}{R_1^{2\alpha_2+1}}\int_{R_1}^{R_2}\frac{\Psi^2_{q_2,\alpha_2;11}(\lambda R_2,\lambda\rho)}{\Delta^{11}_{q_2,\alpha_2;11}(\lambda R_1,\lambda R_2)}g_2(\rho)\rho^{2\alpha_2-1}d\rho,$$

$$G_{23} = \frac{c_{12}}{R_2^{2\alpha_2+1}}\int_{R_1}^{R_3}\frac{\Psi^1_{q_2,\alpha_2;12}(\lambda R_1,\lambda\rho)}{\Delta^{11}_{q_2,\alpha_2;11}(\lambda R_1,\lambda R_2)}g_2(\rho)\rho^{2\alpha_2-1}d\rho - \frac{c_{22}}{shR_2}\int_{R_2}^{R_3}\frac{F^{(\mu),3}_{\nu_3;11}(chR_3,ch\rho)}{\Delta^{(\mu)}_{\nu_3;11}(chR_2,chR_3)}g_3(\rho)sh\rho\,d\rho,$$

$$G_{34} = \frac{c_{13}}{shR_3}\int_{R_2}^{R_3}\frac{F^{(\mu),2}_{\nu_3;12}(chR_2,ch\rho)}{\Delta^{(\mu)}_{\nu_3;11}(chR_2,chR_3)}g_3(\rho)sh\rho\,d\rho - c_{23}\int_{R_3}^{+\infty}\frac{e^{-q_4(\rho-R_3)}}{\alpha^3_{12}q_4+\beta^3_{12}}g_4(\rho)d\rho.$$

Let us introduce the functions:

$$\Delta_{\nu,(\alpha);j}(q) = U^{11}_{\nu,\alpha_1;21}(q_1R_1)\Delta_{q_2,\alpha_2;1j}(\lambda R_1,\lambda R_2) - U^{11}_{\nu,\alpha_1;11}(q_1R_1)\Delta_{q_2,\alpha_2;2j}(\lambda R_1,\lambda R_2),$$

$$\Delta^{(\mu)}_{q_2,\alpha_2;jk}(q) = \Delta_{q_2,\alpha_2;j2}(\lambda R_1,\lambda R_2)\Delta^{(\mu)}_{\nu_3;1k}(chR_2,chR_3) - \Delta_{q_2,\alpha_2;j1}(\lambda R_1,\lambda R_2)\Delta^{(\mu)}_{\nu_3;2k}(chR_2,chR_3),$$

$$A^{(\mu)}_{\nu,(\alpha);j}(q) = \Delta_{\nu,(\alpha);2}(q)\Delta^{(\mu)}_{\nu_3;1j}(chR_2,chR_3) - \Delta_{\nu,(\alpha);1}(q)\Delta^{(\mu)}_{\nu_3;2j}(chR_2,chR_3),$$

$$B^{(\mu)}_{\nu,(\alpha);j}(q) = V^{32}_{22}(q_4R_3)\Delta^{(\mu)}_{q_2,\alpha_2;j1}(q) - V^{32}_{12}(q_4R_3)\Delta^{(\mu)}_{q_2,\alpha_2;j2}(q),$$

$$\delta^{(\mu)}_{\nu_3;j}(q) = V^{32}_{22}(q_4R_3)\Delta^{(\mu)}_{\nu_3;j1}(chR_2,chR_3) - V^{32}_{12}(q_4R_3)\Delta^{(\mu)}_{\nu_3;j2}(chR_2,chR_3),$$

$$\Theta^{(\mu)}_{q_2,\alpha_2;j}(r,q) = \delta^{(\mu)}_{\nu_3;1}(q)\Psi^2_{q_2,\alpha_2;2j}(\lambda R_2,\lambda r) - \delta^{(\mu)}_{\nu_3;2}(q)\Psi^2_{q_2,\alpha_2;1j}(\lambda R_2,\lambda r),$$

$$\Theta^{(\mu)}_{\nu_3;1}(r,q) = V^{32}_{22}(q_4R_3)F^{(\mu),3}_{\nu_3;11}(chR_3,chr) - V^{32}_{12}(q_4R_3)F^{(\mu),3}_{\nu_3;21}(chR_3,chr),$$

$$\Theta_{\nu,(\alpha);j}(r,q) = U^{11}_{\nu,\alpha_1;21}(q_1R_1)\Psi^1_{q_2,\alpha_2;1j}(\lambda R_1,\lambda r) - U^{11}_{\nu_1,\alpha_1;11}(q_1R_1)\Psi^1_{q_2,\alpha_2;2j}(\lambda R_1,\lambda r),$$

$$\Theta^{(\mu)}_{\nu_3;2}(r,q) = \Delta_{\nu,(\alpha);2}(q)F^{(\mu),2}_{\nu_3;12}(chR_2,chr) - \Delta_{\nu,(\alpha);1}(q)F^{(\mu),2}_{\nu_3;22}(chR_2,chr),\ \ j,k = 1,2.$$

Let us suppose the condition of univalent solution for boundary problem (1), (2) has been accomplished: for any non-zero vector $\vec{q} = \{q_1, q_2, q_3, q_4\}$ the identification item of algebraic system (8) is

$$\Delta_{\nu,(\alpha)}^{(\mu)}(q) \equiv V_{22}^{32}(q_4R_3)A_{\nu,(\alpha);1}^{(\mu)}(q) - V_{21}^{32}(q_4R_3)A_{\nu,(\alpha);2}^{(\mu)}(q) = \\ = U_{\nu_1,\alpha_1;21}^{11}(q_1R_1)B_{\nu,(\alpha);1}^{(\mu)}(q) - U_{\nu_1,\alpha_1;11}^{11}(q_1R_1)B_{\nu,(\alpha);2}^{(\mu)}(q) \neq 0. \tag{9}$$

Let us determine the main solutions for the problem (1), (2):

1) originated by the inhomogeneity of conjugation conditions for Green's function

$$R_{\nu,(\alpha);11}^{(\mu),1}(r,q) = -\frac{B_{\nu,(\alpha);2}^{(\mu)}(q)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} I_{\nu,\alpha_1}(q_1r), \; R_{\nu,(\alpha);21}^{(\mu),1}(r,q) = -\frac{B_{\nu,(\alpha);1}^{(\mu)}(q)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} I_{\nu,\alpha_1}(q_1r),$$

$$R_{\nu,(\alpha);12}^{(\mu),1}(r,q) = -\frac{c_{11}}{R_1^{2\alpha_2+1}} \frac{\delta_{\nu_3;2}^{(\mu)}(q)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} I_{\nu,\alpha_1}(q_1r), \; R_{\nu,(\alpha);22}^{(\mu),1}(r,q) = -\frac{c_{11}}{R_1^{2\alpha_2+1}} \frac{\delta_{\nu_3;1}^{(\mu)}(q)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} I_{\nu,\alpha_1}(q_1r),$$

$$R_{\nu,(\alpha);13}^{(\mu),1}(r,q) = -\frac{c_{11}}{R_1^{2\alpha_2+1}} \frac{c_{22}}{sh^2R_2B_{(\mu)}(q_3)} \frac{V_{22}^{32}(q_4R_3)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} I_{\nu,\alpha_1}(q_1r),$$

$$R_{\nu,(\alpha);23}^{(\mu),1}(r,q) = -\frac{c_{11}}{R_1^{2\alpha_2+1}} \frac{c_{22}}{sh^2R_2B_{(\mu)}(q_3)} \frac{V_{12}^{32}(q_4R_3)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} I_{\nu,\alpha_1}(q_1r),$$

$$R_{\nu,(\alpha);12}^{(\mu),2}(r,q) = -\frac{\delta_{\nu_3;2}^{(\mu)}(q)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu,(\alpha);2}(r,q), \; R_{\nu,(\alpha);22}^{(\mu),2}(r,q) = -\frac{\delta_{\nu_3;1}^{(\mu)}(q)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu,(\alpha);2}(r,q),$$

$$R_{\nu,(\alpha);23}^{(\mu),2}(r,q) = -\frac{c_{22}}{sh^2R_2B_{(\mu)}(q_3)} \frac{V_{12}^{32}(q_4R_3)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu,(\alpha);2}(r,q),$$

$$R_{\nu,(\alpha);13}^{(\mu),2}(r,q) = \frac{c_{22}}{sh^2R_2B_{(\mu)}(q_3)} \frac{V_{22}^{32}(q_4R_3)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu,(\alpha);2}(r,q),$$

$$R_{\nu,(\alpha);11}^{(\mu),2}(r,q) = -\frac{U_{\nu,\alpha_1;21}^{11}(q_1R_1)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{q_2,\alpha_2;1}^{(\mu)}(r,q), \; R_{\nu,(\alpha);21}^{(\mu),2}(r,q) = \frac{U_{\nu,\alpha_1;11}^{11}(q_1R_1)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{q_2,\alpha_2;1}^{(\mu)}(r,q),$$

$$R_{\nu,(\alpha);13}^{(\mu),3}(r,q) = \frac{V_{22}^{32}(q_4R_3)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu_3;2}^{(\mu)}(r,q), \; R_{\nu,(\alpha)23}^{(\mu),3}(r,q) = -\frac{V_{12}^{32}(q_4R_3)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu_3;2}^{(\mu)}(r,q),$$

$$R_{\nu,(\alpha);22}^{(\mu),3}(r,q) = \frac{\Delta_{\nu,(\alpha);1}(q)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu_3;1}^{(\mu)}(r,q), \; R_{\nu,(\alpha);12}^{(\mu),3}(r,q) = \frac{\Delta_{\nu,(\alpha);2}(q)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu_3;1}^{(\mu)}(r,q),$$

$$R_{\nu,(\alpha);11}^{(\mu),3}(r,q) = \frac{c_{12}}{R_2^{2\alpha_2+1}} \frac{U_{\nu,\alpha_1;21}^{11}(q_1R_1)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu_3;1}^{(\mu)}(r,q), \; R_{\nu,(\alpha);21}^{(\mu),3}(r,q) = \frac{c_{12}}{R_2^{2\alpha_2+1}} \frac{U_{\nu,\alpha_1;11}^{11}(q_1R_1)}{\Delta_{\nu,(\alpha)}^{(\mu)}(q)} \Theta_{\nu_3;1}^{(\mu)}(r,q),$$

$$R^{(\mu),4}_{\nu,(\alpha);13}(r,q)=\frac{A^{(\mu)}_{\nu,(\alpha);2}(q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}e^{-q_4(r-R_3)},\ R^{(\mu),4}_{\nu,(\alpha)23}(r,q)=\frac{A^{(\mu)}_{\nu,(\alpha);1}(q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}e^{-q_4(r-R_3)},$$

$$R^{(\mu),4}_{\nu,(\alpha);11}(r,q)=\frac{c_{12}}{R_2^{2\alpha_2+1}}\frac{c_{13}}{sh^2R_2B_{(\mu)}(q_3)}\frac{U^{11}_{\nu,\alpha_1;21}(q_1R_1)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}e^{-q_4(r-R_3)},$$

$$R^{(\mu),4}_{\nu,(\alpha)21}(r,q)=\frac{c_{12}}{R_2^{2\alpha_2+1}}\frac{c_{13}}{sh^2R_2B_{(\mu)}(q_3)}\frac{U^{11}_{\nu,\alpha_1;11}(q_1R_1)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}e^{-q_4(r-R_3)},$$

$$R^{(\mu),4}_{\nu,(\alpha);12}(r,q)=\frac{c_{13}}{sh^2R_2B_{(\mu)}(q_3)}\frac{\Delta_{\nu,(\alpha);2}(q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}e^{-q_4(r-R_3)},$$

$$R^{(\mu),4}_{\nu,(\alpha);22}(r,q)=\frac{c_{13}}{sh^2R_2B_{(\mu)}(q_3)}\frac{\Delta_{\nu,(\alpha);1}(q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}e^{-q_4(r-R_3)};$$

2) originated by the inhomogeneity of the system of influence function

$$H^{(\mu)}_{\nu,(\alpha);11}(r,\rho,q)=\frac{q_1^{2\alpha_2}}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}\begin{cases}I_{\nu,\alpha_1}(q_1r)\left[B^{(\mu)}_{\nu,(\alpha);1}(q)\Psi^1_{q_2,\alpha_2;21}(\lambda R_1,\lambda\rho)-B^{(\mu)}_{\nu,(\alpha);2}(q)\Psi^1_{q_2,\alpha_2;11}(\lambda R_1,\lambda\rho)\right],\\ 0<r<\rho<R_1,\\ I_{\nu,\alpha_1}(q_1\rho)\left[B^{(\mu)}_{\nu,(\alpha);1}(q)\Psi^1_{q_2,\alpha_2;21}(\lambda R_1,\lambda r)-B^{(\mu)}_{\nu,(\alpha);2}(q)\Psi^1_{q_2,\alpha_2;11}(\lambda R_1,\lambda r)\right],\\ 0<\rho<r<R_1;\end{cases}$$

$$H^{(\mu)}_{\nu,(\alpha);12}(r,\rho,q)=\frac{c_{21}}{R_1^{2\alpha_2+1}}\frac{\Theta^{(\mu)}_{q_2,\alpha_2;1}(\rho,q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}I_{\nu,\alpha_1}(q_1r),$$

$$H^{(\mu)}_{\nu,(\alpha);13}(r,\rho,q)=\frac{c_{11}}{R_1^{2\alpha_2+1}}\frac{c_{22}}{sh^2R_2B_{(\mu)}(q_3)}\frac{\Theta^{(\mu)}_{\nu_3;1}(\rho,q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}I_{\nu,\alpha_1}(q_1r),$$

$$H^{(\mu)}_{\nu,(\alpha);14}(r,\rho,q)=\frac{c_{11}}{R_1^{2\alpha_2+1}}\frac{c_{22}c_{23}}{sh^2R_2B_{(\mu)}(q_3)}\frac{e^{-q_4\rho}}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}I_{\nu,\alpha_1}(q_1r),$$

$$H^{(\mu)}_{\nu,(\alpha);21}(r,\rho,q)=\frac{c_{11}}{R_1^{2\alpha_1+1}}\frac{\Theta^{(\mu)}_{q_2,\alpha_2;1}(r,q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}I_{\nu,\alpha_1}(q_1\rho),$$

$$H^{(\mu)}_{\nu,(\alpha);24}(r,\rho,q)=-\frac{c_{22}}{sh^2R_2B_{(\mu)}(q_3)}\frac{\Theta^{(\mu)}_{q_2,\alpha_2;2}(r,q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}e^{-q_4(\rho-R_3)},$$

$$H^{(\mu)}_{\nu,(\alpha);23}(r,\rho,q)=-\frac{c_{22}}{sh^2R_2B_{(\mu)}(q_3)}\frac{\Theta_{\nu,(\alpha);2}(r,q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}\Theta^{(\mu)}_{\nu_3;1}(\rho,q),$$

$$H^{(\mu)}_{\nu,(\alpha);22}(r,\rho,q)=\frac{\lambda^{2\alpha_2}}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}\begin{cases}\Theta_{\nu,(\alpha);2}(r,q)\Theta^{(\mu)}_{q_2,\alpha_2;1}(\rho,q),\ R_1<r<\rho<R_2,\\ \Theta_{\nu,(\alpha);2}(\rho,q)\Theta^{(\mu)}_{q_2,\alpha_2;1}(r,q),\ R_1<\rho<r<R_2;\end{cases}$$

$$H^{(\mu)}_{\nu,(\alpha);31}(r,\rho,q)=\frac{c_{11}}{R_1^{2\alpha_1+1}}\frac{c_{12}}{R_2^{2\alpha_{12}+1}}\frac{\Theta^{(\mu)}_{\nu_3;1}(r,q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}I_{\nu,\alpha_1}(q_1\rho),$$

$$H^{(\mu)}_{\nu,(\alpha);32}(r,\rho,q)=\frac{c_{12}}{R_2^{2\alpha_{12}+1}}\frac{\Theta^{(\mu)}_{q_2,\alpha_2;2}(r,q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}\Theta^{(\mu)}_{\nu_3;1}(\rho,q),$$

$$H^{(\mu)}_{\nu,(\alpha);34}(r,\rho,q)=\frac{c_{23}}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}\Theta^{(\mu)}_{\nu_3;2}(r,q)e^{-q_4(\rho-R_3)},$$

$$H^{(\mu)}_{\nu,(\alpha);33}(r,\rho,q)=\frac{B_{(\mu)}(q_3)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}\begin{cases}\Theta^{(\mu)}_{\nu_3;2}(r,q)\Theta^{(\mu)}_{\nu_3;1}(\rho,q),\ R_2<r<\rho<R_3,\\ \Theta^{(\mu)}_{\nu_3;2}(\rho,q)\Theta^{(\mu)}_{\nu_3;1}(r,q),\ R_2<\rho<r<R_3;\end{cases}$$

$$H^{(\mu)}_{\nu,(\alpha);41}(r,\rho,q)=\frac{c_{11}}{R_1^{2\alpha_1+1}}\frac{c_{12}}{R_2^{2\alpha_2+1}}\frac{c_{13}}{sh^2R_3B_{(\mu)}(q_3)}\frac{e^{-q_4(r-R_3)}}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}I_{\nu,\alpha_1}(q_1\rho),$$

$$H^{(\mu)}_{\nu,(\alpha);42}(r,\rho,q)=\frac{c_{12}}{R_2^{2\alpha_2+1}}\frac{c_{13}}{sh^2R_3B_{(\mu)}(q_3)}\frac{\Theta^{(\mu)}_{q_2,\alpha_2;2}(\rho,q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}e^{-q_4(r-R_3)},$$

$$H^{(\mu)}_{\nu,(\alpha);43}(r,\rho,q)=\frac{c_{13}}{sh^2R_3B_{(\mu)}(q_3)}\frac{\Theta^{(\mu)}_{\nu_3;2}(\rho,q)}{\Delta^{(\mu)}_{\nu,(\alpha)}(q)}e^{-q_4(r-R_3)},$$

$$H^{(\mu)}_{\nu,(\alpha);44}(r,\rho,q)=\frac{1}{q_4\Delta^{(\mu)}_{\nu,(\alpha)}(q)}\begin{cases}e^{-q_4(\rho-R_3)}\left[\Delta_{\nu,(\alpha);1}(q)\Phi^3_{22}(q_4R_3,q_4r)-\Delta_{\nu,(\alpha);2}(q)\Phi^3_{12}(q_4R_3,q_4r)\right],\\ R_3<r<\rho<+\infty,\\ e^{-q_4(r-R_3)}\left[\Delta_{\nu,(\alpha);1}(q)\Phi^3_{22}(q_4R_3,q_4\rho)-\Delta_{\nu,(\alpha);2}(q)\Phi^3_{12}(q_4R_3,q_4\rho)\right],\\ R_3<r<\rho<+\infty.\end{cases}$$

As the result of invariant solution of algebraic system (8) and substitution of obtained values A_i $(i=1,2,3)$ and B_j $(j=2,3,4)$ in equation (3) we will get the unique solution of the boundary problem (1), (2)

$$u_j(r)=\sum_{k,m=1}^{3}R^{(\mu),j}_{\nu,(\alpha);km}(r,q)\omega_{km}+\int_0^{R_1}H^{(\mu)}_{\nu,(\alpha);j1}(r,\rho,q)g_1(\rho)\rho^{2\alpha_1+1}d\rho+$$

$$+\int_{R_1}^{R_2} H^{(\mu)}_{\nu,(\alpha);j2}(r,\rho,q)g_2(\rho)\rho^{2\alpha_2-1}d\rho+\int_{R_2}^{R_3} H^{(\mu)}_{\nu,(\alpha);j3}(r,\rho,q)g_3(\rho)sh\rho d\rho+$$

$$+\int_{R_3}^{+\infty} H^{(\mu)}_{\nu,(\alpha);j4}(r,\rho,q)g_4(\rho)d\rho,\quad j=\overline{1,4}. \qquad (10)$$

<u>II. Method of integral transformations</u>. The hybrid differential operator $M^{(\mu)}_{\nu,(\alpha)}$ was built in [5] ([5], c. 83), the direct $H^{(\mu)}_{\nu,(\alpha);3}$ and the reverse $H^{-(\mu)}_{\nu,(\alpha);3}$ 1st-genus Hankel (Kontorovych-Lebedev) 2nd genus Legendre 2nd genus Fourier hybrid integral transformation was determined ([5], c. 86), which was originated on multiplication I_3 with hybrid differential operator (HDO) $M^{(\mu)}_{\nu,(\alpha)}$, and the main identity of integral transformation of hybrid differential operator (HDO) $M^{(\mu)}_{\nu,(\alpha)}$ was proved ([5], c. 87).

The direct $H^{(\mu)}_{\nu,(\alpha);3}$, and reverse $H^{-(\mu)}_{\nu,(\alpha);3}$ hybrid integral transformation and the main identity of integral transformation of hybrid differential operator (HDO) $M^{(\mu)}_{\nu,(\alpha)}$ make up the mathematical instrument to solve the boundary problem (1), (2).

Let us record the system (1) in matrix form

$$\begin{bmatrix} \left(B_{\nu,\alpha_1}-q_1^2\right)u_1(r) \\ \left(B_{\alpha_2}-q_2^2\right)u_2(r) \\ \left(\Lambda_{(\mu)}-q_3^2\right)u_3(r) \\ \left(\dfrac{d^2}{dr^2}-q_4^2\right)u_4(r) \end{bmatrix} = -\begin{bmatrix} g_1(r) \\ g_2(r) \\ g_3(r) \\ g_4(r) \end{bmatrix}. \qquad (11)$$

Integral operator $H^{(\mu)}_{\nu,(\alpha);3}$ will be presented in the form of operator matrix-row

$$H^{(\mu)}_{\nu,(\alpha);3}[\cdots]=\Bigg[\int_0^{R_1}\ldots V^{(\mu)}_{\nu,(\alpha);1}(r,\beta)\sigma_1 r^{2\alpha_1+1}dr \quad \int_{R_1}^{R_2}\ldots V^{(\mu)}_{\nu,(\alpha);2}(r,\beta)\sigma_2 r^{2\alpha_2-1}dr$$
$$\int_{R_2}^{R_3}\ldots V^{(\mu)}_{\nu,(\alpha);3}(r,\beta)\sigma_3 shrdr \quad \int_{R_3}^{+\infty}\ldots V^{(\mu)}_{\nu,(\alpha);4}(r,\beta)\sigma_4 dr\Bigg]. \qquad (12)$$

In the equation (12) $V^{(\mu)}_{\nu,(\alpha);j}(r,\beta)$ $(j=\overline{1,4})$ the components of spectral vector-function $V^{(\mu)}_{\nu,(\alpha)}(r,\beta)$ ([5], c. 86), a σ_j $(j=\overline{1,4})$ – weight coefficients of weight function $\sigma(r)$ ([5], p. 84).

Let us apply the operator matrix-row (12) to the system (11) according to the rule of matrix multiplication. As the result of main identity, we will obtain the algebraic equation

$$\left(\beta^2+q^2\right)\tilde{u}(\beta)=\tilde{g}(\beta)+\sum_{k=1}^{3}d_k\left[Z^{(\mu),k}_{\nu,(\alpha);12}(\beta)\omega_{2k}-Z^{(\mu),k}_{\nu,(\alpha);22}(\beta)\omega_{1k}\right],$$

where $q=\max\{q_1;q_2;q_3;q_4\}$, $\tilde{u}(\beta)=\sum_j^4 \tilde{u}_j(\beta)$, $\tilde{g}(\beta)=\sum_j^4 \tilde{g}_j(\beta)$, $d_1=\dfrac{c_{12}c_{13}R_1^{2\alpha_1+1}}{c_{21}c_{22}c_{23}R_1^{2\alpha_1+1}}\dfrac{shR_2}{shR_3}$,

$d_2=\dfrac{c_{13}}{c_{22}c_{23}}\dfrac{shR_2}{shR_3}$, $d_3=\dfrac{1}{c_{23}}$, $Z_{\nu,(\alpha);j2}^{(\mu),k+1}(\beta)=\left(\alpha_{j2}^k\dfrac{d}{dr}+\beta_{j2}^k\right)V_{\nu,(\alpha);k+1}^{(\mu)}(r,\beta)\Big|_{r=R_k}$ $(j=\overline{1,2},\ k=\overline{1,3})$.

Hence, the function is as follows

$$\tilde{u}(\beta)=\frac{\tilde{g}(\beta)}{\beta^2+q^2}\tilde{g}(\beta)+\sum_{k=1}^{3}d_k\left[\frac{Z_{\nu,(\alpha);12}^{(\mu),k}(\beta)}{\beta^2+q^2}\omega_{2k}-\frac{Z_{\nu,(\alpha);22}^{(\mu),k}(\beta)}{\beta^2+q^2}(\beta)\omega_{1k}\right]. \tag{13}$$

Integral operator $H_{\nu,(\alpha);3}^{-(\mu)}$ as the one that is reverse to $H_{\nu,(\alpha);3}^{(\mu)}$ will be represented in the form of operator matrix-column

$$H_{\nu,(\alpha);3}^{-(\mu)}[\cdots]=\begin{bmatrix}\dfrac{2}{\pi}\displaystyle\int_0^{+\infty}\ldots V_{\nu,(\alpha);1}^{(\mu)}(r,\beta)\Omega(\beta)d\beta\\ \dfrac{2}{\pi}\displaystyle\int_0^{+\infty}\ldots V_{\nu,(\alpha);2}^{(\mu)}(r,\beta)\Omega(\beta)d\beta\\ \dfrac{2}{\pi}\displaystyle\int_0^{+\infty}\ldots V_{\nu,(\alpha);3}^{(\mu)}(r,\beta)\Omega(\beta)d\beta\\ \dfrac{2}{\pi}\displaystyle\int_0^{+\infty}\ldots V_{\nu,(\alpha);4}^{(\mu)}(r,\beta)\Omega(\beta)d\beta\end{bmatrix}. \tag{14}$$

Let us apply the operator matrix-column (14) according to the rule of multiplication of the matrixes on the matrix-element $[\tilde{u}(\beta)]$, where function $\tilde{u}(\beta)$ is determined with the formula (13). As the result of simple transformations, we will obtain the unique solution of the boundary problem (1), (2)

$$u_j(r)=\frac{2}{\pi}\int_0^{+\infty}\tilde{u}(\beta)V_{\nu,(\alpha);j}^{(\mu)}(r,\beta)\Omega(\beta)d\beta=\sum_{k=1}^{3}d_k\left[\left(\frac{2}{\pi}\int_0^{+\infty}\frac{Z_{\nu,(\alpha);12}^{(\mu),k}(\beta)}{\beta^2+q^2}V_{\nu,(\alpha);j}^{(\mu)}(r,\beta)\Omega(\beta)d\beta\right)\omega_{2k}-\right.$$

$$\left.-\left(\frac{2}{\pi}\int_0^{+\infty}\frac{Z_{\nu,(\alpha);22}^{(\mu),k}(\beta)}{\beta^2+q^2}V_{\nu,(\alpha);j}^{(\mu)}(r,\beta)\Omega(\beta)d\beta\right)\omega_{1k}\right]+$$

$$+\int_0^{R_1}\left(\frac{2}{\pi}\int_0^{+\infty}\frac{V_{\nu,(\alpha);j}^{(\mu)}(r,\beta)}{\beta^2+q^2}V_{\nu,(\alpha);1}^{(\mu)}(\rho,\beta)\Omega(\beta)d\beta\right)g_1(\rho)\sigma_1\rho^{2\alpha_1+1}d\rho+$$

$$+\int_0^{R_1}\left(\frac{2}{\pi}\int_0^{+\infty}\frac{V_{\nu,(\alpha);j}^{(\mu)}(r,\beta)}{\beta^2+q^2}V_{\nu,(\alpha);2}^{(\mu)}(\rho,\beta)\Omega(\beta)d\beta\right)g_2(\rho)\sigma_2\rho^{2\alpha_2-1}d\rho+$$

$$+\int_0^{R_1}\left(\frac{2}{\pi}\int_0^{+\infty}\frac{V^{(\mu)}_{\nu,(\alpha);j}(r,\beta)}{\beta^2+q^2}V^{(\mu)}_{\nu,(\alpha);3}(\rho,\beta)\Omega(\beta)d\beta\right)g_3(\rho)\sigma_3 sh\rho d\rho+$$

$$+\int_0^{R_1}\left(\frac{2}{\pi}\int_0^{+\infty}\frac{V^{(\mu)}_{\nu,(\alpha);j}(r,\beta)}{\beta^2+q^2}V^{(\mu)}_{\nu,(\alpha);4}(\rho,\beta)\Omega(\beta)d\beta\right)g_4(\rho)\sigma_4 d\rho,\quad j=\overline{1,4}. \tag{15}$$

Having compared the solutions (10) and (15) as the result of unification, we will get the following formulae to compute the improper integrals.

$$\frac{2}{\pi}\int_0^{+\infty}S^{(\mu)}_{\nu,(\alpha);j}(r,\beta)V^{(\mu)}_{\nu,(\alpha);k}(\rho,\beta)d\beta=\sigma_k^{-1}H^{(\mu)}_{\nu,(\alpha);jk}(r,\rho,q),\quad j,k=\overline{1,4}, \tag{16}$$

$$\frac{2}{\pi}\int_0^{+\infty}S^{(\mu)}_{\nu,(\alpha);j}(r,\beta)Z^{(\mu),k}_{\nu,(\alpha);12}(\beta)d\beta=d_k^{-1}R^{(\mu),j}_{\nu,(\alpha);2k}(r,\rho,q),\quad j=\overline{1,4},\ k=1,2\,, \tag{17}$$

$$\frac{2}{\pi}\int_0^{+\infty}S^{(\mu)}_{\nu,(\alpha);j}(r,\beta)Z^{(\mu),k}_{\nu,(\alpha);22}(\beta)d\beta=-d_k^{-1}R^{(\mu),j}_{\nu,(\alpha);1k}(r,\rho,q),\quad j=\overline{1,4},\ k=1,2\,, \tag{18}$$

$$S^{(\mu)}_{\nu,(\alpha);j}(r,\beta)=\left(\beta^2+q_1^2\right)^{-1}V^{(\mu)}_{\nu,(\alpha);j}(r,\beta)\Omega^{(\mu)}_{\nu,(\alpha)}(\beta),\quad j=\overline{1,4}.$$

The conclusion of accomplished investigation is the statement.

Main theorem. *Let the vector-function* $f(r)=\{B_{\nu,\alpha_1}[g_1],B_{\alpha_2}[g_2],\Lambda_{(\mu)}[g_3],g_4''\}$ *is continuous on the set* I_3*, and the functions* $g_j(r)$ *satisfy the limitation conditions*

$$\lim_{r\to 0}r^{2\alpha_1+1}\left(V^{(\mu)}_{\nu,(\alpha);1}(r,\beta)\frac{dg_1(r)}{dr}-g_1(r)\frac{dV^{(\mu)}_{\nu,(\alpha);1}(r,\beta)}{dr}\right)=0,$$

$$\lim_{r\to +\infty}r^{2\alpha_1+1}\left(V^{(\mu)}_{\nu,(\alpha);4}(r,\beta)\frac{dg_4(r)}{dr}-g_4(r)\frac{dV^{(\mu)}_{\nu,(\alpha);4}(r,\beta)}{dr}\right)=0,$$

and conjugation conditions (2). If the condition of univalent solution for the boundary (1), (2) problem (9) is adhered, then there appear the formulae (16) – (18) to calculate the multi-parameter improper integrals due to Eigen-elements of hybrid differential operator $M^{(\mu)}_{\nu,(\alpha)}$, *indicated in the research project ([5], c. 83).*

Conclusions. The obtained formulae (16) – (18) contribute to reference literature in the sphere of computation of improper integrals from superposition of special functions of mathematical physics.

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ОБЧИСЛЕННЯ НЕВЛАСНИХ ІНТЕГРАЛІВ ЗА ВЛАСНИМИ ЕЛЕМЕНТАМИ ГІБРИДНОГО ДИФЕРЕНЦІАЛЬНОГО ОПЕРАТОРА ГАНКЕЛЯ 1-ГО РОДУ – (КОНТОРОВИЧА-ЛЄБЄДЄВА) 2-ГО РОДУ – ЛЕЖАНДРА 2-ГО РОДУ – ФУР'Є НА ПОЛЯРНІЙ ОСІ

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Резюме. *Методом порівняння розв'язку крайової задачі на полярній осі з трьома точками спряження для сепаратної системи диференціальних рівнянь Ганкеля, Конторовича-Лєбєдєва, Лежандра та Фур'є для модифікованих функцій, побудованого, з одного боку, методом функцій Коші, а з другого – методом відповідного гібридного інтегрального перетворення, обчислено поліпараметричну сім'ю невласних інтегралів за власними елементами гібридного диференціального оператора Ганкеля 1-го роду – (Конторовича-Лєбєдєва) 2-го роду – Лежандра 2-го роду – Фур'є.*

Ключові слова: *невласні інтеграли, власні елементи, гібридний диференціальний оператор, інтегральне перетворення, головні розв'язки.*

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