Вісник Тернопільського національного технічного університету
Scientific Journal of the Ternopil National Technical University
2017, № 2 (86)
ISSN 1727-7108. Web: visnyk.tntu.edu.ua

UDC 621.791.052

MATHEMATICAL MODEL OF BOUNDARY CRACK PROPAGATION AT BENDING OF SYMMETRIC THIN-WALLED FLANKS

Taras Dovbush; Nadiy Khomuk; Anatoliy Dovbush; Natalia Rubinets

Ternopil Ivan Puluj National Technical University, Ternopil, Ukraine

Summary. *There was studied engineering methods for determining stress intensity factors for complex symmetrical cross-sectional flank open at the edge of the crack by the action of stretching and bending effort. Analysis of specific stages crack propagation edge coupled Z-shaped flanks was carried out. Mathematical models and change depending obtained axial cross-sectional moment of inertia dual Z-like flank and intensity ratios derived by two methods – through nominal tension in the net – the intersection and in the development of boundary cracks were built. Analytical analysis proved the stress intensity factor for thin-walled coaxial elements Z-shaped flank, which perceive tensile strain and pure bending; correction functions which account for changing the geometry of the cross section in the development of cracks. A generalized correction function coupled, symmetric Z-flanks similar size 200x87x6 mm, which simplifies the calculation of residual life of structural elements of the system was obtained.*

Key words: *stress intensity factor, correction function, the length of the crack.*

Received 08.06.2017

Problem setting. The assessment of tensile strength of construction elements at acceptable stresses or acceptable rates for strength margin results in their excessive weight due to increased usage of metals. The evaluation of tensile strength of bearing elements due to their crack resistance is currently considered to be more up-dated. The essence of the method assumes they determine the cycles' number before crack origination at certain stress with further investigation of its propagation in particular material.

Analysis of the latest researches and publications. The issues [1, 2] define the durability of bearing elements in agricultural machinery under dynamic stresses upon thin-walled elements with cracks and real operation influence. The definition of stress intensity rate in some open and closed bends being used to determine the stress margin was presented in [3].

Research objective is to create the mathematical model of under-critical boundary crack propagation in thin-walled cold-bent flanks of dual symmetric Z-shape cross-cut intersections between bearing elements as well as to define analytically the correction functions accounting the alteration of thin-walled flanks geometry at fatigue crack propagation. It will allow determining of stress intensity rates for dual symmetric Z-shape cross-cut intersections being under tensile and flexure strains, making graphs of correction functions accounting the alteration of geometry of dual symmetric Z-shape cross-cut flanks during fatigue crack propagation along the wall by its approximation as well as obtaining of generalized correction functions for dual symmetric Z-shape flanks with dimensions of 200x87x6 mm.

Tasks assume analytical determination of correction functions accounting the alteration of geometry of dual symmetric Z-shape cross-cut intersections under stress propagation, in particular for the shelf and wall as well as making graphs of correction functions, their approximation and obtaining of generalized correction functions for dual symmetric Z-shape flanks with dimensions of 200x87x6 mm.

Research results. The creation of calculation models for stress-strain conditions of bearing construction elements due to real stress and modified method of minimum of strain energy potential [4, 5] facilitated identification of dangerous intersections of probable crack origination.

Corresponding author: *Natalia Rubinets; e-mail: natalyarubinets@gmail.com*

The open-end cross-cut intersection assumes its boundary to be probable crack origin where the stress concentrators are located due to available welded joints.

As calculation model to define the stress intensity rate K_I we will select the boundary fatigue crack that propagates in thin-walled dual symmetric Z-shape intersections inside the central side rail of solid fertilizers dispenser of ПРТ-9-type.

The central side rail of solid fertilizers dispenser of ПРТ-9-type is made of two Z-shape cross-cut intersections joint by plates. The scheme of cross-cut intersection of the side rail is displayed on Fig. 1.

For the selected frame of axes (Fig. 1a) we determine the geometric parameters for no-defect cross-cut intersection:

$$z_{C_0} = \frac{2 \cdot b \cdot h \cdot t_1 + h^2 \cdot t_2 + a \cdot \left(h + \frac{t_1}{2} + \frac{t_3}{2} \right) \cdot t_3}{4 \cdot b \cdot t_1 + 2 \cdot h \cdot t_2 + a \cdot t_3}; \quad (1)$$

$$I_Y = 2 \cdot \left[\frac{b \cdot t_1^3}{12} + b \cdot t_1 \cdot \left(z_{C_0} \right)^2 \right] + 2 \cdot \left[\frac{h^3 \cdot t_2}{12} + h \cdot t_2 \cdot \left(z_{C_0} - \frac{h}{2} \right)^2 \right] +$$

$$+ 2 \cdot \left[\frac{b \cdot t_1^3}{12} + b \cdot t_1 \cdot \left(h - z_{C_0} \right)^2 \right] + \frac{a \cdot t^3}{12} + a \cdot t^3 \cdot \left(h + \frac{t_1}{2} + \frac{t_3}{2} - z_{C_0} \right)^2. \quad (2)$$

Let us accept the fracturing of cross-cut intersection of dual Z-type flanks starts from the boundary crack in the shelf of a cross-cut intersection (Fig. 1b). The crack propagates along the shelf towards the wall and the next shelf.

Theoretically the thin-walled flank of Z-shape cross-cut intersection will lose the bearing capability when the total crack length reaches (Fig. 1b):

$$L = L_1 + L_2 + L_3, \quad (3)$$

where L_1 – the length of boundary crack at the first stage of its propagation
($0 \le L_1 \le b$; $L_2 = L_3 = 0$);

L_2 – the length of boundary crack at the second stage of its propagation
($0 \le L_2 \le h$; $L_1 = b$; $L_3 = 0$);

L_3 – the length of boundary crack at the third stage of its propagation
($0 \le L_3 \le b$; $L_1 = b$; $L_2 = h$).

To describe the propagation of the boundary crack we will investigate the thin-walled cross-cut intersection, which consists of two z-shape flanks under the condition the crack propagates only in one of them (Fig. 1). The investigated flank is stressed only by M yaw moment against Y axis. At pressure of yaw moment upon the thin-walled flank its horizontal shelves will be expanded but the vertical wall will be under flexure strain. The stressed condition in Z-shape cross-cut intersection with the crack can be approximately modeled as separate plates of the same thickness and width with the boundary crack at the same stress.

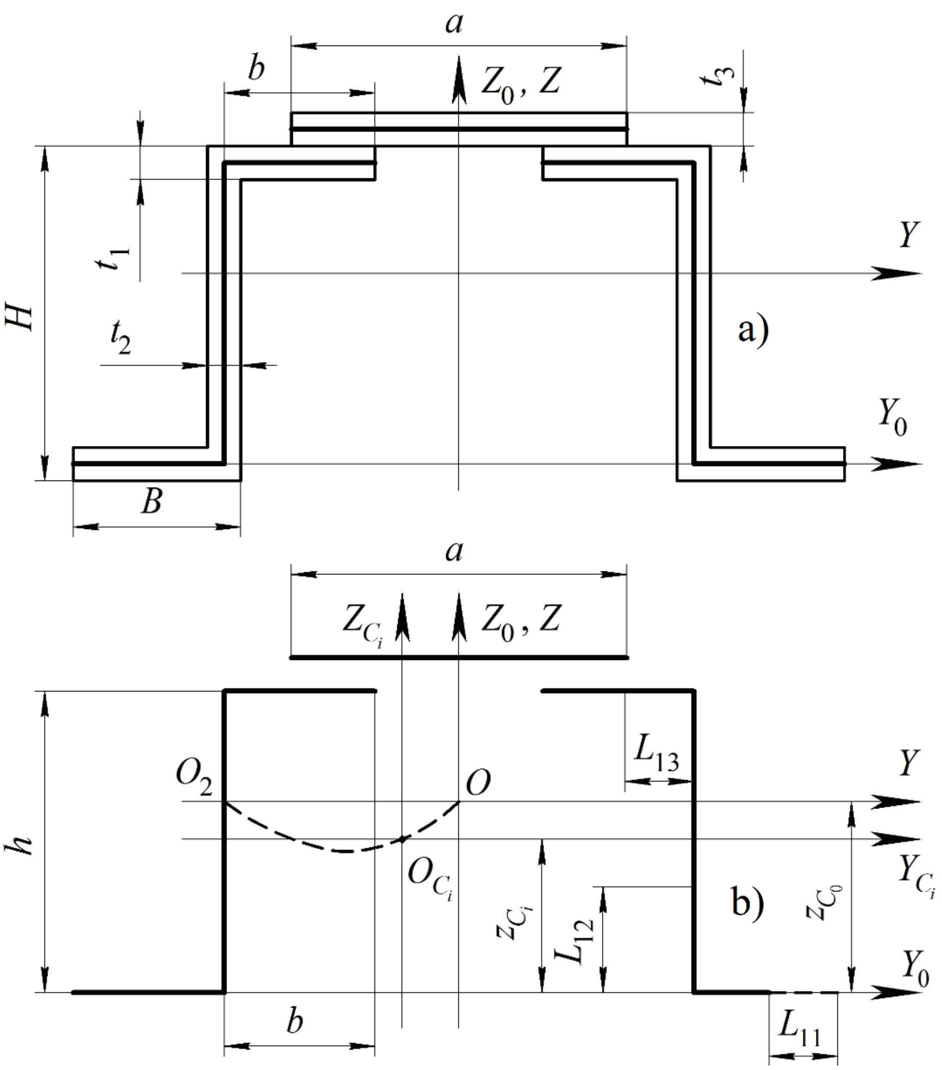

Figure 1. Scheme to determine the geometrical characteristics of thin-walled flank with an edge crack:
a) schematization of two Z-shaped cross sections;
b) schematization of two Z-shaped cross sections with an edge crack;
*O–O*2 – trajectory shift the center of gravity in the development of cracks

The general expressions describing the changes in axis of mass z_{C_i} and I_{Yi} centroidal moment of inertia at cross-cut intersection consisting of two Z-shape flanks during the crack propagation:

$$z_{C_i} = \left[(h-L_2)\cdot t_2 \cdot \frac{h+L_2}{2} + (b-L_3)\cdot t_1 \cdot h + a \cdot t_3 \cdot \left(h + \frac{t_1}{2} + \frac{t_3}{2} \right) + \right. \\ \left. + (h-L_2)\cdot t_2 \cdot \frac{h+L_2}{2} + \left(b-L_3 \right)\cdot t_1 \cdot h \right] / \left[\left(b-L_1 \right)\cdot t_1 + (h-L_2)\cdot t_2 + \right. \\ \left. + (b-L_3)\cdot t_1 + a\cdot t_3 + (b-L_1)\cdot t_1 + (h-L_2)\cdot t_2 + \left(b-L_3 \right)\cdot t_1 \right]; \qquad (4)$$

$$I_{Yi} = \frac{(b-L_1)\cdot t_1^3}{12} + (b-L_1)\cdot t_1 + z_{Ci}^2 + \frac{(h-L_2)^3\cdot t_2}{12} + (h-L_2)\cdot t_2 \cdot \left(z_{Ci} - \frac{h+L_2}{2} \right)^2 +$$

$$+\frac{a\cdot t_3^3}{12}+a\cdot t_3\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}-z_{Ci}\right)^2+\frac{(b-L_1)\cdot t_1^3}{12}+(b-L_1)\cdot t_1\cdot z_{Ci}^2+\frac{(h-L_2)^3\cdot t_2}{12}+$$

$$+(h-L_2)\cdot t_2\cdot\left(z_{C1}-\frac{h+L_2}{2}\right)^2+\frac{(b-L_3)\cdot t_1^3}{12}+(b-L_3)\cdot t_1\cdot(z_{Ci}-h)^2. \quad (5)$$

According to the formulae we determine the stress-strain condition of the dual Z-shape cross-cut intersection on the each stage of crack propagation keeping in mind that the thin-walled cane operates only in bend direction (there is no rotation of frame of axes).

We will accept that at the first stage of crack propagation $L=L_1,\ L_2=L_3=0,$ then axis of mass and centroidal moment of inertia can be defined due to the formulae:

$$z_{C_1}=\frac{2\cdot b\cdot h\cdot t_1+h^2\cdot t_2+a\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}\right)\cdot t_3}{2\cdot b\cdot t_1+2\cdot(b-L)\cdot t_1+2\cdot h\cdot t_2+a\cdot t_3}; \quad (6)$$

$$I_{Y_1}=\frac{b\cdot t_1^3}{6}+\frac{1}{6}\cdot(b-L)\cdot t_1^3+\frac{h^3\cdot t_2}{6}+\frac{a\cdot t_3^3}{12}+$$

$$+\frac{2\cdot(b-L)\cdot t_1\cdot\left[2\cdot b\cdot h\cdot t_1+h^2\cdot t_2+a\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}\right)\cdot t_3\right]^2}{\left[2\cdot b\cdot t_1+2\cdot(b-L)\cdot t_1+2\cdot h\cdot t_2+a\cdot t_3\right]^2}+$$

$$+a\cdot t_3\cdot\left[h+\frac{t_1}{2}+\frac{t_3}{2}-\frac{2\cdot b\cdot h\cdot t_1+h^2\cdot t_2+a\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}\right)\cdot t_3}{2\cdot b\cdot t_1+2\cdot(b-L)\cdot t_1+2\cdot h\cdot t_2+a\cdot t_3}\right]^2+$$

$$+2\cdot b\cdot t_1\cdot\left[-h+\frac{2\cdot b\cdot h\cdot t_1+h^2\cdot t_2+a\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}\right)\cdot t_3}{2\cdot b\cdot t_1+2\cdot(b-L)\cdot t_1+2\cdot h\cdot t_2+a\cdot t_3}\right]^2+$$

$$+2\cdot h\cdot t_2\cdot\left[-\frac{h}{2}+\frac{2\cdot b\cdot h\cdot t_1+h^2\cdot t_2+a\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}\right)\cdot t_3}{2\cdot b\cdot t_1+2\cdot(b-L)\cdot t_1+2\cdot h\cdot t_2+a\cdot t_3}\right]^2. \quad (7)$$

The normal stresses in cross-cut intersections of the flank can be determined according to the formulae:

- in non-cracked intersection

$$\sigma_1^{(t)}=\frac{M\cdot z_{C_0}}{I_Y}; \quad (8)$$

- in cracked intersection

$$\sigma_{nom}^{(t)} = \frac{M \cdot z_{C_1}}{I_{Y_1}}. \qquad (9)$$

On the given stage of cross-cut crack propagation the relative parameter is determined as $\varepsilon_1 = \dfrac{L_1}{b+h+b}$. Due to (2, 5 – 9), we get the dependency for stress intensity rate:

$$K_I^{(t)} = \sigma_{nom}^{(t)} \cdot (1-\varepsilon_1)\sqrt{L_1 \cdot \pi} \cdot \left(1+0{,}128\varepsilon_1 - 0{,}288\varepsilon_1{}^2 + 1{,}525\varepsilon_1{}^3\right); \qquad (10)$$

$$K_I^{(t)} = \sigma_1^{(t)} \cdot \sqrt{L_1 \cdot \pi} \cdot F_1(\varepsilon_1), \qquad (11)$$

where $F_1(\varepsilon_1)$ – non-dimensional coefficient of geometry alteration for the shelf of thin-walled flank during crack propagation.

$$F_1(\varepsilon_1) = \Big\langle \left(12{,}2 \cdot b^3 + 18{,}3 \cdot b^2 \cdot h + 9{,}15 \cdot b \cdot h^2 + 1{,}525 \cdot h^3\right) \times$$

$$\times \left[4 \cdot h \cdot b \cdot t_1 + 2 \cdot h^2 \cdot t_2 + 2 \cdot a \cdot h \cdot t_3 + a \cdot t_3 \cdot (t_1 + t_3)\right] \times$$

$$\times \Big\{ b^2 \cdot \left(48 \cdot h^2 \cdot t_1 + 16 \cdot t_1^4\right) + 4 \cdot h^4 \cdot t_2^2 + 8 \cdot a \cdot h^3 \cdot t_2 \cdot t_3 + a^2 \cdot t_3^4 +$$

$$+ a \cdot h^2 \cdot t_2 \cdot t_3 \cdot (12 \cdot t_1 + 12 \cdot t_3) + a \cdot h \cdot t_2 \cdot t_3 \cdot \left(6 \cdot t_1^2 + 12 \cdot t_1 \cdot t_3 + 8 \cdot t_3^2\right) +$$

$$+ b \cdot t_1 \cdot \Big[32 \cdot h^3 \cdot t_2 + 24 \cdot a \cdot h^2 \cdot t_3 \cdot \left(16 \cdot t_1^2 + 24 \cdot t_1 \cdot t_3 + 16 \cdot t_3^2\right) +$$

$$+ h \cdot \left(8 \cdot t_1^2 \cdot t_2 + 24 \cdot t_1 \cdot t_3 + 24 \cdot a \cdot t_3^3\right) \Big] \Big\} \times (0{,}78 + \varepsilon_1) \cdot \left[h + b \cdot (2 - 2 \cdot \varepsilon) - 1 \cdot h \cdot \varepsilon_1\right] \times$$

$$\times \left(0{,}84 - 0{,}97 \cdot \varepsilon_1 + \varepsilon_1{}^2\right) \Big\rangle \Big/ \Big\langle (2 \cdot b + h)^4 \cdot (h + t_1) \cdot (4 \cdot b \cdot t_1 + 2 \cdot h \cdot t_2 + a \cdot t_3) \times$$

$$\times \Big\{ a^2 \cdot t_3^4 + b^2 \cdot t_1^2 \cdot \left[h^2 \cdot (48 - 96 \cdot \varepsilon_1) + 16 \cdot t_1^2 (-1 + \varepsilon_1)^2\right] +$$

$$+ a \cdot h^3 \cdot t_3 \cdot (8 \cdot t_2 - 24 \cdot t_1 \cdot \varepsilon_1) + h^4 \cdot t_2 \cdot (4 \cdot t_2 - 16 \cdot t_1 \cdot \varepsilon_1) +$$

$$+ a \cdot h \cdot t_3 \cdot \left[8 \cdot t_2 \cdot t_3^2 - 8 \cdot t_1^3 \cdot \varepsilon_1 + t_1^2 \cdot (6 \cdot t_2 - 12 \cdot t_3 \cdot \varepsilon_1)\right] + h^2 \cdot \Big[t_1^3 \cdot \varepsilon_1 \cdot (-4 \cdot t_2 + 4 \cdot t_1 \cdot \varepsilon_1) +$$

$$+ a \cdot t_3 \cdot \left(12 \cdot t_1 \cdot t_2 \cdot + 12 \cdot t_2 \cdot t_3 - 24 \cdot t_1^2 \cdot \varepsilon_1 - 24 \cdot t_1 \cdot t_3 \cdot \varepsilon_1\right) \Big] +$$

$$+ b \cdot t_1 \cdot \Big\{ a \cdot t_3 \cdot \left[t_1 \cdot t_3 \cdot (24 - 24 \cdot \varepsilon_1) + t_1^2 \cdot (16 - 16 \cdot \varepsilon_1) + t_3^2 \cdot (16 - 16 \cdot \varepsilon_1)\right] +$$

$$+a\cdot h^3\cdot t_3\cdot(24-48\cdot\varepsilon_1)+h^3\cdot[t_2\cdot(32-32\cdot\varepsilon_1)-48\cdot t_1\cdot\varepsilon_1]+$$

$$+h\cdot\left[a\cdot t_1\cdot t_3(24-48\cdot\varepsilon_1)+a\cdot t_3^2\cdot(24-48\cdot\varepsilon_1)+\right.$$

$$\left.+t_1^2\cdot t_2\cdot(8-8\cdot\varepsilon_1)+t_1^3\cdot\varepsilon_1\cdot(-16+16\cdot\varepsilon_1)\right]\}\}\rangle. \qquad (12)$$

Let us investigate the second stage of crack propagation inside the mentioned-above intersection. The geometric parameters are:

$$z_{C_2}=\frac{2\cdot b\cdot h\cdot t_1+(b+h-L)\cdot(-b+h+L)\cdot t_2+a\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}\right)\cdot t_3}{2\cdot b\cdot t_1+2\cdot(b+h-L)\cdot t_2+a\cdot t_3}; \qquad (13)$$

$$I_{Y_2}=\frac{b\cdot t_1^3}{6}+\frac{1}{6}(b+h-L)^3\cdot t_2+\frac{a\cdot t_3^3}{12}+a\cdot t_3\times$$

$$\times\left\{h+\frac{t_1}{2}+\frac{t_3}{2}-[2\cdot b\cdot h\cdot t_1+(b+h-L)\cdot(-b+h+L)\cdot t_2+\right.$$

$$\left.+a\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}\right)\cdot t_3\right]\Big/\left[2\cdot b\cdot t_1+2\cdot\left(b+h-L\right)\cdot t_2+a\cdot t_3\right]\Big\}^2+$$

$$+2\cdot b\cdot t_1\cdot\left[-h+\frac{2\cdot b\cdot h\cdot t_1+(b+h-L)\cdot(-b+h+L)\cdot t_2+a\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}\right)\cdot t_3}{2\cdot b\cdot t_1+2\cdot(b+h-L)\cdot t_2+a\cdot t_3}\right]^2+$$

$$+2\cdot(b+h-L)\cdot t_2\times$$

$$\times\left[\frac{1}{2}\cdot(b-h-L)+\frac{2\cdot b\cdot h\cdot t_1+(b+h-L)\cdot(-b+h+L)\cdot t_2+a\cdot\left(h+\frac{t_1}{2}+\frac{t_3}{2}\right)\cdot t_3}{2\cdot b\cdot t_1+2\cdot(b+h-L)\cdot t_2+a\cdot t_3}\right]^2. \qquad (14)$$

According to obtained data the normal stresses in cross-cut intersection are

$$\sigma_{nom}^{(b)}=\frac{M\cdot(z_{C_2}-L_2)}{I_{Y_2}}; \qquad (15)$$

and relative parameters are

$$\varepsilon_2=\frac{b+L_2}{b+h+b}. \qquad (16)$$

Having applied the dependence for definition of normal stresses in cross-cut intersection of non-cracked flank (8), cracked cross-cut intersection (9), geometric parameters (13) and (14) and due to (15) and (16) we will get the dependence for definition of stress intensity rate of normal fracture $K_I^{(b)}$:

$$K_I^{(b)} = \sigma_{nom}^{(b)} \cdot (1-\varepsilon_2)^2 \sqrt{\pi \cdot L_2} \times$$

$$\times \left[1{,}222 - 1{,}4 \cdot \varepsilon_2 + 7{,}33 \cdot (\varepsilon_2)^2 - 13{,}08 \cdot (\varepsilon_2)^3 + 14(\varepsilon_2)^4\right] = \sigma_1^b \sqrt{\pi \cdot L_1} \cdot F_2(\varepsilon_2), \quad (17)$$

where $F_2\ (\varepsilon_2)$ – non-dimensional correction coefficient accounting the geometry alteration for the wall of thin-walled flank during fatigue crack propagation.

$$F_2(\varepsilon_2) = \Big\{ 28 \cdot \big[16 \cdot b^2 \cdot \left(3 \cdot h^2 t_1^2 + t_1^4\right) + 4 \cdot h^4 \cdot t_2^2 +$$

$$+ 8 \cdot a \cdot h^3 \cdot t_2 \cdot t_3 + a^2 \cdot t_3^4 + 12 \cdot a \cdot h^2 \cdot t_2 \cdot t_3 \big] +$$

$$+ 8 \cdot b \cdot t_1 \cdot \big[4 \cdot h^3 \cdot t_2 + 3 \cdot a \cdot h^3 \cdot t_3 + a \cdot t_3 \cdot \left(2 \cdot t_1^2 + 3 \cdot t_1 \cdot t_3 + 2 \cdot t_3^2\right) +$$

$$+ h \cdot \left(t_1^2 \cdot t_2 + 3 \cdot a \cdot t_1 \cdot t_3 + 3 \cdot a \cdot t_3^2\right) \big] \Big\} \cdot (-1 + \varepsilon_2)^2 \cdot \left(0{,}509 - 1{,}068 \cdot \varepsilon_2 + {\varepsilon_2}^2\right) \times$$

$$\times \left(0{,}157 + 0{,}133 \cdot \varepsilon_2 + {\varepsilon_2}^2\right) \times \Big\langle\ b - (2 \cdot b + h) \cdot \varepsilon + \{2 \cdot b \cdot h \cdot t_1 + 1/2 \cdot a \cdot t_3 \cdot (2 \cdot h + t_1 + t_3) +$$

$$+ t_2 \cdot (-b + h + 2 \cdot b \cdot \varepsilon_2 + h \cdot \varepsilon_2) \cdot [b + h - (2 \cdot b + h) \cdot \varepsilon_2] \} \div$$

$$\div \{2 \cdot b \cdot t_1 + a \cdot t_3 + 2 \cdot t_2 \cdot [b + h - (2b + h) \cdot \varepsilon_2] \} \Big\rangle \div \Big\langle\ (h + t_1) \cdot (4 \cdot b \cdot t_1 + 2 \cdot h \cdot t_2 + a \cdot t_3) \times$$

$$\times \Big\{ 2 \cdot b \cdot t_1^3 + a \cdot t_3^3 + 2 \cdot t_2 \cdot [b + h - (2 \cdot b + h) \cdot \varepsilon_2] \Big\} +$$

$$+ \Big\{ 6 \cdot t_2 \cdot [b + h - (2 \cdot b + h) \cdot \varepsilon_2] \cdot \left[2 \cdot b^2 \cdot t_1 + a \cdot t_3 - (2 \cdot b + h) \cdot \varepsilon_2\right] +$$

$$+ b \cdot [2 \cdot h \cdot t_1 + a \cdot t_3 - 2 \cdot (2 \cdot b + h) \cdot t_1 \cdot \varepsilon_2] \Big\} \Big\rangle^2 \div$$

$$\div \left[a \cdot t_3 - 2 \cdot h \cdot t_2 \cdot (-1 + \varepsilon_2) + 2 \cdot b \cdot (t_1 + t_2 - 2 \cdot t_2 \cdot \varepsilon_2)\ \right]^2 +$$

$$+ \Big\{ 12 \cdot a \cdot t_3 \cdot \big[b^2 \cdot t_2 + t_2 \cdot [h - (2 \cdot b + h) \cdot \varepsilon_2] + [h + t_1 + t_3 -$$

$$- (2b + h) \cdot \varepsilon_2] + b \cdot \Big\{ t_1^2 + t_1 \cdot (t_2 + t_3) + t_2 \cdot [2 \cdot h + t_3 - 2 \cdot (2 \cdot b + h) \cdot \varepsilon_2] \Big\} \big] \Big\}^2 \div$$

$$\div\left[a\cdot t_3-2\cdot h\cdot t_2\cdot(-1+\varepsilon_2)+2\cdot b\cdot(t_1+t_2-2\cdot t_2\cdot\varepsilon_2)\right]^2+$$

$$+24\cdot b\cdot t_1\cdot\left\{\left[h-\left\{2\cdot b\cdot h\cdot t_1+1/2\cdot a\cdot t_3\cdot(2\cdot h+t_1+t_3)+\right.\right.\right.$$

$$\left.+t_2\cdot(-b+h+2\cdot b\cdot\varepsilon_2+h\cdot\varepsilon)\cdot\left[b+h-(2\cdot b+h)\cdot\varepsilon_2\right]\right\}\div$$

$$\left.\left.\div\left\{2\cdot b\cdot t_1+a\cdot t_3+2\cdot t_2\cdot\left[b+h-(2\cdot b+h)\cdot\varepsilon_2\right]\right\}^2\right]\right\}. \qquad (18)$$

The correction functions in graphic design for the dual Z-shape thin-walled flank are shown on Fig. 2.

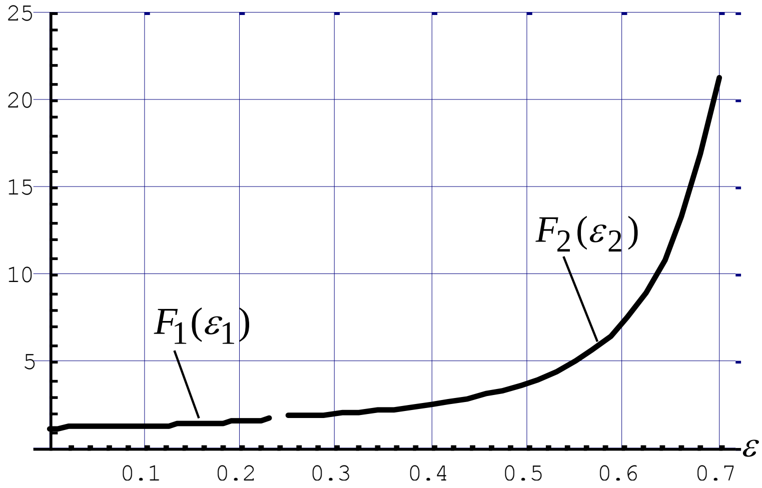

Figure 2. Dependence functions between $F_i(\varepsilon_i)$ and defects ε for dual Z-shaped thin-walled flank with dimensions 200×87×6 mm

Having approximated the functions $F_1(\varepsilon_1)$ and $F_2(\varepsilon_2)$ (Fig. 2), we will get the correction function of dual symmetric Z-shape cross-cut intersection with dimensions 200x87x6 mm.

$$F(\varepsilon)=1{,}192+1{,}362\cdot\varepsilon-8{,}747\cdot\varepsilon^2-42{,}105\cdot\varepsilon^3+1686{,}130\cdot\varepsilon^4-$$

$$-9652{,}620\cdot\varepsilon^5+23140{,}7\cdot\varepsilon^6-23508\cdot\varepsilon^7+4462{,}25\cdot\varepsilon^8+5242{,}86\cdot\varepsilon^9. \qquad (19)$$

Conclusions. The authors elaborated the mathematical model of boundary crack propagation during the flexure strain of thin-walled element of dual symmetric Z-shape flank. There were obtained the dependences for stress intensity rate and correction functions that can be applied to determine the stress-strain conditions for metallic bearing elements of dual symmetric Z-shape intersections at each stage of crack propagation during flexure strain. The obtained dependences can help to define the stress margin of frame construction and suggest the ways to upgrade it.

References

1. Rybak T.I. Poshukove konstruiuvannia na bazi optymizatsii resursu mobilnykh silskohospodarskykh mashyn. Ternopil, VAT. TVPK "ZBRUCh" Publ., 2003. 332 p. [In Ukrainian].
2. Andreikiv A.E. Ustalostnoe razrushenye y dolhovechnost' konstruktsyy. "Scientific thought" Publ., 1992. 120 p. [In Ukrainian].
3. Pidhurskyy M.I. Metody vyznachennya KIN dlya defektnykh elementiv vidkrytoho profilyu. Visnyk TDTU, 2006, vol. 11, no. 3, pp. 92 – 108. [In Ukrainian].
4. Rybak T.I. Udoskonalennya metodyky otsinyuvannya resursu roboty nesuchykh system sil's'kohospodars'kykh mashyn. Mashynobuduvannya, avtomatyzatsiya vyrobnytstva ta protsesy mekhanichnoyi obrobky. Visnyk TNTU, 2012, no. 4(68), pp. 107 – 113. [In Ukrainian].
5. Popovich P.V. Enerhetychnyy sposib rozkryttya statychnoyi nevyznachenosti nesuchykh ramnykh sterzhnevykh system mobil'nykh sil's'kohospodars'kykh mashyn. "Innovatsiyni napryamky rozvytku tekhnichnoho servisu mashyn". Visnyk KhNTUSH im. P. Vasylenka, 2012, no. 120, pp. 198 – 203. [In Ukrainian].
6. Stashkiv M. Vyznachennya KIN dlya kutovoyi naskriznoyi trishchyny u tonkostinnomu sterzhni pryamokutnoho profilyu pry diyi z·hynal'noho momentu. Visnyk TDTU, 2003, vol. 8, no. 3, pp. 32 – 38. [In Ukrainian].

Список використаної літератури

1. Рибак, Т.І. Пошукове конструювання на базі оптимізації ресурсу мобільних сільськогосподарських машин [Текст] / Рибак Т.І. – ВАТ ТВПК "Збруч". – 2003. – 332 с.
2. Андрейкив, А.Е. Усталостное разрушение и долговечность конструкций [Текст] / А.Е. Андрейкив, А.И. Дарчук. – К.: Наук. думка. – 1992. – 120 с.
3. Підгурський, М.І. Методи визначення КІН для дефектних елементів відкритого профілю [Текст] / М.І. Підгурський, М Я. Сташків // Вісник ТДТУ. – 2006. – Т. 11, № 3. – С. 92 – 108.
4. Рибак, Т.І. Удосконалення методики оцінювання ресурсу роботи несучих систем сільськогосподарських машин [Текст] / Т.І. Рибак, Є.Й. Ріпецький, Т.А. Довбуш // Машинобудування, автоматизація виробництва та процеси механічної обробки. Вісник ТНТУ. – Тернопіль: ТНТУ. – 2012. – №4(68). – С. 107 – 113.
5. Енергетичний спосіб розкриття статичної невизначеності несучих рамних стержневих систем мобільних сільськогосподарських машин. "Інноваційні напрямки розвитку технічного сервісу машин" [Текст] / П. Попович, М. Сташків, Т. Довбуш, Г. Дудка // Вісник ХНТУСГ ім. П. Василенка. – Харків: ХНТУСГ. – 2012. – Вип. 120. – С. 198 – 203.
6. Сташків, М. Визначення КІН для кутової наскрізної тріщини у тонкостінному стержні прямокутного профілю при дії згинального моменту [Текст] / М. Сташків // Вісник ТДТУ. – 2003. – Т. 8, № 3. – С. 32 – 38.

УДК 621.791.052

МАТЕМАТИЧНА МОДЕЛЬ РОЗВИТКУ КРАЙОВОЇ ТРІЩИНИ ПРИ ЗГИНІ СИМЕТРИЧНИХ ТОНКОСТІННИХ ПРОФІЛІВ

Тарас Довбуш; Надія Хомик; Анатолій Довбуш; Наталія Рубінець

Тернопільський національний технічний університет імені Івана Пулюя, Тернопіль, Україна

***Резюме**. Розглянуто інженерні методи визначення коефіцієнтів інтенсивності напружень для складних симетричних поперечних перетинів відкритого профілю при розвитку крайової тріщини за дії зусиль розтягу та згину. Проведено аналіз характерних етапів розповсюдження крайової тріщини спарених Z-подібних профілів. Побудовано математичні моделі та отримано залежності зміни осьового моменту інерції поперечного перетину спареного Z-подібного профілю, виведено коефіцієнти інтенсивності напружень за двома методами – через номінальні напруження в нетто-перетині та в процесі розвитку крайової тріщини. Аналітично виведен: коефіцієнти інтенсивності напружень для елементів тонкостінного спареного Z-подібного профілю, які сприймають деформації розтягу і чистого згину; поправочні функції, які враховують зміну геометрії поперечного перетину в процесі розвитку тріщини. Отримано узагальнюючу поправочну функцію спарених симетричних Z-подібних профілів розмірами 200x87x6 mm, яка спрощує розрахунок залишкового ресурсу роботи елементів конструктивної системи.*

***Ключові слова**: коефіцієнт інтенсивності напружень, поправочна функція, довжина тріщини.*

Отримано 08.06.2017