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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ. МАТЕМАТИКА

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IDENTIFICATION OF PARAMETERS OF DIPOLE MODEL OF THE LED RADIATION SOURCE

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Summary. *The results of identification of the parameters of a mathematical model of the radiation source LED are given. For the mathematical modeling abstract physical object is used – dipole and assumptions that diagram radiation of LED is created both by the system of electric dipoles and by the optical system LED. Vectors of electrical strength of dipoles are defined for modeling of low-radiation energy transfer through layered biological media. Mathematical model finds further use in automation to achieve optimal exposure surface of biological objects in layered biological media. The results are used in the research of biological objects stimulation with light intensity below the standard level in the prevention, rehabilitation and medical diagnostics. The decreasing of invasiveness of biological objects, increasing the resolution of assessment of their reaction as well as possibilities for automation of researches have been received.*

Key words*: LED, system of dipoles, mathematical model, radiation, basic strength, layered bio media, irradiation of bio objects.*

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The problem and its connection with scientific and practical tasks. In biophysical researches and medical diagnostic procedures the energy impact of electromagnetic fields on biological objects is often used. Particularly promising is the use for such influence the energy flow with extremely low light intensity [1, 2]. The benefits of low intensity impact on the one hand, and the probabilistic nature of biological objects response – on the other hand, require the general totality of reviews for reliable evaluation (statistics) of the bio object’s state according to the parameters of its response to stimulation. In addition, there is a need to find the optimal quantitative measurement of intensity I of irradiation, and qualitative – its type (i.e., such as stationary $(\lambda_0 \pm \Delta\lambda) >> L$, resonance $(\lambda_0 \pm \Delta\lambda) \approx L$, optical $(\lambda_0 \pm \Delta\lambda) << L$), spectrum width $c/(\lambda_0 \pm \Delta\lambda)$, coordinate location in space and energy efficiency of the source; λ – wavelength of irradiation, L – size of the object. It is also necessary to justify the method for determining the intensity for cases of intermediate values criteria such as type of irradiation, to consider interference, diffraction in the spreading of energy in bio media, to determine the intensity of radiation on the surface of target bio object in bio media. These facts in combination cause considerable difficulty (in terms of complexity theory) of interactive method of investigation. There is a need for an automated search of the optimal irradiation by means of setting technical means through analysis, believability estimation of the parameters of irradiation and response.

Certain tasks of the similar problem, their particular cases have not been investigated systematically, on a common theoretical basis. In ophthalmology, astronomy, biophysics in order to estimate the quality of the image on the surface of the target object of irradiation (but not the intensity of irradiation) in case of intermediate values of the criteria of the type of irradiation the replacement of the light ray to the light beam is used [3, 4]. Control of low irradiation of the surface of bio-object (not image [5]) is used in quantum physics [6]. Development of a method to determine low-light irradiation of the surface of the target object, which is in the bio media, is essential for bioengineering.

Analysis of recent researches. For irradiation of biological object and bio media, in particular with low intensity, the use of LEDs increases [1, 7]. This LED is studied as: (a) spontaneous radiation of light due to the contact of semiconductors; (b) forms, structures and materials of the distribution channel of the light waves; (c) a combination of directions (a) and (b). These approaches in detail and generally are used in the analysis of electromagnetic phenomena based on wave theory of electromagnetic field have been used to calculate the efficiency of the distribution of spontaneous radiation in periodically layered structures [8]. Computational modeling of the spread of radiation dipoles in periodically layered structures as diffraction grate and resonator has been used providing a certain description of radiation. Radiometric approach for modeling the spatial distribution of intensity of encapsulated LEDs based on chip LED dipole model presented in [9], where the analytical nature of the relationship between the character of radiation and parameters of LED chips of encapsulator and reflectors have been received. Thus, there are two main ways to achieve low irradiation intensity of the target bio object – (a-c), or by control the characteristics of the given LED. The second way for Biomedical Engineering is preferred. For the design of adaptive, optimal irradiation the development of the mathematical model of LED source is required. The ratio spatial distribution of light energy in space (diagram LED) is always known [10]. Therefore, there is an opportunity to use it as a source of the wave function for mathematical model that will allow studying the effect of the layered environment – the effects of diffraction, interference, refraction and intensity of irradiation of the object in such an environment. There are different theoretical approaches, rendering realistic physical models of energy waves or photons of light. In studies in this area Maxwell's equation is used. The areas with the pointing vector have been received that indicate waviness that can be interpreted by the lines of energy flow [11].

The aim of the research. Mathematical modeling of the pointed (low extended) radiation source on the chip with additional optics (lens LED) for the use of the received model in the automation to achieve optimal irradiation of the surface of the bio object that is situated in the layered bio media.

Setting objectives. To justify the choice of a mathematical model of the radiation source LED using abstract physical object – dipole. To use the assumption that the radiation diagram of the LED system is generated by the system of electric dipoles and its optical system. To identify the basic vectors of the strength field of LED source that are necessary for modeling of transfer of low-radiation energy of light through the layered bio media.

The main material. The field $V(x, y, z, t)$ from the source at the point P_0 in media according to the distribution of material, initial and boundary conditions is a possibility based on the principles of preserving of properties of this field to present by means of equation. In particular, in a vacuum, without currents and charges, this equation takes the form of [12]

$$\nabla^2 V(x, y, z, t) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} V(x, y, z, t) . \tag{1}$$

Formula (1) reflects the conservation of the properties of the field within the boundaries, that is, save of the changes of the values of field strength. Dependent only on the coordinate solution $U(x,y,z)$ of this equation (Helmholtz)

$$\nabla^2 U(x,y,z) = \frac{\omega}{c^2} U(x,y,z)$$

does not depend on time, the general solution («Integral») of equation (1) - running monochromatic wave

$$V(x,y,z,t) = U(x,y,z)e^{-i\omega t}. \quad (2)$$

According to Huygens-Fresnel principle at the point *P* of the field inside the surface *S (A, B, C)* which covers the volume *v* there is a superposition of waves fig. 1 (a, b).

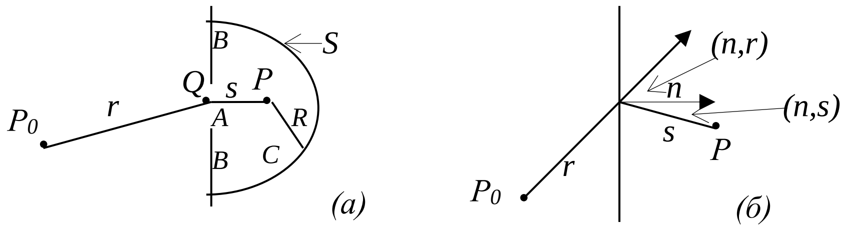

Figure 1. The geometry of the light field

Solution of homogeneous equation of the wave in an arbitrary point *P* of the field consists of the value of unknown quantity and its first derivative at all points of an arbitrary closed around point *P* surface (Kirchhoff's theorem).

Using Green's theorem for the field $U(x,y,z) = e^{iks}/s$ (*s*=*QP*) and integrating with the exception in terms of a marginal sense of point *P* by its asymptotic coverage of the surface S' allows us to derive a formula for the solution [12]

$$U(P) = \frac{1}{4\pi}\iint_S \left\{ U \frac{\partial}{\partial n}\left(\frac{e^{iks}}{s}\right) - \frac{e^{iks}}{s}\frac{\partial U}{\partial n} \right\} dS \quad (3)$$

(Helmholtz-Fresnel formula).

The properties of the field defined by the theorem for non- monochromatic wave exist in the regularity of the field, which allowed to use the Fourier transform [12]

$$V(x,y,z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U_\omega(x,y,z)e^{-i\omega t} dt, \quad (4)$$

$$U_\omega(x,y,z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} V(x,y,z,t)e^{i\omega t} dt, \quad (5)$$

and get the expression of each ω-field component

$$U_{\omega}(P)=\frac{1}{4\pi}\iint_{S}\left\{U_{\omega}\frac{\partial}{\partial n}\left(\frac{e^{iks}}{s}\right)-\frac{e^{iks}}{s}\frac{\partial U_{\omega}}{\partial n}\right\}dS\,. \tag{6}$$

By means of inverse transformation the expression of changes of the field in time has been received

$$V(P,t)=\frac{1}{4\pi}\iint_{S}\left\{[V]\frac{\partial}{\partial n}\left(\frac{1}{s}\right)-\frac{1}{cs}\frac{\partial s}{\partial n}\left[\frac{\partial V}{\partial t}\right]-\frac{1}{s}\left[\frac{\partial V}{\partial n}\right]\right\}dS\,. \tag{7}$$

In brackets «delay values» – the values of the functions in $t-sc$. Formula (7) represents a generalized Kirchhoff's theorem [12].

If P is outside the surface S, then the integral (7) is zero. The last member in (7) represents the contribution of distributed sources with intensity $\frac{1}{s}\left[\frac{\partial V}{\partial n}\right]$ per unit area, and the first two members – the contribution of the dipoles with intensity $\frac{V}{4\pi}$ per unit area, directed normally to the surface (sources and dipoles are fictitious [12]).

Due to the large frequency (3,75 – 7,5) × 1,013 sec^{-1}, and the nature of the energy impact of physical field on bio object [2] during the researches, the instantaneous values of the field are not used. The value has its intensity – the average according to time energy that passes through a unit area and is determined by the vectors of electric and magnetic intensity

$$I=c\left|\langle E\times H\rangle\right|/4\pi\,. \tag{8}$$

In a remote from the source area the waves are represented by intensities

$$\vec{E}_0=\vec{e}(r)e^{ik_0\ell(r)},\ \vec{H}_0=\vec{h}(r)e^{ik_0\ell(r)}, \tag{9}$$

where r – is the distance from the dipole, $\ell(r)$ – «optical length», $\vec{e}$ and $\vec{h}$ – are relatively small constant vectors, but at a significant distance (r >> λ_0) of the dipole, these vectors are, with appropriate normalization of dipole moment, independent $k_0=\omega/c=2\pi/\lambda_0$ position of vector function (which can generally be complex).

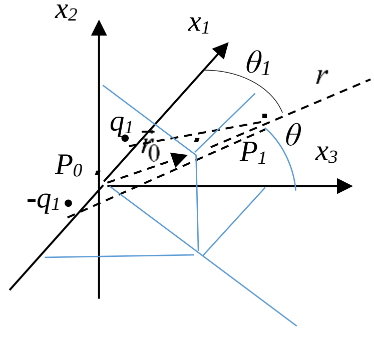

Figure 2. Scheme of dipoles source of LED light radiation (on x_2, x_3 dipoles are not shown)

With the help of (3) as a test solution of Maxwell's equation lead to the set of dependencies among $\vec{e}(r)$, $\vec{h}(r)$ and ℓ [12]. For large k_0 (small λ_0) it is necessary that ℓ was the solution of differential equation, which is independent of amplitude vectors $\vec{e}(r)$ and $\vec{h}(r)$:

$$\nabla\ell\times\vec{h}+\varepsilon\vec{e}=0,\ \nabla\ell\times\vec{e}-\mu\vec{h}=0, \tag{10, 11}$$

$$\vec{e}\cdot\nabla\ell=0,\ \vec{h}\cdot\nabla\ell=0. \tag{12, 13}$$

Simultaneous equations (10) and (11) are treated as six homogeneous linear equations for the scalar Cartesian components e_{x_1}, h_{x_1} …, of $\vec{e}$ and $\vec{h}$:

$$\mu^{-1}[(\vec{e}\cdot\nabla\ell)\nabla\ell-\vec{e}(\nabla\ell)^2]+\varepsilon\vec{e}=0, \tag{14}$$

$$\varepsilon^{-1}[(\vec{h}\cdot\nabla\ell)\nabla\ell-\vec{h}(\nabla\ell)^2]+\mu\vec{h}=0. \tag{15}$$

These equations have nontrivial solutions only when $(\nabla\ell)^2 = n^2(x, y, z)$, where $n=(\varepsilon\mu)^{(1/2)}$ means refractive index. Function ℓ – eikonal, of the surface $\ell(r)$ = constant – geometric wave surfaces (spheres) or geometric wave fronts (fig. 3).

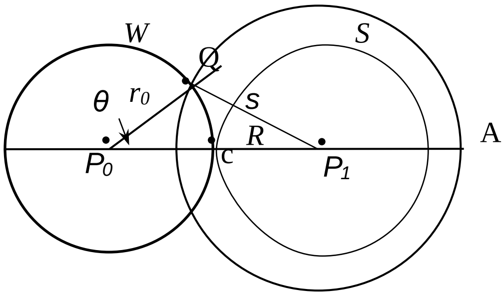

Figure 3. *W* – a wave of sphere front (from a point source in P_0), *S* – front of characteristic sphere, Q(x', y', z') – a typical point in characteristic sphere, *s* – the distance from a point (x', y', z') to P_1, c – center of the LED lens

By means of averaging over time of the Poynting vector $<\mathbf{S}>=(c/8\pi)\mathrm{Re}(\vec{e}\times\vec{h}^*)$ using (4, 5, 8) has been received [12]

$$\langle\mathbf{S}\rangle=\frac{c}{8\pi\mu}\{(\vec{e}\cdot\vec{e}^*)\nabla\ell-(\vec{e}\cdot\nabla\ell)\vec{e}^*\}=(2c/n^2)\langle w_e\rangle\nabla\ell, \tag{16}$$

where $\langle w_e\rangle$ – the averaged over time energy density of the electric field; as $\langle w_e\rangle=\langle w_h\rangle$ then the density of the energy field $\langle w\rangle=\langle w_h\rangle+\langle w_e\rangle=2\langle w_e\rangle$, $c/n=v$ – the speed of its transfer, $(\nabla\ell)/n=\vec{s}$ – is the vector of the unit length in the direction of this transfer.

Then

$$\langle\mathbf{S}\rangle=v\langle w\rangle\vec{s}. \tag{17}$$

Applied content of the left side (17) – is the average observed over time Poynting vector. Then, by means of expression

$$\langle w \rangle \vec{s} = \langle \mathbf{S} \rangle / v \tag{18}$$

is an opportunity to determine the vectors of intensity $\vec{e}, \vec{h}$, as

$$\left(\vec{e} \cdot \vec{e}^{\,*}\right)\vec{s} = \frac{8\pi}{\varepsilon v} \langle \mathbf{S} \rangle , \tag{19}$$

where $\langle \mathbf{S} \rangle \equiv I_0$ – is the power of the radiated light (if it is known [12]). In practice, there is an opportunity to define according to the characteristics of LED, which are listed in its passport data, radiation diagram (fig. 4 – for LED for HL-508H238WC-MD [13]) and the radiated light at nominal direct current and so on.

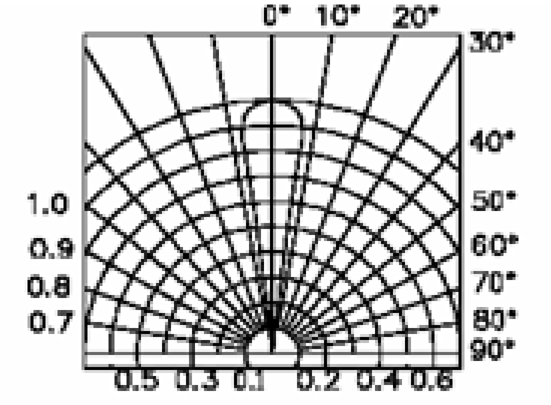

Figure 4. Diagram of the relative distribution of radiation spread

A special measuring adapter is governed by the standard [10] that provides conditions of measurement of the light power (standard CIE, condition B: distance 10^{-1} m, solid angle 10^{-2} sr, under the direct current of 2 mA). The result of this measurement of LED HL-508H238WC-MD using laboratory system that provides protection from exposure of outside sources, equipped with measuring luxmeter LX 1010V. At nominal term of the direct current of 20 mA, of the solid angle 3.410^{-2} sr (at a distance of $1.2 \cdot 10^{-1}$ m lit area of $4.9 \cdot 10^{-4} \mathrm{m}^2$) received 130 lx, i.e, power of light I_0=18.7 cd, or $9.34 \cdot 10^{-4}$ W. This in turn determines the value of 1.9 W/m^2 of power of energy flow. For angle $\theta = 0^\circ$ (fig. 2), $\varepsilon = 1$ $n = 1$ then according to the formula (19) has been received that $|e|$=$4 \cdot 10^{-4}$ v/m. By using (14, 15) intensity at other angles has been received, including the magnetic field component. This result also will help to determine the value of such vectors in structures of living tissues of the bio medium and Poynting vector at the site of irradiation with regard to interference and diffraction.

Conclusions. It is reasonable to choose mathematical representation of a fictitious object – dipole as a mathematical model of the radiation source of LED. As the diagram of radiation LED system is generated by the system of electric dipoles and its optical system, then this known diagram is data to determine the elementary vectors of strength field of LED radiation source. These vectors and parameters of bio medium and characteristics of their boundaries will determine the quantitative value of energy transfer of low light radiation through the layered bio media and its intensity on the surface of bio-object, which is in this environment.

References

1. Bachynskyy M., Stoyanov Y., Yavorskyy B. Mathematical modeling of LED radiation in the system of medical diagnostics. Digest of the Ternopil National Technical University. 2016, no. 4 (84), pp. 124 – 130.
2. Presman A.S. E'lektromagnitnye polya i zhivaya priroda. Moscow, Nauka Publ., 1968. 287 p. [In Russian].
3. The Human Eye: Physical Properties, Trans-imitance, and Acuity. Available at: http://www.telescope optics.net/eye.htm.
4. Jianhong S., Yoon-Mo J. Weberized. Mumford-Shah Model with Bose-Einstein Photon Noise: Light Adapted Segmentation Inspired by Vision Psychology, Retinal Physiology, and Quantum Statistics On the Foundations of Vision Modelling. IMA Tech. Preprint, vol. 4, no. 1949.
5. Chaves J. Introduction to Nonimaging Optics, Second Edition. CRC Press, Science, 2015, 786 p.
6. Brunner N., De Balthazar C., Alvarez R. Detection of Weak Optical Signals by the Human Visual System: Perspectives in Neuroscience and in Quantum Physics. Geneva, HUG Electrical Neuroimaging Group, 2008, 12 p.
7. Nair G.B., Dhoble S.J. A perspective perception on the applications of light-emitting diodes. Wiley Online Library, Luminescence, 2015, vol. 30, pp. 1167 – 1175. Available at: http://onlinelibrary.wiley.com/doi/10.1002/bio.2919/full (accessed 13.03.2015).
8. Delbeke D., Bienstman P., Bockstaele R., Baets R. Rigorous electromagnetic analysis of dipole emission in periodically corrugated layers: the grating-assisted resonant-cavity light-emitting diode. J. Opt. Soc. Am. A, 2002, vol. 19, no. 5, pp. 871 – 880.
9. Moreno I. Spatial distribution of LED radiation. Proc. SPIE 6342, The International Optical Design Conference, 2006, 634216, doi: 10.1117/12.692273. Available at: http://fisica.uaz.edu.mx/~imoreno/ Publicaciones/SPIE2006.pdf.
10. Handbook of LED Metrology. INSTRUMENT SYSTEMS GmbH, version 1.1, 2016, 42 p.
11. Prosser R.D. The interpretation of Diffraction and Interference in Terms of Energy Flow. International Journal of Theoretical Physics, 1976, vol. 15, no. 3, pp. 169 – 180.
12. Born M., Wolf E. Principles of optics. A. Eaton & Co. Ltd. Great Britain, Exeter, 1986, 831 p.
13. HONGLITRONIC, Part: HL508H238WCMD, 2012, 5 p. Available at: http://leds.com.ua/assets/products/ datasheets/121.pdf.

Список використаної літератури

1. Bachynskyy, M.V. Mathematical modeling of LED radiation in the system of medical diagnostics [Текст] / M.V. Bachynskyy, Y.M. Stoyanov, B.I. Yavorskyy // Вісник Тернопільського національного технічного університету. – Тернопіль, 2016. – Вип. 4 (84). С. 124 – 130.
2. Пресман, А.С. Электромагнитные поля и живая природа [Текст] / А.С. Пресман; М.: Наука, 1968. – 287 с.
3. The Human Eye: Physical Properties, Trans-imitance, and Acuity. Available at: http://www.telescope optics.net/eye.htm.
4. Jianhong S., Yoon-Mo J. Weberized. Mumford-Shah Model with Bose-Einstein Photon Noise: Light Adapted Segmentation Inspired by Vision Psychology, Retinal Physiology, and Quantum Statistics On the Foundations of Vision Modelling. IMA Tech. Preprint, vol. 4, no. 1949.
5. Chaves J. Introduction to Nonimaging Optics, Second Edition. CRC Press, Science, 2015, 786 p.
6. Brunner N., De Balthazar C., Alvarez R. Detection of Weak Optical Signals by the Human Visual System: Perspectives in Neuroscience and in Quantum Physics. Geneva, HUG Electrical Neuroimaging Group, 2008, 12 p.
7. Nair G.B., Dhoble S.J. A perspective perception on the applications of light-emitting diodes. Wiley Online Library, Luminescence, 2015, vol. 30, pp. 1167 – 1175. Available at: http://onlinelibrary.wiley.com/doi/10.1002/bio.2919/full (accessed 13.03.2015).
8. Delbeke D., Bienstman P., Bockstaele R., Baets R. Rigorous electromagnetic analysis of dipole emission in periodically corrugated layers: the grating-assisted resonant-cavity light-emitting diode. J. Opt. Soc. Am. A, 2002, vol. 19, no. 5, pp. 871 – 880.
9. Moreno I. Spatial distribution of LED radiation. Proc. SPIE 6342, The International Optical Design Conference, 2006, 634216, doi: 10.1117/12.692273. Available at: http://fisica.uaz.edu.mx/~imoreno/Publicaciones/SPIE2006.pdf.
10. Handbook of LED Metrology. INSTRUMENT SYSTEMS GmbH, version 1.1, 2016, 42 p.
11. Prosser R.D. The interpretation of Diffraction and Interference in Terms of Energy Flow. International Journal of Theoretical Physics, 1976, vol. 15, no. 3, pp. 169 – 180.
12. Born M., Wolf E. Principles of optics. A. Eaton & Co. Ltd. Great Britain, Exeter, 1986, 831 p.
13. HONGLITRONIC, Part: HL508H238WCMD, 2012, 5 p. Available at: http://leds.com.ua/assets/products/datasheets/121.pdf.

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ІДЕНТИФІКАЦІЯ ПАРАМЕТРІВ ДИПОЛЬНОЇ МОДЕЛІ ДЖЕРЕЛА ВИПРОМІНЮВАННЯ У СВІТЛОДІОДІ

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Резюме. *Наведено результат ідентифікації параметрів математичної моделі джерела випромінювання світлодіода. Для математичного моделювання використано фізичний абстрактний об'єкт – диполь, і припущення, що діаграму випромінювання світлодіода формує система електричних диполів та його оптична система. Визначено елементарні вектори напруженостей диполів світлодіодного джерела, необхідних для моделювання перенесення енергії низькоінтенсивного випромінювання через шарувате біосередовище. Математична модель знайде подальше використання при автоматизації досягнення оптимального опромінення поверхні біооб'єкта в шаруватому біосередовищі. Результати дослідження використано в дослідженнях засобів подразнення біооб'єктів світлом з інтенсивністю нижче стандартного рівня під час профілактики, реабілітації та медичної діагностики. Досягнуто зменшення інвазивності біооб'єктів, підвищення роздільної здатності оцінки їх реакції та забезпечення автоматизації досліджень.*

Ключові слова: *світлодіод, система диполів, математична модель, випромінювання, елементарні напруженості, шарувате біосередовище, опромінення біооб'єкта.*

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