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LOAD TRANSFER FROM THE INFINITE STRINGER TO THE TWO JAMMED ALONG ONE EDGE IDENTICAL STRIPES WITH INITIAL (RESIDUAL) STRESSES

MYKOLA DIKHTYARUK

KHMELNYTSKY NATIONAL UNIVERSITY, KHMELNYTSKY, UKRAINE

Summary. *The article is devoted to problems of contact interaction of infinite elastic stringer and two identical jammed along one edge pre-stressed stripes.*

The research has been conducted in the framework of linearized theory of elasticity in a general form for the theory of large (finite) initial deformations and for two variants of the theory of small initial deformations with arbitrary structure of elastic potential. The study is carried out in the coordinates of the initial strain state associated with Lagrangian coordinates (the natural state). In addition, it is assumed that the interaction with the stringer occurs when the initial stresses appeared in the stripes, and the external load caused in the stripes small disturbance of the basic stress-strain state.

Based on the assumption that for the stringer, which is loaded at the same time by the vertical and horizontal forces, a fair model of the bending of the beam in combination with uniaxial stress of the rod, the problem is formulated mathematically as a system of integrated differential equations relatively to the unknown contact stresses. Using Fourier transforms, the system is solved in a closed form. Expressions of the stresses are represented by Fourier integrals of rather simple structure. The influence of initial stresses on the distribution of the contact stresses has been studied and the effects of concentrated load have been identified.

Key words: *the linearized elasticity theory, initial (residual) stresses, contact problems, integral Fourier transformations.*

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Type Codes:

E_1 – *elasticity modulus of stringer;*

ν_1 – *Poisson's ratio stringer;*

λ_i – *elongation ratios, defining displacement of initial (residual);*

y_i – *coordinates of the initial strain state;*

x_i – *Lagrange coordinates;*

t – *the length of the strip in the initial deformed state;*

h – *thickness of stringer.*

Introduction. One of the most actual problems of modern construction and engineering is to improve the reliability and durability of engineering structures and machines. In all real structures and parts of machines there are almost always original or residual stresses. The causes of their appearance may be different. The most often the initial stresses in the details and structures are created especially during their manufacturing or in the process of assembly to compensate those tensions, arising in structural elements, in order to improve the strength characteristics of the structures. They can also appear both in the process of exploitation and under the influence of mechanical factors, such as irreversible plastic deformations, and also because of the reasons of non-mechanical nature. The presence of initial stresses affects the whole stressed-deformed state of the bodies, so can affect the strength of structures, lead to loss of internal stability; promote local destruction of material, etc. Taking into account the impact of the initial (residual) stresses in the calculation of structural elements, machines and constructions allows us to consider more effectively the strength of material resources by correct evaluation of strength margin.

The research of the influence of the initial (residual) stresses began to be actively conducted in our country and abroad only in the late XX century. It should be noted that in general, strict formulation of such problems requires the involvement of the staff of nonlinear elasticity, which greatly complicates the construction of analytical solutions. However, provided the big (ending) and low initial stresses (deformations) it is possible to be limited to consideration of the theory of linearized elasticity [1,2].

In this paper, using ratios of linearized elasticity theory [2] the solution of contact task about contact interaction of infinite stringer with two pre-stressed stripes has been introduced. The research is done in general for compressible and incompressible bodies for the theory of large (finite) initial deformations and for two versions of the theory of small initial deformations at arbitrary structure of elastic potential. Following [2,3,5] we will conduct all researches in the coordinates of the initial stressed potential y_i, that are related with the ratios $y_i = \lambda_i x_i, (i = 1,2)$.

Problem setting and basic correlations. Let the endless elastic strips are made of the same compressible or incompressible materials with potential of arbitrary structures. In these stripes identical initial or residual stresses operate, the thickness of the stripe is t. On the edges y= ± t they are jammed and are in conditions of flat deformation.

We will assume that they are interconnected with the endless elastic stringer where the modulus of elasticity of the material is E_1 and Poisson coefficient is ν_1. After that let us consider the reinforced in such a way stripes with the initial stresses are loaded with the horizontal force $Q_0\delta(y_1)$ acting in the middle point of the stringer and here $\delta(y_1)$ – is a known single Dirac delta function. We will conduct the research of this problem in the coordinates of the initial (residual) strained state oy_1y_2 (Fig.1).

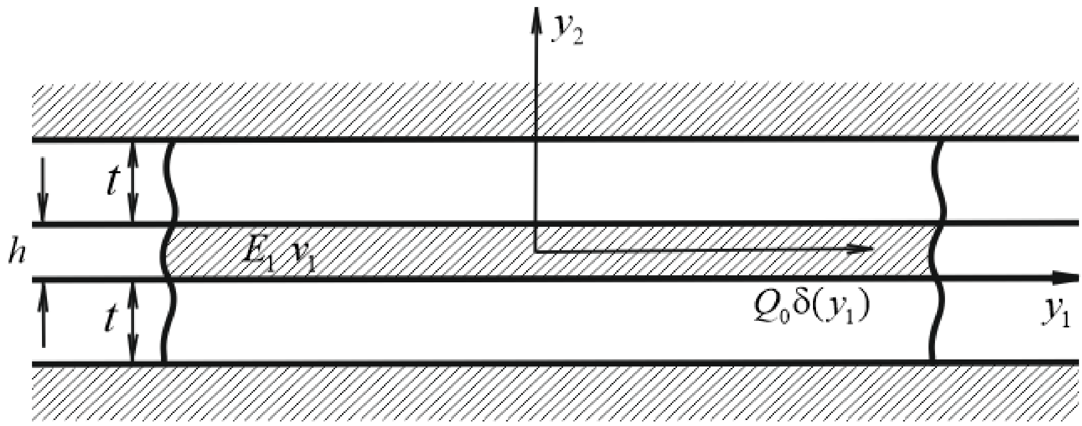

Figure 1. Force operation on the stripes

The law of distribution of normal and tangential contact stresses along the line of stringer connection with the previously stressed stripes should be defined. When considering this problem, following [2], we believe that the interaction occurs in the performance of the known four provisions1-4, which are basic in the theory of contact interaction of bodies with initial stresses. Moreover, we will assume that under the influence of the applied load and only tangential contact stresses, stringer is stretched or is compressed as the core which is located in the uniaxial stressed state 4]. Furthermore we will consider that along the horizontal axis vertical elastic movements are constant, due to the small thickness of stringer. Let us mark intensities of normal and tangential contact stresses by $p(y_1)$ and $q(y_1)$, and vertical and horizontal movements of the stringer accordingly by $v^{(1)}(y_1)$ and $u^{(1)}(y_1)$. Then we can write that:

$$\frac{\partial u^{(1)}(y_1)}{\partial y_1} = \frac{1}{E_1 h}\int_{-\infty}^{y_1}\left[2q(t) - Q_0\,\delta(t)\right]dt\ , \qquad (-\infty < y_1 < \infty) \tag{1}$$

$$\frac{\partial v^{(1)}(y_1)}{\partial y_1} = 0 \qquad \forall y_1 \in (-\infty < y_1 < \infty) \tag{2}$$

In case of full contact it should be noted, that along the line of contact the conditions should be done:

$$\frac{\partial v^{(1)}(y_1)}{\partial y_1} = \frac{\partial u_2^{(2)}(y_1)}{\partial y_1}, \qquad \frac{\partial u^{(1)}(y_1)}{\partial y_1} = \frac{\partial u_1^{(2)}(y_1)}{\partial y_1}, \qquad (-\infty < y_1 > \infty) \tag{3}$$

Where $u^{(1)}(y_1)$, $v^{(1)}(y_1)$ – components of the vector of displacements in the elastic stringer, $u_1^{(2)}(y_1)$, $u_2^{(2)}(y_1)$ – components of the vector of displacements in the elastic stripes with the initial stresses.

The method of solution. Considering the contact conditions (3) together with (1), (2), and also the expressions for the vertical and horizontal displacements of the boundary points free of jamming. The last were received according to the principle of superposition in case of equal and unequal roots of the defining equation [2] for compressible and incompressible bodies and following [3,5] have such a form:

$$u_1(y_1) = \int_{-\infty}^{\infty} h_{11}(|y_1 - t|)p(t)dt + \int_{-\infty}^{\infty} h_{12}(y_1 - t)q(t)dt.$$

$$u_2(y_1) = \int_{-\infty}^{\infty} h_{21}(y_1 - t)\,p(t)dt + \int_{-\infty}^{\infty} h_{22}(|y_1 - t|)q(t)dt. \tag{4}$$

Having considered (1)-(4) relatively to unknown contact stresses, we will obtain the following system of integrated differential equations:

$$\frac{d}{dy_1}\left[\int_{-\infty}^{\infty} h_{11}(|y_1 - t|)p(t)dt + \int_{-\infty}^{\infty} h_{12}(y_1 - t)q(t)dt\right] = 0$$

$$\frac{d}{dy_1}\left[\int_{-\infty}^{\infty} h_{21}(y_1 - t)p(t)dt + \int_{-\infty}^{\infty} h_{22}(|y_1 - t|)q(t)dt\right] = \int_{-\infty}^{y_1}\left[2q(t) - Q_0\,\delta(t)\right]dt. \tag{5}$$

where h_{ij} $(i, j = 1, 2)$– are the functions of the impact for the elastic stripe with initial (residual) stresses, the expressions of which are defined in [3] and in accordance have the form: from the action of single normal force for even roots $n_1 = n_2$

$$h_{11}(y_1)=\frac{1}{\pi}\int_0^{\infty}H_{11}(\alpha)\cos\alpha\, y_1 d\alpha \qquad (6)$$

$$h_{12}(y_1)=\frac{1}{\pi}\int_0^{\infty}H_{12}(\alpha)\sin\alpha\, y_1 d\alpha\,.$$

for uneven roots $n_1 \neq n_2$

$$h_{11}(y_1)=\frac{1}{\pi}\int_0^{\infty}\widetilde{H}_{11}(\alpha)\cos\alpha\, y_1 d\alpha \qquad (7)$$

$$h_{12}(y_1)=\frac{1}{\pi}\int_0^{\infty}\widetilde{H}_{12}(\alpha)\sin\alpha\, y_1 d\alpha.$$

Here $h_{ij}(\alpha)$, $i,j=1,2$ are the functions of the impact, characterizing the moving of the boundary points of the verge $y_2=0$ of the infinite elastic stripe with initial (residual) stresses from a single normal force of the core $H_{ij}(\alpha)$ in accordance have the form

for $n_1=n_2$

$$\begin{aligned}H_{11}(\alpha)=H_1(\alpha,0)=n_0\Big[&s_0 sh^2\alpha\varphi_1+s_1s_0sh^2\alpha\varphi_1-\alpha\varphi_1\xi(\alpha)+(\alpha\varphi_1)^2-\\&-\bar{s}_1\xi(\alpha)+\varphi_1\Big]\Delta_1^{-1}(\alpha).\end{aligned} \qquad (8)$$

$$H_{12}(\alpha)=H_2(\alpha,0)=i\frac{m_1n_0}{\sqrt{n_1}}\left[s_0s\xi(\alpha)-s_0(\alpha\varphi_1)-\bar{s}_1s_1\xi(\alpha)+s_1(\alpha\varphi_1)\right]\Delta_1^{-1}(\alpha).$$

for $n_1 \neq n_2$

$$\begin{aligned}H_{11}(\alpha)=\widetilde{H}_1(\alpha,0)=n_0\big[&-s_1ch2\alpha\varphi_1+s_0\xi_1(\alpha)-s_1s_0(\alpha\varphi_1)\xi_1(\alpha)+s_0(\alpha\varphi_1)^2sh^2\alpha\varphi_1-\\&-s_0ch^2\alpha\varphi_1+s_1\xi_1(\alpha)+\alpha\varphi_1\xi_4(\alpha)\big]\times\Delta_2^{-1}(\alpha).\end{aligned} \qquad (9)$$

$$H_{12}(\alpha)=\widetilde{H}_2(\alpha,0)=i\frac{n_0m_1}{\sqrt{n_1}}\left[s_0s_1\xi_3(\alpha)-s_0(\alpha\varphi_1)\xi_1(\alpha)+s_1(\alpha\varphi_1)\xi(\alpha)-s_1\xi_1(\alpha)\right]\times\Delta_2^{-1}(\alpha).$$

Let us write down the functions of the impact for elastic stripe with initial (residual) stresses from the action of the single tangential force

for even roots $n_1=n_2$

$$h_{21}(y_1)=\frac{1}{\pi}\int_0^{\infty}H_{21}(\alpha)\sin(\alpha)y_1 d\alpha.$$

$$h_{22}(y_1)=\frac{1}{\pi}\int_0^{\infty}H_{22}(\alpha)\cos\alpha\, y_1 d\alpha. \qquad (10)$$

for uneven roots $n_1 \neq n_2$

$$h_{21}(y_1)=\frac{1}{\pi}\int_0^\infty \widetilde{H}_{21}(\alpha)\sin(\alpha)y_1 d\alpha .$$

$$h_{22}(y_1)=\frac{1}{\pi}\int_0^\infty \widetilde{H}_{22}(\alpha)\cos\alpha\, y_1 d\alpha. \tag{11}$$

The cores $H_{ij}(\alpha)$ and $\widetilde{H}_{ij}(\alpha)$ in accordance have the form for $n_1 = n_2$

$$H_{21}(\alpha)=m_0\left[-(s+1)(s_1\xi(\alpha)-\alpha\varphi_1)+ch^2\alpha\varphi_1-s_1sh^2\alpha\varphi_1-s\right]=$$
$$=m_0\left[-(s+1)(s_1sh\,\alpha\varphi_1ch\,\alpha\varphi_1-\alpha\varphi_1)+ch^2\alpha\varphi_1-s_1sh^2\alpha\varphi_1-s\right]\cdot\Delta_1^{-1}(\alpha).$$

$$H_{22}(\alpha)=i\frac{m_0m_1}{\sqrt{n_1}}\left[s\cdot s_1ch^2\alpha\varphi_1+(\alpha\varphi_1)^2-\alpha\varphi_1\xi(\alpha)-s_1^2sh^2(\alpha\varphi_1)-s\cdot s_1\right]\cdot\Delta_1^{-1}(\alpha). \tag{12}$$

for $n_1 \neq n_2$

$$\widetilde{H}_{21}(\alpha)=m_0\left[-ss_1(\alpha\varphi_1)\xi_2(\alpha)-s\xi_3(\alpha)+s(\alpha\varphi_1)\xi_2(\alpha)+\xi_3(\alpha)\right]\cdot\Delta_2^{-1}(\alpha).$$

$$\widetilde{H}_{22}(\alpha)=i\frac{m_0m_1}{\sqrt{n_1}}\left[1-s_1ch(2\alpha\varphi_2)+ss_1\xi_1(\alpha)+s\alpha\varphi_1\xi_4(\alpha)+ss_1(\alpha\varphi_1)^2sh^2\alpha\varphi_1-\right.$$
$$\left.-ss_1\,ch^2\alpha\varphi_{21}-s_1^2(\alpha\varphi_1)\xi_4(\alpha)+\xi_3(\alpha)\ \right]\cdot\varDelta_2^{-1}(\alpha). \tag{13}$$

It should be noted that there are the following asymptotic formulas:
when $\alpha \to 0$

$H_{11}(\alpha)=\widetilde{H}_{11}(\alpha)=0(1);\ \ H_{12}(\alpha)=\widetilde{H}_{12}(\alpha)=0(\alpha);\ \ H_{22}(\alpha)=\widetilde{H}_{22}(\alpha)=0(1)$

when $\alpha \to \infty$

$$H_{11}(\alpha)=\widetilde{H}_{11}(\alpha)=0\left(\frac{1}{\alpha}\right);\ \ H_{12}(\alpha)=\widetilde{H}_{12}(\alpha)=0\left(\frac{1}{\alpha}\right);\ \ H_{22}=\widetilde{H}_{22}(\alpha)=0\left(\frac{1}{\alpha}\right). \tag{14}$$

Let us multiply the first and the second equation (5) by $e^{-i\alpha y_1}$ and integrate in y_1 from $-\infty$ to $+\infty$, applying the convolution theorem after elementary tabs we will get a system of algebraic equations relatively to Fourier transforming $\widetilde{p}(\alpha)$ i $\widetilde{q}(\alpha)$

$$\mathrm{H}_{11}(\alpha)\,\widetilde{p}(\alpha)-i\mathrm{H}_{12}(\alpha)\widetilde{q}(\alpha)=0 \tag{15}$$

$$E_1t_0\alpha^2iH_{21}(\alpha)-\left[E_1t_0\alpha^2H_{22}(\alpha)+2\right]\widetilde{q}(\alpha)=Q_0$$

Here we will enter the following notations:

$$\widetilde{p}(\alpha)=\int_{-\infty}^{\infty}p(y_1)e^{i\alpha y_1}dy_1;\qquad \widetilde{q}(\alpha)=\int_{-\infty}^{\infty}q(y_1)e^{i\alpha y_1}dy_1; \tag{16}$$

Functions $\tilde{p}(\alpha)$, $\tilde{q}(\alpha)$ – the Fourier transforming essence from the function $p(y_1)$, $q(y_1)$ – are the contact stresses on the line of the contact of elastic stringer and the stripes with initial (residual) stresses; Q_0 – external horizontal load; functions H_{ij} $(i=1,2)$ according to the equal and uneven roots of the defining equation [2] are specified by the formulas (12) (13).

To find transformants $\tilde{p}(\alpha)$ and $\tilde{q}(\alpha)$ we will write down the determinants of the system (15)

$$H^*(\alpha)=\begin{vmatrix} H_{11}(\alpha) & -iH_{12}(\alpha) \\ 0 & E_1 t_0 \alpha^2 H_{22}(\alpha)+2 \end{vmatrix}= \tag{17}$$

$$=H_{11}(\alpha)\left[E_1 t_0 \alpha^2 H_{22}(\alpha)+2\right]=H_{11}^*(\alpha)H_{22}^*(\alpha)$$

$$\Delta_1^*(\alpha)=\begin{vmatrix} 0 & -H_{12}(\alpha)\cdot i \\ E_1 t_0 \alpha^2 iH_{21}(\alpha)-Q_0 & E_1 t_0 \alpha^2 H_{22}(\alpha)+2 \end{vmatrix}=H_{21}^*(\alpha)\cdot H_{12}^*(\alpha). \tag{18}$$

$$\Delta_2^*(\alpha)=\begin{vmatrix} H_{11}(\alpha) & 0 \\ 0 & E_1 t_0 \alpha^2 iH_{21}(\alpha)-Q_0 \end{vmatrix}=H_{11}^*(\alpha)H_{21}^*(\alpha)$$

Here, we will introduce the notation:

$$H_{11}^*(\alpha)=H_{11}(\alpha) \qquad H_{12}^*(\alpha)=-iH_{12}(\alpha); \tag{19}$$

$$H_{21}^*(\alpha)=E_1 t_0 \alpha^2 iH_{21}(\alpha)-Q_0 \qquad H_{22}^*(\alpha)=E_1 t_0 \alpha^2 H_{22}(\alpha)+2$$

Applying the formula of Kramer and inverse Fourier transform we will get the solution of the system of integrated differential equations (5). This solution gives the expressions to the required contact stresses in a form of

$$q(y_1)=\frac{Q_0}{\pi}\mu\int_0^\infty \frac{H_{11}^*(\alpha)}{H^*(\alpha)}\cos\alpha y_1 d\alpha$$

$$p(y_1)=\frac{Q_0}{\pi}\mu\int_0^\infty \frac{H_{11}^*(\alpha)}{H^*(\alpha)}\sin\alpha y_1 d\alpha \tag{20}$$

Here the values $H^*{}_i, H_{ij}^*$ $(i,j=1,2)$ are expressed in terms of known functions H_{ij} $(i,j=1,2)$, that are determined by formulas (12) – (13) for even and uneven roots of the defining equation [2] in case of the specific structure of elastic potentials. Let us experiment improper integrals on the convergence that are included in the formulas (20).

Having considered the values $H^*{}_{ij}(\alpha)$ (19) and the value $H_{ij}(\alpha)$ (12) – (13), and also the asymptotic formulas for $H_{ij}(\alpha)$ (14), without considering the bulky elementary transformations, we will find the following asymptotic expressions:

when $\alpha \to 0$

$$\frac{H_{12}^{*}(\alpha)}{H^{*}(\alpha)} \sim 0(\alpha\ ); \tag{21}$$

when $\alpha \to \infty$

$$\frac{H_{12}^{*}(\alpha)}{H^{*}(\alpha)} \sim 0(\alpha^{-1}); \tag{22}$$

From asymptotes (21) – (22) follows that in (20) the second integral, which expresses the law of distribution of normal contact stresses coincides quite quickly.

According to the first integral $\int_0^\infty \frac{H_{11}^{*}(\alpha)}{H^{*}(\alpha)}\cos\alpha y_1 d\alpha$, which expresses the law of distribution of tangential contact stresses, its convergence could be accelerated,

Having presented $\frac{H_{11}^{*}(\alpha)}{H^{*}(\alpha)}$ in the form

$$\frac{H_{11}^{*}}{H^{*}(\alpha)} \approx \frac{C_2}{c_2+\alpha} + 0\left(\ell^{-2t\alpha}\right), \tag{23}$$

(t>0) when $\alpha \to \infty$ here C_2 and c_2 – are the constants, which essentially depend on the value of the roots of the defining equation [2], and on the particular form of elastic potential and are determined from the formulas[D] (3.76) – (3.77), (3.86) – (3.89) i (1.80) – (1.100) for the specific compressible and incompressible structural materials. With the continuity of contact tangent stresses, taking into account the thickness of the elastic cover h and elastic constants of the material λ and μ , out of which the elastic cover is made.

Constant C_2 can be presented in the form of:

$$C_2 = \frac{2c+1}{E_1 t_0 (c\ +1)}, c = \frac{\lambda+\mu}{2\mu} \qquad (c_2 = 2\mu C_2) \tag{24}$$

Having considered (23) and (24) we will get:

$$\int_0^\infty \frac{H_{11}^{*}(\alpha)}{H^{*}(\alpha)}\cos\alpha y_1 d\alpha = -C_2(\cos c_2 y_1 ci c_2 y_1 + \sin c_2 y_1 si c_2 y_1 + \int_0^\infty \left[\frac{(c_2+\alpha)H_{11}^{*}(\alpha) - C_2 H^{*}(\alpha)}{(c_2+\alpha)H^{*}(\alpha)}\right]\cos\alpha\, y_1\, d\alpha \tag{25}$$

here

$$si(c_2 y_1) = -\int_{c_2 y_1}^{\infty} \frac{\sin\alpha}{\alpha} d\alpha; \qquad ci(c_2 y_1) = -\int_{c_2 y_1}^{\infty} \frac{\cos\alpha}{\alpha} d\alpha$$

it is accordingly integral sine and cosine.

Now the last integral in (25) considering (23) coincides quite quickly as:

$$\frac{(c_2+\alpha)H_{11}^{*}(\alpha) - C_2 H^{*}(\alpha)}{(c_2+\alpha)H^{*}(\alpha)} \sim 0(\alpha) \tag{26}$$

when $\alpha \to 0$

So having taken into account (25) for the contact tangent stresses(20) from the action of horizontal external force $Q_0\delta(y_1)$, we will get:

$$q(y_1) = -\frac{Q_0}{2\pi}\cdot\left[c_2(\cos c_2 y_1 ci(c_2 y_1) + \sin c_2 y_1 si(c_2 y_1)) - \int_0^\infty \frac{2\mu(c_2+\alpha)H_{11}^*(\alpha) - c_2 H^*(\alpha)}{(c_2+\alpha)H^*(\alpha)}\cos\alpha y_1 d\alpha\right] \quad (27)$$

$$(-\infty < y_1 > \infty)$$

Numerical analysis. In the last formula with the first number with precision of the constant the known solution of Melanie is provided [4]. Based on the formula (27) the numerical analysis is done [3,5,6], the results of which are presented in graphs (fig. 2, 3).

All graphics and numerical results are obtained in Maple -8, for harmonious potential and potential of Bartenev Khazanovych, in case of equal roots of the defining equation [2].

The graphs (fig. 2, 3) illustrate the impact of the initial (residual) stresses in elastic stripes on the law of distribution of stresses in contact stringer from the action of tangential force $Q_0\delta(y_1)$ for dimensionless values $\frac{h}{Q}q(t)$.

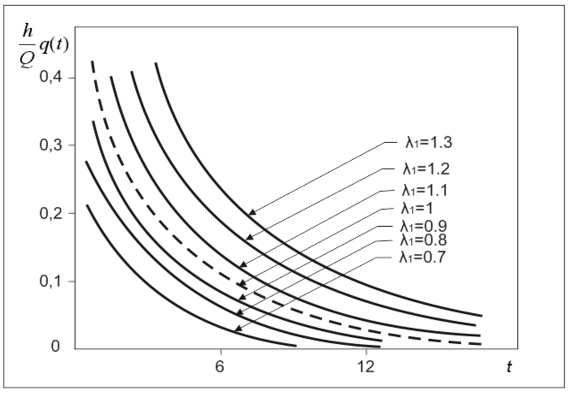

Figure 2. Contact stress distribution under the stringer, the case of harmonic potential

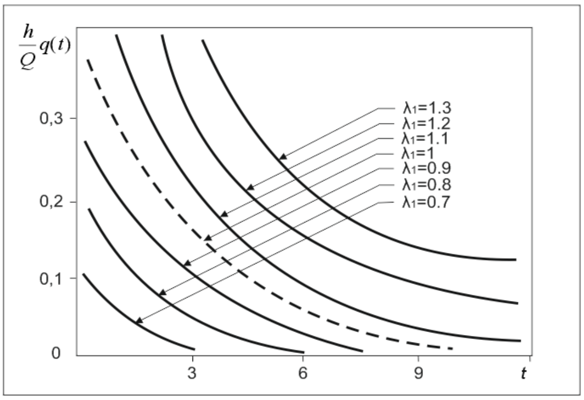

Figure 3. Contact stress distribution under the stringer, the case of Bartenev-Khazanovych

Here $\frac{h}{Q}q(t)$ – dimensionless tangential contact stresses. The value $\lambda_1 = 1$ (the dotted line on the graphs) – correspond to the classical theory of elasticity coincides with the results of the work [7]; $\lambda_1 = 0{,}7;0{,}8;0{,}9$ – correspond to the initial stresses of compression; $\lambda_1 = 1{,}1;1{,}2;1{,}3$ – correspond to the initial stresses of stretching t – dimensionless coordinate of the initial stressed state in the elastic stripes with the initial stresses.

Conclusions. In this paper, within the framework of linearized elasticity theory the flat contact task is considered about the transmission of the load from the endless stringer for two identical stripes with initial stresses. The researches have been conducted in general terms for the initial theory of large initial deformations for the arbitrary structure of the elastic potential. The impact of the presence of the initial (residual) stresses in the stripes on the distribution law of contact stresses along the line of contact with the infinite stringer has been studied.

The impact of the initial stresses on the stress-strain state along the line of contact stringer with elastic stripes is that: initial stresses in the stripes lead in the case of compression to the reduction of the stresses in the contact area, and in the case of stretching – to its increasing, and for the movement - on the contrary. This result can be used effectively to regulate contact efforts when considering the structures for durability. Moreover for the contact stresses initial stresses are more dangerous in case of stretching, and for the moving – in case of compression.

Comparing fig. 2 and fig. 3, which show the distribution of contact stresses, we can see that more significant influence of quantitative character initial (residual) stresses form in highly elastic materials compared with hard materials.

Mechanical effect that is similar to the earlier conducted researches has been discovered [2, 3, 5], which means that in case when λ_1, approach to the values of surface resistance of the material, phenomena of resonance character appear both in the stripes and in the stringer. They lie in the fact that the stress and the movement in the bodies that interact dramatically change their meaning.

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ПЕРЕДАЧА НАВАНТАЖЕННЯ ВІД НЕСКІНЧЕННОГО СТРИНГЕРА ДО ДВОХ ЗАТИСНЕНИХ ПО ОДНОМУ КРАЮ ОДНАКОВИХ СМУГ З ПОЧАТКОВИМИ (ЗАЛИШКОВИМИ) НАПРУЖЕННЯМИ

МИКОЛА ДІХТЯРУК

Хмельницький національний університет, Хмельницький, Україна

Резюме. *Дослідження виконано у рамках лінеаризованої теорії пружності в загальному вигляді для теорії великих (кінцевих) початкових деформацій та двох варіантів теорії малих початкових деформацій при довільній структурі пружного потенціалу. Дослідження проведено у координатах початкового деформованого стану, що пов’язані з лагранжевими координатами (природного стану). Крім того, припускається, що взаємодія зі стрингером відбувається після того, як у смугах виникли початкові напруження, а зовнішнє навантаження викликає в смугах мале збурення основного напружено-деформованого стану. На основі допущення, що для стрингера, навантаженого одночасно вертикальними і горизонтальними силам, справедлива модель згину балки в поєднанні з одновісним напруженням стрижня. Задача математично формулюється у вигляді системи інтегро-диференціальних рівнянь відносно невідомих контактних напружень. За допомогою перетворень Фур’є система розв’язується в замкненій формі. Вирази напружень представляються інтегралами Фур’є досить простої структури. Досліджено вплив початкових напружень на розподіл контактних напружень і виявлено ефекти зосередженого навантаження.*

Ключові слова: *лінеаризована теорія пружності, початкові (залишкові) напруження, контактні задачі, інтегральні перетворення Фур’є.*

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