Вісник Тернопільського національного технічного університету
Scientific Journal of the Ternopil National Technical University
2016, № 1 (81)
*ISSN 1727-7108. Web: **visnyk.tntu.edu.ua***

UDC 534-21: 537.226.86

RESONANCE RADIAL OSCILLATIONS OF A PIEZOCERAMIC CYLINDERS AND SPHERES TAKING INTO ACCOUNT ELECTROMECHANICAL LOSSES

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Summary. *Hollow piezoceramic radially polarized cylinders and spheres widely used as sound transducers and receivers are considered. The most characteristic run mode of piezoceramic transmitters is resonant mode. In this paper, a universal approach to solving of problems of forced thickness oscillations of piezoceramic cylinders and spheres is expanded on research of amplitude of the electromechanical state with due regard to mechanical, dielectric, piezoelectric energy losses. General equation system of steady oscillation of cylinders and spheres is reduced to Hamiltonian equation system which is solved by superposition of solutions of initial problems. The suggested approach enables to research into oscillation of the transmitters under electrical and mechanical load and arbitrary boundary conditions. For calculation of electromechanical state of transmitters at resonance modes the energy dissipation counts towards by the introduction of complex physical constants with predetermined values of tangents of losses. The oscillations of spheres and cylinders with electrode free outer surfaces under loading by electric potential difference are studied. The amplitude of movement of the outer surface depending on the loading frequency and geometrical dimension of the figures is considered. The amplitude of movement in the neighbourhood of the first resonance at different values of tangents of losses is studied; oscillations forms on the first three resonances are built.*

Key words*: piezoceramic transmitters, radially polarized cylinder and sphere, forced oscillations, energy losses, tangents of losses, resonance frequencies.*

Received 01.03.16

Introduction. Piezoceramic radially polarized cylinders and spheres are widely used in different scientific and technical fields as sound transducers and receivers, which operate at wide frequency range [1 – 4]. The most characteristic run mode of piezoceramic transmitters is resonant mode [5]. Fixed oscillations of piezoceramic layers with curved surfaces have been investigated in works [6, 7]. Propagation of oscillations in magneto-electro-elastic hollow cylinders have been studied in work [8]. Free oscillations of piezoceramic hollow sphere have been examined in work [9].

To calculate electromechanical condition of piezoceramic transmitters at resonance run modes dissipation of material energy introducing complex material coefficients [1 – 3] with specific meanings of tangents of losses should be taken into account. Experimental determinative methods of real and imaginary complex material constants have been described in works [10 – 13]. Investigations are done on series of samples of different forms and direction of polarization. Considerable material, experimental and mathematical base is necessary for finding nine unknown tangents of losses. Results depend on mode of load, conditions of production and operation of a sample, operating temperature, quality factor, which may differ at resonance and anti-resonance in several times.

§1. Problem statement. The influence of viscoelastic characteristics on radial oscillations of polarized in thickness cylinders and spheres at loading by difference of potentials $2V(t)$ was investigated. Coordinate r is changed within $R-h \le r \le R+h$, where R − radius of middle surface, $2h$ − thickness of wall. Oscillations of body in general case are described by equations of movement [4]

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{N}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho \frac{\partial^2 u}{\partial t^2}, \tag{1.1}$$

quasi-static equation of Maxwell for electric variables

$$\frac{\partial D_r}{\partial r} + N\frac{D_r}{r} = 0, \tag{1.2}$$

which are added by material relations at radial polarization

$$\sigma_{rr} = c_{33}\frac{\partial u}{\partial r} + Nc_{13}\frac{u}{r} + e_{33}\frac{\partial \varphi}{\partial r};$$

$$\sigma_{\theta\theta} = c_{13}\frac{\partial u}{\partial r} + N[c_{11} - \frac{1}{2}(N-1)(c_{11} - c_{23})]\frac{u}{r} + e_{31}\frac{\partial \varphi}{\partial r};$$

$$D_r = e_{33}\frac{\partial u}{\partial r} + Ne_{31}\frac{u}{r} - \varepsilon_{33}\frac{\partial \varphi}{\partial r}. \tag{1.3}$$

At $N = 1$ equations (1.1), (1.2) are correspond to cylindrical coordinates, at $N = 2$ − spherical system of coordinates. The given scheme at $N = 0$ gives also denouement about oscillations of thickness of piezoceramic layer.

Energy dissipation can be taken into consideration introducing complex material constants in physical correlation [3-5] (1.3) in mono-harmonic approaching within viscoelastic model. Complex modules are introduced by following correlations:

$$\widehat{c}_{ij} = c_{ij}^{E}(1 + ic'_{ij}), \qquad \widehat{e}_{ij} = e_{ij}(1 + ie'_{ij}), \qquad \widehat{\varepsilon}_{ij} = \varepsilon_{ij}^{S}(1 + i\varepsilon'_{ij}), \tag{1.4}$$

where c'_{ij}, e'_{ij}, ε'_{ij} – tangents of angles of mechanical, dielectric and piezoelectric losses.

Energy dissipation influence is the most noticeable at resonance run modes owing to smallness of loss tangents. Thus, to compare forced oscillations of transmitters taking into account energy dissipation and without it and investigate oscillations of piezoceramic bodies at resonance frequencies is our aim. Because of difficulty of defining full set of material characteristics together with loss tangents, we will consider the case when tangents of mechanic, dielectric and piezoelectric losses are equal and occur in the range from 0.5% to 2%.

Forced oscillations of cylinders and spheres. Harmonic oscillations $f(r,t) = \mathrm{Re}\, f^{a}(r)\exp i\omega t$, arising at load of piezo-element owing to difference of electric potential $\mathrm{Re}\, 2V_0 \exp i\omega t$, applied to electroded outer surfaces is going to be considered.

Approach, suggested in works [6, 7] will be used to investigate forced oscillations. By transformation of dependences (1.1) (1.2) together with (1.3), operating system of Hamiltonian equations along spatial coordinate for amplitude meanings of parameters of electromechanical state is got

$$\frac{\partial r^N \sigma_{rr}^a}{\partial r} = \frac{c_{13*}}{c_{33*}}\frac{N}{r}r^N\sigma_{rr}^a - \frac{e_{31*}}{\varepsilon_{33*}}\frac{N}{r}r^N D_r^a +$$

$$+ r^N\left[-\rho\omega^2 + \frac{N^2}{r^2}\left(e_{13}\frac{e_{13*}}{\varepsilon_{33*}} - c_{13}\frac{c_{13*}}{c_{33*}} + c_{11} - \frac{N-1}{2}(c_{11} - c_{12})\right)\right]u_r^a,$$

$$\frac{\partial r^N D_r^a}{\partial r} = 0 \,,$$

$$\frac{\partial u_r^a}{\partial r} = \frac{1}{c_{33*} r^N} \cdot r^N \sigma_{rr}^a + \frac{e_{33}}{\varepsilon_{33} c_{33*} r^N} \cdot r^N D_r^a - \frac{c_{31*}\, N}{c_{33*}\, r} u_r^a \,,$$

$$\frac{\partial \varphi^a}{\partial r} = \frac{e_{33}}{\varepsilon_{33} c_{33*} r^N} \cdot r^N \sigma_{rr}^a - \frac{c_{33}}{\varepsilon_{33} c_{33*}\, r^N} \cdot r^N D_r^a + \frac{e_{31*}\, N}{\varepsilon_{33*}\, r} u_r^a \qquad (2.1)$$

according to functions $r^N \sigma_{rr}$, $r^N D_r$, u_r, φ. Signs are used here

$$c_{13*} = c_{13} + \frac{e_{13} e_{33}}{\varepsilon_{33}}, \quad c_{33*} = c_{33} + \frac{e_{33} e_{33}}{\varepsilon_{33}}, \quad e_{13*} = e_{13} - \frac{c_{13} e_{33}}{c_{33}}, \quad \varepsilon_{33*} = \varepsilon_{33} + \frac{e_{33} e_{33}}{c_{33}}.$$

System (2.1) can be reduced to Hamilton equation system introducing chosen canonic variables and characteristic function of Hamilton.

For equation system (2.1), boundary conditions are put on surfaces $r_0 = R - h$ i $r_1 = R + h$ using one of alternative couples

$$\sigma_{rr}(r_0, t) = \overset{0}{\sigma}_{rr}(t) \vee u_r(r_0, t) = \overset{0}{u}_r(t),$$

$$\sigma_{rr}(r_1, t) = \overset{1}{\sigma}_{rr}(t) \vee u_r(r_1, t) = \overset{1}{u}_r(t). \qquad (2.2)$$

When needed, limit conditions of impedance (mixed) type can be established.

Further denouement can be searched as a vector

$$\mathbf{Y} = (Y_1,\, Y_2,\, Y_3,\, Y_4) = (r^N \sigma_{rr}^a,\ r^N D_r^a,\ u_r^a,\ \varphi^a)\,. \qquad (2.3)$$

Equation system of electric elasticity (2.1) and conditions (2.2) are reduced to non-dimensional form

$$r = R + x\,, \quad \bar{x} = \frac{x}{h}, \quad \bar{t} = \frac{t}{t_h}, \quad \bar{u} = \frac{u}{h}, \quad \bar{\sigma}_{rr} = \frac{\sigma_{rr}}{c_{00}},$$

$$\bar{\sigma}_{\theta\theta} = \frac{\sigma_{\theta\theta}}{c_{00}}, \quad \bar{\varphi} = \frac{\varphi}{h}\sqrt{\frac{\varepsilon_{00}}{c_{00}}}, \quad \bar{D}_r = \frac{D_r}{\sqrt{c_{00}\varepsilon_{00}}}, \quad \bar{\omega} = \omega h \sqrt{\frac{\rho_{00}}{c_{00}}},$$

$$\bar{\rho} = \frac{\rho}{\rho_{00}}, \quad \bar{c}_{ij} = \frac{c_{ij}}{c_{00}}, \quad \bar{e}_{ij} = \frac{e_{ij}}{\sqrt{c_{00}\varepsilon_{00}}}, \quad \bar{\varepsilon}_{33} = \frac{\varepsilon_{33}}{\varepsilon_{00}},$$

$$\varepsilon = \frac{h}{R}. \qquad (2.4)$$

where $\rho_{00} = \rho$, $c_{00} = c_{33}^{E}$, $\varepsilon_{00} = \varepsilon_{33}^{S}$, $t_h = h\sqrt{\rho / c_{33}^{E}}$. Here ε − parameter of curvature. Taken non-dimension allows to transfer from spatial coordinate r to non-dimensional coordinate x , $-1 \le x \le 1$. Further signs of non-dimension are omitted.

System (2.1) in non-dimensional form is of the following form

$$\frac{\partial Y_1}{\partial x} = \frac{c_{13*}}{c_{33*}} \frac{N\varepsilon}{1+\varepsilon x} Y_1 - \frac{e_{31*}}{\varepsilon_{33*}} \frac{N\varepsilon}{1+\varepsilon x} Y_2 +$$

$$+\left(\frac{1+\varepsilon x}{\varepsilon}\right)^N \left[-\rho\omega^2 + \frac{N^2\varepsilon^2}{(1+\varepsilon x)^2} \left(e_{13} \frac{e_{13*}}{\varepsilon_{33*}} - c_{13} \frac{c_{13*}}{c_{33*}} + c_{11} - \frac{N-1}{2}\left(c_{11} - c_{12}\right) \right) \right] Y_3 ,$$

$$\frac{\partial Y_2}{\partial x} = 0 ,$$

$$\frac{\partial Y_3}{\partial x} = \frac{\varepsilon^N}{c_{33*}(1+\varepsilon x)^N} Y_1 + \frac{e_{33}\varepsilon^N}{\varepsilon_{33} c_{33*}(1+\varepsilon x)^N} Y_2 - \frac{c_{31*}}{c_{33*}} \frac{N\varepsilon}{1+\varepsilon x} Y_3 ,$$

$$\frac{\partial Y_4}{\partial x} = \frac{e_{33}}{\varepsilon_{33} c_{33*}(1+\varepsilon x)^N} Y_1 - \frac{c_{33}}{\varepsilon_{33} c_{33*}(1+\varepsilon x)^N} Y_2 + \frac{e_{31*}}{\varepsilon_{33*}} \frac{N\varepsilon}{1+\varepsilon x} Y_3 . \quad (2.5)$$

Amplitude of oscillations of electrical potential on outer surfaces is known

$$\varphi^a (R \pm h) = \pm V_0 \rightarrow Y_4(\mp 1) = \mp V_0 \quad (2.6)$$

Let mechanic load of outer surfaces be absent

$$\sigma^a{}_{rr} (R \pm h) = 0 \rightarrow Y_1(\mp 1) = 0 . \quad (2.7)$$

To do boundary value problem (2.5) – (2.7), generally accepted problem-solving methods (discrete orthogonalization method, collocation method, superposition of denouement of initial problems [6, 7]) can be used.

Oscillations of radial polarized bodies of ceramic PZT-4 with following material parameters are examined:

$$c_{11}^{E} = 13{,}9 \cdot 10^{10}\, Н/м^2 , \quad c_{12}^{E} = 7{,}78 \cdot 10^{10}\, Н/м^2 ,$$
$$c_{13}^{E} = 7{,}43 \cdot 10^{10}\, Н/м^2 , \quad c_{33}^{E} = 11{,}5 \cdot 10^{10}\, Н/м^2 , \quad e_{31} = -5{,}2 Кл/м^2 ,$$
$$e_{33} = 15{,}1 Кл/м^2 , \qquad \varepsilon_{33}^{S} = 562 \cdot 10^{-11}\, Ф/м , \rho = 7500\, кг/м^3 .$$

Bodies with correlation $\varepsilon = h / R = 0.25$ are investigated.

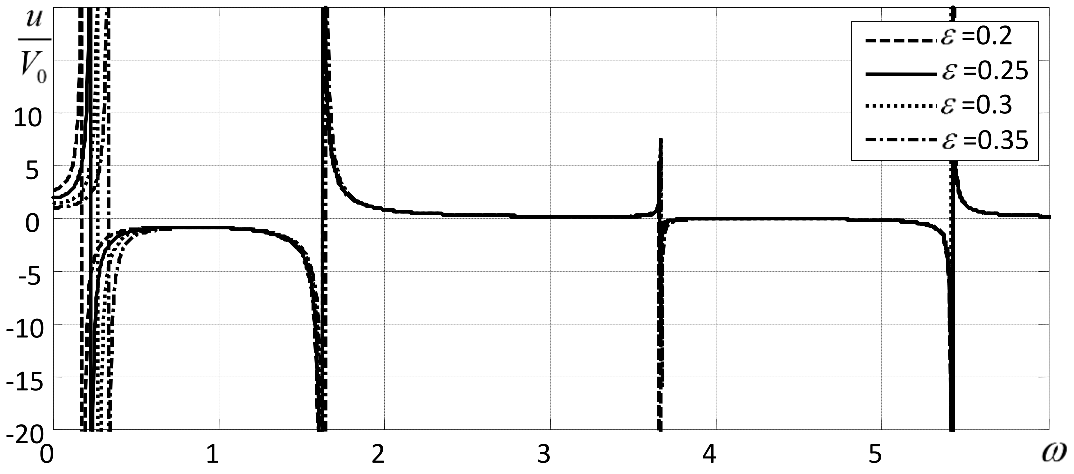

Figure 1. The amplitudes of displacements of the outer surface of the cylinder with different parameters $\varepsilon = h/R$ without losses

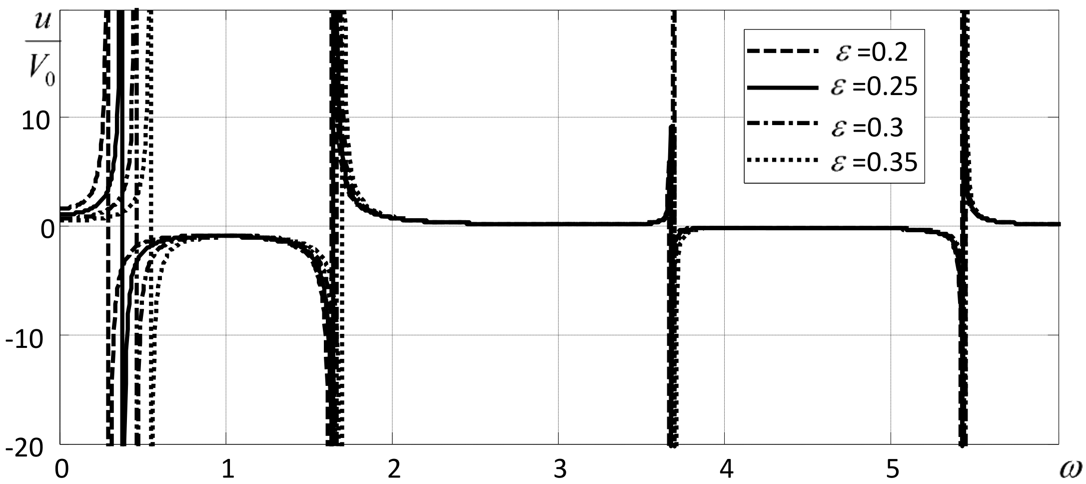

Figure 2. The amplitudes of displacements of the outer surface of the cylinder with different parameters $\varepsilon = h/R$ without losses

Dependence of outer surface of cylinders' displacements $N = 1$ (Fig. 1) and spheres $N = 2$ (Fig. 2) with different parameters $\varepsilon = h / R$ on frequency of applied load without energy dissipation is going to be considered. Received meanings of displacement should be multiplied by thickness of wall and by non-dimensional amplitude meanings applied to difference of potentials V_0 to take physical values.

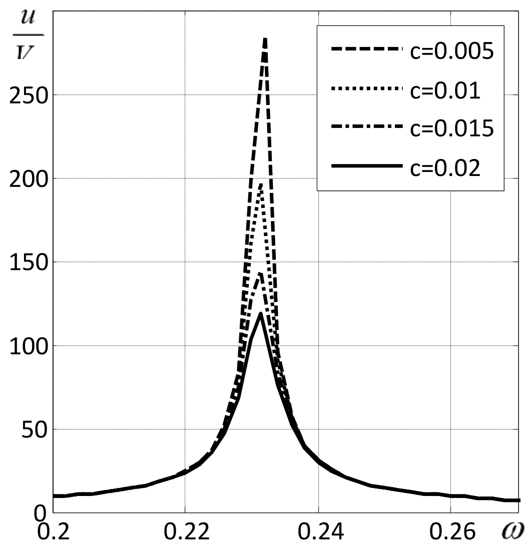

Figure 3. Modules of the amplitude of displacements of the cylinder $\varepsilon = 0.25$ near the first resonance at different values of loss tangents

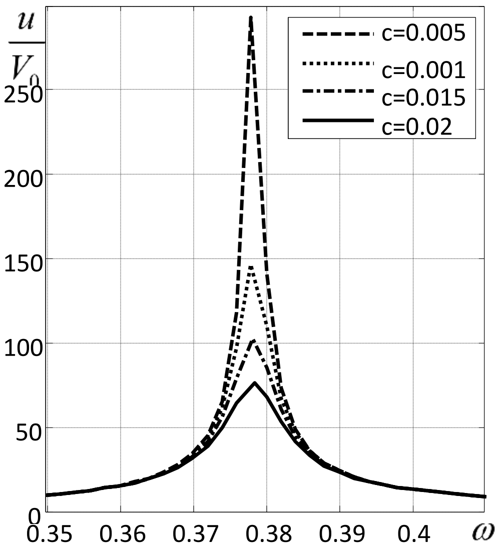

Figure 4. Modules of the amplitude of displacements of the sphere $\varepsilon = 0.25$ near the first resonance at different values of loss tangents

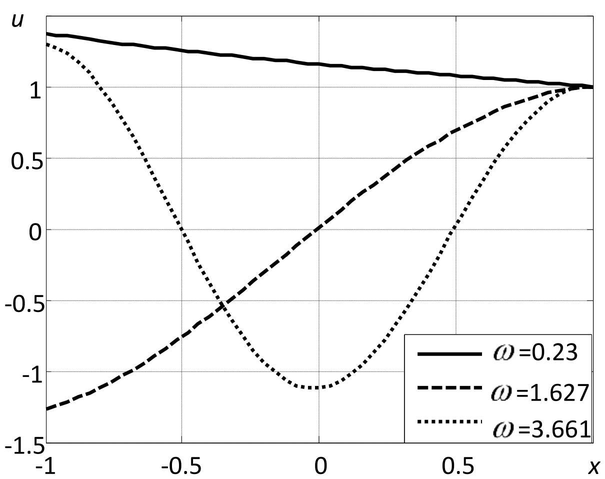

Figure 5. The first three forms of cylinder vibration ($\varepsilon = 0.25$)

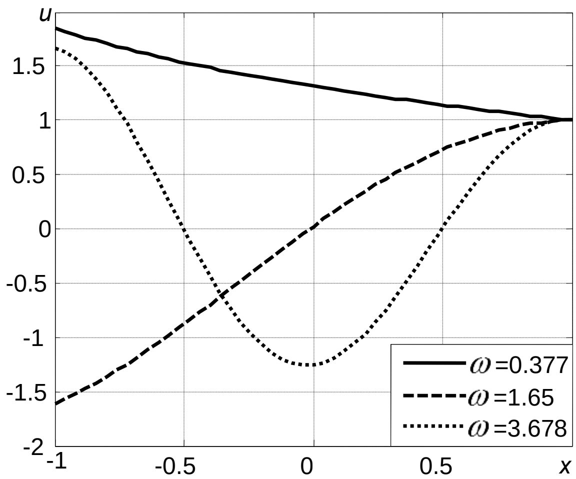

Figure 6. The first three forms of sphere vibration ($\varepsilon = 0.25$)

Interval of frequency change involves first four own frequencies. Amplitude of displacements on resonances takes theoretically unlimited meanings. Higher resonances for different bodies are close to each other, the most difference is observed at first resonance. At decreasing ε the first resonance approaches to zero. There are following resonance frequencies at $\varepsilon = 0.25$: for cylinder $\Omega = (0.235, 1.627, 3.662)$, for sphere $\Omega = (0.381, 1.651, 3.677)$.

Detailed curves of modules of the amplitude of displacements taking into account energy dissipation with different tangents meanings of losses $c'_{ij} = e'_{ij} = \varepsilon'_{ij} = c$ were shown in Fig. 3 and Fig. 4. Distinct dependence between the amplitude of oscillations and tangents of losses is observed. Dependence nonlinearity is more observable for sphere.

Oscillation forms at first three resonances are shown in Fig. 5, 6. Displacement correlations of body points to the amplitude of oscillations of outer surface were illustrated in these figures.

Conclusions. Suggested generalized approach to solving of problems of radial oscillations of piezoceramic cylinders and spheres gives opportunity to effectively investigate forced oscillations of piezoceramic bodies with different geometrical parameters, define resonance frequencies, investigate dependence of the amplitude of parameters of electromechanical state on tangents of losses.

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УДК 534-21:537.226.86

РЕЗОНАНСНІ РАДІАЛЬНІ КОЛИВАННЯ П’ЄЗОКЕРАМІЧНИХ ЦИЛІНДРІВ ТА КУЛЬ З УРАХУВАННЯМ ЕЛЕКТРОМЕХАНІЧНИХ ВТРАТ

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Резюме. *Поширено універсальний підхід до розв’язання задач про вимушені товщинні коливання п’єзокерамічних циліндрів та куль на дослідження амплітудних значень електромеханічного стану в околі резонансних частот із урахуванням механічних, діелектричних, п’єзоелектричних втрат енергії. Розглянуто амплітудні значення переміщень зовнішньої поверхні залежно від частоти навантаження та геометричних розмірів тіл, досліджено амплітудні значення переміщень в околі першого резонансу при різних значеннях тангенсів втрат, побудовано форми коливань на перших трьох резонансах.*

Ключові слова: *п’єзокерамічні перетворювачі, радіально поляризований циліндр та куля, вимушені коливання, втрати енергії, тангенси втрат, резонансні частоти.*

Отримано 01.03.16