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DETERMINING THE STABILITY REGION IN THE PLANE OF PARAMETERS AND QUALITY INDICATORS OF LINEAR DISCRETE AUTOMATIC CONTROL SYSTEMS BY D-PARTITIONING METHOD

Leonid Movchan; Anatolii Lupenko; Volodymyr Zakordonets; Serhii Babiuk

Ternopil Ivan Puluj National Technical University, Ternopil, Ukraine

Summary. *The problem of constructing the boundary of the stability region (BSR) of linear discrete automatic control systems in the plane of the system parameters, which are linearly included in the coefficients of the characteristic equation, and quality indicators of the transient process (stability degree, fluctuation degree or attenuation factor) by the D-partition method is considered. The shifted and fictitious characteristic equations for BSR construction in the area of parameters and quality indicators are introduced. It is shown that the quality indicators are non-linearly included in the coefficients of the characteristic equation, therefore it is impossible to construct the BSR of discrete automatic control system using the classical D-partition method. Constructing of digital control system BSR of spaceship state using one coordinate in the plane of the system parameter- stability degree is considered. The BSR is obtained using the previously proposed by the authors method of constructing the region of stability in the plane of two parameters, one of which is nonlinearly included in the system equation. At the same time, the construction of the entire D-partition curve, special straight lines, and the use of Neimark hatching is excluded, and computer realization of the limit of stability region is ensured. The obtained BSR family in the plane of the parameter and at different values of another system parameter which is nonlinearly included in the coefficients of the shifted characteristic equation makes it possible to estimate, and for the parameter values on the boundary of the stability region of the BSR family, to determine the stability degree.*
Key words: *D-partition, boundary of the stability region in the parameter space, quality indicator, characteristic equation.*

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Formulation of the problem. During the study of real automatic control systems (ACS), solving the problem of determining the stability of a linear discrete system at fixed parameters is not sufficient. It is important to construct the boundary of the stability region (SR) in the space of parameters whose influence on the stability of the system is being studied [1, 2, 3].

Quality of transient processes characterised by such indicators as transient time (system performance), oscillation, overshoot, etc., which are directly determined by the transient curve, is important.

It is important in practice to determine the values of the parameters that meet the required indicators of the quality of the transition process and to estimate the limits of change in these parameters as well.

Analysis of available research results. The known numerical methods of constructing the stability region (SR) of linear BRS by searching points in the parameter space using computer are not economical in terms of machine implementation with increased accuracy of determining the SR and the correctness of the result is not always guaranteed [4, 5].

In general, the boundary of the stability region is determined using the D-partitioning method.

In [6] and others, the authors present a method that allows, for a wide class of discrete ACS, to determine in the plane of the SR parameters the subregions whose control time and

Corresponding author: Leonid Movchan; e-mail: wladim21@gmail.com

variability are less than some predetermined values. The values of the system parameters on the boundary of the obtained subregions (lines of equal values of the degree of stability and the degree of variability) of stability correspond to the boundary values of the transient quality indicators. But at the same time, the problem of determining the correspondence between the parameters of the coefficients of the characteristic equation and the quality indicators within the subregions of the entire stability region is not solved.

Objectives of the research. To build a boundary of the stability region in the space of parameters of discrete ACS characteristic equation and transient quality indicators, which allows to estimate and determine the values of parameters that correspond to the specified values of the degree of stability and the degree of oscillation. The solution of the problem is carried out similarly to the previous work of the author [7], taking into account the significant differences between continuous and discrete systems.

Statement of the problem. It is known that a linear discrete system is described by difference equations:

$$a_0 y\left[(n+k)T\right] + a_1 y\left[(n+k-1)T\right] + \ldots + a_{k-1} y\left[(n+1)T\right] + a_k y\left[nT\right] = \\ = b_0 x\left[(n+m)T\right] + b_1 x\left[(n+m-1)T\right] + \ldots + b_m x\left[nT\right] \tag{1}$$

The characteristic equation of a discrete system is as follows:

$$D(z) = a_0 z^k + a_1 z^{(k-1)} + \ldots + a_{k-1} z + a_k = 0 \tag{2}$$

or

$$D(q) = a_0 e^{kqT} + a_1 e^{(k-1)qT} + \ldots + a_{k-1} e^{qT} + a_k = 0 \tag{3}$$

The characteristic equation of the system (2), which has all roots on the complex z-plane within a circle of unit radius, i.e. $|z_i| < 1$, and the characteristic equation (3) has all roots q_i on the complex *q*-plane with a negative real part, corresponds to all values of the parameters located in the stability region.

The parameters located on the boundary of the stability region correspond to the characteristic equation (2), which has at least one root of z-plane on a circle with unit radius, i.e. $|z_i| = 1$, and the characteristic equation (3) has at least one root on the imaginary axis of the complex *q*-plane.

The stability of a discrete ACS is a necessary but not sufficient condition for its practical applicability. It is important to consider the quality of transient processes, which are characterised by such indicators as transient time, which characterises the system performance, fluctuation, re-adjustment, and others, which are determined by the direct method immediately by the transient curve.

Indirect methods of studying linear discrete systems are used to approximate the quality of the transient process. Here, distinguish the regions in the plane of the roots of the characteristic equation that meet the specified quality indicators.

The criterion for an indirect assessment of the nature of the transient process is the degree of stability η, the distance from the imaginary axis to the nearest root in the *q*-plane, and the degree of fluctuation $\mu = |\omega/\eta|$, the largest value of the absolute ratio of the imaginary part of the root of the *q*-plane characteristic equation to its real part.

For a given acceptable value of the degree of stability, the area of the roots of the characteristic equation is limited to the lines of constant value of the degree of stability in q-plane by shifting to the left by distance η of the imaginary axis in q-plane, or to the curves of constant attenuation in z-plane, which are circles with radius $z = e^{-\eta T}$ centred at the origin.

For a given value of the degree of fluctuation μ, the roots are located in the region bounded by the lines in the q-plane (obtained by rotating the imaginary axis by an angle $\left(\frac{\pi}{2} - \varphi\right)$, where $\varphi = \operatorname{arctg} \mu$ and the logarithmic spiral in z-plane.

To determine whether the roots of the characteristic equation of the discrete ACS belong to the region bounded by the lines of constant attenuation value and to construct BRS in the plane of the parameter and the degree of stability, we introduce a new variable $q = q^* - \eta$ into the characteristic equation (3) and a new variable $z = e^{-\eta T} \cdot z^*$ into equation (2). As a result, we obtain the so-called shifted characteristic equations in the form:

$$D(z^*) = A_0 z^{*k} + A_1 z^{*(k-1)} + \ldots + A_{k-1} z^* + A_k = 0 \tag{4}$$

$$D(q^*) = A_0 e^{kq^*T} + A_1 e^{(k-1)q^*T} + \ldots + A_{k-1} e^{q^*T} + A_k = 0 \tag{5}$$

where $A_e = a_e e^{-(k-e)\eta T}$, $z^* = e^{q^*T}$.

To construct the stability region in the plane of the parameter and the degree of fluctuation, it is necessary to introduce a new variable $q = jq^* e^{-j\varphi} = q^* e^{j\left(\frac{\pi}{2} - \varphi\right)} = q^* e^{j\beta}$ into the characteristic equation corresponding to the rotation of the imaginary axis in the q-plane by an angle $\beta = \left(\frac{\pi}{2} - \varphi\right)$, where $\varphi = \operatorname{arctg}\left(\frac{\omega}{\eta}\right) = \operatorname{arctg} \mu$, and $\beta = \operatorname{arctg}\left(\frac{\eta}{\omega}\right) = \operatorname{arctg} \frac{1}{\mu}$. We obtain the so-called fictitious characteristic equation, which is relatively complicated for the construction of the SR. Therefore, we first perform a bilinear transformation of the characteristic equation (2) by replacing $z = \frac{1+s}{1-s}$ and obtain the characteristic equation of a closed discrete system of the same order as equation (2):

$$D(s) = B_0 s^k + B_1 s^{k-1} + \ldots + B_{k-1} s + B_k = 0, \tag{6}$$

where

$$B_0 = \sum_{i=0}^{k} a_i, B_e = \sum_{i=1}^{k} a_i \sum_{y=1}^{k} a_i \binom{i}{y}\binom{k-i}{e-y}(-1)^y, y = 1, 2, \ldots, n-1, B_k = \sum_{i=0}^{k} (-1)^i a_i, i\binom{i}{e} = \frac{i}{e!(i-e)!}$$

are binomial coefficients

The binomial transformation displays the plane of the circle of unit radius of the complex z-plane of the roots of the characteristic equation $D(z)$ into the left half-plane of the complex s-plane.

To construct the stability region in the plane of the parameter and the degree of fluctuation, or attenuation coefficient ξ $(\xi = \sin \beta)$, we substitute $s = js^* e^{j\varphi} = s^* e^{j\left(\frac{\pi}{2} - \varphi\right)} = s^* e^{j\beta}$ into the characteristic equation and obtain a fictitious characteristic equation:

$$D(s^*) = C_0 s^{*k} + C_1 s^{*(k-1)} + ... + C_{k-1} s^* + B_k = 0, \quad (7)$$

where $C_i = e^{(k-i)j\beta} B_i$, $\beta = \text{arctg}\frac{1}{\mu} = \arcsin\xi$.

To construct a stability region in the plane of the parameter and one of the quality indicators, given another quality indicator, a substitution $s = \left(s^* - \eta\right)e^{j\beta}$ is introduced into the characteristic equation (6) and a shifted fictitious characteristic equation is obtained:

$$D(S^*) = C_0 S^{*k} + C_1 S^{*(k-1)} + ... + C_{k-1} S^* + C_k = 0, \quad (8)$$

where $C_e = e^{(k-e)\beta} B_i \frac{1}{(k-e)!}\left[\frac{\partial^{k-1}}{\partial S^{k-1}}\right] \cdot S = -\eta.$

Since the quality indicators (degree of stability η, degree of oscillation μ, attenuation coefficient ξ) are included in the coefficients of the characteristic equations (4), (7), (8) nonlinearly, it is impossible to construct the boundary of the stability region in the plane of the parameter and these indicators by the classical D-partitioning method. Therefore, in order to obtain the stability region, similarly to that for continuous systems, but taking into account the peculiarities of discrete systems, we use the method of constructing discrete systems in the plane of two parameters, which eliminates the need to construct the entire D-partitioning curve and special curves, the use of Neumark hatching, and provides machine implementation of the construction of BRS [8].

In the classical method of constructing the BRS using the D-splitting method, ω in the characteristic equation (5) is changed from 0 to π and the entire D-splitting curve is obtained. In contrast to the classical method, we will change the parameter η from the predefined value $\eta_{min} = 0$ to η_{max} ($\eta_{min} = 0 \leq \eta \leq \eta_{max}$), which is of practical importance.

For each fixed value of the degree of stability η when determining the BRS in the plane of one parameter by the D-partitioning method, the characteristic equation will be as follows:

$$D(z^*) = KH(z^*) - L(z^*)$$

where K is the parameter whose influence on the system stability is being studied.

Then the equation of the boundary of the D-partitioning region in the plane of one parameter is described by the formula:

$$D(e^{j\varpi}) = KH(e^{j\varpi}) - L(e^{j\varpi}),$$

and taking into account that $e^{j\varpi} = \cos\varpi + j\sin\varpi$

$$D(e^{j\varpi}) = D(\varpi) = K\left(H(\varpi) + jH_2(\varpi) - L_1(\varpi) + jL_2(\varpi)\right), \quad (9)$$

where

$$L_1(\varpi) = d_\kappa \cos k\varpi + d_{\kappa-1}\cos(k-1)\varpi + ... + d_1\cos\varpi + d_0,$$
$$L_2(\varpi) = d_\kappa \sin k\varpi + d_{\kappa-1}\sin(k-1)\varpi + ... + d_1\sin\varpi,$$
$$H_1(\varpi) = C_r \cos r\varpi + C_{r-1}\cos(r-1)\varpi + ... + C_1\cos\varpi + C_0,$$
$$H_2(\varpi) = C_r \sin r\varpi + C_{r-1}\sin(r-1)\varpi + ... + C_1\sin\varpi.$$

From equation (9), we obtain an expression for determining the parameter *K*:

$$K(\varpi)=\frac{L_1(\varpi)H_1(\varpi)+L_2(\varpi)H_2(\varpi)}{H_1^2(\varpi)+H_2^2(\varpi)}+j\frac{L_2(\varpi)H_1(\varpi)-L_1(\varpi)H_2(\varpi)}{H_1^2(\varpi)+H_2^2(\varpi)}=$$
$$=U(\varpi)+jV(\varpi).$$

Since *K* is a viable physically significant value, the problem of determining the BRS by D-partitioning method in the plane of one parameter can be reduced to the problem of determining the stability interval, which can be the interval of the real axis (K',K'') located in the plane of the parameter [8].

The boundary values K' and K'' correspond to the points of intersection of the D-folding curve with the real axis $K(\varpi)$. Therefore, the values of the frequencies ϖ' and ϖ'', which correspond to the boundary values of the parameter K, are determined from the equation:

$$V(\varpi)=\frac{L_2(\varpi)H_1(\varpi)-L_1(\varpi)H_2(\varpi)}{H_1^2(\varpi)+H_2^2(\varpi)}=0$$

or

$$L_2(\varpi)H_1(\varpi)-L_1(\varpi)H_2(\varpi)=0$$

Then the boundary values of the parameter *K* of the stability segment (K',K'') are determined by the expressions:

$$K'=K(\omega')=\frac{L_1(\varpi')H_1(\varpi')+L_2(\varpi')H_2(\varpi')}{H_1^2(\varpi')+H_2^2(\varpi')},$$
$$K''=K(\varpi'')=\frac{L_1(\varpi'')H_1(\varpi'')+L_2(\varpi'')H_2(\varpi''')}{H_1^2(\varpi'')+H_2^2(\varpi'')}.$$

In general, a D-partition curve can cross the real axis $\upsilon(\omega)$ in the plane of one parameter a certain number of times. A potential stability interval is a stability segment located in the area where all values of the parameter correspond to the roots of the characteristic equation $|z_i|<1$.

The totality of the obtained segments of the stability region (K',K'') for all values of the quality indicator ($0 \le \eta \le \eta_{max}$) defines the boundary of the stability region in the plane $[\eta,K]$. This eliminates the construction of the entire D-partitioning curve, special lines, the use of Neumark hatching, and also provides a machine implementation of the construction of BRS.

Analysis of numerical values. To illustrate the practical application of the proposed approach, let us consider the construction of a BRS in the space of a parameter that is linearly included in the coefficients of the characteristic equation and the stability index η on the example of a discrete spacecraft control system [9]. The system is designed to control the state of the spacecraft along one coordinate.

The transfer function of a closed-loop digital control system is as follows [9]:

$$W(Z)=\frac{T^2K_P(Z+1)}{2J_rZ^2+\left(2K_rT-4J_r+T^2K_P\right)Z+\left(2J_r-2K_rT+T^2K_P\right)},$$

where the nominal parameters of the control system are: state sensor gain $K_P=1,65\cdot10^6$; velocity sensor gain $K_r=3{,}17\cdot10^5$; moment of inertia of the spacecraft $J_r=41822$; T is the quantisation (sampling) period.

The shifted characteristic equation looks as follows:

$$2J_re^{-2\eta T}Z^{*2}+\left(2K_rT-4J_r+T^2K_P\right)e^{-\eta T}Z^{*2}+\left(2J_r-2K_rT+T^2K_P\right)=0, \tag{10}$$

and the equation of the boundary of the D-partitioning domain in the parameter plane K_P is:

$$D\left(e^{j\varpi}\right)=KH\left(e^{j\varpi}\right)-L\left(e^{j\varpi}\right), \tag{11}$$

where $H\left(e^{j\varpi}\right)=D\left(Ne^{j\varpi}+1\right),\;\; L\left(e^{j\varpi}\right)=Ae^{j2\varpi}+Be^{j\varpi}+C,\; D=\dfrac{T^2}{2J_r},\; N=e^{-\eta T},$

$$A=-e^{-2\eta T},\;\; B=\left(2-\frac{K_vT}{J_r}\right)e^{-\eta T},\; C=\frac{K_rT}{J_r}-1.$$

From equation (11), and taking into account that $e^{j\varpi}=\cos\varpi+j\sin\varpi$, we obtain an expression for determining the parameter *K*:

$$K(\varpi)=\frac{L(\varpi)}{H(\varpi)}=\frac{L_1(\varpi)+jL_2(\varpi)}{D\left(H_1(\varpi)+jH_2(\varpi)\right)}=\frac{L_1(\varpi)H_1(\varpi)+L_2(\varpi)H_2(\varpi)}{D\left(H_1^2(\varpi)+H_2^2(\varpi)\right)}+$$
$$+j\frac{L_2(\varpi)H_1(\varpi)+L_1(\varpi)H_2(\varpi)}{D\left(H_1^2(\varpi)+H_2^2(\varpi)\right)}=U(\varpi)+jV(\varpi),$$

where $L_1(\varpi)=A\cos2\omega+B\cos\omega+C,\;\; L_2(\varpi)=A\sin2\omega+B\sin\omega,$

$$H_1(\varpi)=N(\cos\omega+1),\;\; H_1(\varpi)=N\sin\omega.$$

The values of frequencies that correspond to the limit values of the parameter K_P, for each predetermined value of the degree of stability η ($0<\eta<\eta_{max}$), are determined from equation $V(\omega)=0$, i.e.:

$$L_2(\varpi)H_1(\varpi)-L_1(\varpi)H_2(\varpi)=0,$$

which, after elementary transformations, looks as follows:

$$\left(2A\cos\varpi+1+B-CN\right)\sin\varpi=0,$$

the solution of which is $\varpi'=0,\quad \varpi''=\arccos\dfrac{CN-1-B}{2A}$.

The boundary parameters K' i K'' of the stability segments for each value of η are determined from the expressions:

$$K'=U\left(\omega'\right)=\frac{L_1\left(0\right)H_1\left(0\right)+L_2\left(0\right)H_2\left(0\right)}{D\left(H_1^2\left(0\right)+H_2^2\left(0\right)\right)},$$

$$K''=U\left(\varpi''\right)=\frac{L_1\left(\varpi''\right)H_1\left(\varpi''\right)+L_2\left(\varpi''\right)H_2\left(\varpi'''\right)}{D\left(H_1^2\left(\varpi''\right)+H_2^2\left(\varpi''\right)\right)}.$$

The flowchart of the algorithm for determining BRS in the plane of the parameter K_P, and the degree of stability, which is nonlinearly included in the coefficients of the shifted characteristic equation, is shown in Fig. 1.

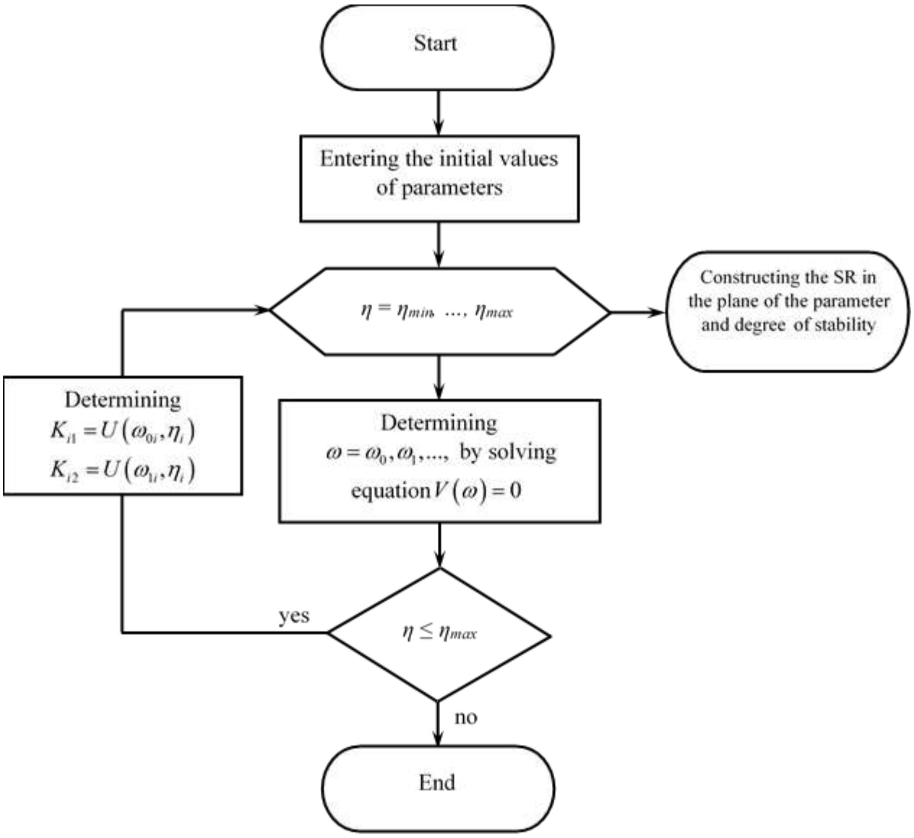

Figure 1. Flowchart of the algorithm for determining the BRS

Fig. 2 shows the boundaries of the stability regions in the plane of the parameter K_P and the degree of stability for different values of the quantisation period ($T=0{,}05$ c,

$T = 0,075$ c, $T = 0,1$ c). The values of the lower boundary of the SR correspond to the frequency $\omega' = 0$, and of the upper boundary to the values ω'' when η changes from *0* to η_{max}.

To validate our results, we determine the roots of the shifted characteristic equation for one of the values of η = 1 and $T = 0,05$ within the stability region (point C), on the boundary of the stability region (point A), and outside the RS region (point B).

At point A, the value of the state sensor gain is $K_p = 9,496 \cdot 10^6$, and the roots of the characteristic equation are $\underline{z}_{1,2} = 0,999989 e^{\pm j40^\circ}$, whose modulus is $|\underline{z}| \approx 1,0$. Thus, the roots are located on a circle of unit radius, so point A is the boundary of the stability region.

At point B, the value of is $K_p = 10 \cdot 10^6$, and the roots of the characteristic equation are $\underline{z} = 1,003 e^{\pm j45^\circ}$, whose modulus is $|\underline{z}_{1,2}| = 1,003 > 1,0$. The point B is located outside the stability region and the system is unstable.

At the point C $K_p = 0,9 \cdot 10^6$, the roots of the characteristic equation $|\underline{z}_{1,2}| = 0,991 e^{\pm j41^\circ}$ whose modulus is $|\underline{z}_{1,2}| = 0,991 < 1,0$. The point is located in the stability region for values $T = 0,05$ c and $\eta = 1$ and the system is stable.

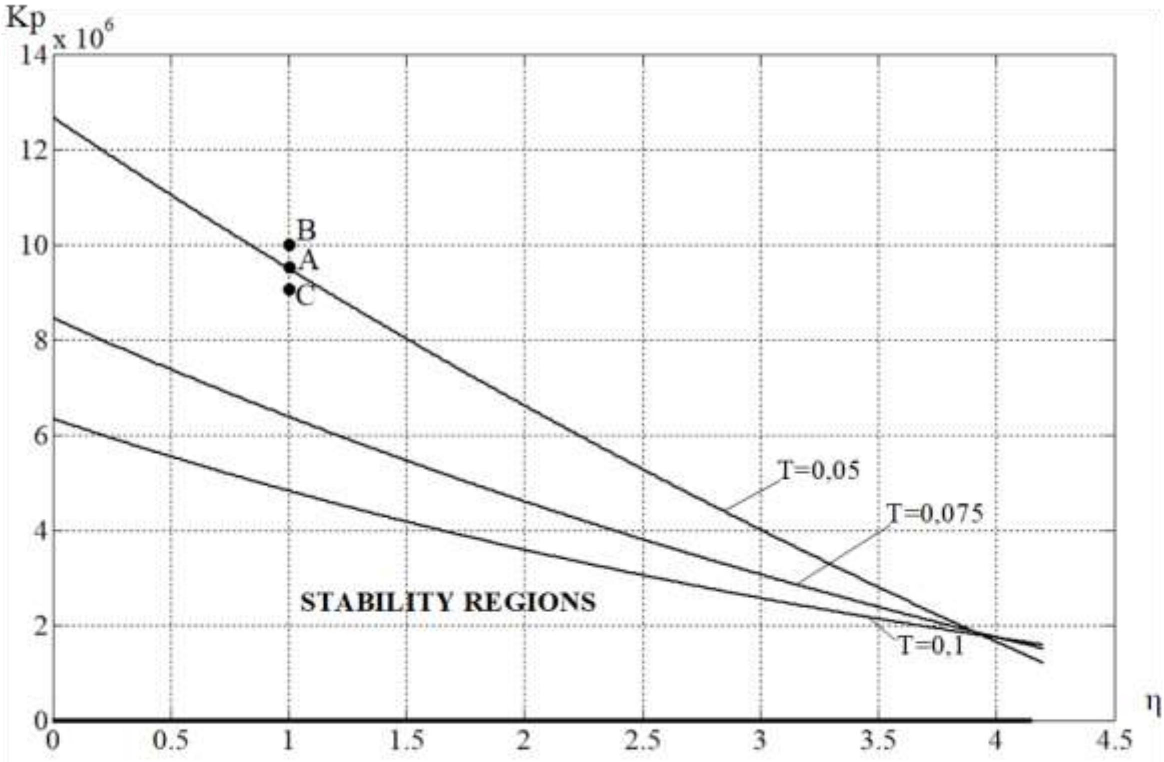

Figure 2. Boundaries of the stability regions in the parameter plane K_p and the degree of stability for different values of the quantisation period

Conclusion. The problem of constructing the boundary of the stability region of a discrete ACS in the plane of one of the parameters, which is linearly included in the coefficients of its characteristic equation, and the transient quality indicator (degree of stability, degree of oscillation or attenuation coefficient), which is nonlinearly included in the coefficients of the characteristic equation, is defined. The article presents the shifted and fixed characteristic equations obtained for the construction of the above-mentioned BRS.

To solve this problem, the previously proposed approach of constructing the BRS in the space of parameters is used, one of which is nonlinearly included in the coefficients of the characteristic equation, and allows obtaining the stability region without constructing the entire D-partitioning curve and special lines, while excluding the use of Neumark hatching.

An example of constructing the boundary of stability region of a digital spacecraft state control system along one coordinate is considered.

The boundaries of the stability region in the plane of one of the parameters and the degree of stability at different values of the other parameter, which is nonlinearly included in the characteristic equation, are obtained.

The results of the present research can be used to construct the BRS of any digital system that has at least one parameter that is nonlinearly included in the coefficients of the characteristic equation.

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ВИЗНАЧЕННЯ ОБЛАСТІ СТІЙКОСТІ В ПЛОЩИНІ ПАРАМЕТРІВ ТА ПОКАЗНИКІВ ЯКОСТІ ЛІНІЙНИХ ДИСКРЕТНИХ СИСТЕМ АВТОМАТИЧНОГО КЕРУВАННЯ МЕТОДОМ D-РОЗБИТТЯ

Леонід Мовчан; Анатолій Лупенко; Володимир Закордонець; Сергій Бабюк

Тернопільський національний технічний університет імені Івана Пулюя, Тернопіль, Україна

Резюме. *Розглянуто питання побудови межі області стійкості (МОС) дискретних лінійних систем автоматичного керування в площині параметрів системи, які лінійно входять до характеристичного рівняння, та показників якості (ступеня стійкості, ступеня коливності, або коефіцієнта загасання) методом D-розбиття. Представлено розміщені та фіксовані характеристичні рівняння, які використовуються при побудові моделі області стійкості в області параметрів і показників якості. Показано, що показники якості нелінійно входять до коефіцієнтів характеристичного рівняння, тому побудова області стійкості класичним методом D-розбиття неможлива. Розглянуто побудову моделі області стійкості цифрової системи керування станом космічного корабля по одній координаті в площині параметра системи та ступеня стійкості. Ця область стійкості отримана, використовуючи раніше запропоновану авторами методику побудови області стійкості в площині двох параметрів, один з яких нелінійно входить до коефіцієнтів рівняння системи. При цьому включається побудова всієї кривої D-розбиття, особливих прямих і використання штриховки по Неймарк., а також забезпечується комп'ютерна реалізація побудови МОС. Отримане сімейство меж областей стійкості в площині параметра та показника ступеня стійкості при різних значеннях іншого параметра системи, який нелінійно входить до коефіцієнтів зміщеного характеристичного рівняння, дозволяє оцінити, а для значень параметрів на сімействі МОС, визначити показники якості перехідного процесу. Для підтвердження достовірності отриманих результатів для одного зі значень ступеня стійкості визначено корені характеристичного рівняння в межах області стійкості та за межею області стійкості. Показано, що межі області стійкості, відповідає характеристичне рівняння, корені якого розміщені на колі з одиночним радіусом. Значенням параметрів, що знаходяться в області стійкості відповідають корені характеристичного рівняння, моделі яких менші одиниці (система стійка), а значенням параметрів за межею області стійкості відповідають корені з модулями, більшими за одиницю, що характерно для нестійких систем.*

Ключові слова: *D-розбиття, межа області стійкості в площині параметрів показника якості, характеристичне рівняння.*

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