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COMPUTER MODELING OF CARDIAC RHYTHM BASED ON VECTOR OF STATIONARY RANDOM SEQUENCES

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Summary. The article is devoted to a computer modeling method of electrocardiogram rhythm based on a mathematically justified model in the form of a vector of stationary random sequences. The developed computer modeling method allows for generating realizations of vector electrocardiogram rhythm signal (vector components of stationary random sequences) for different types of electrocardiogram signals, both normal and with various types of rhythm pathologies. The modeling of electrocardiogram rhythms was carried out based on the obtained statistical information in the form of estimates of the mathematical expectation and variance of the components of the vector of stationary random sequences. It has been shown that the obtained estimates of statistical characteristics of the modeled vector components (components that describe the electrocardiogram rhythm) are within confidence intervals, which is an indication of the correctness of the experiments conducted using the developed computer simulation method. The accuracy of the computer simulation method for generating realizations of the vector components of stationary random sequences has been investigated, and the error of the computer simulation does not exceed 13% for the investigated vector components.

Key words: computer modeling, statistical estimation methods, vector of stationary random sequences, electrocardiogram signal, heart rhythm.

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Introduction. The development of modern diagnostic systems allows for significant improvement in the state of functional diagnosis of cardiovascular diseases. The process of conducting cardiac diagnostics based on automated diagnostic systems involves processing the electrocardiogram signal (ECG), which is divided into several stages. The first stage allows obtaining diagnostic information based on the analysis of the morphological features of the patient's ECG by analyzing the shape and amplitude of the diagnostic zones of the ECG. The second stage involves generating diagnostic information based on the obtained rhythm characteristics (temporal relations of the durations of diagnostic zones (segments) of the ECG). Methods related to the analysis of heart rhythm are considered to be effective methods for studying the state of the cardiovascular system, its adaptive-regulatory capabilities, and the psychological state of the patient [1, 2]. The development of procedures for automating the analysis and processing of obtained diagnostic features, including those that characterize the regularities of heart rhythm, is important for the construction of modern information-measuring and diagnostic computerized systems [3, 4].

Currently, there are many developed automated diagnostic systems for processing ECGs, which investigate its rhythm. In recent years, systems that use artificial intelligence and machine learning [5] have become widely used, allowing for diagnostic conclusions to be formed based on the training of neural network algorithms. In addition, there are systems that use stochastic mathematical models that take into account information obtained through processing ECGs using statistical methods [8].

Approaches to processing ECGs based on a mathematical model in the form of a vector of cyclic rhythmically related random processes are known [9–13]. All of these studies consider new diagnostic features and new possibilities for cardio-diagnostic systems. The use of mathematical models of cyclostationary signals for processing ECGs is discussed in works [14–15]. Modeling of diagnostic features of ECGs is described in works [16–19].

In automated diagnostic systems, methods of processing heart signals and algorithms based on their adequate mathematical models are created. Since there are a significant number of methods that focus specifically on evaluating morphological diagnostic features, while rhythm analysis methods have received less development, not all mathematical models consider the stochasticity and variability of the rhythm. This is due to the insufficient level of unifying ideas in the construction of mathematical models and rhythm analysis methods, which would take into account both the stochastic nature of the signal, which manifests itself in morphological features, and the possibility of considering the stochastic and dynamic nature of the rhythm in mathematical models, which can be manifested in the temporal durations of diagnostic segments during the unfolding of the heart's work process over time.

The mentioned arguments indicate the relevance of developing both new mathematical models and methods of processing electrocardiogram signals to improve the level of informativeness in automated analysis of heart rhythm for the needs of automated cardiodynamics.

An important aspect of building modern efficient cardiac diagnostic systems is the development of software and hardware components on which diagnostic systems are based. Therefore, the functional capabilities of a cardiac diagnostic system are determined by the methods embedded in it, which are implemented based on the appropriate software [20]. Mathematical methods of processing are determined by the corresponding mathematical models, which form the basis of the mathematical support for modern cardiac diagnostic systems. Important tasks of cardiac rhythm research are those related to its computer modeling, in particular for the formation of test signals that include information about the peculiarities of rhythm pathologies, which are necessary for checking the effectiveness of diagnostic systems. The purpose of this study is to develop and investigate a method of computer modeling based on a known mathematical model of a rhythmic cardiac signal with increased resolution in the form of a vector of stationary random sequences [21].

1. The mathematical model of the cardiac rhythm signal with high resolution in the form of a vector of stationary and stationary-related random sequences.

According to the work [21], the electrocardiogram signal is represented as a cyclic random process $\{\xi(\omega, t_{ml}(\omega')) \in \Theta_\xi, \omega' \in \Omega', \omega \in \Omega, t_{ml}(\omega') \in D(\omega')\}$ of the discrete argument, given on stochastically independent probability spaces (Ω, F, P) , (Ω', F', P') and defined on a random discrete domain $D(\omega') = \{t_{ml}(\omega') \in R, m \in Z, l = \overline{1, L}, L \geq 2\}$, and for which each his ω' -realization $\{\xi_{\omega'}(\omega, t_{ml}^{\omega'}), \omega \in \Omega, t_{ml}^{\omega'} \in D_{\omega'}\}$ belongs to the class $\Theta_\xi = \{\xi_\lambda(\omega, t_{ml}^\lambda), \omega \in \Omega, t_{ml}^\lambda \in D_\lambda, \lambda \in \Lambda\}$ isomorphic to the order and values of the cyclic random processes of discrete argument. Each ω' -realization $D_{\omega'} = \{t_{ml}^{\omega'} \in R, m \in Z, l = \overline{1, L}, L \geq 2\}$ random domain definition $D(\omega') = \{t_{ml}(\omega') \in R, m \in Z, l = \overline{1, L}, L \geq 2\}$ conditional cyclic random process $\xi(\omega, t_{ml}(\omega'))$ is a discrete subset of real numbers whose elements satisfy the following conditions: $t_{m_1 l_1}^{\omega'} t_{m_2 l_2}^{\omega'}$, if $m_2 < m_1$, or if $m_2 = m_1, l_2 < l_1$, in other cases $t_{m_1 l_1}^{\omega'} t_{m_2 l_2}^{\omega'}$ ($m_2, m_1 \in Z, l_2, l_1 = \overline{1, L}, 0 < t_{m, l+1}^{\omega'} - t_{ml}^{\omega'} < \infty$).

The rhythmic structure of a conditional cyclic random process of a discrete argument is completely given by the random function of the rhythm $T(t_{ml}(\omega'), n)$, $\omega' \in \Omega'$, $t_{ml}(\omega') \in R$, $n \in Z$, which is given in the probability space (Ω', F', P') , for which each its own ω' -realization $T_{\omega'}(t_{ml}^{\omega'}, n)$, $t_{ml}^{\omega'} \in D_{\omega'}$ satisfies the conditions of the rhythm function, namely: 1) a group of conditions: 1.a) $T_{\omega'}(t_{ml}^{\omega'}, n) > 0$, if $n > 0$ ($T_{\omega'}(t_{ml}^{\omega'}, 1) < \infty$); 1.b) $T_{\omega'}(t_{ml}^{\omega'}, n) = 0$, if $n = 0$; 1.c) $T_{\omega'}(t_{ml}^{\omega'}, n) < 0$, if $n < 0$, $t_{ml}^{\omega'} \in D_{\omega'} \subset R$; 2) for any of $t_{m_1 l_1}^{\omega'} \in D_{\omega'}$ and $t_{m_2 l_2}^{\omega'} \in D_{\omega'}$, for which $t_{m_1 l_1}^{\omega'} t_{m_2 l_2}^{\omega'}$, for the function $T_{\omega'}(t_{ml}^{\omega'}, n)$ a strict inequality holds $T_{\omega'}(t_{m_1 l_1}^{\omega'}, n) + t_{m_1 l_1}^{\omega'} T_{\omega'}(t_{m_2 l_2}^{\omega'}, n) + t_{m_2 l_2}^{\omega'} > T_{\omega'}(t_{ml}^{\omega'}, n)$, $\forall n \in Z$; 3) function $T_{\omega'}(t_{ml}^{\omega'}, n)$ is the smallest in terms of modulus ($|T_{\omega'}(t_{ml}^{\omega'}, n)| \leq |T_{\omega'}^{\gamma}(t_{ml}^{\omega'}, n)|$) among all such functions $\{T_{\omega'}^{\gamma}(t_{ml}^{\omega'}, n), \gamma \in \Gamma\}$, which satisfy the above conditions 1 and 2. Heart rhythm analysis is reduced to statistical analysis of elements of a random domain of definition $D(\omega')$ conditional cyclic random process of a discrete argument $\xi(\omega, t_{ml}(\omega'))$, or to a statistical analysis of its random rhythm function $T(t_{ml}(\omega'), n)$. Random rhythm function $T(t_{ml}(\omega'), n)$ is completely determined through the elements of the random domain $D(\omega')$ according to the formula:

$$T(t_{ml}(\omega'), n) = t_{m+n,l}(\omega') - t_{m,l}(\omega'), m, n \in Z, l = \overline{1, L}, t_{m,l}(\omega') \in D(\omega'). \quad (1)$$

In the general case, the rhythmic signal with increased resolution is justified and described by a vector of stationary and stationary-correlated random sequences $E_L(\omega', m) = \{T_l(\omega', m), \omega' \in \Omega', l = \overline{1, L}, m \in Z\}$, the elements of which can be elements from a vector $V_L(\omega', m)$ (when it is necessary to investigate the time distances between similar phases of the electrocardiogram signal in two adjacent cycles), which provided logical grounds for the consistency of the constructive stochastic model of the heart rhythm and the stochastic model of the electrocardiogram, and also enabled the study of the time stochastic dynamics of the heart rhythm with high resolution based on mathematical statistics methods. The dimensionality (number of components) L of vector $E_L(\omega', m)$ determines the resolution of the cardiac rhythm signal and equals the number of analyzed time intervals between the predefined phases in the electrocardiosignal. By refining and specifying the probabilistic characteristics of the vector, it becomes possible to investigate the temporal stochastic dynamics of the cardiac rhythm with increased resolution based on mathematical statistical methods $E_L(\omega', m)$. A mathematical model of the heart rate signal with increased resolution in the form of a vector of stationary and stationary-related random sequences was applied [21], for which the invariance of its family of distribution function to time shifts by any integer $k \in Z$ is characteristic, namely, for the distribution function $F_{pT_{l_1} \dots T_{l_p}}(x_1, \dots, x_p, m_1, \dots, m_p)$ for order p ($p \in N$) from the family of vector distribution functions $E_L(\omega', m)$ stationary and stationary connected random sequences have the following equality:

$$F_{pT_{l_1} \dots T_{l_p}}(x_1, \dots, x_p, m_1, \dots, m_p) = F_{pT_{l_1} \dots T_{l_p}}(x_1, \dots, x_p, m_1 + k, \dots, m_p + k), \quad x_1, \dots, x_p \in R, m_1, \dots, m_p \in Z, l_1, \dots, l_p \in \{\overline{1, L}\}, k \in Z. \quad (2)$$

The works [21, 22] investigate the structure of probability characteristics of the high-resolution rhythmocardiogram, which arise from the invariance properties of the corresponding probability characteristics of the vector of stationary and stationary-related

random sequences, and complement the known probability characteristics of the vector rhythmocardiogram based on a known model in the form of a vector of random variables.

2. The method of computer modeling of the heart rhythm, as a component of the vector of stationary random sequences.

Consider realizations of a five-component vector $\Xi_{5\omega'}(\omega', m) = \{T_{l\omega'}(m), l = \overline{1,5}, m = \overline{1,245}\}$ durations of the diagnostic zones of the electrocardiogram, the model of which is a vector $\Xi_5(\omega', m) = \{T_l(\omega', m), \omega' \in \Omega', l = \overline{1,5}, m = \overline{1,245}\}$. The stationary random sequences are obtained based on processing the ECG presented in Figure 1 a using a method described in [20] to generate the components of the vector. A fragment of the discrete rhythm function obtained using methods described in [20] is presented in Figure 1, b. The dashed line represents a continuous rhythm function, which is an estimation of the discrete rhythm function and characterizes the rhythm of the investigated ECG [20]. The components of the investigated vector can be obtained by selecting from the discrete rhythm function those components that correspond to the durations of the diagnostic zones of the investigated ECG. The first component $T_1(\omega', m)$ is a random stationary sequence that describes a tooth P (diagnostic zone) in electrocardiograms for all its 245 registered cycles. Realization graph $T_{1\omega'}(m)$ of this component is presented in Figure 2, a. The second component $T_2(\omega', m)$ of this vector is a random stationary sequence describing the duration of the diagnostic zone between P and Q of tooth in electrocardiograms. Implementation graph $T_{2\omega'}(m)$ of this component is presented in Figure 2, b. The third component $T_3(\omega', m)$ of this vector is a random stationary sequence describing the duration of the diagnostic zones QRS complex in electrocardiosignals. Realization graph $T_{3\omega'}(m)$ of this component is presented in Figure 3, a. The next components are the fourth and fifth, they are described in their section $T_4(\omega', m)$ – tooth T in the electrocardiosignal $T_5(\omega', m)$ describes the duration of the diagnostic zone between T and P tooth of the next cycle in the electrocardiogram. Realization graphs $T_{4\omega'}(m), T_{5\omega'}(m)$ these components are presented in Figure 3, b and 4. It can be seen from Figure 1a that there are no rhythm disturbances (arrhythmias) in the studied ECG (ECG is normal), but the values of the rhythm function (Figure 1 b) are not constants

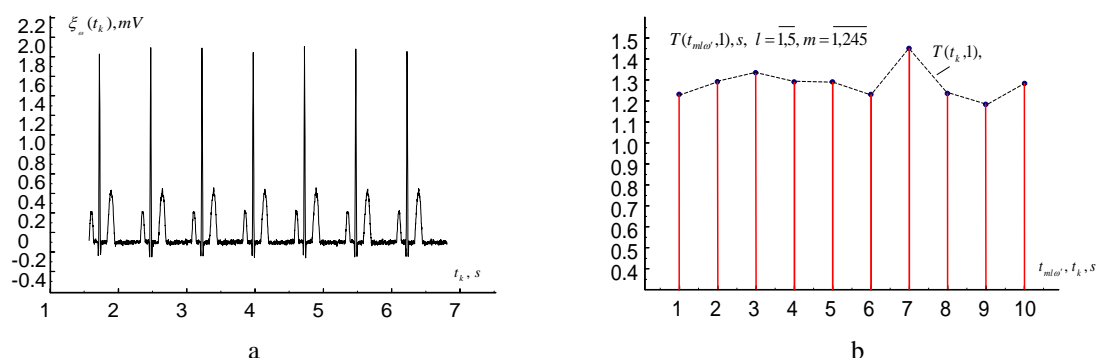


Figure 1. Graphs of fragments of the realization of the examined ECG and the discrete function of the rhythm for ten cycles in the EKS: a) fragment of the implementation of the ECG (II-lead, ECG in the norm); b) a fragment of the discrete rhythm function for ten cycles of ECG (the continuous rhythm function is indicated by a dotted line)

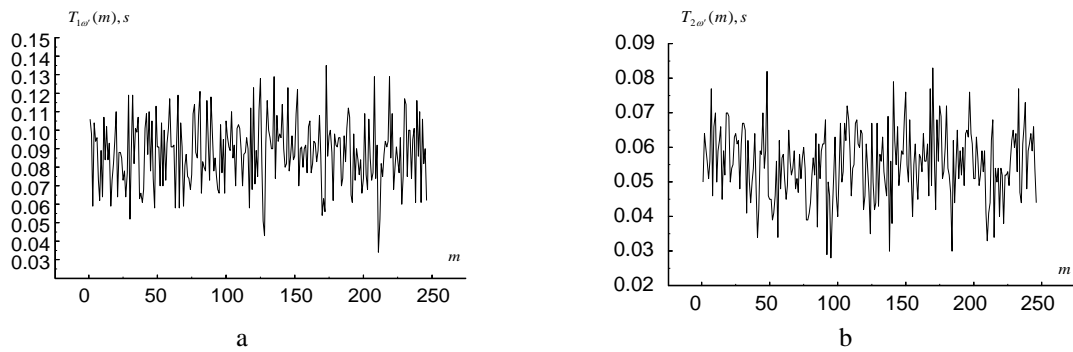


Figure 2. Realization graphs of the first and second components of the vector: a) realization $T_{1\omega'}(m)$ of the first component of the vector $T_1(\omega', m)$, which describes durations of P – teeth in the electrocardiogram; b) realization $T_{2\omega'}(m)$ of the second component of the vector $T_2(\omega', m)$, which describes the duration of the diagnostic zone between P -teeth and Q – teeth in the electrocardiogram

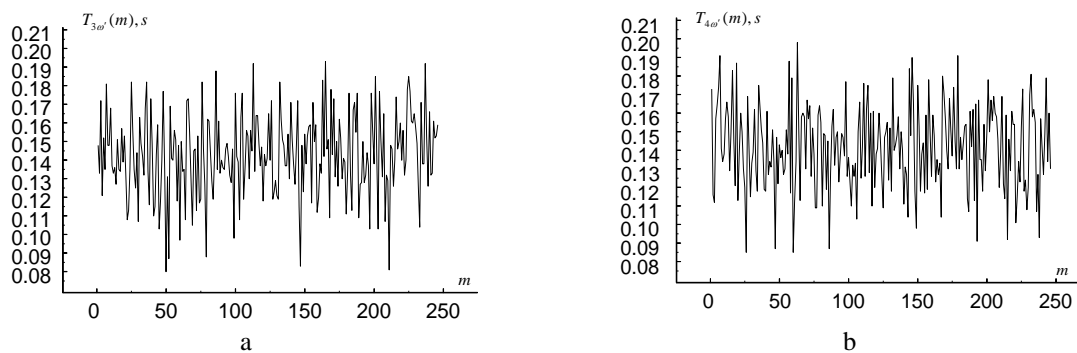


Figure 3. Realization graphs of the third and fourth components of the vector: a) realization $T_{3\omega'}(m)$ of the third component of the vector $T_3(\omega', m)$, which describes durations QRS-complex in the electrocardiosignal; b) realization $T_{4\omega'}(m)$ the fourth component of the vector $T_4(\omega', m)$, which describes durations of the T – teeth in the electrocardiosignal

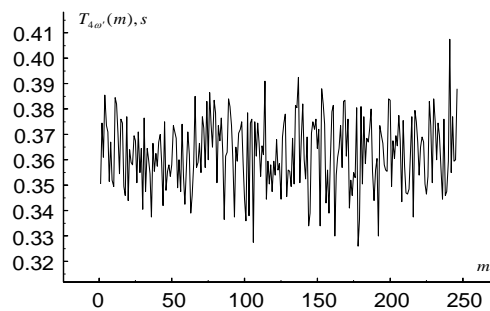


Figure 4. Realization graph $T_{5\omega'}(m)$ the fifth component of the vector $T_5(\omega', m)$, which describes the durations between T -teeth and P -teeth of the next cycle in the electrocardiogram

For model of the components of the vector, we will provide statistical estimates (mathematical expectation and variance) of the duration of one cycle of the electrocardiogram signal in the form:

$$\hat{e}_{T_{\text{cycle}}\{\omega'\}}(l) = \sum_{i=1}^L \hat{e}_{T_{\omega'}}(l) \hat{d}_{T_{\text{cycle}}\{\omega'\}}(l) = \sum_{i=1}^L \hat{d}_{T_{\omega'}}(l), l = \overline{1,5}, \quad (3)$$

where $\hat{e}_{T_{\omega'}}(l)$ – assessment of mathematical expectation, $\hat{d}_{T_{\omega'}}(l)$ – estimates of the variance of each of the five components of the vector $\Xi_{5\omega'}(\omega', m) = \{T_{l\omega'}(m), l = \overline{1,5}, m = \overline{1,245}\}$.

Statistical estimates of the mathematical expectation and variance of the components of the corresponding components of the vector of stationary random sequences are determined as follows:

- estimates of mathematical expectations $\hat{c}_1^l = \{c_{T_{\omega'}}(l), l = \overline{1,5}\}$ for each of the five components of the vector ($L = 5$):

$$c_{T_{\omega'}}(l) = \frac{1}{M} \sum_{m=1}^M T_{l\omega'}(m), l = \overline{1,5}, m = \overline{1,245}; \tag{4}$$

- estimates of variances $\hat{D}_1^l = \{d_{T_{\omega'}}(l), l = \overline{1,5}\}$ for each of the five components of the vector ($L = 5$):

$$d_{T_{\omega'}}(l) = \frac{1}{M-1} \sum_{m=1}^M (T_{l\omega'}(m) - c_{T_{\omega'}}(l))^2, l = \overline{1,5}, m = \overline{1,245}, \tag{5}$$

where $M = 245$ – the number of counts corresponding to the registered cycles of the studied realization of the ECG.

The obtained statistical estimates (mathematical expectation and variance) of the components of the vector are presented in figures 5 a, b.

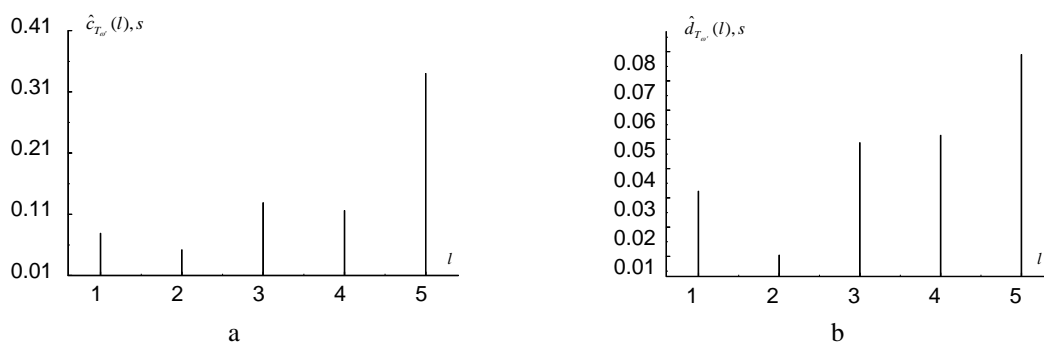


Figure 5. The graph of the realizations of the averaged statistical estimates of mathematical expectations and variances for the components of the vector: a) mathematical expectations for the five components of the vector $T_l(\omega', m)$; b) variances for the five components of the vector $T_l(\omega', m)$, which describe the duration of the diagnostic zones in the electrocardiogram

Based on the obtained statistical information for each component of the vector, we will model the durations of diagnostic zones of ECG. Since it is known that the durations of diagnostic zones of ECG have a normal distribution [21], we will obtain the results of modeling the rhythm for each component of the stationary sequences vector, presented in Figures 6–8.

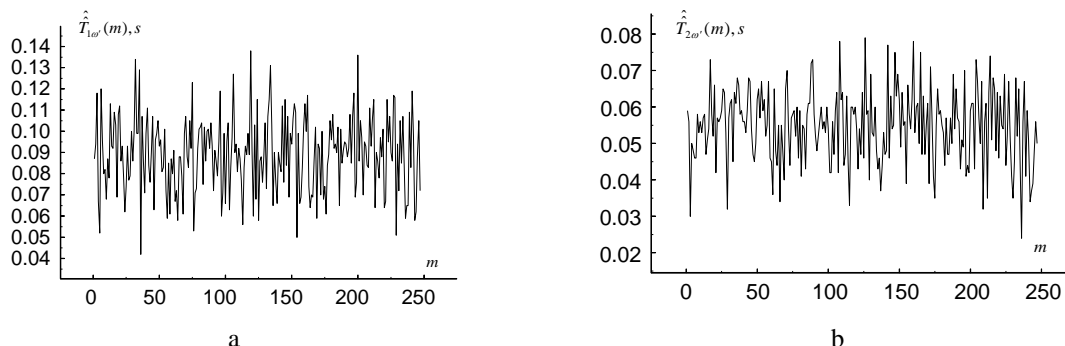


Figure 6. The realization graph of the simulated first and second components of the vector: a) realization $\hat{T}_{1\omega'}(m)$ the first component of the vector $T_1(\omega', m)$, which describes durations P – teeth in the electrocardiogram; b) realization $\hat{T}_{2\omega'}(m)$ of the second component of the vector $T_2(\omega', m)$, which describes the duration of the diagnostic zone between P – teeth and Q – teeth in electrocardiosignal

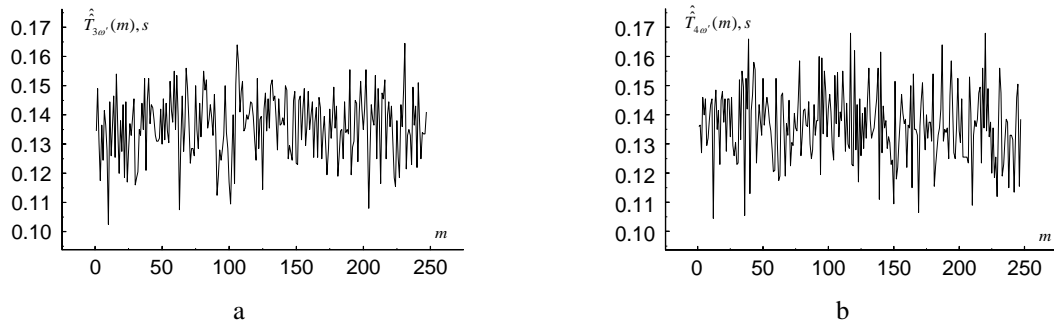


Figure 7. The graph of realizations of the simulated third and fourth components of the vector: a) realization $\hat{T}_{3\omega'}(m)$ of the third component $T_3(\omega', m)$ of the vector, which describes durations QRS-complex in the electrocardiogram; b) realization $\hat{T}_{4\omega'}(m)$ of the fourth component $T_4(\omega', m)$ of the vector, which describes durations T – teeth in the electrocardiogram signal

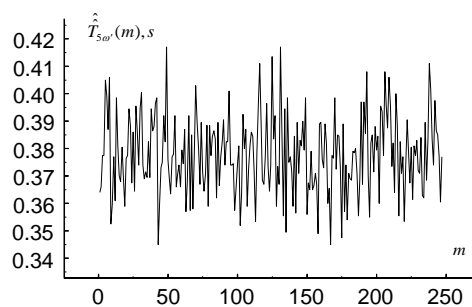


Figure 8. Graph of realization of the simulated $\hat{T}_{5\omega'}(m)$ of the fifth component $T_5(\omega', m)$ of the vector, which describes the durations between T -teeth and P -teeth of the next cycle in the electrocardiogram

We will estimate the mathematical expectation and variance for the modeled components of the vector and show that they fall within the confidence intervals (with a probability of 0.95) obtained by taking into account the input estimates for the investigated ECG presented in Figure 5. The confidence intervals were determined by the formula

$$\begin{aligned}
 v_{\max}(l) &= \hat{c}_{T\omega'}(l) + 3\sqrt{\hat{d}_{T\omega'}(l)}, \quad l = \overline{1,5}. \\
 v_{\min}(l) &= \hat{c}_{T\omega'}(l) - 3\sqrt{\hat{d}_{T\omega'}(l)}
 \end{aligned}
 \tag{6}$$

The obtained statistical estimates of the simulated vector of stationary sequences are presented in Figure 9.

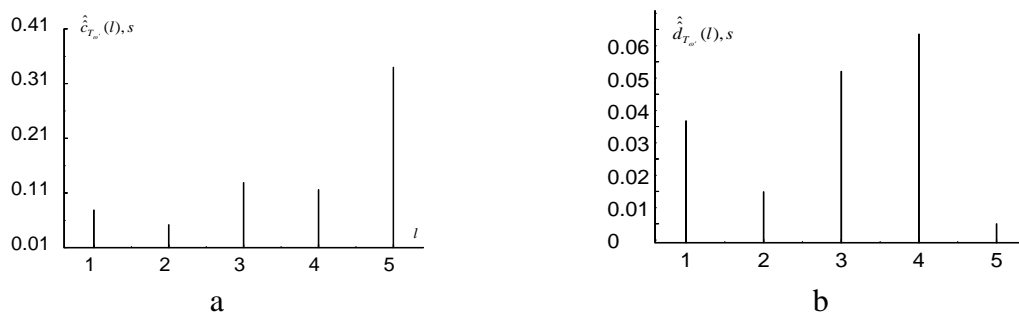


Figure 9. The graph of the averaged statistical estimates of mathematical expectations and variances of simulated vector components: a) mathematical expectations for five vector components $T_l(\omega', m)$; b) variances for the five components of the vector $T_l(\omega', m)$, which describe the simulated durations of the diagnostic zones in the electrocardiogram signal

The resulting confidence intervals are presented in Figure 10.

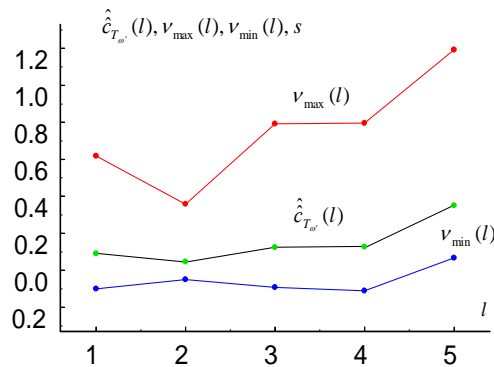


Figure 10. Graph of realization of averaged statistical estimates of mathematical expectations based on simulated values $\hat{c}_{T_{\omega'}}(l)$ and the obtained limits of confidence intervals $v_{\max}(l), v_{\min}(l)$ based on statistical data obtained from the input data Figure 5

The obtained estimates of the mathematical expectations of the modeled stationary sequence vector are within the confidence intervals, indicating the correctness of the developed method of computer simulation of the components of the stationary sequence vector

We will now evaluate the errors of the computer modeling based on the developed method. For this purpose, we will determine the absolute and relative errors of the obtained statistical estimates for the modeled components of the stationary sequence vector.

The absolute and relative errors of the modeling were determined as follows:

$$\Delta(l) = \left| \hat{c}_{T_{\omega'}}(l) - \hat{c}_{T_{\omega'}}(l) \right|, \quad l = \overline{1,5}. \quad (7)$$

$$\delta(l) = \frac{\Delta(l)}{\hat{c}_{T_{\omega'}}(l)}$$

The results of the obtained absolute and relative errors of rhythm modeling are presented in Figure 11:

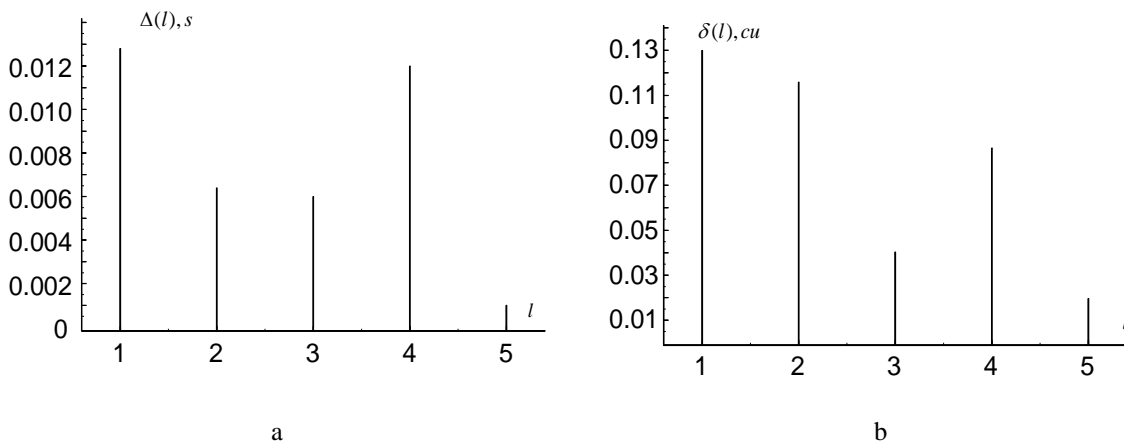


Figure 11. Absolute and relative errors of computer modeling: a) absolute error of computer modeling; b) relative error of computer modeling of vector components

The obtained results show that the maximum relative modeling error for the vector components in the modeling of the ECG rhythm does not exceed 13% for the investigated realizations, indicating sufficient accuracy of computer modeling. In the work [20], a structured diagram (Figure 12) of a diagnostic complex is presented. We will show that an additional block for computer modeling of the vector rhythm of the ECG signal (stationary random sequences) based on the obtained statistical estimates is introduced in this structural diagram of the diagnostic complex.

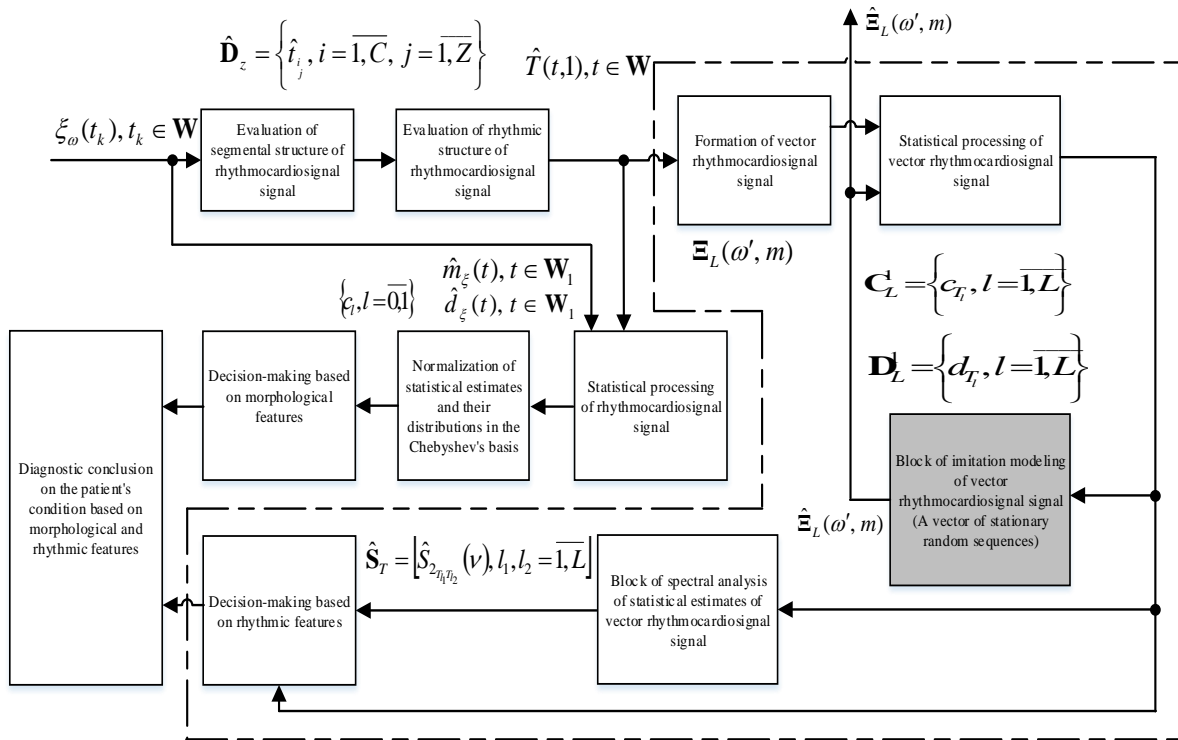


Figure 12. Structural diagram of the modernized diagnostic complex

Conclusions and prospects for further research. This work justifies the use of a known mathematical model of the vector rhythmocardiogram in the tasks of computer modeling of the heart rhythm in the form of a vector of stationary random sequences. Statistical processing methods of the components of the vector rhythmocardiogram based on the mathematical model in the form of a vector of stationary random sequences were applied. The obtained statistical estimates were used during the computer modeling of the components of the vector rhythmocardiogram. The results of the statistical estimates of the modeled components of the vector rhythmocardiogram with a probability of 0.95 are within the confidence intervals obtained based on the statistical estimates of the input data. The accuracy of modeling the components of the vector rhythmocardiogram was evaluated and it was established that the relative error of computer modeling does not exceed 13%.

In future work, it is planned to conduct studies of ECG with various types of rhythm pathologies such as tachycardia, bradycardia, arrhythmia, and others with the aim of identifying marker estimates of those vector components where significant differences in heart rhythm disorders are manifested. In addition, it is planned to conduct computer modeling of vector rhythmocardiogram components taking into account their autocorrelation and cross-correlation functions.

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КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ СЕРЦЕВОГО РИТМУ НА ОСНОВІ ВЕКТОРА СТАЦІОНАРНИХ ВИПАДКОВИХ ПОСЛІДОВНОСТЕЙ

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Резюме. *Присвячено методу комп'ютерного імітаційного моделювання ритму електрокардіосигналу на основі обґрунтованої математичної моделі у вигляді вектора стаціонарних випадкових послідовностей. Розроблений метод комп'ютерного імітаційного моделювання дозволяє формувати реалізації векторного ритмокардіосигналу (компоненти вектора стаціонарних випадкових послідовностей, які описують ритм розгортання у часі процесу роботи серця) для різних типів електрокардіосигналів як в нормі, так і з різними видами патологій ритму. Проведено моделювання ритму електрокардіосигналів на основі врахування отриманої статистичної інформації у вигляді оцінок математичного сподівання та дисперсії компонент вектора стаціонарних випадкових послідовностей. Показано, що отримані оцінки статистичних характеристик (математичних сподівань та дисперсій) змодельованих реалізацій компонент вектора (компонент, які описують ритм електрокардіосигналу) знаходяться в межах інтервалів довіри, отриманих на основі статистичних оцінок вхідних досліджуваних реалізацій вектора стаціонарних випадкових послідовностей, що є ознакою коректності проведених експериментів по застосуванню розробленого методу комп'ютерного імітаційного моделювання. Проведено дослідження точності методу комп'ютерного імітаційного моделювання реалізацій компонент вектора стаціонарних випадкових послідовностей. Отримана похибка комп'ютерного імітаційного*

моделювання не перевищує 13% для досліджуваних компонент вектора стаціонарних випадкових послідовностей. Показано в загальній структурній схемі діагностичного кардіокомплексу, де розміщено блок комп'ютерного імітаційного моделювання реалізації векторного кардіосигналу на основі математичної моделі вектора стаціонарних випадкових послідовностей та що є вхідною інформацією та вихідною для моделювання.

Ключові слова: математичне моделювання, методи статистичного оцінювання, вектор стаціонарних випадкових послідовностей, електрокардіосигнал, ритмокардіосигнал, серцевий ритм.

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