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INVESTIGATION OF THE STRESS-STRAIN STATE OF BEAMS WITH DIFFERENT TYPES OF WEB PERFORATION

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Summary. *This article is devoted to evaluating the effectiveness of I-beams with different web perforations: hexagonal, round, oval and elliptical. The technology of manufacturing of castellated beams is described. For the purpose of verification the analytical calculation of the beam with hexagonal web perforation and for comparison the calculation by the finite element method is given. To correctly assess the stress-strain state, the mesh of finite elements in the area of openings was concentrated. The results of maximum normal stresses and strains obtained by different methods were compared with each other and the efficiency of using the finite element method to determine the stress-strain state of castellated beams was proved.*

In the castellated beams there is a complex stress-strain state, which was confirmed in this work for the most characteristic shapes of openings. Beams with hexagonal, round, oval (horizontal and vertical), elliptical and elliptical (rotated by 45°) openings are considered in the article, their geometric parameters and characteristics as well as advantages and disadvantages are described. Beams with round openings are currently the most widely used. In addition, the parameters that affect the efficiency of castellated beams with oval (horizontal and vertical) and elliptical rotated by 45° openings were identified.

In this work, it was found that the shape of the openings significantly affects the stress-strain state of the castellated beams, especially for hexagonal openings, which are mainly used so far. The stress distribution in the first opening for each of the considered types of perforations and the nature of the change of σ_{max} *in other openings is shown. The stress-strain state of castellated beams was studied using the finite element method.*

The results of this study are of practical value because they can be used when arranging the sections and openings of castellated beams.

Key words*: castellated beams, hexagonal, round, elliptical, oval perforation, stress-strain state, finite element method.*

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Statement of the problem. Castellated beams are used as bearing constructions of buildings and structures and take a place between uniform beams and trusses. Such beams are used in slabs and roofs of multi-storey office buildings, shopping centres, sports facilities, multi-storey garages, load-bearing bridge structures, supporting structures of bridge cranes, aircraft- and shipbuilding [1].

Due to significant increase in the moment of inertia of the section owing to increase in height, castellated beams are used to cover large spans – up to 36–42 m, which is several times more than conventional rolled beams [2]. Castellated arch beams are especially effective. In addition to economic effect, castellated beams look aesthetic [2]. Sometimes utilities are passed through the openings, which saves construction space.

Analysis of recent research. Beams with hexagonal or octagonal [1, 3], round [4], oval [5], sinusoidal [6] openings have become widespread in modern construction. Irregular topology is sometimes used.

The variety of castellated beams is quite large and depends on the type of perforation, relative height of openings and relative width of sections between openings. Each of openings has its advantages and disadvantages. For example, round perforation in comparison with hexagonal or octagonal ones creates lower concentration of stresses, but their manufacturing technology increases metal waste. Also, existing beam manufacture technologies impose

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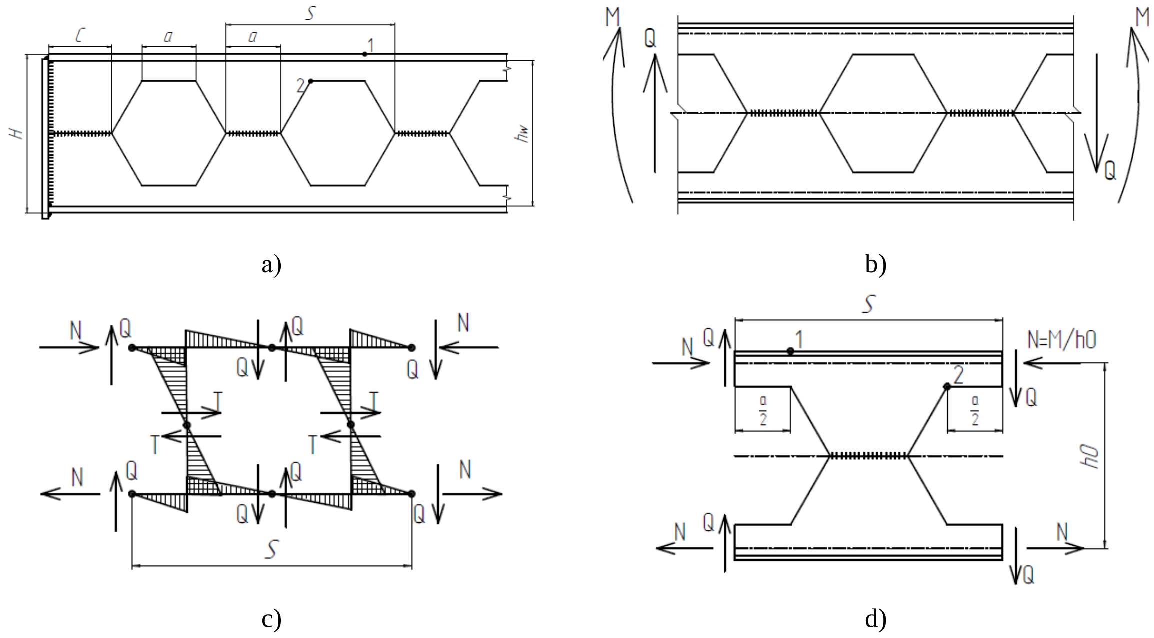
certain restrictions on the width of the web-post between the openings. This reduces the possibility of varying the stiffness of castellated beams.

In this regard, the aim of the research is to select the most rational option for perforation based on the analysis of the influence of structural shapes of beams on their stress-strain state and stability.

Main part. Calculation of beams with perforated web is documented in regulations of many countries [1]. However, most methods involve the calculation of castellated beams with hexagonal openings (Fig. 1, a, b). Beams are calculated on strength, rigidity, lateral torsional and local buckling. Thus, the calculation of the strength of the beam is carried out according to the following model. I-beam with perforated web is structurally intermediate between a solid beam and a Vierendeel frame with rigid knots – a frame system with longitudinal elements and webs. Such multiple statically indeterminate systems can be calculated with sufficient accuracy as a Vierendeel frame according to the approximate Vierendeel method [1]. It is assumed that in the middle of solid area of the web-posts and area of the flanges at the level of the centres of the openings there are points with zero moments (Fig. 1, c). Then we can imagine that at these points there are conventional hinges, where only the transverse and longitudinal forces act as interaction forces (Fig. 1, c, d).



a) b)

c) d)

Figure 1. Calculation model of a beam with hexagonal openings:
a) basic symbols; b) the calculated area of the I-beam; c) model during calculation by the Virendel method; d) section of the beam that is constantly repeated

According to the requirements of [7], the strength regulation of castellated beams with hexagonal openings is carried out for the flange and near the top of the opening (see Fig. 1). For the flange, the design stress is checked by the yield strength R_y (1), and in the opening area, where the stress is much higher, the strength is normalized by the ultimate strength limit R_u (2). Since the tensile strength for plastic steels is significantly higher, the concentration of stresses near the opening is taken into account in the specific way.

Stresses at points 1 and 2 (for each of the openings) are determined by the dependences:

$$\sigma_1 = \frac{M \cdot h_1}{I_x} + \frac{\frac{Q}{2} \cdot a}{2 \cdot W_{1,max}} \leq \frac{R_y \cdot \gamma_c}{\gamma_n} \tag{1}$$

$$\sigma_2 = \frac{M \cdot d_1}{I_x} + \frac{\frac{Q}{2} \cdot a}{2 \cdot W_{1,min}} \leq \frac{R_u \cdot \gamma_c}{\gamma_n \cdot \gamma_u} \tag{2}$$

where *M* is the bending moment in the cross-section of the beam; *Q*/2 is a transverse force perceived by T-sections; I_X is the moment of inertia of the cross-section of the beam with an opening relative to the axis X–X; $W_{1,max}$ and $W_{1,min}$ are the largest and smallest section modulus of the upper T-section; R_y and R_u are design resistance of rolling; h_1 is the height of T-section; d_1 is the half-height of the opening; γ_c is work ratio; γ_n is reliability coefficient for compliance; γ_u is the coefficient of reliability in the calculations of ultimate resistance.

To determine deflections and ultimate loads, Rzhanitsyn's theory of composite rods is used [8]. According to this theory, the castellated rod (Fig. 2) consists of two rigid layers connected by an elastic layer.

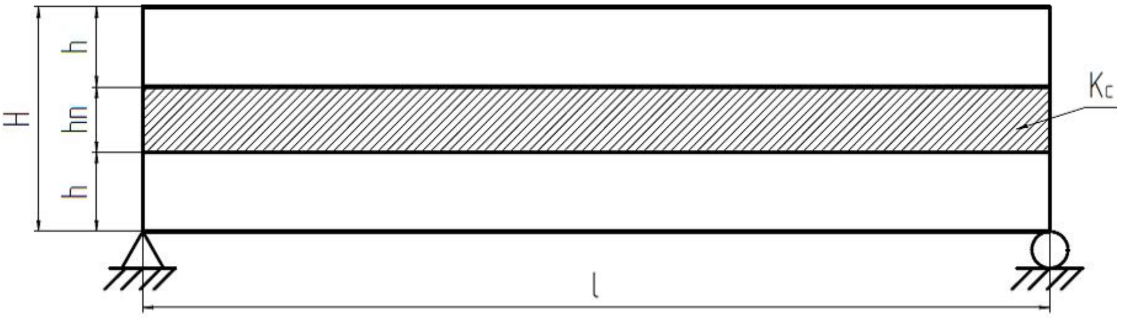


Figure 2. Calculation scheme for estimating the stiffness of the castellated beam

The elastic layer counteracts the bilateral shear and divergence of the castellated rod layers. Joint work of three layers also defines bearing capacity of a beam. The role of the elastic layer is performed by web-post between the perforations, which determine its coefficient of linear stiffness K_C. Experimental data [9] show that the K_C coefficient depends on the beam material, web thickness and support conditions. Also, the coefficient K_C depends on the height of the openings, width and shape of the web-post, so the correctly determined value of the coefficient of linear stiffness of the castellated beam is crucial to estimate its stiffness.

With accuracy sufficient for engineering calculations, the stiffness of the beam with hexagonal openings is determined by:

$$f_{max} = \frac{5 \cdot M_{max} \cdot l^2}{48 \cdot 0{,}95 \cdot EI_x}, \tag{3}$$

where EI_x is the stiffness of the initial beam in relation to axis *x*

The stiffness of the web-post also affects the overall and local stability of the castellated beams.

Due to the complexity of obtaining reliable analytical solutions for determining the strength, rigidity and stability of beams with different perforations of the web, another method is used – the finite element method (FEM), which is based on a plate-rod calculation model. In this model, the castellated beam is represented as a system of single- and multi-joint plates connected to each other by elastic continuous joints. In [4, 10] it is noted that the most accurate solutions for castellated beams are obtained by FEM.

For this reason, in the first stage, a comparative analysis of the strength and stiffness characteristics of beams with the correct hexagonal perforation, obtained analytically and by

FEM was carried out. Beams with this type of perforation are the most studied [5]. Finite element studying of the beam was performed in SolidWorks, Ansys and Lira software packages. The calculation was carried out for the elastic behaviour of the beam. The strength of the castellated beam and its maximum deflection were studied.

A rolled I-beam № 50Б1 with a cutting coefficient ε = 0.75 was selected for cutting. The relative length of the beam is l/H = 16.25; the relative height of the openings is $\beta = \frac{d}{h_w} = 0.667$ (see Fig. 1). Comparative calculations were performed for a beam with a span of 12 m loaded with a concentrated force of 100 kN in the middle of the span.

For test comparison of analytical calculations, the calculation of the above characteristics by the finite element method was conducted using calculation software packages SolidWorks, Lira and Ansys (Table 1). A fragment of a castellated beam with a mesh of finite elements is shown in Fig. 3.

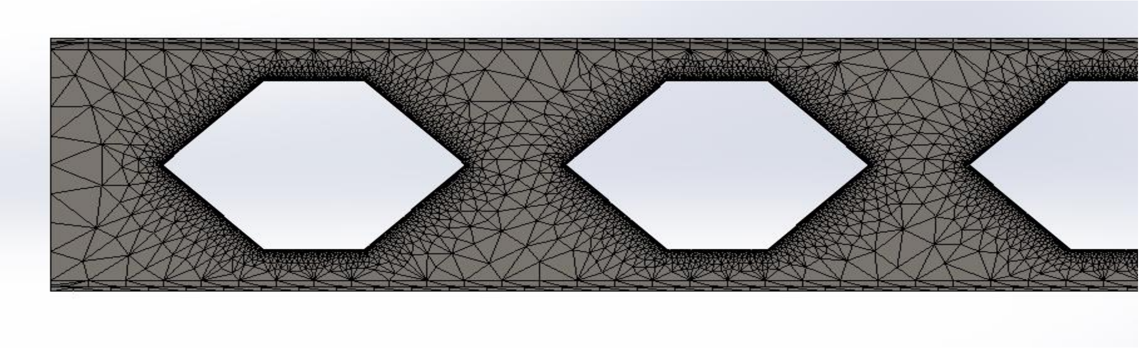


Figure 3. Fragment of a modeled castellated beam with a mesh of finite elements

Calculations of the specified characteristics of strength and stiffness of a castellated beam are carried out. The results of comparative studies of stresses in the flange of the considered castellated beam (see Fig. 1) are presented in Table 1. The numbering of the openings starts from the support hinge of the beam. Also in Table 1, the maximum deflections of the beam obtained by different methods are presented.

Table 1

Stresses and strains in the castellated beam obtained analytically and by FEM

	Stresses in flange σ_1, MPa			
№	Analytical	ANSYS	SolidWorks	Lira
1	33.1	34.2	30.57	29.0
2	540	54.5	50.88	49.0
3	74.9	75.5	67.32	69.1
4	95.7	96.2	90.37	83.0
5	116.6	118.5	109.5	108.5
6	137.5	138.5	134.9	133
Maximum deflection f_{max}, mm				
	26.0	24.7	24.7	24.1

Comparison of the results presented in Table 1 shows that in a constellated beam with hexagonal openings, which is loaded with a concentrated force in the middle of the span, the stresses are maximum in the middle of the span, decreasing gradually to the endings of the beam. For beams with hexagonal openings, for which the method of analytical calculation of stresses in flanges is well developed, their good coincidence with the data obtained using ANSYS software package modelling (error does not exceed 3.3%) is observed. The stress calculations using FEM at SolidWorks and Lira showed discrepancies of up to 9.9% and 13.3%, respectively.

Based on the obtained results, a comparative analysis of stress-strain state with the following perforation of the beam web (Fig. 4) was conducted: round (A) (first used since 1987 and is considered as one of the most promising in the world); traditional hexagonal perforation with a curvature radius *r* = 5 mm (B), a significant disadvantage of which is considered to be the concentration of stresses in the corners of the openings; oval horizontal (B) and vertical (D) perforations; elliptical perforation (D) and (E) (the latter rotated by an angle of 45˚), which is characterized by a certain limitation in the composition of the beam due to the shape of the opening (sharp narrowing at the top, unlike other types of openings).

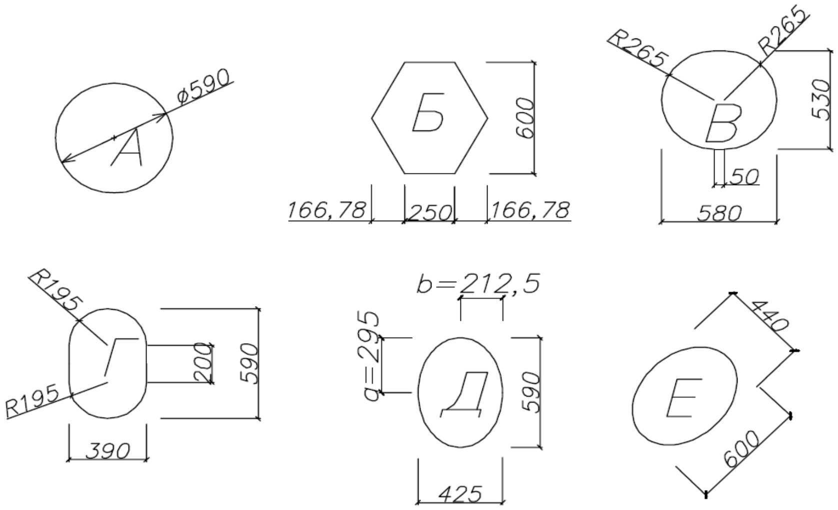


Figure 4. Geometric characteristics of the openings

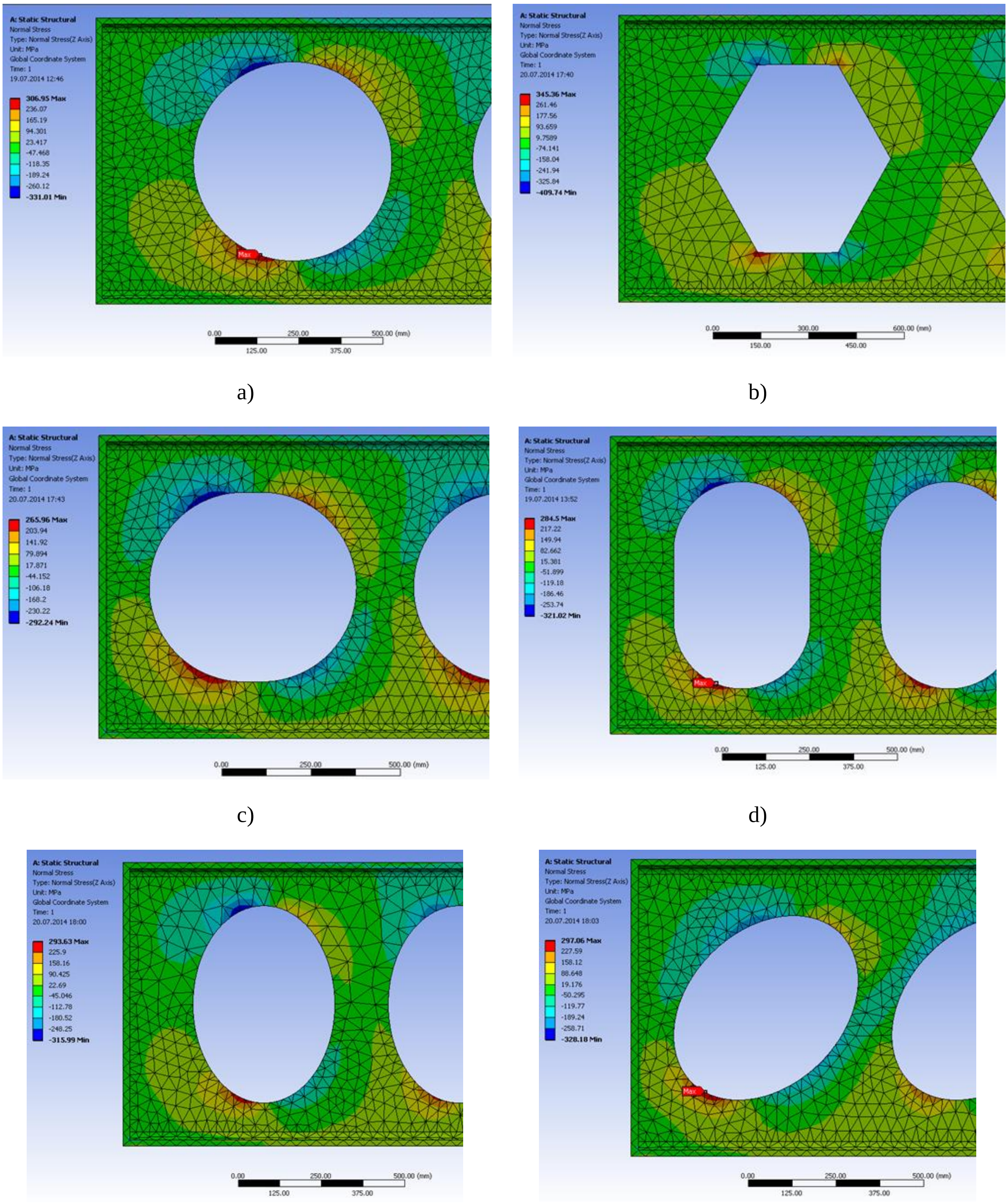
Table 2 presents data on the height of the beam and openings, the distance from the beginning of the beam to the first opening (c), the distance between the axes of the openings (S), the width of the web-post between the openings, the number, width and area of openings for each type of perforation.

Table 2

Geometric characteristics of beam web perforations

No	Type of perforation	Characteristics									
		Beam height, H, mm	c, mm	S, mm	Web-post width, mm	Number of openings	Height of opening, d, mm	Relative height of opening,	Width of opening, mm	Relative width of web-post	Area of openings, cm^2
1	Round (A)	847	270	830	240	14	590	0.7	590	0.41	38256.2
2	Hexagonal (B)	897	250	834	250	14	600	0.67	583.56	0.43	34419.4
3	Oval (horizontal) (B)	847	120	740	160	16	530	0.63	580	0.28	39521.6
4	Oval (vertical) (G)	847	160	580	190	20	590	0.7	390	0.49	39480.0
5	Elliptical (D)	847	190	585	160	20	590	0.7	425	0.38	39367.8
6	Elliptical (rotated by 45°) (E)	847	113	621.8	120	19	526.06	0.62	501.79	0.24	39375.6

It should be noted that all types of perforations (Fig. 5) have the same opening area except for hexagonal openings, which is due to the peculiarities of their manufacturing technology. Thus, hexagonal perforation is characterized by one cutting line, in contrast to others, where there are two cutting line. Therefore, there is a correlation between the height of the resulting beam (height of the opening) and the area of the openings. Therefore, for comparative analysis, the same total area of openings is taken as a basis.



a) b)

c) d)

e) f)

Figure 5. The distribution of normal stresses in the first opening of the studied beams with perforations in the form of: a) circle; b) hexagon; c) oval (horizontal); d) oval (vertical); e) ellipse; f) ellipse that is rotated by 45°

Because of great variety of shapes and locations of openings, it is possible to choose the most optimal among them only after analyzing the impact of various elements of design on strain-stress state and stability of the beams [1]. Therefore, in addition to the same total area of the openings, it is necessary to ensure their same relative height and relative thickness of the web-post between the openings, which is not always possible, taking into account the technology of their manufacture (see Table 2). Such requirements for the construction of web-post are important because, as indicated in [11], reducing the relative width of the web-post from 1.0 to 0.2 with the same size and shape of the openings reduces the stiffness of the beam by about 24%. This is due to the role of shear stresses, the increase of which is caused by a decrease in web area. In this article it is also noted that with the increase of relative length of the beam l/H the effect of shear decreases and at $l/H \geq 23$ the deflections of beams with wide and narrow web-post differ within 5%.

Fig. 5 shows the distribution of stresses in the support zone of the beam for all types of perforations. Stress calculation was performed using the ANSYS software package [12, 13].

Analysis of stresses in the opening zones shows that for all types of perforations under the action of distributed load there is a decrease in stresses from the support zone to the middle of the beam (in contrast to the action of concentrated force in the middle of the beam). Figure 6 shows the change in maximum stresses in the openings for round and oval horizontal perforations.

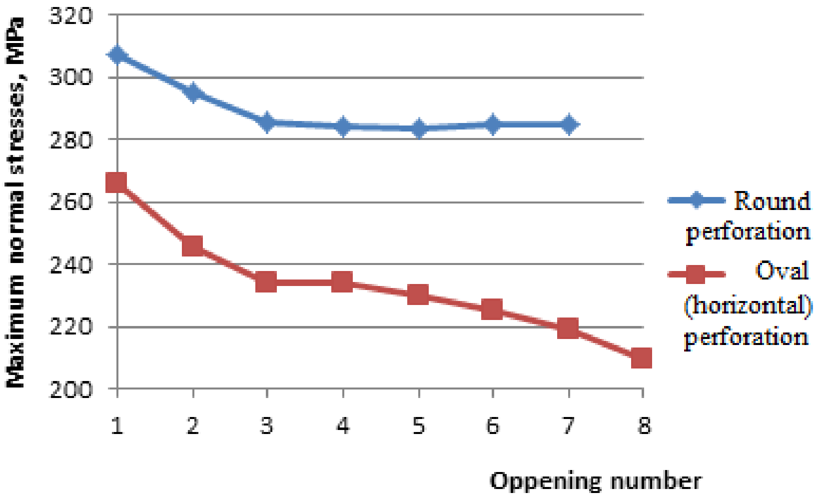


Figure 6. Change of maximum normal stresses in round openings and in oval horizontal openings

Both types of perforation are characterized by gradual decrease in σ_{max} from the first to the middle opening. But for oval horizontal perforation these stresses are significantly lower (1.15 ÷ 1.30 times). This is not only due to the shape of the opening, but also to its horizontal location. This arrangement reduces the relative height of the opening ($\beta = 0.63$) compared to the round perforation ($\beta = 0.70$), because it is obvious that in the area closer to the neutral axis of the beam, the stresses are lower. On the other hand, with oval horizontal perforation, the maximum deflections are greater compared to round openings. This is caused by significant reduction in the relative width of the web-post between the oval horizontal openings (see Table 2).

Conclusions. Comparative analysis of analytical calculation methods and FEM for castellated beams with hexagonal openings has been carried out. It is shown that during the study of stress-strain state of the beams the discrepancy of the results does not exceed 3.3% for the software package ANSYS, 9.9% for SOLID-Works and 13.3% for Lira package. The stress distribution in beams with hexagonal, round, oval (horizontal and vertical) and elliptical

(vertical and located at an angle of 45˚) web perforations is studied. These types of openings in the beams have the same area. On the basis of the comparative analysis of normal stresses it is established that the smallest maximum stresses are observed for oval horizontal perforation.

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УДК 624.014

ДОСЛІДЖЕННЯ НАПРУЖЕНО-ДЕФОРМІВНОГО СТАНУ БАЛОК З РІЗНИМИ ВИДАМИ ПЕРФОРАЦІЇ СТІНКИ

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Резюме. *Присвячено оцінюванню ефективності двотаврових балок з різною перфорацією стінки: шестикутною, круглою, овальною та еліпсоподібною. Описано технологію виготовлення перфорованих балок. З метою верифікації наведено аналітичний розрахунок балки з шестикутною перфорацією стінки та для порівняння – методом скінчених елементів. Для коректним оцінюванням напружено-деформівного стану проведено згущення сітки скінчених елементів у зонах вирізів. Отримані різними методами результати максимальних нормальних напружень та деформацій порівняні між собою й доведено ефективність використання методу скінченних елементів для визначення напружено-деформівного стану перфорованих балок.*

У перфорованих балках спостерігається складний напружено-деформівний стан, що було підтверджено в даній роботі для найбільш характерних форм вирізів. Розглянуто балки з шестикутними, круглими, овальними (горизонтальними та вертикальними), еліпсоподібними (повернутими на 45°) вирізами, описано їхні геометричні параметри та характеристики, переваги й недоліки. Найбільшого застосування на сьогодні набувають балки з круглими вирізами. Крім того, виявлено параметри, які впливають на ефективність балок з овальною (горизонтальною та вертикальною) і еліпсоподібною повернутою на 45° перфорацією. Встановлено, що форма вирізу суттєво впливає на напружено-деформівний стан перфорованих балок. Особливо це стосується шестикутних вирізів, які переважно використовувалися досі. Показано розподіл напружень у першому вирізі для кожного з розглянутих видів перфорацій, і характер зміни σ_{max} *в інших вирізах. Методом скінчених елементів досліджено напружено-деформівний стан перфорованих балок.*

Результати даного дослідження мають практичну цінність, оскільки можуть бути використані під час компонування перерізів та вирізів перфорованих балок.

Ключові слова: *перфоровані балки, шестикутна, кругла, еліпсоподібна, овальна перфорація, напружено-деформівний стан, метод скінчених елементів.*

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