THE PROCESS OF SHELL FORMATION AND OPTIMIZATION BY THE CRITERION OF ACCURACY OF THEIR GEOMETRIC SHAPE

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Abstract: The research is devoted to the process of deformation of a two-dimensional grid. The difficulty of describing the deformation process is that the situation behaves when deformed, which is different from the sheet solid material. This is due to it possible rotation relative to the perpendicular wires of the grid from one structure to another in the nodes. In addition, as shown in more detail below, the conditions that must satisfy the solution are different from the standard ones: the surface that should lie after deformation is given, while the solutions applied to the grid in nodes remain unknown in the formulation of the problem.

The deformation depends on the effort applied to the ends of the mesh element. The stiffness matrix of the element is located using the DIP-FEM package, for which a model of the element is created and at a given unit effort for each direction, the application of the displacement load is calculated.

Keywords: antenna, grid, wires, energy, deformation, clamp, punch, node

1. Introduction

The production of new generation antenna systems is based on new technological and design ideas, the implementation of which requires appropriate scientific and technical support, and is possible in close cooperation of production with scientific potential.

The main design and technological ideas that are implemented in antenna systems of the new generation are:

- preservation in the production of antennas of the basic principles of aviation technologies and with the exception of technological processes of the influence of subjective factors on product quality; optimization of structures according to the criteria - rigidity-accuracy-mass;
- use without stacking assembly and adjustment of antennas on objects;
- use of vector diffraction methods in optimizing the electro-dynamical characteristics of the antenna system at the design stage;
- antenna positioning control system, speed control of its movement, diagnostics of a condition at operation, self-testing, is carried out on the basis of digital information processing.

The study of the process of shell formation consists of predicting and verifying the deformation of a two-dimensional grid. The difficulty of describing the deformation process is that the grid behaves qualitatively differently when deforming than a sheet of solid material. This is due to its structure, in particular, the ability to rotate mutually perpendicular wires of the grid relative to each other at the nodes. In addition, the conditions that the solution must satisfy differ from the standard ones: the surface on which the grid should lie after deformation is given, while the forces applied to the grid at the nodes remain unknown when formulating the problem.

The process of deformation of the grid

This section is devoted to the development and verification of the deformation of a two-dimensional grid. The difficulty of describing the deformation process is that the grid behaves qualitatively differently than a sheet of solid material. This is due to its structure, in particular the ability to rotate mutual perpendicular wires of the grid relative to each other at the nodes. In addition, as discussed in more detail below, the conditions that must satisfy the solution are different from the standard: given the surface on which the grid should lie after deformation, while the forces applied to the grid at the nodes remain in the formulation of the problem unknown [1].

Therefore, the most general approach was used in solving the problem, namely the theorem on the minimum potential energy of the system in the equilibrium position. The friction in the system is, of course, neglected; it can be taken into account when improving the developed methodology. Minimization of potential energy is carried out numerically by iterative method [2]. Problem statement, solution methods and software implementation of the method are described below.

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Formulation of the problem

The surface of rotation is given, the equation of which is described by the following system:

$$\begin{cases} z=f(\rho), \\ \rho=\sqrt{x^2+y^2} \end{cases}; \quad (1)$$

The surface is given in a cylindrical coordinate system. At ρ_1, greater than some values of R_0, f (ρ) = 0. The grid, which in the undeformed state lay in the plane z = 0 and had a step s_1, is deformed so that it lies on a given surface. Forming is carried out by means of a rim which axis radius is equal to R_0 (Fig. 1).

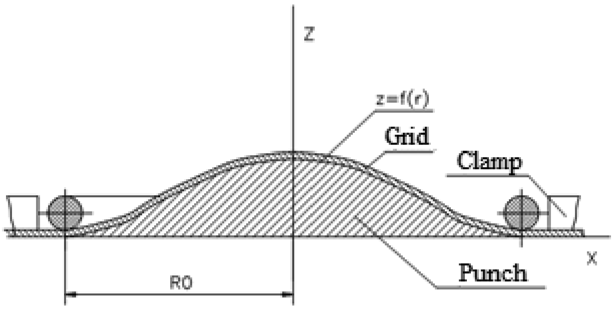

Fig. 1 Formation of a grid on a punch

Friction in the system is completely neglected. Task: to find the position of the grid elements after deformation [5].

Method of solution

Before developing a solution methodology, you should clarify exactly how set the position of the grid. The easiest way to do this is by attaching to the nodes of the grid (that is the points of intersection of mutually perpendicular wires of the grid). To do this, we assume that in the nodes are allowed only the rotation of the wires of the grid relative to each other, while shifts are not allowed. In fact, shifts (minor), of course, exist, but in the first approximate they can be neglected, so under the assumptions made, the position of the grid will be well known if the positions of its nodes are known. Therefore, the problem is to find the position of the nodes of the grid after deformation [3].

Description of the position of the grid

When solving the problem due to symmetry, it is advisable to consider only the part of the grid that is in the first quadrant. To find the position of the nodes of the grid as a whole, it is sufficient to consider that the grid has symmetry of C_4 relative to the z axis. The grid node will be numbered with two numbers-i and j (Fig. 2). The problem will be solved if the vectors of displacements of all nodes during deformation are found. Denote the displacement vector of the node (i, j) as u_{ij}. These vectors are finite numbers. Along with the vectors u_{ij}, we also introduce the vector-mapping function u (x, y). The meaning of this function is that each point (x, y) of the plane z = 0 corresponds to the vector u (x, y), and sets the mapping of the plane z = 0 on the surface, each point of which is a point (x, y) planes z = 0, shifted by the vector u (x, y). We consider the mapping to be mutually unique, and the function u (x, y) - itself to be continuously differentiated in the whole domain. We impose another condition on the function u (x, y): if x_j and y_i - coordinates of the node (i, j), then it should be: u (x_j, y_i) = u_{ij}. In other words, define the function u (x, y) so that it reflects the grid on the surface (1).

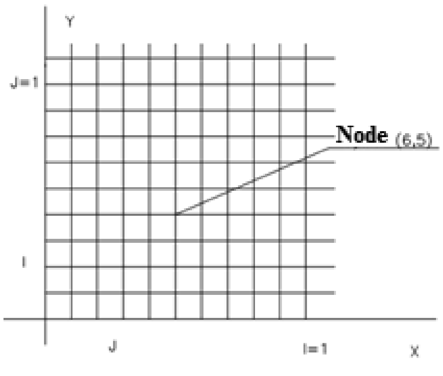

Fig. 2 Grid node numbering

Suppose that before the deformation of the grid, some of its wires were parallel to the x-axis, we, had the equation y = y_0. Whereas this follows directly from the definition of the function u (x, y), after deformation the wire will lie on a curve whose equation in vector form has the form:

$$\begin{cases} R(x) = R_x(x)\cdot i + R_y(x)\cdot j + R_z(x)\cdot k \\ R_x(x) = x + u_x(x, y_0), \\ R_y(x) = y + u_y(x, y_0), \\ R_z(x) = u_z(x, y_0); \end{cases} ; \quad (2)$$

where i, j, k- orts, respectively, the axes x, y, z of the global coordinate system.

Having R (x), you can also find tangent to the wire at any point.

Grid energy

When calculating the energy of the grid, we use the obvious fact: the energy of the whole grid is equal to the sum of the energies of its parts. The smallest element of the grid is the part of the wire the length of the period [4].

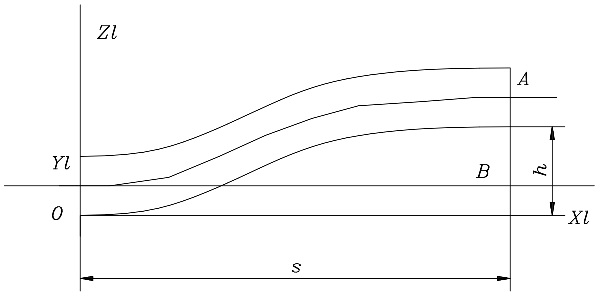

Fig. 3 Grid element

The energy of the whole grid can be found by calculating the energy of each element and adding the obtained values for all elements. Obviously, the energy of the element depends only on the deformation of the element itself. Deformation, in turn, depends on the effort applied to the ends of the element. Place the local coordinate system of the element as shown in Fig. 3. In this case, the point O (left end of the rod) will be considered rigidly fixed. Suppose that the force P and the moment M are applied at the end of the element. We combine these two force factors into one six-dimensional vector F_i, and $F_{1,2,3}= P_{x, y, z}$, and $F_{4,5,6} = M_{x, y, z}$. When applying these force factors, point A will move to some vector Δ and will return to some vector φ. The last two vectors are also combined into one vector D_i, and the correspondence $D_{1,2,3} = \Delta_{x, y, z}$, and $D_{4,5,6} = \varphi_{x, y, z}$. Within the application of Hooke's law, the relationship between the components F_i and D_i is linear. In the most general form, this connection is expressed by the formula:

$$F_i = \sum_{j=1}^{6} C_{i\ j} \cdot D_j \quad (3)$$

where C_{ij}-square matrix of the sixth order, which is called the matrix stiffness.
The inverse matrix is called the flexibility matrix: Suppose we have an equilibrium

$$B_{i\ j} = (C^{-1})_{i\ j} \quad (4)$$

state at some given forces F_i. Let's change these forces of dF_i. We will have an increase in displacements dD_i:

$$dD_i = \sum_{j=1}^{6} B_{i\ j} \cdot dF_j \quad (5)$$

When changing the displacements of the force applied to the element at point A perform some work. According to the law of conservation of energy, this work is equal to the change in the potential energy of the element. Therefore for the differential the potential energy of the element will have the following expression:

$$dU = \sum_{i=1}^{6}(\sum_{j=1}^{6} C_{i\,j} \cdot D_j) \cdot dD_i \quad (6)$$

It is easy to see that dU is really a complete differential. This is easily shown using the theorem on the change of the order of differentiation, which states that for the function U of the variables D_i and D_j the equality must hold:

$$\partial^2 U / \partial D_i \ \partial D_j = \partial^2 U / \partial D_j \ \partial D_i \qquad (7)$$

It is easy to see, using (6), that condition (7) can be rewritten as $C_{ij} = C_{ji}$. This means that the stiffness matrix must be symmetrical. As practical calculations show, this is true. Therefore, dU is a complete differential. The function U itself can be found by simple integration and has the following form:

$$U = \frac{1}{2} \cdot \sum_{i=1}^{6} C_{ii} \cdot D_i^{\,2} + \sum_{j>i} C_{ij} \cdot D_i \cdot D_j . \qquad (8)$$

According to the physical content of the function U as the potential deformation energy of the grid element, U must be positively defined. Specific numerical calculations confirmed this for one case. From physical considerations should be true and in general.

Finding the stiffness matrix of the element

The stiffness matrix of the element can be found using the DIP-FEM package. To do this, create a model of the element in the package according to Fig. 3. If the element is alternately applied in such a way that in each case in the vector F_i is different from 0 only one component (alternately from the first to the sixth), and this component is equal to 1. According to the relationship between displacements and forces for the element:

$$D_i = \sum_{j=1}^{6} B_{ij} \cdot F_j$$

B_{ij} is equal to D_i when applying a unit force in the "direction" j. C_{ij} is then located as a matrix inverted to B_{ij}. With a given unit effort for each direction of application, the displacement loads are calculated by the package. Illustrations showing the deformation of the element when applying loads are given in the graphic part of the work. Finding the inverse matrix is done by a program written in Borland Pascal. For a grid with a step of 7.5 mm from a steel wire with a diameter of 0.75 mm. the stiffness matrix is given below. As expected, it turned out to be symmetrical (accuracy to errors caused by rounding).

Table 1

The stiffness matrix of the grid element

i\j	1	2	3	4	5	6
1	2.582E+05	9.566E-10	1.405E+06	7.845E-13	-4.415E+02	2.163E-14
2	9.612E-10	8.962E+04	5.333E-11	3.361E+02	2.061E-13	-3.361E+01
3	1.405E+06	1.318E-11	1.181E+07	-2.354E-13	-8.395E+02	-2.311E-15
4	8.073E-13	3.361E+02	-3.356E-14	1.694E+00	-3.566E-16	-1.346E-01
5	-4.415E+02	2.092E-13	-8.395E+02	-3.414E-16	1.776E+00	-7.324E-17
6	1.891E-14	-3.361E+01	-2.328E-14	-1.346E-01	-7.048E-17	3.611E-01

Finding Di for an element

Consider an element that in the undeformed state is projected onto the plane z = 0 into a segment parallel to the x-axis. Let this element be bounded by nodes (j, i) and (j + 1, i). The coordinates of the nodes in the undeformed state have the following values:

$$\begin{cases} x_0 = (j-1)\cdot s, \\ y_0 = (i-1)\cdot s, \\ x_B = j \cdot s, \\ y_B = (i-1)\cdot s. \end{cases} \qquad (9)$$

In the deformed state, the coordinates of the nodes are:

$$\begin{cases} x_o^i = x_o + u_x(x_o, y_o), \\ y_o^i = y_o + u_y(x_o, y_o), \\ z_o^i = u_z(x_o, y_o), \\ x_A^i = x_A + u_x(x_A, y_A), \\ y_A^i = y_A + u_y(x_A, y_A), \\ z_A^i = u_z(x_A, y_A). \end{cases} \qquad (10)$$

The deformation vector Di of the element must be expressed in the local coordinate system of the element (Fig. 3). To do this, first of all, we find the orts of the local coordinate system (more precisely, their coordinates in the global coordinate system). As can be seen from Fig. 3, the orth of the x_l axis coincides with the tangent to the element at the point O. Therefore, according to (2), we have:

$$\begin{cases} i_l = \dfrac{l}{|l|}, \\ l_x = 1 + \dfrac{\partial \mathrm{u_x}}{\partial \mathrm{x}}, \\ l_y = \dfrac{\partial \mathrm{u_y}}{\partial \mathrm{x}}, \\ l_z = \dfrac{\partial \mathrm{u_z}}{\partial \mathrm{x}}; \end{cases} \tag{11}$$

where the derivatives are taken at the point (x_0, y_0).

Next, we assume that the orth of the z_l axis coincides with the normal to the surface (1) at the point O. Then:

$$\begin{cases} k_l = \dfrac{l}{|l|}, \\ l_x = -\dfrac{\partial \mathrm{z}}{\partial \mathrm{x}} = -\dfrac{x}{\rho}\dfrac{\partial \mathrm{z}}{\partial \rho}, \\ l_y = -\dfrac{\partial \mathrm{z}}{\partial \mathrm{y}} = -\dfrac{y}{\rho}\dfrac{\partial \mathrm{z}}{\partial \rho}, \\ l_z = 1; \end{cases} \tag{12}$$

Finally, the orth of the y_l axis is simply $j_l = [k_l, i_l]$ -as in any right coordinate system.
For an undeformed element, the coordinates of the end of A should be:

$$R_A{}^0 = R_0 + i_l \cdot s + k_l \cdot h \tag{13}$$

where R_0 is the radius vector of the point O.
In fact, R_A has the form:

$$R_A = R_B + k_l \cdot h \tag{14}$$

where R_B is the radius vector of point B; k_l-normal to the surface (1) at point B.
From (13) and (14) the displacement vector is equal to:

$$\begin{cases} \Delta = R_A - R_A^O, \\ D_{1,2,3} = (\Delta \cdot (i_l, j_l, k_l)); \end{cases} \tag{15}$$

At small angles of rotation of the end (in radians) the following relations are also valid:

$$\begin{cases} D_4 = \varphi_x = -\left|k_l^i - k_l\right| \cdot sign\left[\ (\ k_l^i - k_l\ ) \cdot j_l\ \right], \\ D_5 = \varphi_y = -l_{Zl}, \\ D_6 = \varphi_z = l_{Yl}, \end{cases} \tag{16}$$

where l_{Zl}, l_{Yl}-components of the unit vector tangent to the element at point A in the local coordinate system. They can be found by (12), taking the derivatives at point B.

As follows, it was possible to express Di through u (x, y). When implementing the method, derivatives of u (x, y) should (of course, approximately) be expressed in u_{ij}. So, for example, for node (i, j) we will have:

$$\frac{\partial \mathrm{u_x}}{\partial \mathrm{y}} \approx \frac{u^x_{i+1,j} - u^x_{i-1,j}}{2s} \tag{17}$$

This D_i is expressed in u_{ij}, which is necessary.

Energy minimization

According to the method of solving the problem, the minimization of energy is planned to be carried out numerically as a minimization of the function of many variables. In this case, the energy depends on the changes u_{ij}. First we write the zero approximation [6]:

$$\begin{cases} u^x_{ij} = 0, \\ u^y_{ij} = 0, \\ u^z_{ij} = f(s\sqrt{(i-1)^2 + (j-1)^2}); \end{cases} \tag{18}$$

Then, as described above, we find the grid energy E (u_{ij}) i partial derived from energy by

$$a^m_{ij} = \frac{\partial \mathrm{E}}{\partial \mathrm{u^m_{ij}}}$$

To reduce the energy, we take the following approximations u_{ij} in the form:

$$(u^m_{ij})_{k+1} = (u^m_{ij})_k - \lambda \cdot (u^m_{ij})_k \tag{19}$$

where k is the approximation number;
λ-some constant became > 0;
The change in energy is expressed by the formula:

$$dE_k = \sum_{i,j,m} \frac{\partial \mathrm{E}}{\partial \mathrm{u^m_{i,j}}}\, du^m_{i,j} = -\lambda \sum_{i,j,m} (a^m_{i,j})^2_k \tag{20}$$

As can be seen from (20), $dE_k <= 0$, and $dE_k = 0$ only if all derivatives of E are equal to 0.

The value of λ should be chosen experimentally. By subtracting the derivatives at some point in the space u_{ij}, we can improve the solution for using (19) until E_k begins to grow. Then you should find the derivatives at a new point and continue the process. There is a way to find directly, but it requires numerical calculation of derivatives from E to u_{ij}. It is planned to implement it in the future.

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