THEORETICAL AND EXPERIMENTAL ASPECTS OF OPTIMAL DESIGNING OF DYNAMIC VIBRATION ABSORBERS – ROTATING MACHINES SYSTEM

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Abstract: Significant issue in the development of modern machines is the reduction of vibration. Dynamic vibration absorbers are widely used to reduce vibration and noise levels in vehicle cabs, to reduce vibration of rotating machines, to reduce vibration amplitudes of various towers and structures and the like. Along with dynamic vibration absorbers tuned to the resonant frequency of the main design, DVAs used to reduce vibration in a given narrow frequency range are widely applied. These are, for example, turbo-generators, gas compressor units, pumps and other rotating machines with a standard speed over a period of exploitation time. Vibration in such machines is narrow-frequency and is caused by imbalances of rotating elements.

Keywords: *dynamic vibration absorbers, rotating machines, frequency range*

1. Introduction

The design of the DVA was probably first proposed in [1] in 1911 by Fram. Tanks to reduce ship oscillations are named after him. However, the first mentioning of the use of DVA to reduce ship oscillations is found in [2]. The theory of DVA was first proposed in [3] (Ormondroyd & den Hartog (1928)). In [4] (Den Hartog (1985)) optimal values for frequency and damping in DVA were determined. The most well-known DVA equations are in [4] (den Hartog) and [5] (Timoshenko). A large amount of DVAs research is described in [6]. Here the optimal values for variable amplitude are found. The calculation of DVAs in pulsed and random perturbation is considered. Some constructive forms of DVAs are considered.

In addition to dynamic vibration dampers, various devices for absorbing energy of oscillations and shocks are widely used in the construction of devices and machines. The book [7] shows various devices for energy dissipation in structures under seismic loading. These are various types of dampers: dampers with viscous, dry friction, liquid-type dampers. DVAs of such different types are considered, too. Damping is considered on the basis of both a linear viscous model and hysteresis for nonlinear oscillations. Damping is not a simple process. Some basics can be found in [8–11].

Optimization of a DVA for a non-damped single-mass basic system under influence of harmonic excitation belongs to standard problems. A detailed review of the methods for calculating DVA was performed in papers [1–3]. Considerable attention was paid to the calculation and optimization of DVAs when interacting with the rotor [12–16]. Such studies are conducted in “time space” as opposed to studies in the “frequency domain”. Most practical applications of DVAs are based on insufficiently complete mathematical models of complex structures and inefficient designing of DVAs. It does not take into account elastic properties of the structure itself, elastic properties of the node joining DVA to the main structure, and characteristics of the attached elements.

Also significant is the study of DVAs efficacy beyond the main system’s own resonances. After all, the design of the DVA is often required to be effective for some fundamental external disturbance frequency, such as in pumps, turbines, motors etc. The shock mass model was used to correct both the DVAs masses and damping in their joints. Rigidity of the elastic plate elements was corrected based on both the refined determination of the elastic and damping properties of the plate and the refined calculation of the elastic plate clamping parameters. An alternative way to determine the model parameters is to experiment [16–18].

2. Rotating machines with DVA on the example of a pump. Identification

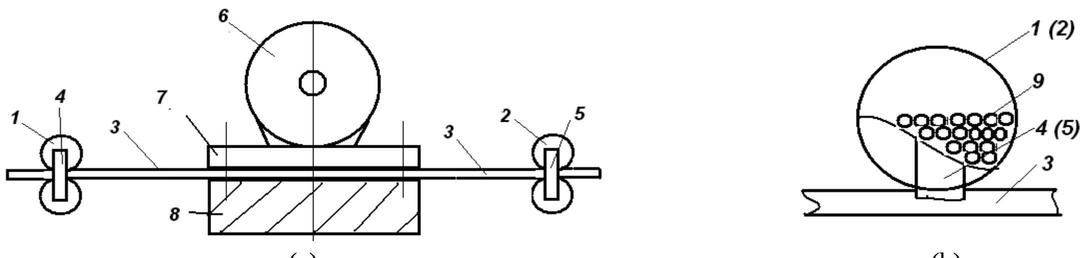
For accurate calculation of the design, it is necessary to develop a detailed theoretical model of the oscillating pump – DVA system and to determine factual parameters of this model. This requires a series of specific experimental studies that allow identification of the required number of parameters and determine their factual values. In this case, a number of model parameters are specified a priori (masses, geometric dimensions), but such parameters as elastic characteristics and damping require additional study.

To determine optimal parameters of a DVA it is necessary to determine its dynamic characteristics depending on the design parameters as well as investigate appropriate characteristics of the pump housing at the points of connection of the DVA. According to the simplified calculation scheme developed in the section, it is necessary to determine a number of model parameters that reflect both properties of the DVA and properties of the pump on the foundation: stiffness coefficients, damping coefficients, parameters of nonlinear stiffness and nonlinear friction of granular mass in the containers.

Let us consider the scheme of a DVA inertial masses of which are made in the form of containers filled with lead balls. Fig. 1 shows a general view of an adjustable wide-frequency dynamic vibration absorber of a rotor machine developed in the course of scientific research.

The following designations are used in Fig. 1: 1 and 2 are vibration absorbing elements (containers filled with damping material); 3 is an elastic plate element; 4 and 5 are connecting devices of the DVA vibration absorbing elements; 6 is a rotary machine; 7 is a support platform; 8 is a foundation; 9 are lead balls freely inserted into containers 1 and 2.

The principle of operation of the dynamic vibration absorber of the rotary machine is as follows: vibration from the rotary machine 6 (see Fig. 1) is transmitted to the vibration absorbing elements 1, 2. Each of them begins absorbing vibrational energy of the system independently in a certain frequency range. By adjusting the distance of the vibration absorbing elements 1, 2 with respect to the axis of the rotary machine 6 by means of the couplings 4, 5 we adjust each of the vibration absorbing elements 1, 2 to the main resonant frequency of the rotary machine.



(a) (b)

Fig. 1 Adjustable wide-frequency DVA of a rotary machine: (a) front view, (b) element of DVA with particles

Changing the number and weight of lead balls 9 freely mounted in the hollow containers of the vibration absorbing elements 1, 2, they provide the corresponding values of damping in the vicinity of a given operating frequency. In this way, operational properties of the integral elastic-damping system are changed and the tuning parameters of the DVA vibration absorption are precisely adjusted, which allows you to solve two main tasks: 1) to provide highly efficient absorption of energy of mechanical vibrations of the system; 2) to prevent occurrence of resonant oscillations in the area of the calculated frequencies which are selected in the vicinity of the frequency of the most intense oscillations of the rotor machine.

The DVA described above has several advantages lying in that the DVA provides adjustable wide-frequency vibration absorption, does not create parasitic resonance perturbations around the operating frequency, and is characterized by improved performance.

The DVA having been developed was adopted as the basic model in the process of analytical and experimental studies of the working processes of fire pumps equipped with DVA.

The calculation model was described by the following system of differential equations:

$$m_1\ddot{\omega}_0+\left(k_1D_K+k_AD_A+k_{A2}D_{A2}\right)\dot{\omega}_0+\left(k_1+k_A+k_{A2}\right)\omega_o-k_AD_A\omega_A-k_A\dot{\omega}_a-k_{A2}\omega_{A2}=F\ ;$$

$$m_A\ddot{\omega}_A+k_AD_A\dot{\omega}_A+k_A\omega_A-k_AD_A\dot{\omega}_0-k_A\omega_0=0\ ; \qquad (1)$$

$$m_{A2}\ddot{\omega}_{A2}+k_{A2}D_{A2}\dot{\omega}_{A2}+k_{A2}\omega_{A2}-k_{A2}D_{A2}\dot{\omega}_0-k_{A2}\omega_0=0\ .$$

where, m_1, m_A, and m_{A2} are masses, respectively, of the basic structure, the first and second DVA; k_1, k_{A1}, and k_{A2} are appropriate stiffness values; D_K, D_A, and D_{A2} are viscosity coefficients; ω_0, ω_A, and ω_{A2} are displacements; *F* is harmonic perturbation.

To simulate the motion of particles (crumbs or lead balls) of the filling of the containers, a shock mass model was used [12, 16], see Fig. 2).

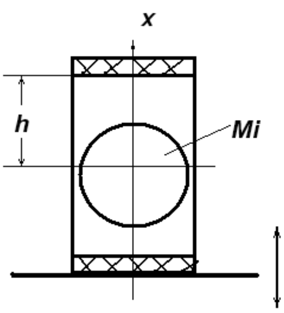


Fig. 2 DVA with shock mass *Mi*

The dynamic equilibrium equations for the mass *Mi* are as follows:

$$m_i \ddot{\omega}_i + m_i g + k_G(x)(\omega_i - \omega_0) + C_G(x)(\dot{\omega}_i - \dot{\omega}_0) = 0\,,\ |\omega_i - \omega_0| > |h - R|\,,$$

$$m_i \ddot{\omega}_i + m_i g = 0\,,\ |\omega_i - \omega_0| \le |h - R|\,, \tag{2}$$

where m_i is mass of the filler; k_G is stiffness of elastic gaskets; C_G are coefficients of viscous damping of gaskets; ω_i are displacements of equivalent mass; ω_0 is displacement of the container base, R is radius of mass m_i.

Analysis having been conducted confirmed that the experimentally derived characteristics of the processes of oscillation damping were similar to the results derived theoretically.

3. Multi-criteria identification of the main design – DVA system parameters

To determine appropriate parameters and characteristics of the DVA design scheme of which is shown in Fig. 3 it is necessary to perform parametric analysis of its properties.

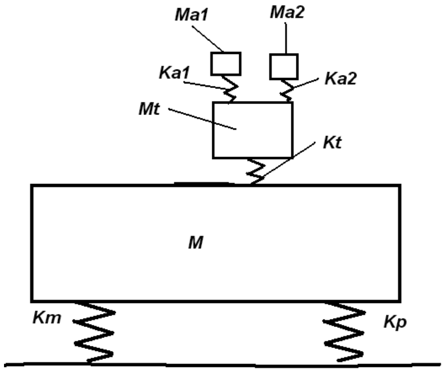


Fig. 3 Design scheme of the DVA under investigation

According to Fig. 3 we select the following for the DVA parameters:

1) Mass of the first DVA – M_{A1};

2) Mass of the second DVA – M_{A2};

3) Distance from the clamping point (shoulder) of the first DVA – L_{A1};

4) Distance from the clamping point (shoulder) of the second DVA – L_{A2};

5) Damping in the first DVA – D_{A1};

6) Damping in the second DVA – D_{A2}.

Stiffness magnitudes of the elastic elements of the DVA will be (without taking into account the compression force) as follows [5]:

$$K_{A1} = \frac{3EI}{L_{A1}^3}\,,\ K_{A2} = \frac{3EI}{L_{A2}^3}\,. \tag{3}$$

We also take into account inertial properties of the elastic element. We use the method given in [5]. We consider that the first form of oscillation of a beam with mass is approximated fairly accurately by the line of deflection of this beam. We obtain a modification of the formula given in [5]:

$$M_{D1} = M_{A1} + M_B \frac{33}{140}\,, \tag{4}$$

where M_B is mass of the beam; M_D is the equivalent mass centered at the end of the beam.

In our case, we obtain equivalent masses concentrated at the locations of the masses of DVA:

$$M_{D1} = M_{A1} + M_B \frac{33\chi_1}{140}\left[1 + 3\left(\frac{1}{\chi_1} - 1 + \left(1 - \frac{1}{\chi_1}\right)^2\right)\right], \tag{5}$$

$$M_{D2} = M_{A2} + M_B \frac{33\chi_2}{140}\left[1 + 3\left(\frac{1}{\chi_2} - 1 + \left(1 - \frac{1}{\chi_2}\right)^2\right)\right], \tag{6}$$

where $\chi_1 = \frac{L_{A1}}{L}$, $\chi_2 = \frac{L_{A2}}{L}$, L is the length of DVA beam.

Instead of the masses of the DVA we consider reduced masses concentrated at the same points. This allows you not to consider additional equations of dynamic equilibrium of massive elastic elements (beams).

Fig. 4 shows a block diagram of a measuring and recording hardware complex, which shows the following designations: 1 is a mounting unit of a DVA; 2 is a DVA; 3 are lead balls; 4 is a container; 5 is an electric motor; 6 are screws; 7 is a vibrating platform.

In further studies we will use the first method of determining of damping based on the calculation of the monotonic decrease in the amplitude of oscillations at some initial perturbation of two types: kinematic and shock. Let us consider first kinematic perturbation which was carried out according to the layout shown in Fig. 5 and used to determine the elastically-damping properties of the plate DVA (Fig. 5).

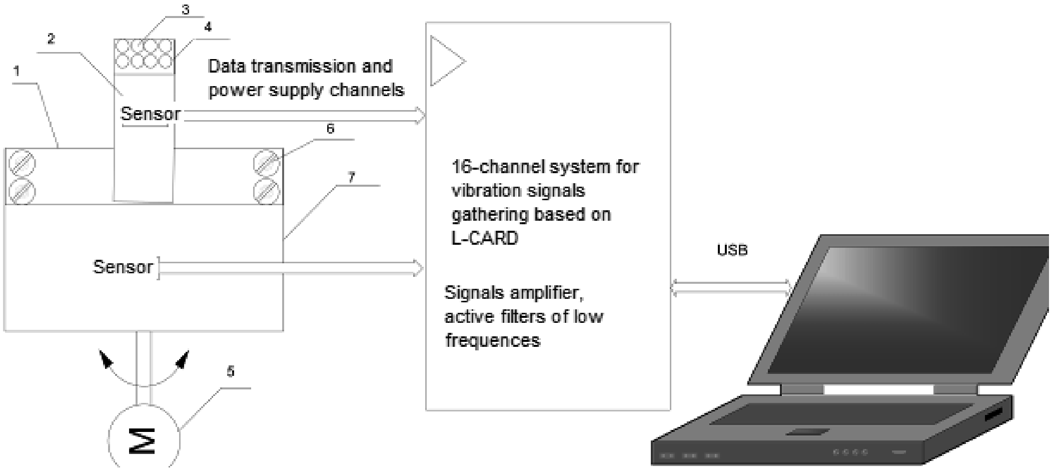


Fig. 4 Layout of measuring and recording hardware complex

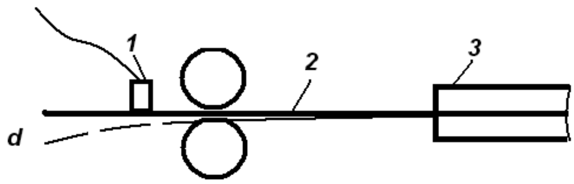


Fig. 5 Setup of the experimental bench used for the studying of kinematic perturbation of DVAs

In Fig. 5 the following designations are accepted: 1 is a vibration sensor, 2 is a DVA; 3 is a clamping; d is the set initial kinematic deviation of the elastic plate 2 of the DVA.

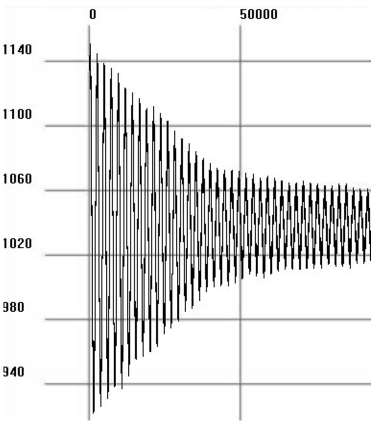
In Fig. 6 shows view of the plate DVA under investigation (equipped with vibration sensor).

Fig. 6 Appearance of the plate DVA under study

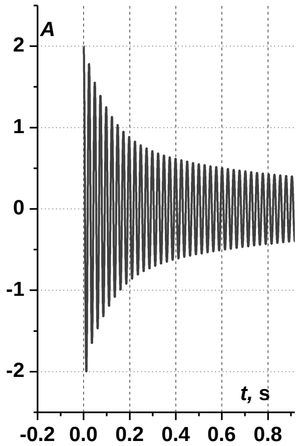
In the first stage of the oscillation process the amplitude of oscillations sharply decreases. This is caused by considerable dissipation of energy due to interaction of the lead balls moving the container with its walls. Further, as the amplitude of oscillations of the bulk decreases, when the acceleration of the container becomes less than gravitational, the balls fall to the bottom of the container and function only as additional mass without creating effect of internal intergranular friction.

Fig. 7 shows the oscillogram of oscillations (experimental and theoretical) of the platform in the presence of filling (lead balls) in containers of the DVA.

The total weight of lead balls was 0.1 kg and the diameter of each ball was 3 mm. Even with such a small (1/15) ratio of the masses of the DVA and the filler, a significant decrease in the amplitude of the DVA oscillations was achieved initially (as long as the DVA acceleration exceeded the gravity acceleration and the balls were movable and provided internal friction). Fig. 7b shows results of the analytical calculations.



(a)



(b)

Fig. 7 Experimental (a) and theoretical (b) oscillograms of the DVA oscillations with filling

In the course of the experiment, such parameters as C_i, k_G, C_G were set in the shock mass equation to maximize the coincidence of theoretical and experimental results. However, it is quite difficult to study the complete system (1), especially in the wide frequency spectrum. Therefore, only system (2) with modified parameters was considered. The damping coefficients based on the experimental results were set as follows: $D_{A2} = 0.0001$ for the filled container; $D_{A2} = 0.00033$ for the empty container.

4. Determination of the dynamic characteristics of the pump housing

The layout of the experiment is shown in Fig. 8. Impact excitation was used to investigate dynamic characteristics of an oscillating engine – pump system. Fig. 9 shows experimental and theoretical vibrorecords.

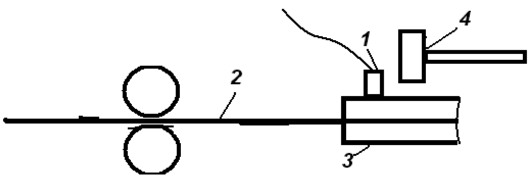
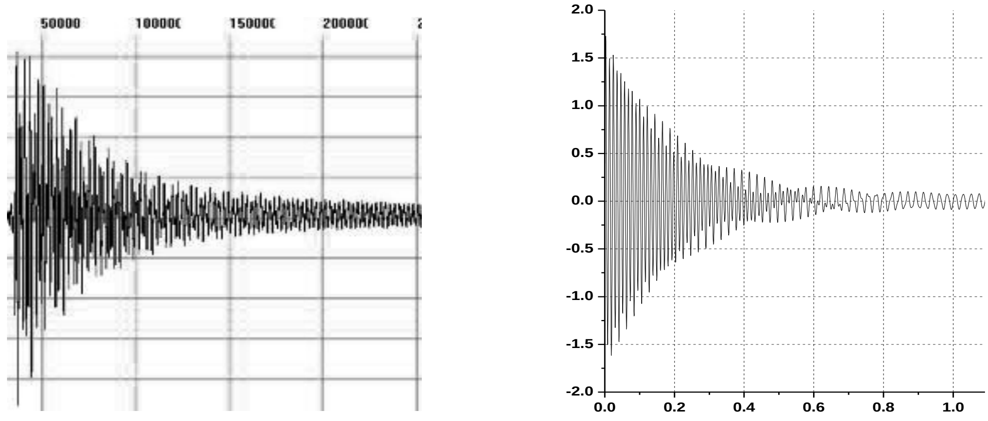


Fig. 8 Layout of the experiment for determining parameters of the oscillatory system in shock perturbation: 1 is a vibration sensor; 2 is a DVA; 3 is a clamping; 4 is an impact device



(a) (b)

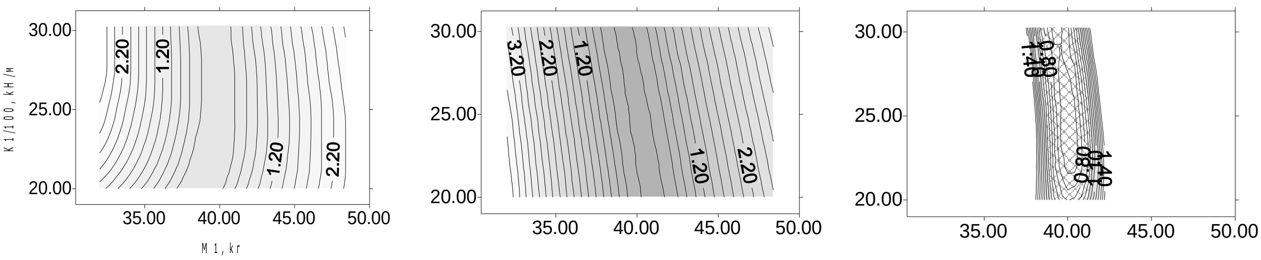
Fig. 9 Experimental oscillogram of oscillations being dampened at pulse perturbation of the pump base (a), theoretical waveform oscillogram of oscillations being dampened at pulse perturbation of the pump base (b)

For further research, it is advisable to consider only the initial stage, since oscillations of the operating DVAs with high frequency and damping of oscillations of the pump housing are observed after that. Comparing the oscillograms the natural oscillation frequency of the pump housing can be estimated as (70/36) × 50 = 90 Hz. The damping is quite high and equals to $D_M = 0.0002$.

Identification of pump parameters based on design schemes. For more precise determination of the model parameters, a number of additional experiments were carried out (concerning determination of the parameters of m_1 and k_1 i.e. mass and stiffness of the main system). At the same time, parameters of DVA, in particular m_A and k_A should have been determined. Although they could be calculated more accurately than the parameters of the main system, it still required considerable effort both to determine elastic properties of the DVA itself and elastic properties of the clamping of the DVA plate. Although a detailed theoretical analysis can be made here [19], however, on the basis of a series of simple experiments it is possible to quite accurately define these parameters as complex quantities included in the system of equations (1) and (2). First, let's find out correctness of these schemes. Fig. 10 shows charts of frequency deviation from the centered values depending on changing of m_{1i} and k_{1i} parameters:

$$R = \left| f(m_{10}, k_{10}) - f(m_{1i}, k_{1i}) \right|,$$

$$m_{1i} = m_{10}(i - N/2),\ i = 1,...N, \quad k_{1i} = k_{10}(i - N/2),\ i = 1,...N\ . \qquad (7)$$



(a) (b) (c)

Fig. 10 Charts of deviations of frequency of oscillations of the DVA from the centered values depending on changing of appropriate parameters, for the following mass of the DVA: $m_A = 1.5$ kg (a) and $m_A = 3.0$ kg (b), and intersection of these charts (c)

Fig. 10 shows that each chart in particular does not define these parameters uniquely, but if we take the sum of these charts $R_\Sigma = R_1 + R_2$ (Fig. 10), this point will be determined unequivocally.

Table 1

Natural oscillation frequencies of a DVA for its different masses

M, kg	0	0.669	1.100	1.521	3.115
f, Hz	69	48	36	32.3	24.4

To determine all the parameters of k_1, m_1, m_A, and k_A we apply the method of genetic minimization for the objective function $F_{c=}\sum_i \left| f_T\left(M_i\right) - f_e\left(M_i\right) \right|$, where $f_T\left(M_i\right) = f_T\left(M_i, k_1, m_A, k_A\right)$ are theoretically obtained values of natural frequency, and $f_e\left(M_i\right)$ are experimental values.

Influence of mass is difficult to track due to the complexity of the pump design, but oscillation frequency can be observed on vibrorecords on fig. 7 in case of shock perturbation. We see that it is in the region of 65 Hz (as theoretically determined). That is, the principal construction's natural frequency is higher than the operating frequency of 50 Hz. This gives information in which neighborhood of DVAs natural frequencies to look for the optimum vibration absorption at the operating frequency.

5. Optimization of DVA at actual parameters of the oscillatory system

Based on the theoretical and experimental analysis the parameters of the basic design were determined: m_1 = 35 kg, f_1 = 65.5 Hz. Let's optimize a DVA at these parameters. Fig. 11 shows ~~the~~ results of optimization at different frequency ranges: *47 Hz < f < 50.5 Hz* is optimization for a wide range of frequencies; *48.5 Hz < f < 50.5 Hz* is optimization in a narrower range.

As shown above, the main parameter of the DVA optimization is its own oscillation frequency within the structure.

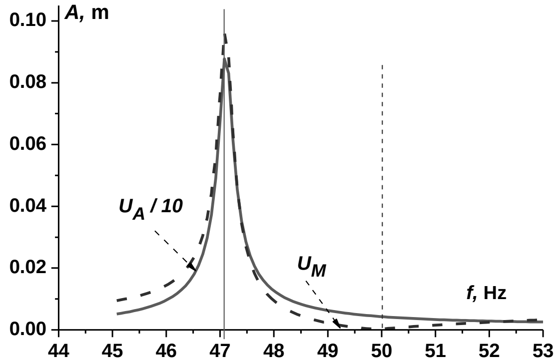


Fig. 11 Frequency response of the main design at different frequency ranges

Experimental verification of the optimality of the DVA system. Based on the the theoretical and theoretical-experimental studies having been conducted, optimal parameters of the DVA system were determined. ~~In~~ Fig. 8.12 shows the vibration diagrams of the basic design at operating frequency $f_R \approx 50$ Hz. Measured deviations from the operating frequency were in the range of 0.1 to 0.15 %. The following algorithm was applied: the mass of the DVA was moved along the plate with some fixed step (1 cm). Based on the kinematic perturbation scheme, the natural frequency of DVA was measured. Then measurements were made on the basis of the pump.

Weight of the DVA was 1.881 kg.

As we can see , at a frequency close to the theoretically determined optimum (47Hz, Fig. 12), the oscillation amplitude of the basic structure decreases by an order of magnitude.

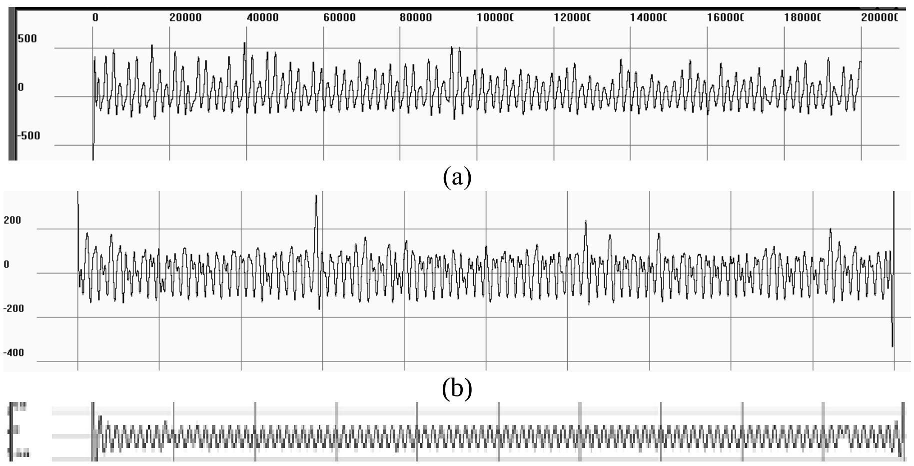


Fig. 12 Vibrorecords of the basic design at different natural frequencies of DVA:
(a) – $f_A = 46Hz$; (b) – $f_A = 46.5Hz$; (c) – $f_A = 47Hz$

Thus, taking into account appropriate characteristics of a rotating machine and ~~a~~ sufficiently large numerical experiment it is possible to select optimal characteristics of a DVA. For the majority of low-power pumps of fire-fighting systems the dominant factor is vibration absorption in the operating frequency range. Frequency response of the optimum DVA in this case will deviate significantly to the left (item 2), too, from the operating frequency of the main structure (oscillating system of motor – pump) which is 50 Hz.

6. DVA strength

Important criteria for DVA designing are its durability and compactness. Strength of the vast majority of structures is determined by strength of their joints. In addition, connections often serve as stress concentrators. In the case of DVA in the form of mass on a console maximum stresses should be expected in the clamping beam-plate which is an elastic element of a DVA.

The influence of different parameters of DVA on its effectiveness was studied above. If the stiffness parameters influence very significantly, but the damping parameters are much less influential. At the same time, one should expect ~~a~~ significant effect of the damping properties of DVAs on the maximum amplitude of their oscillations, and thus on their strength. The maximum strength in the elastic element of the DVA, which is the plate element, will be as follows:

$$\sigma_{MAX} = \frac{M}{W} = \frac{\omega^2 A M_A L_A z_{MAX}}{EI}. \quad (8)$$

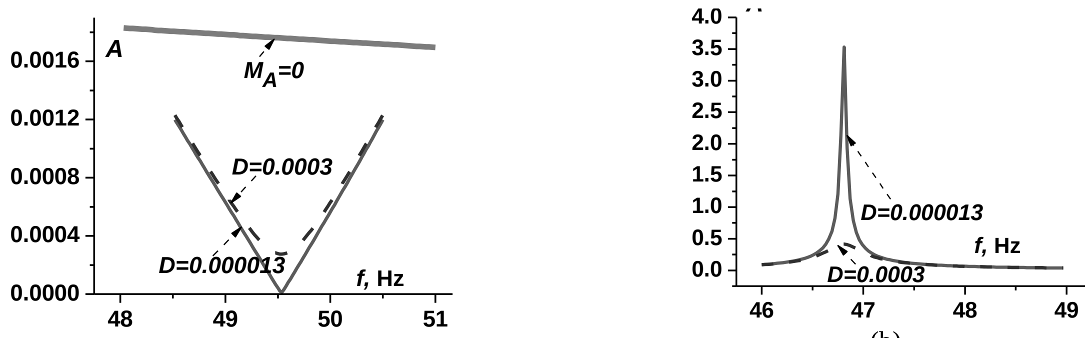
Here M is the moment, W is the moment of section resistance, ω is the circle frequency $(\omega = 2\pi \times 50)$, A is the oscillation amplitude, M_A is the mass of DVA, L_A is the distance of mass from clamping, z_{MAX} is maximum deviation of section of plate from midline (in our case half thickness of plate), EI is the moment of bending resistance of the cross section. If we substitute in (8) $z_{MAX} = \frac{h}{2}$ and $I = \frac{bh^3}{12}$, then we obtain

$$\sigma_{MAX} = \frac{\omega^2 A M_A L_A}{6bh^2 E}. \quad (9)$$

This value shall be less than the allowable multi-cycle allowable strength

$$\sigma_{MAX} < [\sigma]^{-1}, \quad (10)$$

which for the 65G spring steel is approximately equal to 240 MPa. All geometrical parameters of a DVA spring are regulated both by its frequency characteristics and by design requirements. The only independent adjustable parameter is the oscillation amplitude *A* of the DVA mass. Fig. 13a shows frequency response of the basic structure with low (optimized) damping and some larger damping. Fig. 13b shows corresponding frequency response of a DVA (one DVA is considered).



(a) (b)

Fig. 13 Frequency response of the basic structure (a) at low (optimized) damping and some greater damping and frequency response of DVA (b)

It can be noticed that with a slight deterioration of the vibration-absorbing properties of a DVA (only in some narrow frequency range, which will not necessarily correspond exactly to the operating frequency) the amplitude of the DVA vibration is reduced by an order of magnitude. That is, the risk of DVA destruction is significantly reduced. In our DVA design this was achieved by using containers filled with lead balls instead of solid masses of DVA. Large damping in this type of DVA design does not lead to the destruction of the elastic element in critical cases, when the operating frequency approaches the natural frequency of the DVA or when the transient process of the pump acceleration is slow enough and the DVA manages to gain large oscillation amplitudes in the pre-working zone.

7. Conclusions

A theoretical and experimental complex for determining characteristics of both DVA and the basic structure are shown. Two dynamic methods were used to determine dynamic characteristics experimentally: some initial kinematic deviation for the DVA and some initial shock excitation for the basic structure were specified. A sufficiently large damping was shown in the construction of DVA with containers filled with lead beads. There is also a large enough damping of the pump body itself on the foundation. Based on the comparison of experimental data with the theoretical ones specific parameters of the mathematical model were determined.

A method of determining dynamic characteristics of the basic structure at the point of attachment of the DVA was developed. Here, the same DVAs, but with different mass parameters, were used as a test models. This allowed us to determine uniquely dynamic characteristics of the basic structure. It should be noted that in our system the natural frequency of the structure was higher than the frequency of perturbation. This result was obtained in two ways: by direct analysis of vibrorecords during shock perturbation and Based on the multi-mass method developed. Thus, this section determines dynamic parameters of the DVA and the pump housing which enables optimal designing.

Based on the preliminary theoretical and experimental analysis (clause 2.3) theoretical and experimental methods were developed to reduce vibration of the pump due to optimally designed DVA. The following optimization algorithm was proposed:

1) Theoretical and experimental determination of the parameters of both the basic design and DVA.

2) Vibrorecords of the basic structure were derived by changing the DVA frequency in some predetermined theoretically determined frequency range.

3) Optimum ones shall be received from the vibrorecords, be selected.

It was shown that the optimum values of DVA frequencies are not necessarily the same or even close to the operating frequency. The optimal value of the DVA frequency is close to the frequency of f_A = 47 Hz; the optimized DVA reduced the oscillation amplitude of the pump at an operating frequency of 50 Hz.

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