COMPUTER MODELING OF THE STRESS-STRAIN STATE OF THIN-WALLED TUBULAR STRUCTURAL ELEMENTS FOR PREDICTING THE LIMITING STATE

Halyna Kozbur[1], Oleh Shkodzinsky[2], Ihor Kozbur[3], Gashchyn Nadiia[4]

[1]*Ternopil National Ivan Puluj Technical University, Rus'ka str. 56, 46001, Ternopil, Ukraine, kozbur.galina@gmail.com*

[2]*Ternopil National Ivan Puluj Technical University, Rus'ka str. 56, 46001, Ternopil, Ukraine, shkod@tntu.edu.ua*

[3]*Ternopil National Ivan Puluj Technical University, Rus'ka str. 56, 46001, Ternopil, Ukraine, kozbur.igor@tntu.edu.ua*

[4]*Ternopil National Ivan Puluj Technical University, Rus'ka str. 56, 46001, Ternopil, Ukraine, gashchyn_n@tntu.edu.ua*

Abstract: If a thin-walled pipe loaded with internal pressure and axial tension allows the appearance of plastic strains, then the uniform plastic stability loss with the emergence of a local plastic deformation zone is considered the limit state, the corresponding stresses are considered as the limit ones. Correct prediction of the stress-strain state at the moment of strain localization requires taking into account the actual size of the loaded pipe and the calculation of true stresses. The paper proposes the implementation of the methodology of predicting the limit values of true stresses in the pipe at different ratios of internal pressure and axial tension values through the development of an algorithm for its computer modelling. Unlike existing, the methodology takes into account the physical and mechanical properties of the material, the type of stress state and the change in the actual dimensions of the loaded pipe. The algorithm is based on analytical dependences, established by the authors.

For two grades of steels (carbon steel 45 and alloy steel 10MnH2MoV), an increase in the calculated strength threshold is shown with an insignificant additional load of a pipe loaded with pressure and axial tension. Analysis of the numerical results showed that it is possible to establish a balance between the actual geometry of the element and the load, which will solve the problem of finding the optimal ratio of «weight-strength», important for practical applications in aircraft, rocket and mechanical engineering. The developed computing modelling algorithm for finding the limit values of actual stresses makes it possible to calculate a realistic safety factor and make improved engineering solutions at the design and operation stages of structural elements; to increase the efficiency and safety of using pipeline and shell-type saving systems.

Keywords: *computer modelling, large plastic strains, true stresses, the uniform plastic stability loss, complex stress state, localization of strains*

1. Introduction

The accurate prediction of maximum load (burst pressure, fracture pressure) is critical for the engineering design, assessment of integrity and on-line control of oil- and gas pipeline and other vessels under internal pressure [1]. Thus, computer modelling of strain-stress behavior of tubes and containers under combined load conditions is quite important.

As it is shown in [2 – 6], the influence of axial tension on loading capacity of main pipelines is quite essential. In papers [7, 8] some experimental research and theoretical models to determine the stress state impact on the pipes loading capacity has been proposed. Based on the Mises criterion, Updike and Kalnins [9] developed a general mathematical model for stress strain and internal pressure limit values forecasting for axially symmetric thin wall vessels. It was found that calculated fracture pressure causing the local plastic strains has resulted in higher values compared to the experimental ones.

A great number of analytical and empirical equations are known to be used for pipelines and vessels limit pressure predicting. Nevertheless, it is very difficult to calculate the exact value of fracture pressure and to estimate the accurateness of these equations. The problem of determining the real ultimate stress and ultimate strength factors values hasn't been studied properly. Most studies haven't taken into account the material strengthening and the change in actual dimensions of structural elements under load. Nominal stress is assumed lower the liquidity limit at design of heavy loaded shell-type structures like reactors bodies, collector bodies, steam generators and their elements. Though, some plastic deformation is quite allowable for the elements operating under intense short-term single load conditions. At a certain ultimate level of stress the structural element either loses its plastic resistance or undergoes plastic deformation localization. In both cases the structural element loses its further functionality dealing with shape and size change. Thus, one should be able to calculate real stress taking into account these changes to predict stress-strain state of the element at the moment of the uniform plastic stability loss as accurate as possible. In case when plastic deformation is allowed to be occurred the correct assessment of limit stress is especially important preceding the moment of the uniform plastic stability loss resulting in local deformation area appearance and further ductile fracture of the structural element. The analytical review of literary sources has shown that the problem of predicting the real stress values in loaded structural elements at the moment of plastic strains localization hasn't been studied properly. The conventional existing models and methods do not take into account the material properties, a type of stress state and a change of actual size of loaded structural elements in the complex restricting the areas of their use. At the same time, the continuous increase of cost-efficiency, structure efficiency and performance safety requirements has made the task of improvement of analytical-calculation approach to ultimate states of metal materials forecasting under complex stress state very important and quite urgent.

The paper purpose is to introduce a new methodology and computer modelling algorithm for predicting the ultimate states for thin-walled pipes loaded with internal pressure in order to analyze the influence of physical-mechanical properties of the material, pipe geometry and a type of stress condition on the values of actual stress and real force factors at the moment of local plastic strains formation.

2. Problem setting

If a structural element under loading allows some plastic strains occurred then the correct estimation of ultimate stresses at the moment of the uniform plastic stability loss resulting in local strain area appearance and further ductile fracture of the structural element will be necessary. When it comes to the area of large plastic strains, the change of the element size will be important. That is why it will be worth saying about the real strength limit which prediction requires taking into account the actual size of loaded element and true stress-strain curve building. The calculation of ultimate values of true stresses at the moment of strains localization under uniaxial strain conditions is based on the use of Swift-Marceniak criterion and analytical dependence $\tilde{\sigma} = d\tilde{\sigma}/d\varepsilon$ between true stresses and tangent modulus in true stresses use. In the papers [10, 11] the criterion of the uniform plastic stability loss was offered to use for ultimate pressure prediction in thin-walled pipes. Analytically the moment of the uniform plastic stability loss in this case is described by the formula $\tilde{\sigma} = \frac{1}{2} d\tilde{\sigma}/d\varepsilon$ containing a correcting multiplier ½ at the tangent modulus $d\tilde{\sigma}/d\varepsilon$. The authors explained the appearance of a correcting multiplier by the loaded element geometry impact.

The algorithm implementation to find the ultimate values of true stresses occurring at the moment of the uniform plastic stability loss in the pipe loaded simultaneously by internal pressure *q* and strain *N* has been proposed in the study under discussion. The main analytical dependencies of the method under discussion have been obtained in [12]. The following equation was obtained due to the development of summarized methods of true stresses prediction in the pipe at the beginning of localization of uniform plastic strains [13]:

$$\tilde{\sigma} = \mu \cdot d\tilde{\sigma}/d\varepsilon \tag{1}$$

where μ is a correcting multiplier (factor) taking into account the physical-mechanical properties of the material, stress state type and element geometry as a whole set. The limit values of equivalent strains and real equivalent stresses in the pipe walls at the moment of strain localization are found as coordinates of the point of real deformation curve $\tilde{\sigma}(\varepsilon)$ and the curve of the right–hand side of the formula (1) intersection.

3. The model of plastic deformation of thin-walled tubular elements

For computer modeling of the true ultimate strength of the metal of thin-walled cylindrical pipes, two structural materials were considered:

– structure carbon-based high-quality steel 45 [14];
– heat resistant alloy steel 10MnH2MoV [15].

The load was considered simple, so $\sigma_z = k\sigma_\theta$ for different values *k*.

The technique which allows predicting the true stresses at the moment when local plastic strains appeared has been described in [16]. The technique has been used for thin-walled pipe of wall thickness *h*=2,5 mm and external diameter *d*=65 mm made of steel 45. The results of the experiment under discussion have been given as hoop and axial stresses and strains – $(\varepsilon_\theta)_i$, $(\sigma_\theta)_i$ $(\varepsilon_z)_i$, $(\sigma_z)_i$ respectively, for *k*=0; 0,5; 1; 2. The points have been chosen for which the condition was satisfied $\min\left((\varepsilon_z)_i, (\varepsilon_\theta)_i\right) \ge 1\%$ ($i = \overline{1, n}$, where *n* = 30 is a number of observations).

From the set of values $(\varepsilon_\theta)_i$, $(\sigma_\theta)_i$, $(\varepsilon_z)_i$, $(\sigma_z)_i$ equivalent stresses and strains were found by formulas:

$$\left(\sigma_{eq}\right)_i = \frac{p}{2}\left[\frac{\left|(\sigma_z)_i - (\sigma_\theta)_i\right|^p + \left|(\sigma_\theta)_i\right|^p + \left|(\sigma_z)_i\right|^p}{2}\right]^{\frac{1}{p}}, \tag{2}$$

$$\left(\varepsilon_{eq}\right)_i = \frac{p}{2(p+1)}\left[\frac{\left|(\varepsilon_z)_i - (\varepsilon_\theta)_i\right|^p + \left|(\varepsilon_\theta)_i - (\varepsilon_r)_i\right|^p + \left|(\varepsilon_z)_i - (\varepsilon_r)_i\right|^p}{0,5}\right]^{\frac{1}{p}}. \tag{3}$$

Radial stresses were neglected taking into account the assumptions of the theory of thin-walled shells, i.e. $(\sigma_r)_i = 0$. Under simple loading the following relation between hoop and axial strains is valid: $\varepsilon_z = n\varepsilon_\theta$. Radial strains $(\varepsilon_r)_i$ were linked with hoop and axial ones by using of material incompressibility:

$$(\varepsilon_r)_i = 1 - \frac{1}{\left(1+(\varepsilon_z)_i\right)\left(1+(\varepsilon_\theta)_i\right)}. \tag{4}$$

The discrete values of parameter *p* were chosen from the interval (1; 2) with a step 0,01. For the given set of points the Pearson correlation coefficient *r* and variation coefficient have been calculated. The Pearson correlation coefficient maximums were correlated with variation coefficient minimums. As optimum of *p* the value was chosen for which $r_{\max}$ was reached. For steel 45 99% the interval of confidence for $r = r_{\max}$ equals to 0,969±0,038, optimum *p=1,30,* and for steel 10MnH2MoV the interval of confidence 0,927±0,026, optimum *p*=1,59.

Thus summarized stress-strain curve was constructed by the points $\left((\varepsilon_{eq})_i, (\sigma_{eq})_i\right)$, which were obtained for the optimum parameter *p*. The segment of strengthening was approximated by the power model $\sigma_{eq} = A\varepsilon_{eq}{}^{B}$. So we get:

$$\sigma_{eq} = 721,42 \cdot \varepsilon_{eq}{}^{0,23} \quad \text{(for steel 45)}, \tag{5}$$

$$\sigma_{eq} = 687,32 \cdot \varepsilon_{eq}{}^{0,11} \quad \text{(for steel 10MnH2MoV)}. \tag{6}$$

Having calculated the values of true hoop and axial stresses by formulae $(\tilde{\sigma}_z)_i = (\sigma_z)_i\left(1+(\varepsilon_z)_i\right)$, $(\tilde{\sigma}_\theta)_i = (\sigma_\theta)_i\left(1+(\varepsilon_\theta)_i\right)^2$, true equivalent stresses were obtained by using the formula $(\tilde{\sigma}_{eq})_i = \frac{p}{2}\left[\frac{\left|(\tilde{\sigma}_z)_i-(\tilde{\sigma}_\theta)_i\right|^p+\left|(\tilde{\sigma}_\theta)_i\right|^p+\left|(\tilde{\sigma}_z)_i\right|^p}{2}\right]^{\frac{1}{p}}$. The computer program was developed to obtain the true stress-strain dependence $\tilde{\sigma}_{eq}(\varepsilon_{eq})$ [16]. True stress-state curves and experimental points, equations of power regression and determination coefficient R^2 are given on fig. 1.

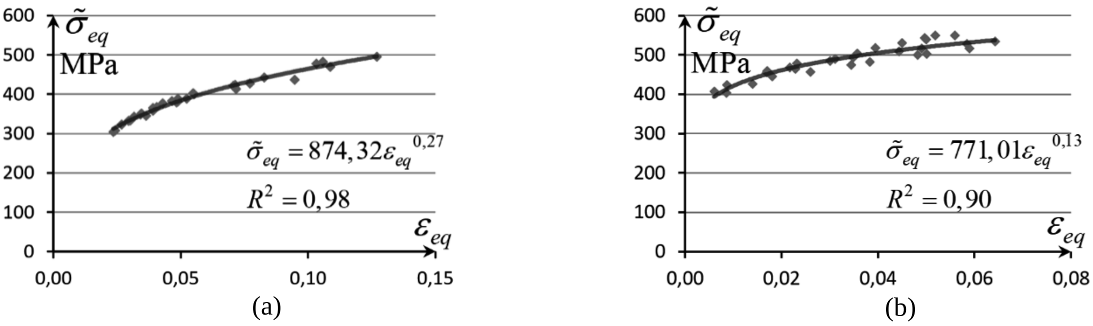


Fig. 1. Approximation of the true stress-strain curve by a power model and coefficients of determination for: a) steel 45; b) steel 10MnH2MoV.

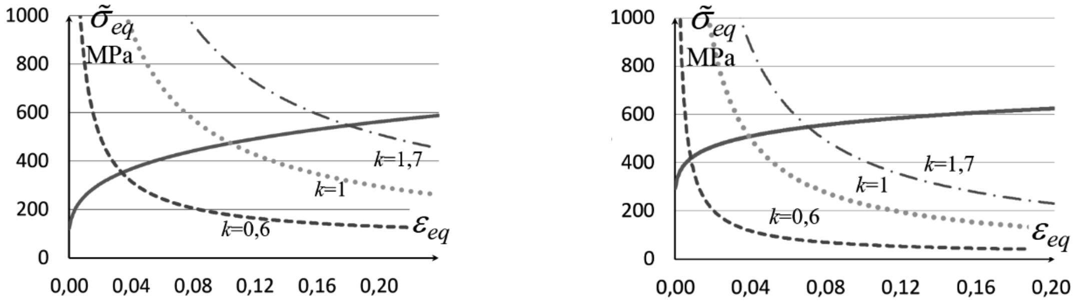
To solve the equation (1) and to find the true ultimate stresses in the system of coordinates $(\varepsilon_{eq};\ \tilde{\sigma}_{eq})$ the graphs of both sides of the equations have been constructed. At *k*>>2 the pipe stress-and-strain state is close to the uniaxial one, so on the figures the calculations results are given for the interval which is important in practical engineering, $k \in (0,5;\ 2)$.

As a power model was chosen for the analytical description of the stress-strain curve, then the derivative $d\tilde{\sigma}_{eq} / d\varepsilon_{eq}$ was defined by the formula $d\tilde{\sigma}_{eq} / d\varepsilon_{eq} = A \cdot B \cdot \varepsilon_{eq}{}^{B-1}$. From (5) and (6) the expression was obtained which made it possible to find tangent modulus:

$$\frac{d\tilde{\sigma}_{eq}}{d\varepsilon_{eq}} = 240,0 \cdot \varepsilon_{eq}^{-0,7255} \text{ (for steel 45),} \tag{7}$$

$$\frac{d\tilde{\sigma}_{eq}}{d\varepsilon_{eq}} = 101,4 \cdot \varepsilon_{eq}^{-0,8685} \text{ (for steel 10MnH2MoV).} \tag{8}$$

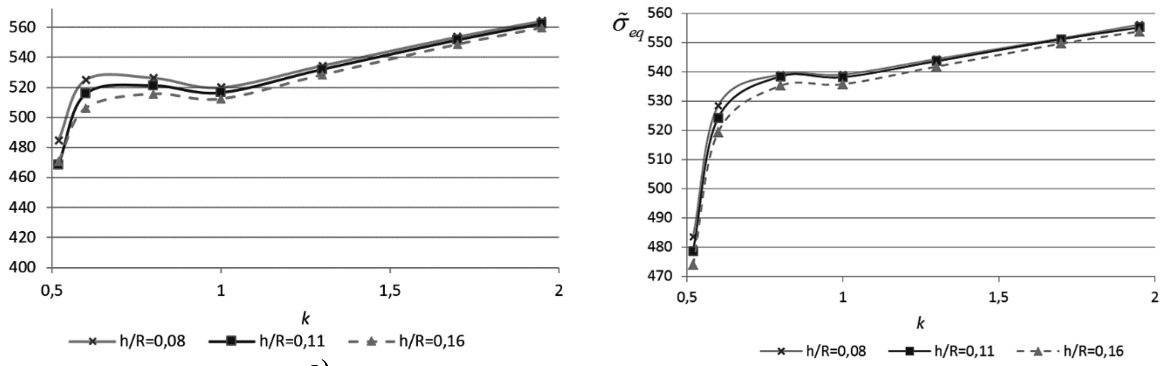
Graphical solution of equation (1) for h/R=0,08, where $R=(d–h)/2$ – radius of the pipe middle surface is given on fig. 2. The value μ has been calculated by the formula obtained in [17] by the known values p, k, h, R. Solid curves show the true generalized stress-strain curves constructed at optimal values of the parameter p for each of the materials. True stress-strain curve is a graph representation of the equation left-hand side (1). Curves of the equation (1) right-hand side are given as dotted graphs – the product of tangent modulus and correcting factor μ taking into account the geometry of a pipe at the moment when local plastic strains appeared. Fig. 2 shows that the type of stressed state k makes a great impact on the pipe strength.



a) b)

Fig. 2. Graphical solution of equation (1) for thin-walled pipes made of: a) steel 45; b) steel 10MnH2MoV. A solid line is a true stress-strain curve

The dependences of limiting true equivalent stresses $\tilde{\sigma}_{eq}$ on k for several values of h/R are shown on fig. 3.



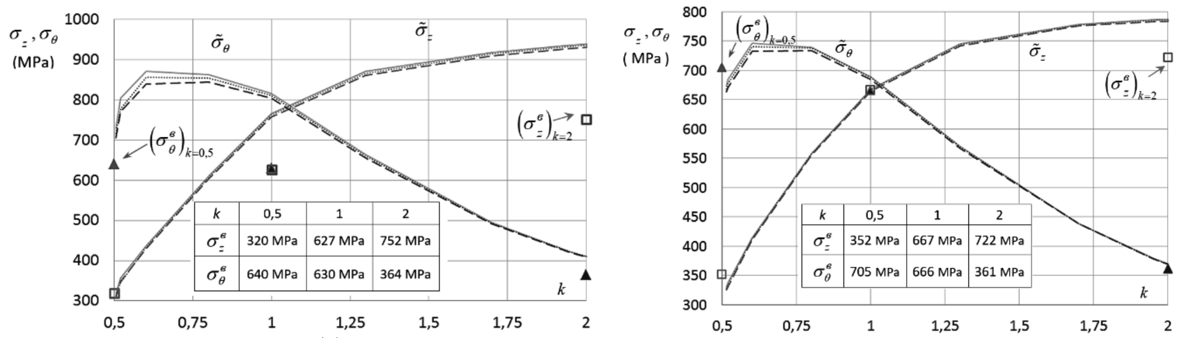
a) b)

Fig. 3. Dependence of limiting stresses $\tilde{\sigma}_{eq}$ on k for certain h/R for: a) steel 45; b) steel 10MnH2MoV

Fig. 3 shows that the smallest calculated durability occurs when pipe is loaded only by internal pressure (k=0,5). Under stress state with a coefficient k=0,5..1 the uniform plastic stability loss occurs at higher values of true equivalent stresses for all patterns, the pipe durability is getting higher. The increase of h/R from 0,08 to 0,11 for k=0,6..0,7 makes the calculated ultimate values of true equivalent stresses 10 MPa (or ≈2%) lower on average. In pipes of smaller parameter h/R the higher level of calculated true equivalent stresses has been found at the moment of deformation localization. At k = 1 the increase of correlation h/R from 0,08 to 0,11 results in average 5 MPa (or ≈1%) decrease of calculated true ultimate equivalent stresses for steel 45. For steel 10MnH2MoV the level of limit equivalent stresses at the correlations h/R=0,08 and h/R=0,11 is the same. Under further increase conditions of correlation h/R from 0,11 to 0,16 the limit calculated equivalent stresses are 1-1,5% smaller for both materials.

Dependence $\tilde{\sigma}_{eq}$ on the type of stressed state at $k = 1..2$ is close to the linear one for all chosen values h/R.

The dependencies of calculated ultimate values of true hoop and axial stresses under combined loading conditions of the pipe by internal pressure and tension for both steel grades are shown on fig. 4. The graphs allow to understand the behavior of dependence of limit true hoop and axial stresses on k in the interval (0,5; 2) taking into account its real geometry of element. In case of pipe loading only with internal pressure the calculated ultimate true axial stress coincides with the referencing one. It can be explained by zero axial strains. The engineering ultimate values of stresses are much lower than true calculated ones. For k=1 the calculated true stresses are 1,2-1,3 higher than referencing ones, and for k = 2 – 1,1-1,2 higher. The two times increase of thin wall index (from 0,08 to 0,16) results in 4-5 % decrease of the level of maximum calculated ultimate hoop stresses (for k=0,6). The dependence for steel 10MnH2MoV is close to the previous one. The two times increase of thin wall index (from 0,08 to 0,16) results in 2-3% decrease of the level of maximum calculated ultimate hoop stresses (for k=0,6). Separate symbols denote the points corresponding the engineering ultimate values of stresses for steel 45 at three types of stressed state (for k=0,5; 1; 2).



(a) (b)

Fig. 4. Dependence of limit values of calculated stresses on k for certain h/R.
Denoted: ——— – h/R=0,08; .... – h/R=0,11; — — . – h/R=0,16.
Separate points show the engineering strength limits for k=0,5; 1; 2

Fig. 4 proves that the calculated true ultimate hoop stresses are on average 1,01..1,3 times higher than engineering ones for k = 1, the true ultimate axial stresses are on average 1,1..1,2 times higher than engineering ones. The two times increase of h/R (from 0,08 to 0,16) makes the level of calculated ultimate stresses 1-3% lower. It shows the importance of improvement of strength properties of structural elements not only due to their mass increase.

The whole algorithm of computer modelling the ultimate strength of thin-walled cylindrical pipe with cupped ends loaded by internal pressure q and axial strain N, and determination of limit values of strength factors is formed from the task sequence (fig. 5):

1) finding the constant p by the initial stress-strains curves built for several values $k = \sigma_z / \sigma_\theta$ and building of generalized stress-state curve;
2) obtaining the dependence between true stresses and strains in equivalent coordinates;
3) determination of the true ultimate stresses in equivalent coordinates;
4) calculating the limit values of internal pressure q and tension N.

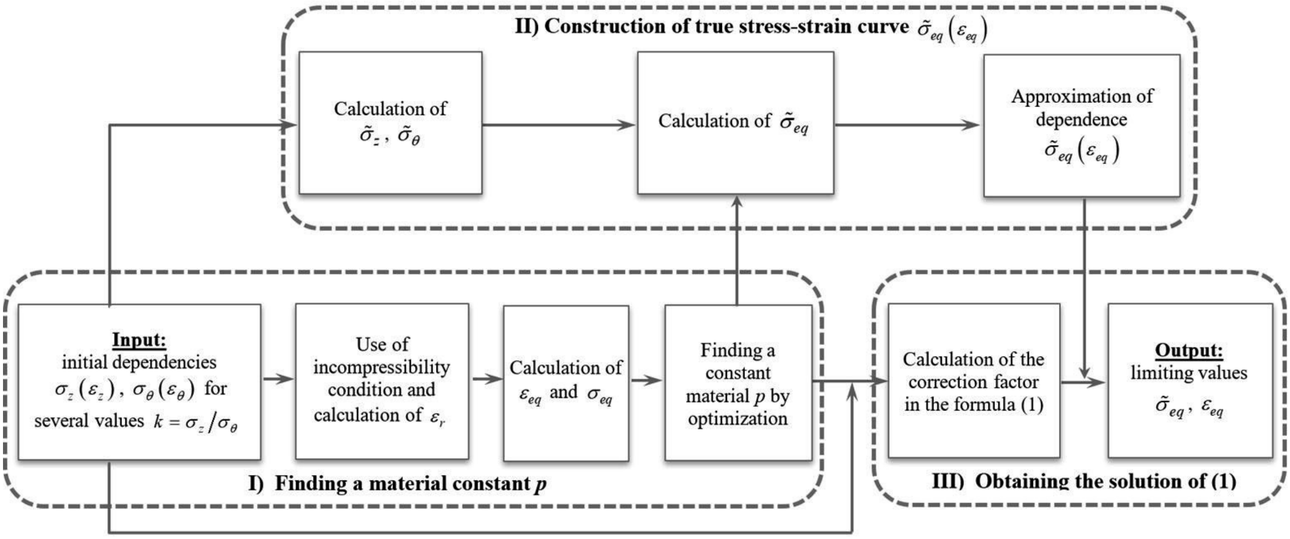


Fig. 5. Algorithm for computer modeling of the stress-strain state of thin-walled cylindrical pipe loaded by internal pressure q and axial tension N

4. Conclusions

The algorithm for computer modeling of the influence of stress state type and thin-walled cylinder geometry on the ultimate values of true stresses at the moment of the uniform plastic stability loss has been developed. The results of the calculations showed that the ratio *h*/*R* increasing to some extent allows the ultimate values of axial strain and internal pressure be increased for both materials. Though the further increase of ratio *h*/*R* increases the allowable value of internal pressure but makes the maximum allowable values of strains lower. These facts prove the existence of optimal balance between the pipe geometry and its strength. The above-mentioned results can be used for calculation of loaded thin-walled pipes aimed at their material consumption saving, safety level increase under performance conditions and choose real safety factor.

Calculating of the limiting values of actual stresses makes it possible to predict the strength of thin-walled pressure vessels (pipelines, collectors of steam generators, reservoirs, etc., in mechanical engineering, aircraft construction, chemical, food, energy and other industries); choose a realistic safety factor and make optimal engineering solutions at the design and operation stages of structural elements; to increase the efficiency and safety of using pipeline and shell-type saving systems.

References

1. Luchko, J., Ivanyk, E. (2017). Diagnostics of the main gas pipelines and assessment of their residual life under the conditions of long-term operation. *Scientific Journal of TNTU*, *87*(3), 48-63. https://doi.org/10.33108/visnyk_tntu2017.03.048

2. Bony, M., Alamilla, J. L., Vai, R., Flores, E. (2010). Failure pressure in corroded pipelines based on equivalent solutions for undamaged pipe. *ASME. J. Pressure Vessel Technol*., *132*(5). doi:10.1115/1.4001801

3. Hillier, M.J. (1965). Tensile plastic instability of thin tubes–I. *International Journal of Mechanical Sciences, 7*(8), 531–538, doi: 10.1016/0020-7403(65)90010-X

4. Tomita, Y., Shindo, A., Nagai, M. (1984). Axisymmetric deformation of hoop elastic-plastic tubes under axial tension and internal pressure. *International Journal of Mechanical Sciences, 26*(6–8), 437–444. doi:10.1016/0020-7403(84)90033-X

5. Dilman, V.L., Ostsemin, A.A. (2000). O vliyanii dvuhosnosti nagrujeniya na nesuschuyu sposobnost trub magistralnyih gazonefteprovodov. *Izv. RAN. Mehanika tverdogo tela, 5*, 179–185 [in Russian]

6. Dilman, V.L., Ostsemin, A.A. (2000). O potere plasticheskoy ustoychivosti tonkostennyih tsilindricheskih obolochek. *Problemyi mashinostroeniya i nadejnosti mashin, 5*, 50–57 [in Russian]

7. Degtyarev, V.P. (1987). *Deformatsii i razrushenie v vyisokonapryajennyih konstruktsiyah*. Moskva: Mashinostroenie [in Russian]

8. Kollinz, Dj. (1984). *Povrejdenie materialov v konstruktsiyah. Analiz, predskazanie, predotvraschenie*. Moskva: Mir [in Russian]

9. Updike, D.P., Kalnins, A. (1998). Tensile plastic instability of axisymmetric pressure vessels. *ASME. J. Pressure Vessel Technol*, *120*(1), 6–11. doi: 10.1115/1.2841888.

10. Zhu, X.-K., Leis, B., (2011). Evaluation of burst pressure prediction models for line pipes. *International Journal of Pressure Vessels and Piping*. doi:10.1016/j.ijpvp.2011.09.007

11. Law, M. (2005). Use of the cylindrical instability stress for blunt metal loss defects in linepipe. *International Journal of Pressure Vessels and Piping*, *82*(12), 925–928. doi:10.1016/j.ijpvp.2005.04.002

12. Kozbur, H. (2020). Prediction technique for thin-walled cylindrical tubes boundary state. *Scientific Journal of TNTU, 94*(2), 145–155. doi: 10.33108/visnyk_tntu2019.02.145

13. Kozbur, H. (2020). Method of predicting necking true stress in a thin-walled tube under a complex stress state. *Strojnícky časopis - Journal of Mechanical Engineering*, *70*(2), 101-116. doi:10.2478/scjme-2020-0024

14. Kaminskiy, A.A., Bastun, V.N. (1985). *Deformatsionnoe uprochnenie i razrushenie metallov pri peremennyih protsessah nagrujeniya*. Kyiv: Naukova dumka [in Russian].

15. Lebedev, A.A. (Ed.), Kovalchuk, B.I., Giginyak, F.F., Lamashevskiy, V.P. (2003). *Mehanicheskie svoystva konstruktsionnyih materialov pri slojnom napryajennom sostoyanii*. Kiev: Izdatelskiy dom “In Yure” [in Russian]

16. Copyright for the work "Computer modeling" The modeling of stress-state-independent deformation curve". Ministry of Economic Development, Trade and Agriculture of Ukraine, copyright registration certificate т 96585 dated 10.03.2020. (authors – Kozbur H., Gladio O.)

17. Kozbur, H. (2020). Method of predicting necking true stress in a thin-walled tube under a complex stress state. *Strojnícky časopis – Journal of Mechanical Engineering, 70*(2), 101-116. doi:10.2478/scjme-2020-0024.