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EVALUATION OF THEORETICAL STRENGTH OF POROUS MATERIALS ACCORDING TO CATASTROPHE THEORY

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***Summary**. With the rapid development of modern science, in particular, applied mechanics, the catastrophe theory proved to be quite effective in the analysis of classical results and the development of modern ones. This theory has developed significantly in the study of a number of issues in the theory of elastic stability, which studies the response of elastic bodies and structures to existing mechanical loads. Catastrophe theory predictions have important technical applications for estimating the critical forces that initiate the loss of stability of elastic bodies and engineering structures. The main basics of the research are analysed in this paper; based on the catastrophe theory, the problems are set; the main types of catastrophes' functions are described; and the simplest of them, in particular the fold catastrophe, is applied. Based on the set analytical relations for the calculations of effective electrical conductivities and elastic modules by the pore concentration of the electrically conductive material, the estimation of the element strength of the composite sample is simulated in the form of a rod.*

***Key words**: strength, catastrophe theory, Morse lemma, catastrophe fold, catastrophe function, volume concentration of pores, critical nominal stress, porous composite, effective electrical conductivity, engineering formulas.*

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Introduction. Prediction of the strength of materials, as well as the performance of products and structures made of them, usually involves the use of a series of analytical studies. The priority is to define critical values of strength, nominal stresses, as well as critical deformations. The values of these parameters gain special importance due to the inhomogeneity of their structure inherent in the respective materials, for example, porosity, various impurities, technological inclusions or other stress concentrators. The vast majority are functionally inhomogeneous materials [1, 2], which are porous. Therefore, the objective of this paper is to develop analytical dependences for determining the theoretical strength of elastic materials with consideration of their porosity.

Analysis of available investigations and publications. The initial sources in the study were the mathematical theory of catastrophes and analytical relations concerning the porosity of materials. Catastrophe theory is an applied mathematical theory, which has been actively developed recently. Nowadays, together with the methods of systems analysis, it has become an effective tool for qualitative research on the reliability and durability of machine parts and structures. This science combines the theory of singularities of smooth surfaces by H. Whitney, the theory of stability and bifurcations of dynamical systems by A. Poincare, A. Lyapunov, A. Andronov. Its appearance and name are the result of research conducted by the French mathematician R. Tom [3]. The studies of V. I. Arnold [4, 5], T. Poston, I. Stewart [6], A. Campo [7, 8], Ye. S. Ziman [9], R. Gilmore [10], J. M. T. Thompson [11] and others influenced greatly the development of methods of catastrophe theory.

Catastrophe theory studies a sudden, abrupt change in the state of a specified system caused by real changes in external influences, and contains significant potential for describing the phenomena of loss of its stability.

General statement of problems based on the catastrophe theory. In mathematical formulation, catastrophe theory studies the qualitative nature of the dependence of solutions $y_i(t,x,c)$ $(i=1,2,...,n,\ x=(x_1,...,x_n), c=(c_1,...,c_n))$ of the equations system

$$F_i(t, y_j, x, \frac{dy_j}{dt}, ..., \frac{dy_j}{dx_k}, ..., \int y_j dx_k, ...) = 0 \quad (1)$$

on parameters c_α called control parameters. In a simplified version, the task of catastrophe theory is to study the dependence of the equilibrium state y_i of a certain potential function $V(y_i, c_\alpha)$ of the corresponding process on the change of parameters c_α.

The catastrophe theory is rather often applied in science and technology [6, 10]. At the same time, it is not developed enough in the direction of research on the destruction of elastic bodies, in particular the theory of strength.

A significant number of tests of materials for strength and fracture indicate that the nature of the behaviour of the phenomena under study is nonlinear [12]. This is due to the presence of critical external factors. Their excess leads to significant deviations of the equilibrium of elastic bodies from the steady state, to periodic changes in the process of accumulation of damage, and to abrupt changes in their states. Such a number of factors can be considered in the catastrophe theory application [6] in order to assess the stability of solid deformable bodies before their destruction.

Brief description of the major catastrophe functions. The local behaviour of a potential function $V(x,c)$, determined by the initial members of its Taylor series, is investigated by reducing it to some canonical form. In addition, a number of theorems of functional analysis are used. Thus, to develop the canonical form of a potential function at a noncritical point (x^0, c^0), that is, the point at which $\nabla V \neq 0$ ($\nabla = \sum_i \frac{\partial}{\partial x_i}$ – Hamilton operator), the theorem on implicit function is used; at the usual critical point ($\nabla V = 0,\ \det V_{ij} = \det(\frac{\partial V}{\partial x_i x_j}) \neq 0$) – Morse lemma [13]; at the degenerated critical point ($\nabla V = 0,\ \det V_{ij} = 0$) – splitting lemma [14]. If the number of control parameters c_α does not exceed 5 ($k \leq 5$), then, according to Tom's theorem [3], there is such a smooth substitution of variables that the potential function can be written in the form:

$$V = \Phi(l,k) + \sum_{j=l+1}^{n} \lambda_j(c) y_j^2(x), \quad (2)$$

if $\Phi(l,k)$ – catastrophe function (catastrophe), l – the number of eigenvalues of the matrices V_{ij}, λ_j – constants, which also depend on the control parameters c_α, k – number of control parameters.

Tom proved that if $k \le 5$, there are seven types of function $\Phi(l,k)$ – elementary catastrophes. Consider the simplest of them – a fold catastrophe.

The potential function, for which a fold catastrophe can occur, should be summarized as follows:

$$V(z,M) = \frac{1}{3}z^3 + Mz + c, \quad (3)$$

if z – state variable, M – control parameter, c – constant. The critical points of this function are derived from the condition $\frac{dV}{dz} = 0$, that is

$$z^2 + M = 0, \quad (4)$$

and twice degenerated critical points – from the condition $\frac{d^2V}{dz^2} = 0$, that is

$$z = 0. \quad (5)$$

Equation (5) indicates the existence of two critical points of the function V if M <0, one of which $z = \sqrt{-M}$ is the minimum point of this function and corresponds to the steady state of the system (Fig. 1).

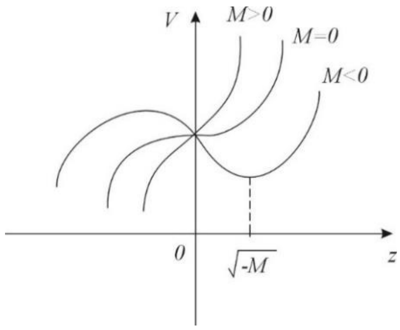

Figure 1. Nature of the change in potential function

The change in the parameter M is accompanied by a smooth change in the depth of the minimum of the function V, which has no critical points if M> 0. That is, the point M = 0 is a point that divides the functions of two qualitatively different classes. If condition (5) is satisfied, then the minimum of the function V disappears and passes into the inflection point with the horizontal tangent. Thus, the system abruptly passes from a stable equilibrium state to an unstable one. Therefore, the condition of the fold catastrophe is a simultaneous execution of equations.

$$\left.\begin{array}{l} M = 0 \\ z = 0 \end{array}\right\}. \quad (6)$$

Based on relations (6), the theoretical strength of a porous composite can be easily estimated by its known effective mechanical characteristics.

Calculation of porous composite strength based on effective modulus of elasticity and fold catastrophe. The vast majority of materials and composites are porous. Therefore, calculations of the theoretical strength of such materials are carried out taking into account the concentration of pores present in them. The calculation can be made applying efficient stiffness characteristics $G_{eff.p.}$– effective shear modulus, $E_{eff.p.}$ – effective Young's module,

$\mu_{eff.p.}$ – effective Poisson's ratio of composites, which should be dependent on the volume concentration of pores.

Determining the effective stiffness characteristics of electrically conductive materials becomes possible by means of non-destructive measurements of their electrical conductivity. However, currently, there are no analytical and experimental approaches, which establish close relationships between conductive and mechanical characteristics of conductive materials.

Assumedly, the volume concentration of the pores is small enough as compared to the total volume of the material. Each pore has an ellipsoidal shape. The authors also believe that under the action of external loads of deformation and displacement of its elements at each point of the volume occupied by it, the basic equation of the linear theory of elasticity is satisfied. An analytical relationship between porosity, on the one hand, and stiffness, on the other, should be determined.

Based on independent solutions of the corresponding boundary problems of electrostatics and micromechanics of composite materials, the porosity indexes are defined. The essence of the proposed approach is to compare them. As a result, relationships are found between the required material characteristics.

To found the analytical relations for calculating the effective electrical conductivity and porosity of the electrically conductive material, the model problem of electrostatics on the conductivity of a continuous conductive medium with an electrically conductive elliptical inclusion should be considered. The authors argue that the continuous current components occur at the boundary of heterogeneous media, i.e.

$$\dot{j}_{n1} = \dot{j}_{n2} \tag{7}$$

and the components of the electric field strength are equal

$$E_{\tau 1} = E_{\tau 2}\,. \tag{8}$$

Assumedly, a homogeneous electrostatic field is specified. The effective electrical conductivity of the specified composition should be found.

The equations of stationary field of such a problem are deduced:

$$div\vec{j} = 0;\; rot\vec{j} = 0\,, \tag{9}$$

$$\vec{j} - \sigma\vec{E} = 0\,, \tag{10}$$

where the parameter σ, which is responsible for the electrical conductivity, takes the value $\{\sigma_1, \sigma_2\}$, and σ_1 – conductivity of the matrix; σ_2 – conductivity of the elliptical fiber.

Based on the theory of functions of a complex variable [15], as well as the results of [16, 17], after the necessary calculations, the complex-valued functions of current $j_1(z)$ and $j_2(z)$ are determined in the matrix and fiber, respectively. Based on the further averaging of field quantities with consideration of Ohm's law and the concentration c of elliptical fibers, the effective electrical conductivity of the material in the directions xx, xy, yy is deduced:

$$\sigma_{xxeff.p.} = \sigma_1 \left\{ 1 - c\frac{m+n}{n} \frac{(\sigma_1 - \sigma_2)\left(\frac{n}{m}\sigma_1 + \sigma_2\right)}{(\sigma_1 + \sigma_2)^2 \frac{(m-n)^2}{mn}\sigma_1\sigma_2} \right\};$$

$$\sigma_{xxef.p} = 0;$$

$$\sigma_{yyeff.p.} = \sigma_1 \left\{ 1 - c\frac{m+n}{m} \frac{(\sigma_1 - \sigma_2)\left(\frac{n}{m}\sigma_1 + \sigma_2\right)}{(\sigma_1 + \sigma_2)^2 \frac{(m-n)^2}{mn}\sigma_1\sigma_2} \right\}. \quad (11)$$

Based on relations (11), the effective electrical conductivities of a medium with pores of a certain concentration can be calculated. Thus, if the conductivity of the pores is assumed to be zero, i.e. $\sigma_2 = 0$, the working formulas for calculating the effective electrical conductivity of porous material with pores of elliptical shape are developed:

$$\sigma_{xxeff.p.} = \sigma_1\left(1 - c\frac{m+n}{n}\right)$$

$$\sigma_{yyeff.p.} = \sigma_1\left(1 - c\frac{m+n}{m}\right) \quad (12)$$

In what follows, the assumption is that pores with a circular cross-section predominate in the porous material. Then, based on relations (12) for the theoretical calculation of the planar concentration of pores c_S, the formula is deduced

$$c_S = \frac{\sigma_m - \sigma_{eff.p.}}{2\sigma_m}, \quad (13)$$

if σ_m – conductivity of the medium, $\sigma_{ef.p.}$ – conductivity of the porous medium.

After the transition from planar c_S to volumetric c_V by the obvious formula

$$c_S = c_V^{2/3} \quad (14)$$

the calculation formula for determining the volume porosity coefficient is deduced

$$c_V = \frac{1}{2\sqrt{2}}\sqrt{\left(1 - \frac{\sigma_{eff.p.}}{\sigma_m}\right)^3} \quad (15)$$

Based on formula (15), the relationship between mechanical and electrical characteristics of materials can be easily found.

Functional dependences between physical – mechanical and electrical characteristics of materials. Similar relationships between the volume coefficient of porosity and the stiffness characteristics of the composite material applying the linear theory of elasticity follow from the results of [18–19]. After comparing the corresponding values of the volumetric porosity coefficients, expressed in terms of effective stiffness and electrical conductivity characteristics, in order to calculate the effective shear modulus of the porous composite material, the ratio is developed

$$G_{eff.p.} = 3\frac{1-\frac{1}{\sqrt{2}}\sqrt{\left(1-\frac{\sigma_{eff.p.}}{\sigma_m}\right)^3}}{3-\frac{1}{2\sqrt{2}}\sqrt{\left(1-\frac{\sigma_{eff.p.}}{\sigma_m}\right)^3}}G_m, \tag{16}$$

if G_m – shear modulus of the material without pores. Taking into account the individual characteristics of the structural elements and the presence of pores, the shear modulus of the composite material is determined by the formula

$$G_{eff.p.} = 3\frac{1-\frac{1}{\sqrt{2}}\sqrt{\left(1-\frac{\sigma_{eff.p.}}{\sigma_m\left(1-2c_a\frac{\sigma_1-\sigma_a}{\sigma_1+\sigma_a}\right)}\right)^3}}{3-\frac{1}{2\sqrt{2}}\sqrt{\left(1-\frac{\sigma_{eff.p.}}{\sigma_m\frac{\sigma_1-\sigma_a}{\sigma_1+\sigma_a}}\right)^3}}G_m, \tag{17}$$

here σ_1 – specific electrical conductivity of the composite matrix; σ_a – specific electrical conductivity of reinforcement elements; c_a – concentration of reinforcing material.

It should be noted that the parameter σ_a in formula (17), as well as all other characteristics of the two-component material with index *a* can be determined by the rule of mixtures

$$A_m = vA_a + (1-v)A_c, \tag{18}$$

if v – reinforcement concentration factor, A_a – characteristic of the matrix,
A_c – characteristics of the main composite material.

To calculate the effective Young's modulus according to this technique, a formula is deduced

$$E_{eff.p.} = 3\frac{1-\frac{1}{2\sqrt{2}}\sqrt{\left(1-\frac{\sigma_{eff.p.}}{\sigma_m}\right)^3}}{1+\frac{5}{2\sqrt{2}}\frac{E_a}{G_a}\frac{4G_a-E_m}{24G_a-5E_m}\sqrt{\left(1-\frac{\sigma_{eff.p.}}{\sigma_m}\right)^3}}E_m, \tag{19}$$

if E_m – Young's modulus of material without pores. Providing $E/G = 2(1+\mu)$, then, formula (19) takes the form

$$E_{eff.p.} = 3\frac{1-\frac{1}{2\sqrt{2}}\sqrt{\left(1-\frac{\sigma_{eff.p.}}{\sigma_m}\right)^3}}{1+\frac{5}{\sqrt{2}}\frac{1-\mu^2}{7-5\mu}\sqrt{\left(1-\frac{\sigma_{eff.p.}}{\sigma_m}\right)^3}}E_m, \quad (20)$$

For the average value $\mu = 0.25$, a simplified formula is deduced

$$E_{eff.p.} = 3\frac{1-c_V}{1+0.96c_V}E_m, \quad (21)$$

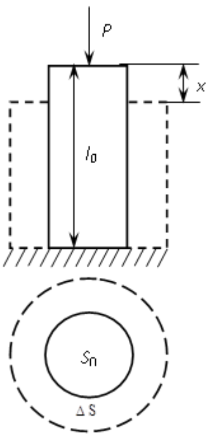

Figure 2. Scheme of loading of a composite element

Based on the above relations for the effective parameters and provisions of the catastrophe theory, the element of the composite sample is simulated in the form of a length rod l_0. The cross-sectional area of such a sample is denoted by S_0. Supposedly, a load of magnitude is applied to this composite element, as shown in Fig. 2. The material under study is assumed to have a modulus of elasticity $E_{eff.p.}$, shear module $G_{eff.p.}$ and Poisson's ratio $\mu_{eff.p.}$ The presence of pores is already taken into account.

Assumedly, x is the magnitude of the longitudinal compression of the sample under the action of force P. Then, to the nearest constant, the energy of the system

$$W = Px + \int_0^x \frac{xdx}{\Pi}, \quad (22)$$

if $\Pi = l/(E_{eff}S)$ – flexibility of the elastic system.

This pliability is considered dependent on compression x in accordance with the expression

$$\Pi = \frac{l_0 - x}{E_{eff.p.}(S_0+\Delta S)} = \Pi_0\frac{1-\frac{x}{l_0}}{1+\frac{\Delta S}{S_0}}, \quad (23)$$

if $\Pi_0 = l_0 / (E_{eff.p.} S_0)$ – initial pliability of the porous composite rod. Considering that $\frac{\Delta S}{S_0} = 2\mu_{eff.p.} \frac{x}{l_0}$, and $x << l_0$, neglecting a member x^2, the energy of the rod is presented in the form

$$\Pi = \Pi_0 \frac{1 - \frac{x}{l_0}}{1 + 2\mu_{eff} \frac{x}{l_0}} = \frac{\Pi_0}{1 + ax}. \tag{24}$$

Here

$$a = \frac{1 + 2\mu_{eff.p.}}{l_0}. \tag{25}$$

Terefore, the energy (22) of the system composite «load-rod» is found

$$W = Px + \frac{1}{2\Pi_0} x^2 + \frac{a}{3\Pi_0} x^3. \tag{26}$$

The substitution is

$$x = z - \frac{1}{2a}. \tag{27}$$

As a result, a potential catastrophe function is developed

$$V = \frac{z^3}{3} - \frac{1}{4a^2} (1 - 4a\Pi_0 P) z + N, \tag{28}$$

and $V = \frac{W\Pi_0}{a}$; N – a permanent member that is independent on z. Equation (28) fully corresponds to the canonical equation [6, 10]

$$V = \frac{1}{3} z^3 + Mz + N \tag{29}$$

for fold catastrophe if

$$M = -\frac{1}{4a^2} (1 - 4a\Pi_0 P). \tag{30}$$

According to the catastrophe theory, the values $M = 0$ and $z = 0$ correspond to the critical state of the rod, i.e. its destruction. Therefore, the critical load is

$$P_{cr.} = \frac{1}{4\Pi_0 a} = \frac{E_{eff.p.}.S_0}{4\left(1+2\mu_{eff.p.}\right)}. \quad (31)$$

Also, the size of the critical compression is

$$x_{cr.} = \frac{1}{2a} = \frac{l_0}{2\left(1+2\mu_{eff.p.}\right)} \quad (32)$$

and critical rated voltage

$$\sigma_{cr.} = \frac{P_{cr.}}{S_0} = \frac{E_{eff.p.}S_0}{4\left(1+2\mu_{eff.p.}\right)}. \quad (33)$$

Expressing the effective stiffness characteristics on the basis of formulas (16)–(17) and (19)–(21) through the usual ones, we obtain appropriate engineering working formulas for determining the critical parameters of porous composite fracture. Based on such formulas, the methods of assessing the load-bearing capacity and durability of structural elements can be improved with consideration of the existing porosity in real composites. In particular, this becomes necessary when studying the effect of hydrogen on stress [20] in the materials and the corresponding analysis of experimental studies of the hydrogen degradation of nickel heat-resistant alloys. [21, 22]. The obtained results for porous materials allow us to evaluate [23–25] the distribution of hydrogen near the fracture-like defect in the porous material, which is important for solving a number of problems of hydrogen energy.

Conclusion. Based on the catastrophe theory, the general decision for solving the problems of the theory of elasticity and mechanics of destruction is made. According to the standpoint of the catastrophe theory with the use of catastrophe folds, the compressive strength of the material is estimated. The basic working formulas to establish rigid effective characteristics depending on electrical conductivity, modulus of elasticity and porosity of composites are developed. The values of critical load and critical nominal stress for a composite cylindrical rod as a sample for experimental studies are found. The application of the given results to the development of hydrogen-saving technologies is indicated.

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ОЦІНЮВАННЯ ТЕОРЕТИЧНОЇ МІЦНОСТІ ПОРИСТИХ МАТЕРІАЛІВ ЗА ТЕОРІЄЮ КАТАСТРОФ

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***Резюме**. Особливого значення в останні роки набуває прикладна математична теорія – теорія катастроф. У поєднанні з класичними та сучасними методами системних досліджень ця теорія стала ефективним практичним інструментом якісного аналізу низки різноманітних процесів та явищ. Значного розвитку теорія катастроф набула у вивченні питань теорії пружної стійкості матеріалів, яка вивчає реакцію пружних тіл і конструкцій на діючі механічні навантаження та уможливлює оцінювання їх міцності й надійності. Прогнози теорії катастроф мають важливе технічне застосування для оцінювання критичних сил, які ініціюють втрату стійкості пружних тіл та інженерних споруд. У роботі коротко проаналізовано першооснови досліджень, зроблено постановку задач на основі торії катастроф, описано основні типи катастроф функцій, а також використано найпростішу з них – катастрофу складки. За встановленими аналітичними співвідношеннями для розрахунків ефективних електропровідностей і пружних модулів за концентрацією пор електропровідного матеріалу змодельовано оцінку міцності елемента зразка пористого композиту у вигляді стержня. Зроблено загальну постановку вирішення завдань теорії пружності та механіки руйнування на основі теорії катастроф. З позицій теорії катастроф із використанням катастрофи складки оцінено міцність матеріалу щодо стійкості на стиск. Записано основні робочі формули для встановлення жорсткісних ефективних характеристик, залежних від електропровідності, модулів пружності та пористості композитів. Встановлено значення критичного навантаження та критичного номінального напруження для пористого композиційного циліндричного стержня як зразка для експериментальних досліджень. Вказано застосування наведених результатів до розроблення воденьзберігаючих технологій, де використовуються пористі матеріали.*

***Ключові слова:** міцність, теорія катастроф, лема Морса, катастрофа складки, функція катастрофи, об'ємна концентрація пор, критичне номінальне напруження**,** пористий композит, ефективні елетропровідності, інженерні формули.*

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