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PREDICTION OF SMA RESIDUAL LIFETIME TAKING INTO ACCOUNT MECHANICAL PROPERTIES UNDER CONSTANT AMPLITUDE LOADING

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Summary. *Shape memory alloys are widely used in medicine, bioengineering, aerospace, mechanical, civil engineering and other areas. Though they haven't been used for a long period of time, nevertheless, there are already known cases of SMA failure in structural elements. Therefore, there arises the question: how long can such structural elements be under operation. To answer this question, it is necessary to predict the residual lifetime of such alloys. The development of science and technology demands new increased requirements for the safety of such important structural elements, and, in particular, to the used devices. Fatigue crack growth (FCG) diagrams are generally significantly scattered, that can be taken into account, for instance, by building the distribution of parameters, which are involved in the equation describing the FCG diagram. Therefore, it is necessary to be able to predict FCG taking into account the scatter of mechanical properties, preliminary determined statistical distribution of crack growth resistance parameters, particularly, parameters of C and n of Paris equation. It is known, that the parameters of Paris equation are mutually dependent. Therefore, considering C as random variable, that changes from the specimen to specimen, it is possible to take into account the existing data scatter. The defects, which are found in the structural elements, have frequently the form of semi–elliptical cracks. FCG rate of pseudo–ellastic NiTi alloy was studied experimentally under uniaxial tension of cyllindrical specimens with a dimater of 8 mm at room temperature on air at servo-hydraulic testing machine STM-100. A methodology for predicting the residual lifetime of SMA cylindrical specimens with a semi-elliptical surface crack is proposed. The methodology is based on solving the system of differential equations describing crack propagation, load parameters and cyclic crack grow resistance, taking into account their statistical scatter and change of crack front shape. There were plotted the cumulative distribution functions of lifetime of specimens with a diameter of 2r for different initial crack depth (b/r = 0.25; 0.36; 0.5; 0.75) and the crack shape factor.*

Key words: *cyclic loading. pseudoelastic shape memory alloy, residual lifetime, Paris equation, modeling, fatigue crack, stress intensity factor.*

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Problem statement. Shape memory alloys (SMA) are widely used in medicine, aviation, bioengineering, mechanical, civil engineering and other areas [1, 2]. Though they haven't been used for a long period of time some cases of their failure in the structures are well known. So, the question is arising: how long can these structural elements be in operation? To answer this question one should be able to predict the residual lifetime of the above-mentioned alloys. The development of science and engineering requires new higher standards of such important structures safety, namely of the used devices.

Analysis of the known results of the study. An example of probabilistic assessment of a steel structural element under fatigue loading conditions has been proposed and described in the article [3]. In this case special attention is paid to the cracks growing from the edge and from the surface. The theoretical model of a fatigue crack growth is based on linear fracture mechanics, and the probabilistic methods, suggested in the paper under discussion, are based on optimized numerical integration.

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The experimental data, obtained after the specimen tests under different loading conditions, often shows a considerable scatter making a significant impact on fracture and predicting the fatigue lifetime of structural elements [4].

The defects in structural elements are often in the form of surface semi-elliptical cracks.

Some engineering technique of load-carrying ability calculation of solid bodies with surface cracks, repaired by contrast layer surfacing has been suggested in the paper [5]. The analytical dependencies of surfacing efficiency on the crack filling depth and on the crack resistance of the material have been specified.

In linear fracture mechanics the power law dependence is mostly used which was suggested in the paper [6], describing the experimental data of fatigue cracks growth (FCG) very well for the substantial range of fatigue cracks growth rate.

It is known that the parameters of Paris equation are interdependent [7]. Taking into consideration *C* as a random variable, which varies from a specimen to a specimen one can take into account the existing data scattering.

The impact of initial shape of a defect on the residual lifetime of continuous casting machines roll made of steel 25Cr1MoV has been discussed in the paper [8]. The surface fatigue crack growth has been simulated in the roll under loading and temperature conditions close to the operational ones taking into account the statistical scatter of parameter *C* of Paris equation. The dependencies of the roll residual lifetime on the initial length of the defect and critical size of the crack have been obtained.

Paper purpose. To predict the residual lifetime of cylindrical specimen made of pseudo-elastic shape-memory alloy taking into account statistical scatter of cyclic crack resistance characteristics and change of crack front shape during its formation.

Problem setting. In the paper, there is proposed the technique of residual lifetime prediction of cylindrical specimens made of shape-memory alloy with a surface semi-elliptical crack. The particular technique is based on solution of the system of equations describing the crack growth, involving loading parameters and cyclic crack resistance characteristics taking into account their statistical scatter and change of crack front shape during its propagation.

Technique of experimental study. The FCG in pseudo-elastic NiTi alloy has been studied experimentally by uniaxial tension of cylindrical specimens of 8 mm diameter with a square cut perpendicular to the specimen axis of 0.6 mm depth at room temperature in the air on servo-hydraulic testing machine STM-100. The loading frequency was 25 Hz, the coefficient of asymmetry of a loading cycle $R = K_{min}/K_{max} = 0.22$ (here K_{min} and K_{max} are the smallest and the largest stress intensity factors). The finish temperature of NiTi alloy austenitic transformation $A_F = -38{,}7°C$, yield stress $\sigma_{0.2} = 447$ MPa [9].

Results of the study. The fatigue cracks growth rate data usually are considerably scattered. This can be taken into account, for example, in building the distribution of parameters involved in the equation describing the FCG diagram. In this case, one can predict FCG taking into account the scatter of mechanical properties, by preliminary determining the statistical distribution of cyclic crack growth resistance parameters, namely constant values *C* and *n* of Paris equation

$$da / dN = C(\Delta K)^n \quad , \tag{1}$$

where C and n are the experimentally determined parameters.

Statistical analysis of crack growth resistance parameters. There were used the experimental data for statistical description of parameter C of Paris equation. After approximation of FCG rate experimental data at R = 0.22 within the range ΔK = (5 – 10) MPa√m, there was obtained the equation of the curve for the second region of FCG diagram. Using the least-squares method, there were determined the parameters of Paris equation, namely, $C = 7 \cdot 10^{-7}$ mm/cycle, n = 2,234

Taking randomly 5 points of FCG diagram, a line with the fixed value of n was built, and, as a result, the sample of C was obtained. For convenience, the elements of sample were substituted into their decimal logarithms. After that the statistical distribution, which corresponds to the parameter lg$C$, was determined.

Figure 1 shows the statistical distribution of parameter lg$C$ of Paris equation for NiTi alloy at 20°C and average value of n = 2.234 at R = 0.22.

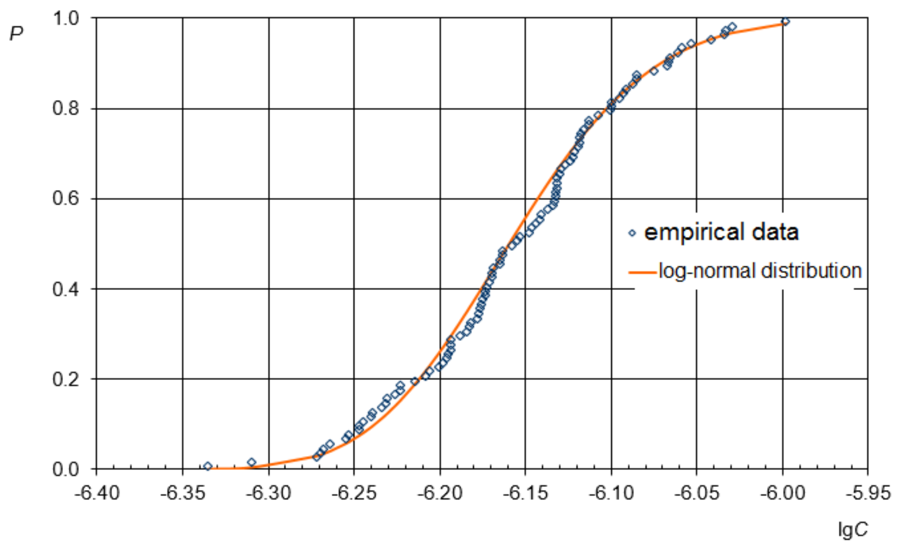


Figure 1. Statistical distribution of parameter lg$C$ of Paris equation for NiTi alloy under average value of n = 2.234 at R = 0.22

Hypothesis on the functions of distribution were tested by the Anderson-Darling (AD) goodness of fit test [10]. Logarithmic-normal distribution was found to be the reasonable one according to the above-mentioned test. The density function of the log-normal distribution has the form:

$$f(x) = \frac{1}{(x-x_0)\cdot\sigma\sqrt{2\pi}} \exp\left[-1\frac{1}{2\sigma^2}\left(\ln\frac{x-x_0}{m}\right)^2\right], \quad (2)$$

where x_0 = –6.8532 is the location parameter; m = 0,67568 is the scale parameter; □ = 0.09639 is the shape parameter.

Lifetime prediction of NiTi specimen with a crack. Fatigue fracture of a cylindrical specimen of 8 mm diameter by uniaxial tension with the coefficient of asymmetry of a loading cycle R = 0.22 with a semi-elliptical surface crack has been simulated. The initial depth of the defect b_0 was chosen in as folows: 1.0 mm, 1.45 mm, 2 mm and 3 mm. The ratio of initial depth of the defect (length of the smaller semi-axis) to the length of major semi-axis of the ellipse b_0/a_0 was equal to 1/3, 1/2, 2/3 and 3/4. Minimal and maximal stress the loading cycle were 24.5 MPa and 110.4 MPa, respectively.

To describe a surface crack, one should determine the crack length and its shape. In practice, the crack has usually semi-elliptical shape. Moreover, its shape remains semi-elliptical during the crack growth. In this case, the crack is described by its length and depth. Having assumed that Paris equation is valid for the deepest point and the point on the crack front surface, the system of differential equations was obtained [11].

FCG was predicted by the Paris equation probabilistically (1). The FCG in two directions was found from the system of differential equations

$$\begin{cases} \dfrac{da}{dN} = C\left(\Delta K^{(a)}\right)^n, \\ \dfrac{db}{dN} = C\left(\Delta K^{(b)}\right)^n, \end{cases} \tag{3}$$

where b is the crack depth; a is the major semi-axis of the ellipse.

In each loading cycle the range of stress intensity factor (SIF) for the surface semi-elliptical crack by uniaxial tension was found by the formula [12]

$$\Delta K = \Delta\sigma\sqrt{\pi s}\, F_I, \tag{4}$$

where $\Delta\sigma = \sigma_{max} - \sigma_{min}$ is the stress range; σ_{min}, σ_{max} are the smallest and the largest stresses of the loading cycle; 2s is the crack length on the surface in circumferential direction; F_I is the correction function, which was determined by the approximation of numerical results obtained by the method of finite elements within the boundaries $0.1 < b/(2\,r) < 0.5$.

The coefficient C of the Paris equation was considered as a random variable. The values of the parameters of lg$C$ distribution, obtained by Levenberg-Marquardt non-linear least squares fit method, served as the input for the FCG calculation. There were performed 100 random simulations of FCG. In each simulation, a random number p^i ($0 \le p^i < 1$, $i = 1,2,\dots,100$) was generated, and a number C^i was calculated by formula $C^i = 10^{F^{-1}(p^i)}$, where $F^{-1}(p^i)$ – inverse function to $F(\lg C)$. The obtained value C^i was substituted into the system of equations (3).

By integrating the system of equations (3) and substituting the randomly generated values of C, FCG has been simulated for the specimen. For each specimen, one hundred FCG simulations were performed. The cumulative distribution functions (CDFs) of fatigue lifetime were obtained (Figure 2).

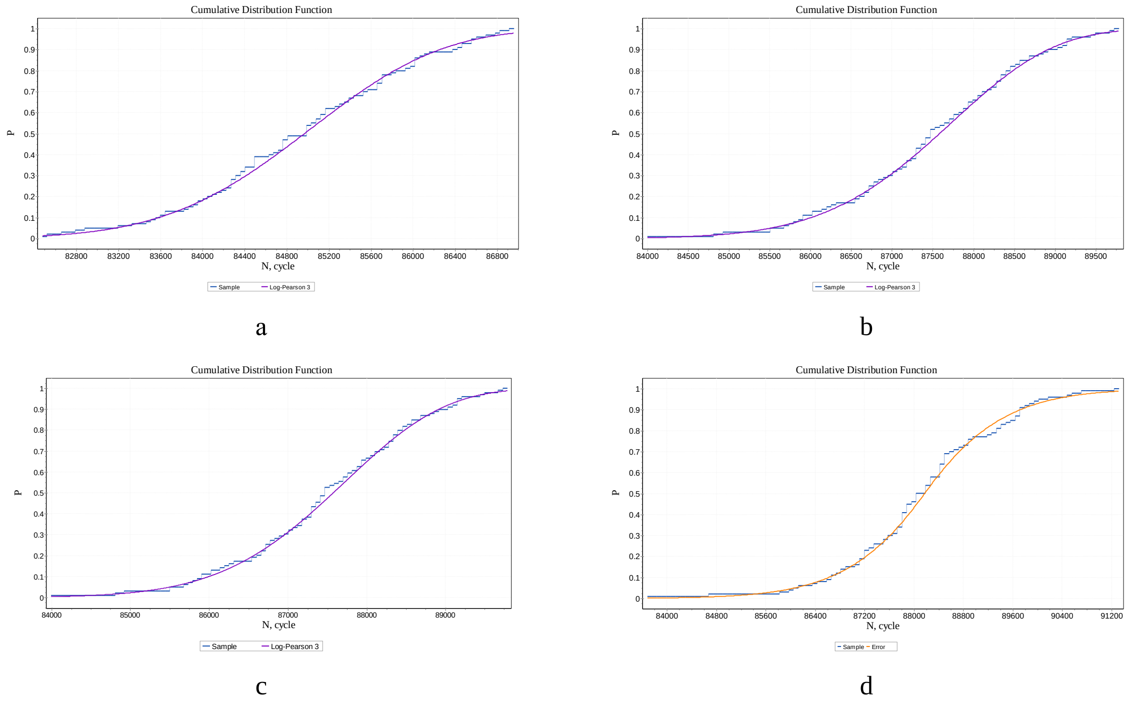


Figure 2. The CDFs of specimen residual lifetime for fixed initial depth b_0 = 1 mm, b_f = 4 mm, and initial crack shape factor b_0/a_0 = 1/3 (a), 1/2 (b), 2/3 (c), 3/4 (d)

The obtained data were used to build CDFs of residual lifetime Nf. Each sample was fitted by the distribution function from the set of available models. The function with the determined parameters, which had the best empirical value according to AD test, was regarded as the most suitable. For initial crack depth b0 = 1 mm, CDFs are shown on Figure 2, and the corresponding parameters are given in tables 1 and 2, respectively

Table 1

Parameters of Log-Pearson 3 function

b_0/a_0	α	β	γ	function
1/3	91.916	–0.00128	11.467	Log-Pearson 3
1/2	21.847	–0.00279	11.44	
2/3	23.374	–0.00271	11.4443	

Table 2

Parameters of Error function

b_0/a_0	k	σ	μ	function
3/4	1.3715	1268.7	88173	Error

For b_0 = 1.45 mm, CDFs are presented on Figure 3, and corresponding parameters are given in tables 3 and 4, respectively.

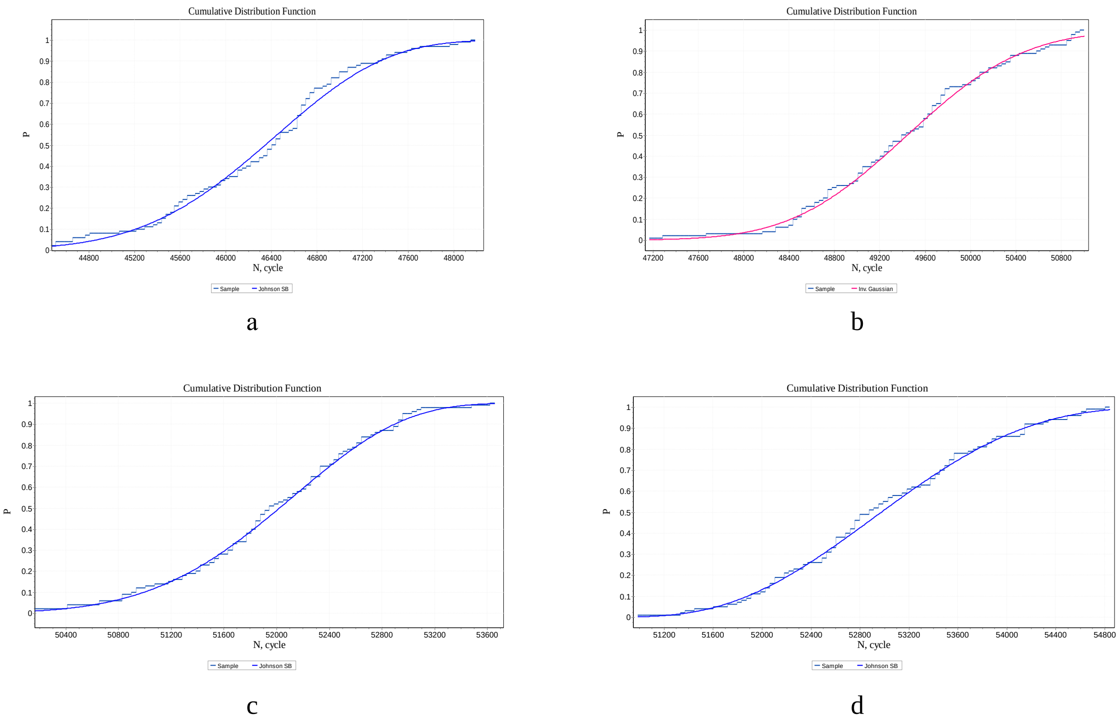


Figure 3. CDFs of specimen residual lifetime for fixed initial crack depth b_0 = 1.45 mm, b_f = 4 mm, and initial crack shape factor b_0/a_0 =1/3 (a), 1/2 (b), 2/3 (c), 3/4 (d)

Table 3

Parameters of Johnson SB function

b_0/a_0	γ	δ	λ	ξ	function
1/3	–1.3892	2.6457	9599.3	40317.0	Johnson SB
2/3	–1.1658	2.1078	6905.1	47639.0	
3/4	0.22735	1.4592	5501.9	50440.0	

Table 4

Parameters of Inverse Gaussian function

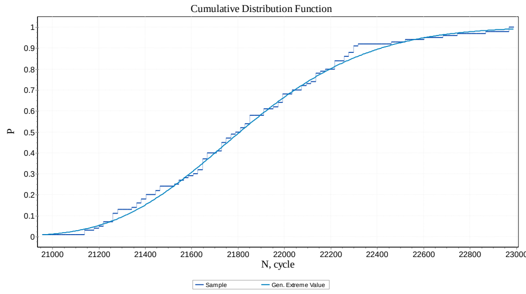
b_0/a_0	λ	μ	function
1/2	1.8453E+8	49444	Inverse Gaussian

For b_0 = 2 mm, CDFs are shown on fig.4, and correspondent parameters are given in Table 5.

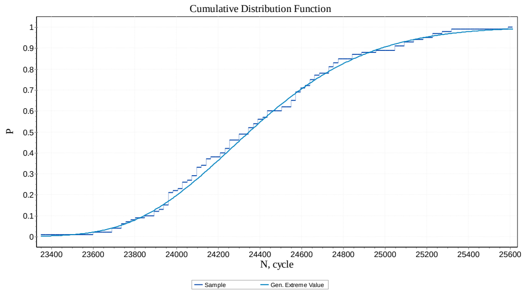
Table 5

Parameters of Generalized Extreme Value function

b_0/a_0	k	σ	μ	function
1/3	–0.14343	404.95	24205	Generalized Extreme Value
1/2	–0.16182	397.4	21667	
2/3	–0.37914	558.52	26675	
3/4	–0.36535	461.37	27858	



a



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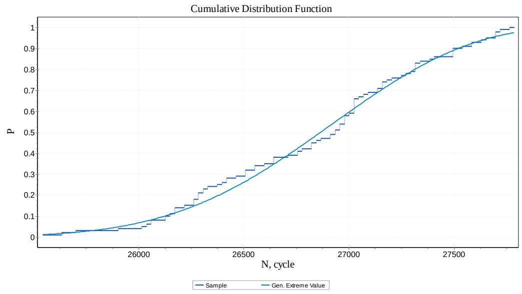

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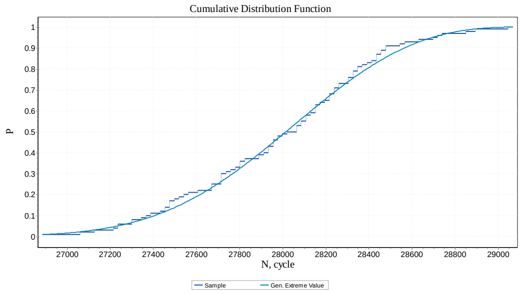

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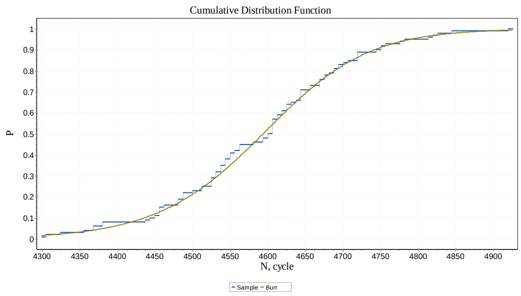
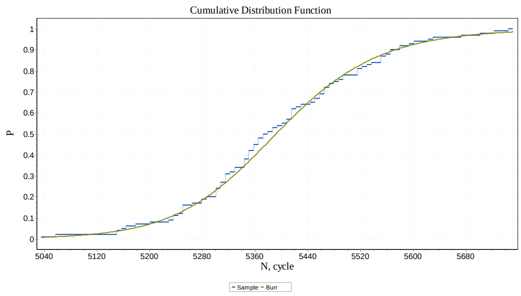
Figure 4. The CDFs of specimen residual lifetime for fixed initial crack depth b_0 = 2 mm, b_f = 4 mm, and initial crack shape factor b_0/a_0 =1/3 (a), 1/2 (b), 2/3 (c), 3/4 (d)

For b_0 = 3 mm, CDFs of residual lifetime are given on Figure 5, and corresponding parameters are presented in tables 6 and 7.



a



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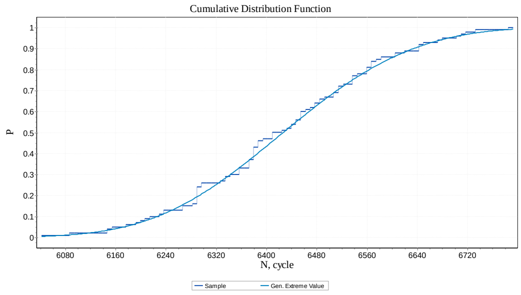

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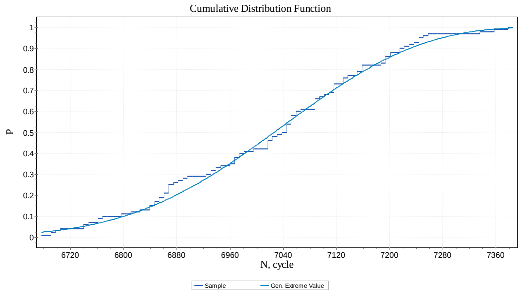

d

Figure 5. The CDFs of specimen residual lifetime for fixed initial crack depth b_0 = 3 mm, b_f = 4 mm, and initial crack ratio b_0/a_0 =1/3 (a), 1/2 (b), 2/3 (c), 3/4 (d)

Table 6

Parameters of Burr function

b_0/a_0	K	α	β	function
1/3	1.3193	60.657	4621.9	Burr
1/2	0.85345	72.874	5375.7	

Table 7

Parameters of Generalized Extreme Value function

b_0/a_0	k	σ	μ	function
2/3	-0.27386	155.55	6372.4	Generalized Extreme Value
3/4	-0.365	171.46	6968.0	

As it can be seen from the mentioned above tables, the distribution, which suits in most cases is Generalized Extreme Value function. The second and third most suitable distributions are Log-Pearson 3 function, and Johnson SB, respectively.

Conclusions. A methodology for predicting the residual lifetime of SMA cylindrical specimens with a semi-elliptical surface crack was proposed. The methodology is based on solving the system of differential equations describing crack propagation, which involves load parameters and cyclic crack grow resistance, taking into account their statistical scatter and change of crack front shape. The cumulative distribution functions of lifetime of specimens with a diameter of $2r$ for different initial crack depth (b/r = 0.25; 0.36; 0.5; 0.75) and the semi-elliptical crack shape factor (1/3; 1/2; 2/3; 3/4) were built.

References

1. Yasniy P. et al. Calculation of constructive parameters of SMA damper. Sci. J. TNTU. 2017. Vol. 88, No. 4. P. 7–15. https://doi.org/10.33108/visnyk_tntu2017.04.007
2. O’Brien B., Weafer F. M., Bruzzi M. S. Shape Memory Alloys for Use in Medicine. Comprehensive Biomaterials II / ed. Ducheyne P. Oxford: Elsevier, 2017. P. 50–78. https://doi.org/10.1016/B978-0-12-803581-8.10084-0
3. Krejsa M. et al. Probabilistic prediction of fatigue damage based on linear fracture mechanics. Frat. ed Integrità Strutt. 2017. Vol. 11. No. 39. P. 43–159. https://doi.org/10.3221/IGF-ESIS.39.15
4. HELLER R. A. Probabilistic Aspects of Fatigue, STP-511, American Society for Testing and Materials, Philadelphia. 1972. 203 p. https://doi.org/10.1520/STP511-EB
5. Shatskyi I. P., Perepichka V. V., Ropyak L. Y. On the Influence of Facing on Strength of Solids with Surface Defects. Met. Noveishie Tekhnol. 2020. Vol. 42. No. 1. P. 69–76. https://doi.org/10.15407/mfint.42.01.0069
6. Paris P., Erdogan F. A Critical Analysis of Crack Propagation Laws. J. Basic Eng. ASME, 1963. Vol. 85. No. 4. P. 528–533. https://doi.org/10.1115/1.3656900
7. Madsen H. O., Krenk S., Lind N. C. Methods of structural safety, Prentice-Hall Inc., Englewood Cliffs, New Jersey. 1986. 407 p.
8. Yasniy O. P., Lapusta Y. Effect of the defect initial shape on the fatigue lifetime of a continuous casting machine roll. Comptes Rendus Mécanique. 2016. Vol. 344. No. 8. P. 596–602. https://doi.org/10.1016/j.crme.2016.04.002
9. Iasnii V. et al. Experimental study of pseudoelastic NiTi alloy under cyclic loading. Sci. J. TNTU. 2018. Vol. 92. No. 4. P. 7–12. https://doi.org/10.33108/visnyk_tntu2018.04.007
10. Anderson T. W., Darling D. A. A Test of Goodness of Fit. J. Am. Stat. Assoc. [American Statistical Association, Taylor & Francis, Ltd.], 1954. Vol. 49. No. 268. P. 765–769. https://doi.org/10.1080/01621459.1954.10501232
11. Wu S.-X. Shape change of surface crack during fatigue growth. Eng. Fract. Mech. 1985. Vol. 22. No. 5. P. 897–913. https://doi.org/10.1016/0013-7944(85)90117-1
12. Nishitani N., Chen D. Stress Intensity Factor for a Semi-Elliptic Surface Crack in a Shaft under Tension. Trans. Japan Soc. Mech. Eng. Ser. A. 1984. Vol. 50. No. 453. P. 1077–1082.

УДК 539.3

ПРОГНОЗУВАННЯ ЗАЛИШКОВОЇ ДОВГОВІЧНОСТІ СПФ З УРАХУВАННЯМ РОЗКИДУ ХАРАКТЕРИСТИК МЕХАНІЧНИХ ВЛАСТИВОСТЕЙ ЗА СТАЛОЇ АМПЛІТУДИ НАВАНТАЖЕННЯ

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Резюме. *Сплави з пам'яттю форми широко застосовують у багатьох галузях відносно недавно, однак вже відомі випадки їх руйнування. Оскільки розвиток науки та техніки висуває нові підвищені вимоги до безпеки таких важливих конструкцій і, зокрема, до пристроїв, які в них використовуватимуться, постає питання: як довго такі елементи конструкцій можна експлуатувати. Для відповіді на це запитання необхідно вміти прогнозувати залишкову довговічність таких сплавів. Діаграми втомного руйнування (ДВР) зазвичай містять значний розкид, котрий можна врахувати, приміром, побудувавши розподіл параметрів, що входять до рівняння, яким описують ДВР. Тому потрібно прогнозувати ріст втомних тріщин з урахуванням розкиду характеристик механічних властивостей, попередньо визначивши статистичний розподіл параметрів циклічної тріщиностійкості, зокрема сталих C і n рівняння Періса. Відомо, що параметри рівняння Періса взаємозалежні. Розглядаючи C як випадкову змінну, яка змінюється від зразка до зразка, можна врахувати існуючий розкид даних. Дефекти, котрі виявляють в елементах конструкцій, часто мають форму поверхневих півеліптичних тріщин. Швидкість росту втомної тріщини псевдопружного NiTi сплаву досліджували експериментально за одновісного розтягу циліндричних зразків діаметром 8 мм при кімнатній температурі на повітрі на сервогідравлічній випробувальній машині СТМ-100. Запропоновано методику прогнозування залишкової довговічності циліндричних зразків з поверхневою півеліптичною тріщиною із СПФ, котра ґрунтується на розв'язанні системи рівнянь, що описують поширення тріщини, куди входять параметри навантаження та характеристики циклічної тріщиностійкості, з урахуванням їх статистичного розкиду та зміни форми фронту тріщини під час її поширення. Побудовано кумулянти розподілу довговічності зразків діаметром 2r для різної відносно початкової глибини тріщини (b/r = 0,25; 0,36; 0,5; 0,75) та коефіцієнта форми півеліптичної тріщини (1/3; 1/2; 2/3; 3/4).*

Ключові слова: *псевдопружний сплав з пам'яттю форми, залишкова довговічність, рівняння Періса, моделювання, втомна тріщина, коефіцієнт інтенсивності напружень.*

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