



UDC 539.3

## ON APPROXIMATION OF MECHANICAL CONDITION DIAGRAMS OF CONIFEROUS AND DECIDUOUS WOOD SPECIES ON COMPRESSION ALONG THE FIBERS

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**Summary.** A detailed analysis of the dependencies for the construction of  $\sigma$ - $u$  diagrams of various coniferous and deciduous wood species for compression along the fibers by the national and foreign scientists was carried out. It is established that polynomial and irregular fractional-rational functions are the most acceptable for approximation of stress-strain diagrams of wood. These dependencies can describe the complete diagrams of deformation of wood on the ascending and partially on the descending branches. It is established that the most appropriate function for describing the stress-strain diagram is a 4th degree polynomial.

**Key words:** wood, stress, strength, deformation, diagram, compression, dependencies.

[https://doi.org/10.33108/visnyk\\_tntu2020.01.057](https://doi.org/10.33108/visnyk_tntu2020.01.057)

Received 16.03.2020

**Statement of the problem.** Recently the products, parts, materials and structures made of wood have become of great interest. This phenomenon is mostly noticed in the foreign countries because of the fact, that wood is a natural material possessing good enough strength and deformation properties as compared with those of other materials and structures (concrete, reinforced concrete and metal). The characteristic feature of different wood species is that of having very fine structure, which does not need to be hidden additionally using other materials.

Lately wood has become of special application in different branches of the national economy, construction engineering and similar areas in particular. At the same time there is a need to carry out additional and detailed investigation of the coniferous and deciduous wood species (both theoretical and experimental). It will make possible to learn better physical-mechanical properties of wood starting from its operation till deformation. With this purpose [1, 2] profound experimental investigations of coniferous (fir-three, pine, white wood) and deciduous (birch, alder, ash-tree) wood under severe mode of testing (growing displacement) have been carried out. The wood of both continuous cross-section and that of pasted, as well as the modified of structural sizes [3, 4], were the subject of testing. The specimens were under the single, short-term compression testing along the fibers of 30x30x120 cross-section, as a rule. About 500 specimens have been tested. After experimental investigations the complete diagrams of the wood deformation since the loading starting till the wood deformation (ascending and descending branches) have been built [1]. That is why the approximation of the obtained diagrams using different dependencies is the pressing problem for both foreign and national scientists.

**Analysis of the available investigations.** Till lately the experimental investigations of wood on compression along the fibers were carried out mostly on the specimens of so-called «plain wood» of 20x20x30 mm cross-section, which does not make possible to estimate properly the stress-strain state of the material. Such investigations must be performed on the specimens of the structural sizes in order to take into account completely the wood under severe testing mode. These investigations, as a rule, were performed on the pine wood of structural

sizes. According to the experimental investigations the authors have built the complete diagrams of the pine wood deformation till the total specimen failure [5], the limits being in the point of yield strain [6]. We were the first to carry out such experimental investigations on the other wood species (birch, alder, fir-tree, pine, ash-tree) [1]. The investigations were performed on the servohydraulic machine CTM-100 [7, 8, 9] with automatic control and the system of data picking up at the Ternopil Ivan Puluji National Technical University. According to the results of experiment the complete deformation diagrams of all investigated wood species were built [1].

Besides, the approximation of such diagrams by different functions during all periods of the wood operation is one of the problems. Foreign and national scientists have already tried to describe such diagrams using different dependencies. But, unfortunately, these diagrams were built only for the experimental data obtained under the soft testing mode (loading being increased). That is, the approximation was in certain area, but not till the total specimen failure. The attempts to make proper approximation under the severe testing mode of the pine wood were done in the paper [6]. That is why the approximation of diagrams of other coniferous and deciduous wood species is still a pressing problem nowadays.

**Statement of the task.** The objective of the article is the approximation of the complete diagrams of different wood species deformation (up till its failure), as well as with the limits in the point of critical and yield strain, and determination of the function for such approximation.

**Presentation of the basic material.** Having analyzed the literature thoroughly and the preliminary approximation of our diagrams, we came to the conclusion, that the most convergent functions are those of polynomial and irregular fraction-rational dependencies [10].

To find the function, which would give the greatest convergence of experimental and theoretical data, it is necessary to compare these dependencies according to our experimental investigations and those of other authors [5, 6].

The first function is the function of cubic parabola [11]

$$\sigma_c = K_1 \cdot u + K_2 \cdot u^3, \quad (1)$$

where  $K_1, K_2$  – are the polynomial coefficients, which are found according to the formulas (2) and (3) correspondingly:

$$K_1 = \frac{1,5 \cdot f_{c,0,d}}{u_{c,0,d}}; \quad (2)$$

$$K_2 = -\frac{f_{c,0,d}}{2 \cdot u_{c,0,d}^3}; \quad (3)$$

The second function is the irregular fraction-rational function proposed in [10]

$$\sigma_c = \frac{E_{co} \cdot u_c - f_{c,0,d} \cdot \left(\frac{u_c}{u_{c1}}\right)^2}{1 + \left(\frac{E_{co}}{f_{c,0,d}} - \frac{2}{u_{c1}}\right) \cdot u_c}, \quad (4)$$

The third function, according to our proposal, is the polynomial of 4<sup>th</sup> degree, obtained by the Lagrangian polynomial:

$$\sigma_c = w_1 \cdot u_c + w_2 \cdot \frac{u_c^2}{u_{c,0,d}} + w_3 \cdot \frac{u_c^3}{u_{c,0,d}^2} + w_4 \cdot \frac{u_c^4}{u_{c,0,d}^3} = \sum_{i=1}^4 w_i \frac{u_c^i}{u_{c,0,d}^{i-1}}, \quad (5)$$

where  $w_1, w_2, w_3, w_4$  – are the coefficients obtained using the Lagrangian polynomial and are calculated using the software Microsoft Excel.

According to three functions (1, 4, 5) let us built the approximation diagrams of our experiments and compare the obtained diagrams with our experimental data with the limits in the point of yield strain (Figure 1). The diagrams of pine wood are assumed to be the basis.

The convergence of experimental data and theoretical values found according to the functions (1, 4, 5) is determined by the methods of mathematic statistics

$$\eta = \sqrt{1 - \frac{\sum \Delta^2}{\sum \alpha^2}}, \quad (6)$$

where  $\Delta = \sigma_{cep} - \sigma_{amp}$  – are the deviations of the experimental average value of diagram from the corresponding (at the given  $u_c$ ) values using the approximation function;

$\alpha = \sigma_{cep} - M$  – is the central deviation;

$M = \frac{\sum \sigma_{cep}}{n}$  – is the average arithmetic stress values.

Calculation of convergence of the experimental data of Gomon S. S. pine wood construction sizes and theoretical values with the limits in the point of yield strain is presented on Table 1.

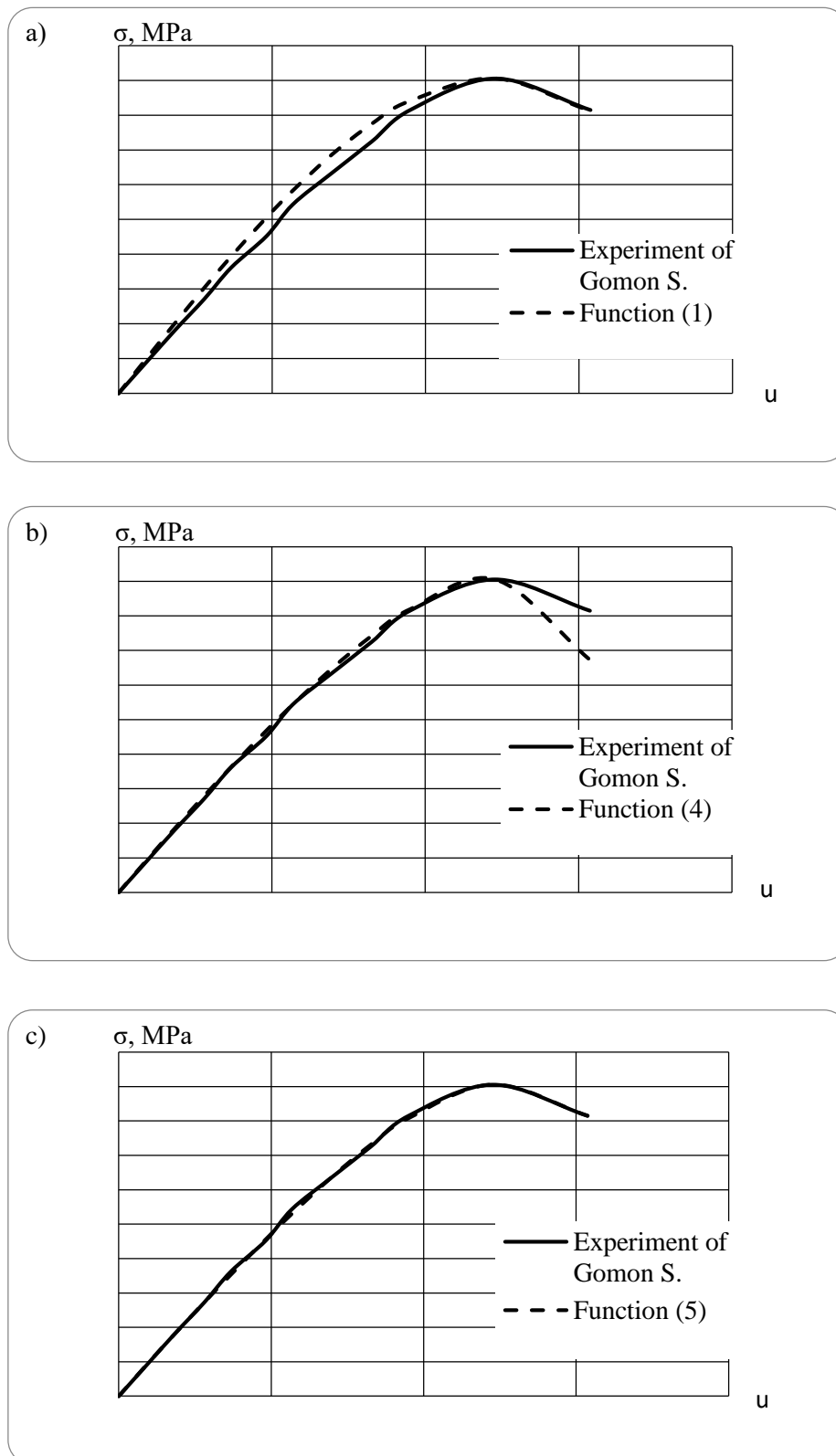
**Table 1**

The convergence of the experimental data of Gomon S. S. pine wood and theoretical values

Function	Cubic parabola $\sigma_c = K_1 \cdot u + K_2 \cdot u^3$	Irregular fraction-rational $\sigma_c = \frac{E_{co} \cdot u_c - f_{c,0,d} \cdot \left(\frac{u_c}{u_{c1}}\right)^2}{1 + \left(\frac{E_{co}}{f_{c,0,d}} - \frac{2}{u_{c1}}\right) \cdot u_c}$	4 <sup>th</sup> degree polynomial $\sigma_c = \sum_{i=1}^4 w_i \frac{u_c^i}{u_{c,0,d}^{i-1}}$
Convergence of theoretical and experimental values of Gomon S. S, $\eta$	0,991	0,986	0,999

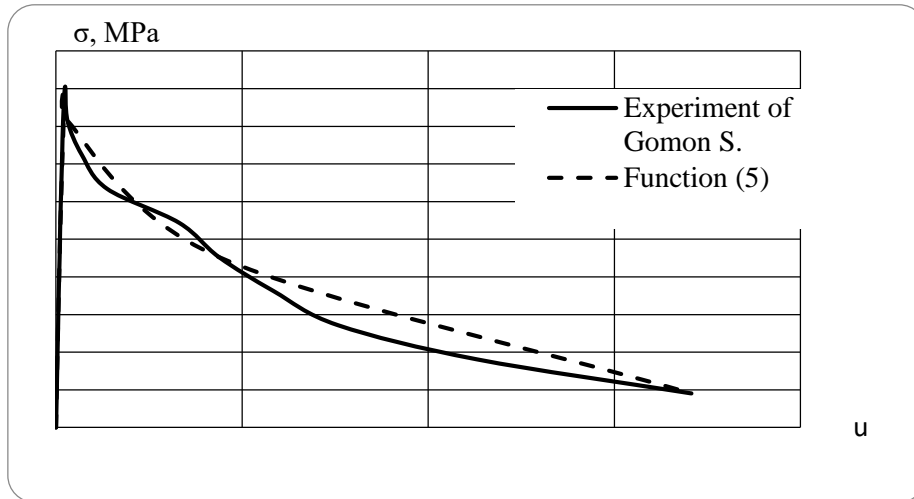
According to the results presented on Table 1 we see, that the convergence of experimental and investigation data of the pine wood is good enough for all dependencies (with limits in the point of yield strains). And the function (5) has a different convergence  $\eta=0,999$ . Besides, it is seen, that the functions (4) and (5) are of ideal convergence on the diagrams with

limit in the point of critical strain. The comparison of functions (1, 4, 5) with our experimental data of other wood species (birch, alder, fir-tree, pine and ash-tree) is of similar nature.



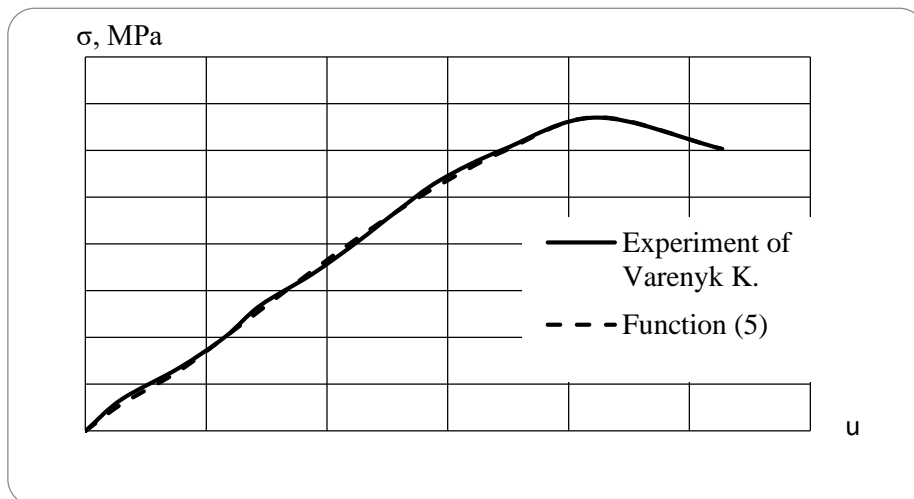
**Figure 1.** Diagrams of pine wood, constructed according to experimental values of Gomon S. and according to the functions of: a) cubical parabola, b) irregular fractional rational, c) polynomial of 4th degree

If we analyze the complete diagram of the pine wood under compression along the fibers, the dependencies (1) and (4) do not work in the lower parts of the descending branch, and the function (5) in this area is of satisfactory convergence (Figure 2), being similar to other wood species.

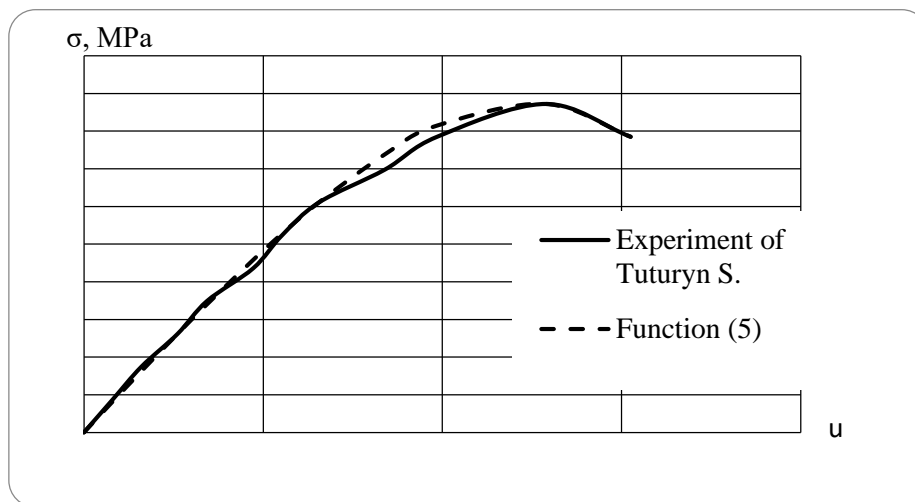


**Figure 2.** Complete diagram of pine wood, which is constructed according to experimental values of Gomon S. and by the function of a polynomial of 4th degree

Let us make convergence of the function (5) with the experimental values of other scientists, the experimental data by Varennyk K. A. [6] (Figure 3) and by Tuturyn S. V. [5] (Figure 4) in the point of yield strain.



**Figure 3.** Diagram of pine wood constructed according to experimental values of Varennyk K. A. [6] and by the function of a polynomial of 4th degree



**Figure 4.** Diagram of pine wood based on experimental values of Tuturyn S. V. [5] and by the function of a polynomial of 4th degree

Calculation of convergence of the investigation data of the pine wood of structural sizes under compression along the fibers of Gomon S. S., Varenyk K. A. [6], Tuturyn S. V. [5], as well as theoretical values according to function (5) worth the limits in the point of yield strains, are presented on Table 2.

**Table 2**

The convergence of experimental data of pine wood of different scientists and theoretical values by function (5)

Scientists' investigation data	Gomon S. S.	Varenyk K. A. [6]	Tuturyn S. V. [5]
Function	4 <sup>th</sup> degree polynomial $\sigma_c = \sum_{i=1}^4 w_i \frac{u_c^i}{u_{c,0,d}^{i-1}}$	4 <sup>th</sup> degree polynomial $\sigma_c = \sum_{i=1}^4 w_i \frac{u_c^i}{u_{c,0,d}^{i-1}}$	4 <sup>th</sup> degree polynomial $\sigma_c = \sum_{i=1}^4 w_i \frac{u_c^i}{u_{c,0,d}^{i-1}}$
Convergence of theoretical and investigation values, $\eta$	0,999	0,999	0,997

Thus, the ideal convergence of the function (5) with the experimental values of other scientists is seen (Table 2).

After approximation of the mechanical condition diagrams of all wood species (complete diagram from the operation starting till the failure, diagrams with limits in the point of critical and yield strains) (Figure 1, 2, 3, 4) and the carried out mathematic statistics (Table 1, 2), it was determined that the most favorable function for the description of these diagrams in all areas is the dependence (5). That is why it is accepted to be used in the further calculations.

**Conclusions.** Approximation of mechanical condition diagrams of different coniferous and deciduous wood species of the structural sizes on compression along the fibers, has been carried out. Polynomial and irregular fraction-rational functions being of the greatest convergence with the experimental investigations, have been analyzed. The dependence (the 4<sup>th</sup> polynomial) was obtained being of the greatest convergence with our experimental data and

investigations of other scientists on the ascending and descending branches of the diagram « $\sigma$ - $u$ », of Varenik K. A. [6], Tuturyn S. V. [5] in particular. It was determined, that approximation of the complete diagrams «stress-strain» (since loading initiation up till the specimens failure) of different coniferous and deciduous wood species is worth being done using the function of the 4<sup>th</sup> polynomial, because the other functions work badly or do not work at all on the descending branch of the diagram « $\sigma$ - $u$ » after the point of yield strains. Further approximation of the complete diagrams of the pasted and modified wood on compression along the fibers must be fulfilled.

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## УДК 539.3

# АПРОКСИМАЦІЯ ДІАГРАМ ДЕФОРМУВАННЯ ХВОЙНИХ ТА ЛИСТЯНИХ ПОРІД ДЕРЕВИНИ СТИСКОМ УЗДОВЖ ВОЛОКОН

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**Резюме.** Проведено детальний аналіз залежностей для побудови діаграм « $\sigma$ - $\epsilon$ » різних хвойних та листяних порід деревини на стиск вздовж волокон вітчизняними та закордонними вченими. Встановлено, що для апроксимації діаграм «напруження-деформації» деревини найбільш прийнятними є поліноміальні та неправильні дробово-раціональні функції. Побудовано діаграми деревини сосни конструкційних розмірів за нашими експериментальними значеннями та відповідно за функціями: а) кубічної параболі, б) неправильною дробово-раціональною; в) полінома 4-го ступеня. Встановлено збіжність теоретичних та експериментальних значень методами математичної статистики. Дані залежності можуть описувати повні діаграми деформування деревини на висхідній та частково на спадній вітках. Побудовано діаграми деревини сосни конструкційних розмірів за експериментальними значеннями Вареника К. А., Тутурина С. В. та за функцією полінома 4-го ступеня. Розраховано, що збіжність теоретичних та експериментальних значень з обмеженням у точці граничних деформацій близька 1. Для інших порід деревини (берези, вільхи, модрини, ялини, ясена) збіжність носить такий же характер. Також визначено, що опис діаграм на висхідній вітці (з обмеженням у точці критичних деформацій) можливо здійснювати за будь-якою із вище наведених залежностей для різних порід деревини. Встановлено, що апроксимацію повних діаграм «напруження-деформації» (від початку завантаження і аж до руйнування зразків) різних хвойних та листяних порід деревини найбільш прийнятно здійснювати за допомогою функції полінома 4-го ступеня, тому що інші функції погано працюють або взагалі не працюють на спадній вітці діаграми « $\sigma$ - $\epsilon$ » після точки граничних деформацій. Отже, найприйнятнішою функцією для опису діаграми «напруження-деформації» на стиск вздовж волокон є поліном 4-го ступеня. Тобто, дана функція буде прийнята в майбутньому для подальшої роботи та розрахунків елементів та конструкцій з деревини. В подальшому необхідно провести апроксимацію повних діаграм клеєної та модифікованої деревини конструкційних розмірів на стиск вздовж волокон.

**Ключові слова:** деревина, напруження, міцність, деформації, діаграма, стиск, залежності.

[https://doi.org/10.33108/visnyk\\_tntu2020.01.057](https://doi.org/10.33108/visnyk_tntu2020.01.057)

Отримано 16.03.2020