Ministry of Science and Education of Ukraine

Ternopil Ivan Puluj National Technical University

Department of Technical
Mechanics and Agricultural Machinery

STRENGTH OF MATERIALS

COURSE BOOK
for practical works

for the students majoring in
Industrial Machinery Engineering,
Applied Mechanics,
Automobile Transport

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"Strength of Materials" is a course book developed in accordance with the curriculum and is intended for practical work and self-studies of the foreign students majoring in Applied Mechanics, Industrial Engineering, Automobile Transport. The book contains theoretical notes of the main material from the subject "Strength of Materials", practical tasks, examples of their solution as well as necessary reference data.

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CONTENTS

INTRODUCTION

Strength of materials is the science of engineering methods for calculating the strength, rigidity and durability of machine and structure elements.

Elements of mechanical engineering and building structures during operation are subjected to the force action of different nature. These forces are either applied directly to the element or transmitted through joint elements. For normal operation of engineering structure or machine, each element must be of such sizes and shapes that it can withstand the load on it, without fracture (strength), not changing in size (rigidity), retaining its original shape (durability).

Strength of materials is theoretical and experimental science. Experiment – theory – experiment – such is the dialectic of the development of the science of solids resistance to deformation and fracture. However, the science of strength of materials does not cover all the issues of deformable bodies mechanics. Other related disciplines are also involved: structural mechanics of core systems, elasticity theory and plasticity theory.

Strength of materials is general engineering science, in which, on the basis of experimental data concernimg properties of materials, on one hand, and rules of theoretical mechanics, physics and higher mathematics, on the other, the general methods of calculating rational sizes and shapes of engineering structures elements, taking into account the size and character of loads acting on them are studied.

Strength of materials tasks are solved by simple mathematical methods, with a number of assumptions and hypotheses, as well as with the use of experimental data.

Strength of materials has independent importance, as the subject, knowledge of which are required for all engineering specialties. It is the basis for studying all sections of structural mechanics, the basis for studying the course of machine parts, etc. Strength of materials is the scientific basis of engineering calculations, without which at rescent time it is impossible to design and create all the variety of modern mechanical engineering and civil engineering structures.

The peculiarity of this course book is its focus on performing the term paper in strength of materials, which includes 14 tasks covering the entire course. The manual summarizes the main material for the topic of each task, outlines the statement of the task, and examples of solutions.

The appendices provide the example of term paper structure (title page, contents, example of solving the task) and reference materials needed for its performance. All this will contribute to deeper course learning and independent performance of the term paper.

How to choose the task

The student chooses term paper assignment according to the last two figures of the credit book number; number of calculation scheme is chosen according to the last figure; option (data from the task statement table) is the second to the last figure.

The term paper in strength of materials contains 14 tasks (the number of task can be changed by the instructor), which cover the entire course. It should be performed in the form explanatory calculation note on A4 sheets.

The title page should be drawn or computer typewritten on the appropriate form.

The first page of the term paper is the title page, the second is the content which includes the list of completed tasks; next are the task statement terms, the tasks solution and references.

The statement of each task with the selected data and the scheme should be recorded on the separate sheet with a frame 40 mm. The task solution should be presented after task statement on sheets with 15 mm frame.

The text of the note should be presented sequentially, concisely, the calculations should be accompanied by brief explanations with reference to the relevant figure. The style of note text presentation should be concise, clear and without ambiguity. The terminology in the text must meet the standard of the scientific technical literature.

The text of the explanatory note should be placed on one side of the A4 (297x210 mm) sheet. The distance from the border to the borders of the text on the left and right must be at least 5 mm, top and bottom are 10 mm. Paragraphs in text begin with a space of five characters in the body of the note. Type the text with 1,5 intervals in clear fonts of at least 2,5 mm in height (14 pt, Times New Roman font) or handwritten in black ink in basic lettering and letters at least 2,5 mm high. Explanatory notes may be written in clear legible handwriting in black ink.

Start counting from the cover page, but do not put the number on the cover page. Page numbering is continuous.

Formulas in the text must be written from the new line in the general form, and under the formula the explanation of each character, indicating size and dimension should be given. Calculate formulas in the following order: writing the desired value in the alphabetical expression, substituting the corresponding numerical values and recording the final result indicating the dimensions.

Make all the diagrams and sketches of the term paper on separate page or two, if necessary, in accordance with the sequence provided by the solution course. Figures should be enumerated according to the task number and accompanied by indexes.

1. BASIC CONCEPTS OF STRENGTH OF MATERIALS

Strength of materials problems

Strength of materials is the science of engineering methods for calculating the strength, rigidity and durability of machines and structures elements.

Structures are all material objects of technology, their parts and details.

Strength is the ability of material or structure to withstand mechanical stress without fracture

$$p_{\max} \le [p],$$

where $p_{\max}$ is maximum stress;

$[p]$ is allowable stress.

Rigidity is the ability of the structure and its elements to withstand elastic deformations, i. e. the ability to perceive external loading without changing the geometric dimensions and shape

$$f_{\max} \le [f],$$

where $f_{\max}$ is maximum deformation (displacement);

$[f]$ is allowable deformation (displacement).

Durability is the ability of the structure or its elements to retain, under the action of given forces, the initial shape of the elastic equilibrium.

The objective of the strength of materisals course:

a) to learn to determine correctly the type of deformation on which the part or structure operates according to the calculation scheme;

b) to determine the most dangerous section by pre-plotting internal force factors;

c) to determine the dimensions of the cross-section with appropriate strength or rigidity and, in some problems, allowable load or maximum stress, and to carry out the strength test.

Calculation objects in strength of materials

All elements of engineering constructions and structures can be reduced to the following typical simplified elements: rods, shells, plates, massive bodies. According to them, the calculations in the strength of materials are carried out.

***Rod* (*bar*)** is a body of prismatic shape where one size (length) is much bigger than the other two (transverse) dimensions.

Thin-walled rods (channels, angles, I-beam) are bodies in which the wall thickness is much smaller than the overall dimensions of the cross-section.

Examples of rods: shafts, axles, beams, pipes, rails, curvilinear elements (screw springs, hooks, chain elements).

Plate is a prismatic (cylindrical) body in which one size (thickness) is much smaller than two others.

Examples of plates: plane bottoms and covers of tanks, chemical production facilities, floor slabs.

Shell is a body restricted by two curvilinear surfaces, the distance between which (thickness) is small in comparison with other dimensions. This is a plate with curved middle surface. Examples: walls of thin-walled tanks, walls of boilers, domes of building structures, hulls of aircrafts, rockets, submarines.

***Solid* (*massive body*)** is the body dimensions of which are of the same order in all (three) directions. Examples: foundations of structures, retaining walls, foundations of powerful presses and machine tools.

Classification of external loads

External loads are classified:

1. By the action nature – static, dynamic.

Static is the load which values, direction and place of application remain constant.

Dynamic are loads that are characterized by rapid changes in their value in time, direction, or place of application.

2. By nature of application (Fig. 1.1):

a) F, Q, R – concentrated forces $[\text{N}, \text{kN}, \text{MN}]$;

b) M, T – moments $[\text{Nm}, \text{kNm}, \text{MNm}]$;

c) q, w – distributed on line $[\text{N/m}, \text{kN/m}]$.

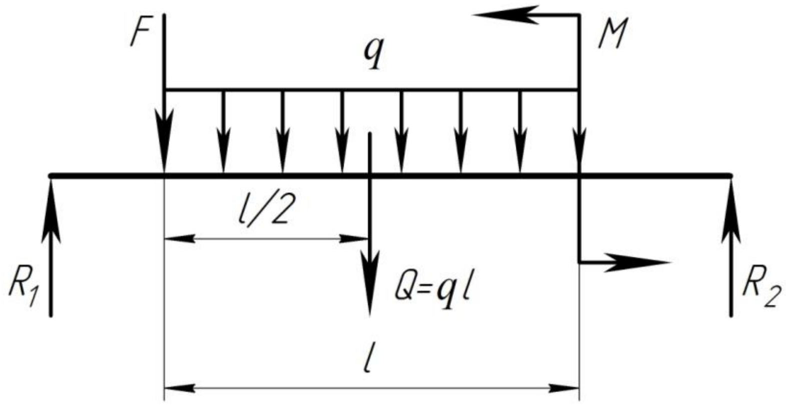


Figure 1.1

Calculation scheme is the real object, free of insignificant features. More than one calculation scheme may be developed for the same object, depending on the load features and operating conditions.

The main types of deformation

All existing bodies under the influence of external forces are able to change their size and shape, i.e. to deform.

In strength of materials we distinguish tensile deformation (compressive), shear, torsion and bending. Different types of deformation in the cross-sections of the body have different internal force factors.

1. **Tensile-compressive** is a type of deformation in which only ***longitudinal* (*axial*) *force*** N occurs in the cross-sections of a straight bar.

The stretching bar is called a rod.

Elements subjected to tensioning are such structural elements as ropes, bolts, cables, truss rods, piston rods. Brick masonry, foundation, columns, punches work on compression.

2. **Shear** is a type of deformation, at which in the cross-section of the rod (bar) only ***shear* (*cutting*) *force*** Q acts. The shear deformation results in material fracture. Rivets, bolts, keys, seams of welded joints undergo shear.

3. **Torsion** is a type of deformation in which only ***torque moment*** M_{TR} , acts in the cross-sections of the rod. The circular cross-section rod (bar) transmiting power during rotational motion is called the shaft. Torsion is often accompanied by bending or other deformation.

4. **Direct lateral bending** is a type of deformation in which ***the bending moment*** M_{BN} and the shear (cutting) force Q occur at the cross sections of the beam. The bending rod (bar) is called the beam. This bending occurs in axes, bridge and floor beams, gear-wheel teeth, leaf springs.

5. **Complex strength** is the combination of two or more simple types of deformation, such as: ***bending + torsion; compression + bending***, etc.

Internal power factors. Section method. Diagram

Internal force factors are internal forces of interaction between particles of the body that occur during the action on the body of external forces, and prevent changes in the distances between the particles and the fracture of the body. They are called ***forces***. External forces applied to the structural element and reactions at the places of supports attachment, that is, active and reactive forces are called ***loads***.

In order to determine the magnitude of the internal forces (force factors) occuring at the section of the rods, the cross-section method is used.

The section perpendicular to the axis of the bar is called ***normal*** or ***shear***; the section drawn at any other angle, is called ***oblique*** or ***inclined***.

The method of sections is that the elastic body (rod), which is in equilibrium under the action of external forces system, is imaginary cut by the plane into 2 parts (Fig. 1.2 *a*). Any of them are neglected. The remaining part is considered as the independent body, which is in equilibrium while applying to it the internal forces of interaction (effort) arising between the two parts of the body under the influence of external forces (Fig. 1.2 *b*). Internal forces replace the impact of the neglected part of the rod (bar) on the rest. It is fundamentally irrelevant which part of the body is neglected.

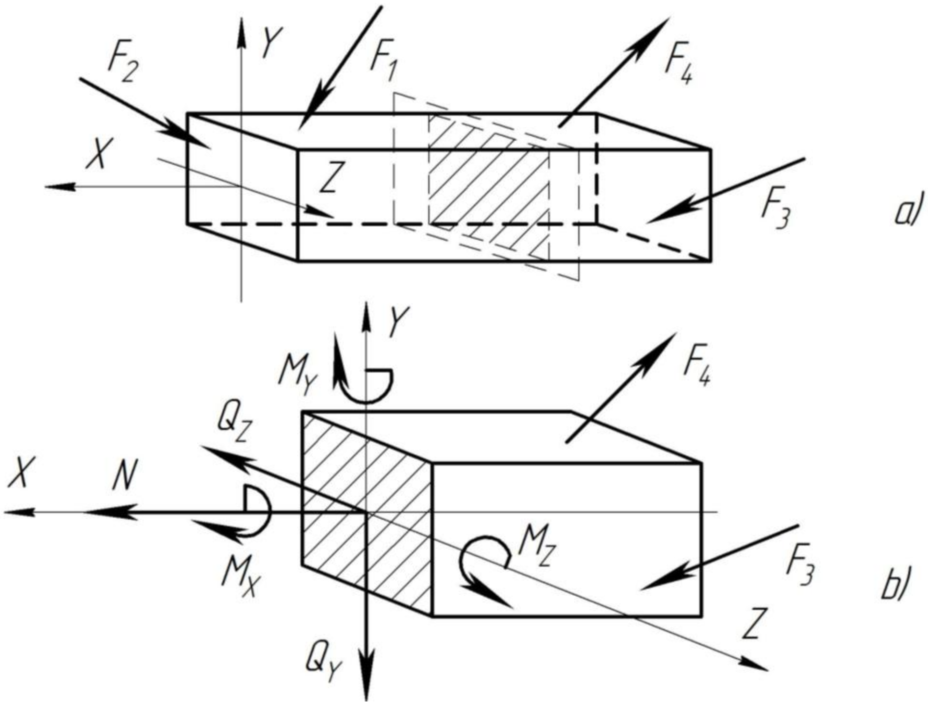


Figure 1.2

In the general case of loading the rod (bar) in its cross-section, six internal force factors occur:

N_X is longitudinal (normal) force acting along the axis of the rod (bar), perpendicular to the section plane;

Q_Y, Q_Z are shear (cutting) forces tangent to the section plane, trying to move one part of the rod (bar) relatively to the other in the directions of OY, OZ axes;

M_Y, M_Z are moments that rotate the section around OY, OZ axes, tending to bend the rod in planes XZ and XY, that is bending moments $M_Y = M_{BN.Y}$; $M_Z = M_{BN \cdot Z}$;

M_X is the moment acting in the section plane and causing the section rotation with respect to the longitudinal axis of the rod (bar) OX, that is, twists the rod (bar), is called its $M_X = M_{TR}$ torque.

Each of the internal force factors is associated with a particular type of deformation.

To determine the internal force factors in general, according to the method of sections, six conditions of equilibrium of forces acting on the remaining part of the rod (bar) (use six equations of static) are written. The algebraic sums of the projections of all forces applied to this part on the axis OX, OY, OZ, and the algebraic sums of the moments of these forces with respect to the same axes are equal zero:

$$\sum X = 0; \qquad N_X + \sum F_{iX} = 0;$$

$$\sum Y = 0; \qquad Q_Y + \sum F_{iY} = 0;$$

$$\sum Z = 0; \qquad Q_Z + \sum F_{iZ} = 0;$$

$$\sum M_X = 0; \qquad M_{TR} + \sum M_X(F_i) = 0;$$

$$\sum M_Y = 0; \qquad M_{BN.Y} + \sum M_Y(F_i) = 0;$$

$$\sum M_Z = 0; \qquad M_{BN.Z} + \sum M_Z(F_i) = 0.$$

Diagram is the graph showing the distribution of internal forces factors or displacements along the axis of the rod. **Diagrams are lined perpendiculary to the axis of the rod (bar).**

Stress

It is a quantity that characterizes the intensity of internal forces. The total stress (Fig. 1.3) is determined by the formula

$$p = \lim_{\Delta A \to 0} \frac{\Delta R}{\Delta A},$$

where ΔR is the internal force, i.e. the force applied to the allocated area;
ΔA is the elementary section area at which the effort ΔR occurs.

The internal force ΔR can be divided into two components: one directed perpendicular to the section ΔN; the other is located in the section plane ΔQ. The stresses that occur at the section of these components are called ***normal*** and ***tangential*** **(*shear*)**.

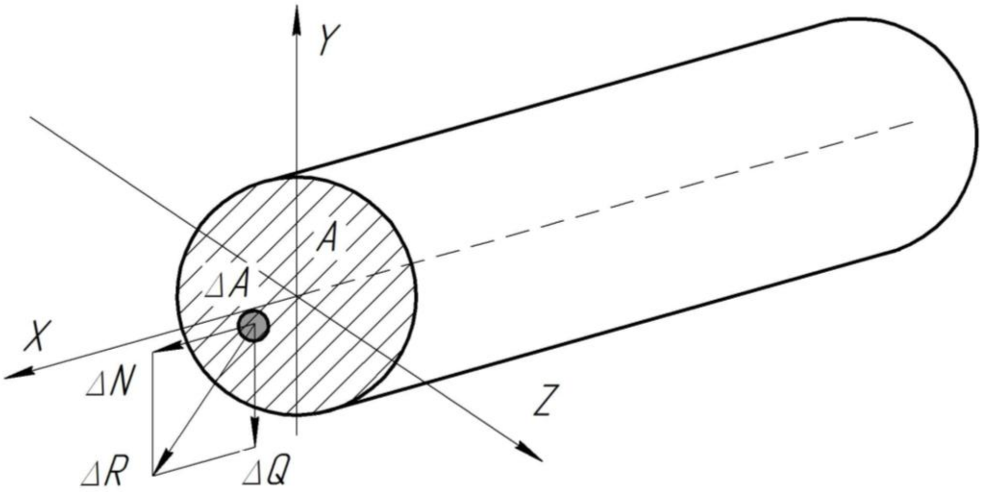


Figure 1.3

The tangential (*shear*) stress is the intensity of the tangent forces at the given point of section

$$\tau = \lim_{\Delta A \to 0} \frac{\Delta Q}{\Delta A}.$$

The normal stress is the intensity of normal forces at the given point of section

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta N}{\Delta A}.$$

The total stress at the point is determined by the formula

$$p = \sqrt{\sigma^2 + \tau^2}.$$

Unit of stresses $1\ \text{Pa} = \text{N}/\text{m}^2$, $1\ \text{MPa} = 10^6\ \text{Pa} = 1\,\text{N}/\text{mm}^2$.

Assumptions (hypotheses) about the properties of structural elements of materials

1. The hypothesis of the material continuity. It is suggested that the material completely fills the body volume, refuting the theory of the discrete structure of substances.

2. The hypothesis of homogeneity and isotropy. It is considered that the mechanical properties of material are the same at any point in the body and in any direction.

3. The hypothesis of the ideal elasticity and natural tension of the material. It is assumed that the deformations caused by the loads are completely disappeare after unloading and that initial forces and stresses are absent.

2. CENTRAL TENSION AND COMPRESSION OF DIRECT RODS (BARS)

Central tension and compression. Drawing the diagrams of normal force

Central (axial) tensile or compression occurs from forces applied along the central axis of the rod (bar). The stress state caused by such forces is called simple or linear.

Tension (compression) is the type of deformation (type of resistance) in which only **longitudinal (axial, normal) force** N or N_X directed along the axis of the rod (bar) and applied at the center of cross-section gravity occurs. It is determined from the equilibrium condition using the ***section method,*** starting from the free end of the rod (bar).

Longitudinal force in the random cross-section of the rod (bar) is equal to the algebraic sum of the projections on its longitudinal axis OX of all external forces applied to the rest part.

Under tension, the longitudinal force is directed from the section and is considered to be ***positive***, under compression it is directed to the section and is considered ***negative***.

In order to estimate the load of the rod (bar), in the case where the longitudinal forces in different cross-sections of the rod (bar) are unequal, the diagrams are drawn. While drawing the diagrams, the rod (bar) is divided into sections. The diagram is drawn in order to use it while calculating the strength. It makes it possible to determine the greatest value of the longitudinal force and the cross-section at which it occurs, that is, the dangerous (in terms of strength) cross-section.

Example. Draw the diagram of normal forces for the rod (bar) shown in Fig. 2.1 *a* (neglect the rod (bar) weight).

Divide the rod (bar) into sections. Area boundaries: beginning and fixing of the rod (bar); cross-sections where the concentrated forces are applied.

Using the section method, we determine the values of the normal forces at each area, starting from the free end.

Normal force is the algebraic sum of all external forces on one side of the intersection. Write down their values (Fig. 2.1 *b*) in each area, considering the rod from the free end:

$$N_1 = F_1 = 20 \text{ kN};$$

$$N_2 = F_1 - F_2 = 20 - 50 = -30 \text{ kN};$$

$$N_3 = F_1 - F_2 + F_3 = -30 + 40 = 10 \text{ kN}.$$

Based on the obtained results, draw the diagram of normal forces N (Fig. 2.1 *c*).

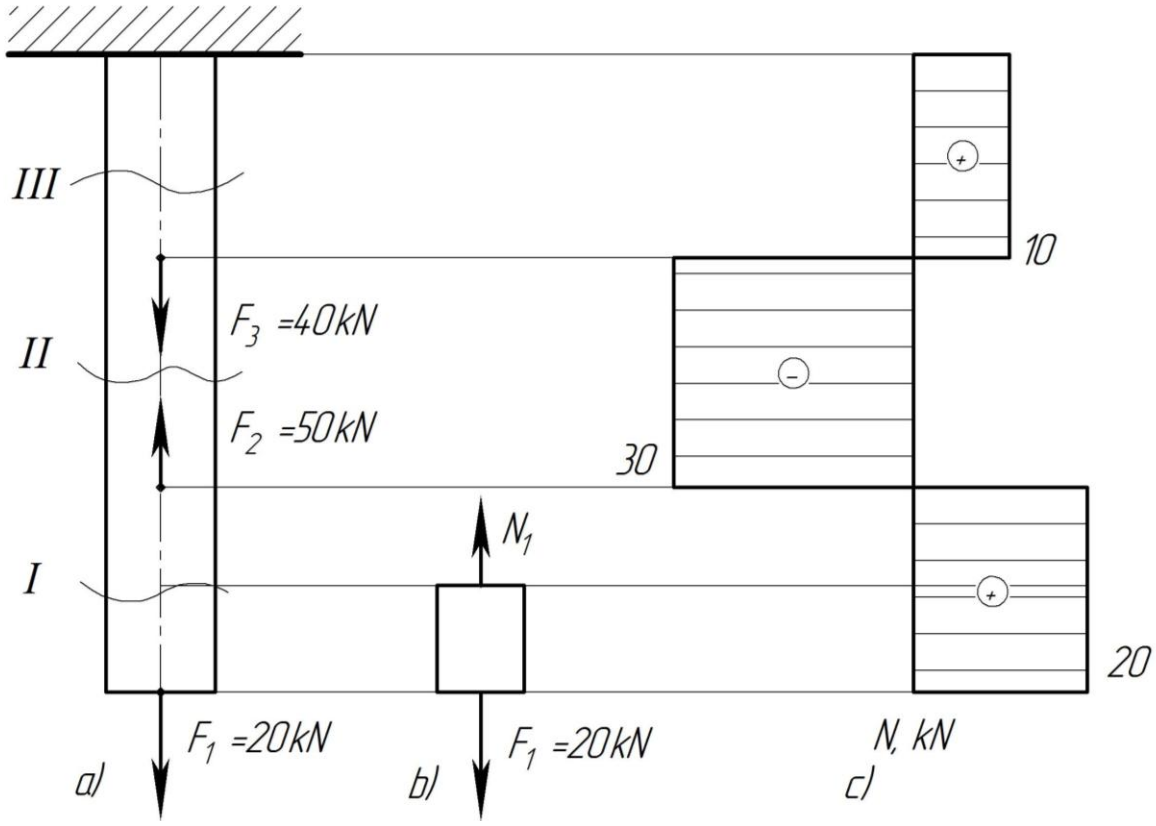


Figure 2.1

Stresses in the rod cross-sections

Under tensile (compression) of the rod, only **normal stresses** occur in its cross-sections.

Under stretching (compression) of a rod (bar) normal stresses on its cross-section are ***distributed evenly***.

There is the relationship between longitudinal (normal) force N and normal stress σ (Fig. 2.2)

$$N = \int_A \sigma \cdot dA .$$

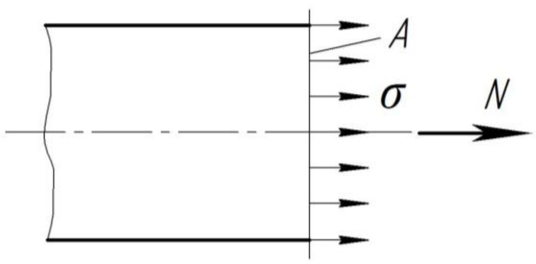


Figure 2.2

Let us assume, that $\sigma = const$, then

$$N = \sigma \cdot A ,$$

hence

$$\sigma = \frac{N}{A} .$$

Normal stresses are positive if they stretch the material of the rod (bar), negative – if they compress.

If the normal stresses in the different cross-sections of the rod (bar) are not the same, it is reasonable to show the law of their changes along the rod (bar) in the form of the graph – the diagram of normal stresses.

The tangential (shear) stresses are positive if the vector τ bypasses the material elements clockwise.

Longitudinal and transverse deformations

Let us consider deformation of the rod (bar) loaded with axial force F (Fig. 2.3):

Δl is total longitudinal elongation of the rod (absolute longitudinal deformation, linear elongation, linear deformation);

Δb is the absolute transverse deformation (linear deformation);

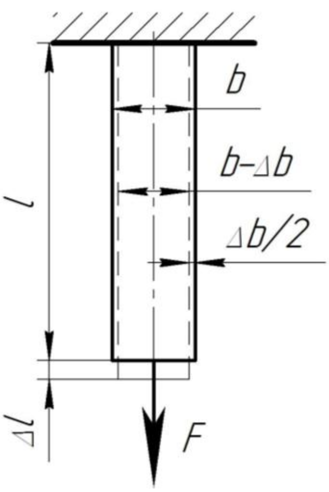


Figure 2.3

ε is relative longitudinal deformation, $\varepsilon = \Delta l / l$;

ε' is relative transverse deformation, $\varepsilon' = \Delta b / b$.

To a certain value, the deformation forces of elastic body are proportional to forces. Under tension (compression) there is a linear dependence between the elongation of the rod and the longitudinal force.

It is experimentally proved that the stresses in the rod material depend on deformation and mechanical characteristics of the material. This dependence is described as **Hooke law under tensile** (compression)

$$\sigma = \varepsilon \cdot E; \qquad \Delta l = \frac{N \cdot l}{E \cdot A},$$

where E is the modulus of elasticity (modulus of elasticity of the first kind, Young’s modulus, normal elastic modulus, longitudinal elastic modulus).

It is proved experimentally that under simple tensile or compression ratio of the transverse deformation to the longitudinal value is constant for this material. This ratio, taken in absolute value, is called the coefficient of transverse deformation or Poisson ratio

$$\mu = \left| \varepsilon' / \varepsilon \right|.$$

E, μ are mechanical characteristics of the material, determining its elastic properties. For steel $E = 2 \cdot 10^5$ MPa ; μ =0,3.

Hooke’s law is valid only for a certain value of normal stress, which is called the limit of proportionality of the given material.

Stress-strain diagram for plastic materials

Mechanical characteristics of materials, i.e. quantities that determine their strength, ductility, as well as elastic constants E and μ are necessary for design engineer to select the material of the part and its calculation for strength and rigidity. These characteristics are obtained experimentally. To do this, laboratory equipment is used on which the static tensile load (compression) is applied to the sample (Fig. 2.4 *b*) and then the forces and strains are measured. To exclude the influence of the absolute dimensions of the investigated sample, so-called conditional stretch diagram in coordinates is drawn: relative elongation ε, normal stress σ. For low carbon steel, the tensile (compression) diagram is shown in Fig. 2.4 *a*.

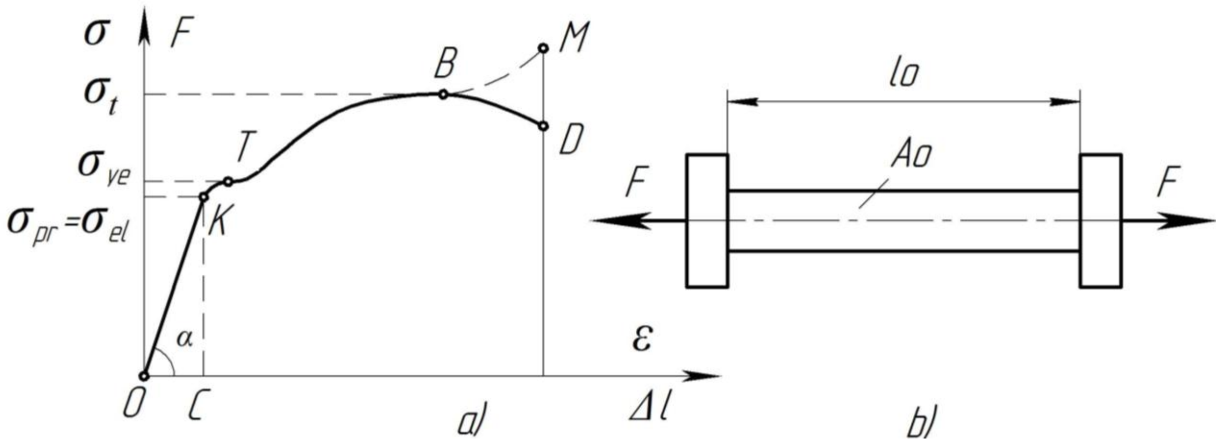


Figure 2.4

The indexes on the diagram are as follows:

σ_{pr} is **the limit of proportionality**, in this section deformation is proportional to the load, the highest stress, at which Hooke law is correct, $\sigma_{pr} = F_{pr} / A_0$;

σ_{el} is **the limit of elasticity**, up to this stress the material retains its elastic properties (no residual deformations occur in the sample at load removal), $\sigma_{el} = F_{el} / A_0$;

σ_{ye} is **the yield strength** is the stress at which the increase of plastic deformation of the sample at constant load occurs, this is the main mechanical characteristic for evaluation the durability of plastic materials (steels), $\sigma_{ye} = F_{ye} / A_0$;

σ_t is **the tensile strength** is the stress at which the fracture of the sample material occurs, that is, the conditional stress that corresponds to the highest load that the sample can withstand up to fracture, $\sigma_t = F_{\max} / A_0$.

Here A_0 is the initial cross-section area of the sample that undergo stretching; F_{pr}, F_{el}, F_{ye} are the increases in the magnitude of the tensile strength, $F_{\max}$ is the maximum load force without regard to the intersection narrowing.

The section of the OK stretching diagram (see Fig. 2.4 *a*) states Hooke law $E = \sigma / \varepsilon$.

Potential deformations energy

Under the static stretching of the rod (sample) within Hooke law application, the force F gradually increases from zero to certain value, the sample deforms by the value Δl (see Fig. 2.3) and thus performs the work *W*. This work is accumulated in the deformed sample as potential deformation energy, that is *W=U*.

If the tensile diagram (see Fig. 2.4 *a*) is drawn in the coordinates (*F*, Δ*l*), then the work is equal to the area of triangle *OCK:*

$$W = U = \frac{1}{2} \cdot F \cdot \Delta l ,$$

where $\Delta l = \dfrac{F \cdot l}{E \cdot A}$; $F = N$.

Then

$$W = U = \frac{F^2 \cdot l}{2F \cdot A} = \frac{\sigma^2 \cdot A \cdot l}{2E} = \frac{\sigma^2 \cdot V}{2E},$$

where F is the force sretching the sample, $F = \sigma \cdot A$;
V is body volume, i.e. the sample, $V = A \cdot l$;
A is the cross-sectional area of the sample.

Specific potential energy is the deformation energy per volume unit

$$U = \frac{W}{V} = \frac{\sigma^2}{2E}.$$

Allowable stresses. Strength calculations

In strength of materials there are three types of normal and tangential (shear) stresses: **working, boundary, allowable.**

Working (actual) stresses are those that actually occur in the structural elements and are determined by calculation or experimentally.

Boundary stresses are those at which material is destroyed or significant residual deformations occur in it.

To ensure the strength of the parts, it is necessary for the stresses occuring during their operation to be less than the boundary. But if the working stresses approximate the boundary ones (though they are less), the strength of the part cannot be guaranteed. Therefore, ***when calculating, the strength, the working stresses are compared not with the boundary, but with the allowable ones.***

The allowable stresses are those in which the safe work of the part is guaranteed. They are indicated by $[\sigma]$ or $[\tau]$ and determined as the fraction of the boundary stresses to guarantee the safety margin:

a) for plastic materials (steels)

$$[\sigma] = \frac{\sigma_{ye}}{n},$$

where $[\sigma]$ is allowable tensile and compressive stress;

n is strength factor;

b) for brittle materials (cast iron)

$$[\sigma]_t = \frac{\sigma_{st}}{n}; \qquad [\sigma]_c = \frac{\sigma_{cs}}{n},$$

where $[\sigma]_t$ is allowable tensile stress;

σ_{st} is tensile strength;

$[\sigma]_c$ is allowable compressive stress;

σ_{cs} is the boundary of compressive strength.

Safety margin reserve factor for plastic materials $n = 1{,}2 \ldots 2{,}5$; for brittle materials $n = 2 \ldots 5$.

Tensile-compression strength condition

$$\sigma = \frac{N}{A} \le [\sigma].$$

While calculating the strength of the parts, there are three main types of problems.

Design calculation which determine the size of the cross-section

$$A \ge \frac{N_{\max}}{[\sigma]},$$

where $N_{\max}$ is the maximum value of the longitudinal force, taken from the diagram N .

Validating calculation by which the working (actual) stresses are determined and compared with the allowable ones

$$\sigma = \frac{N_{\max}}{A} \le [\sigma].$$

Determination of allowable loads

$$[N] \le [\sigma] \cdot A .$$

Task 1
Strength calculation and displacement determination under tensile and compression

For given straight stepped steel rod (Fig. for task 1, Table for task 1), determine the dimensions of the cross-section at all sections, provided that the cross-section is a circle; make the rod sketch; draw the diagram of the working (actual) normal stresses and linear displacements of the rod, if $l = 8$ m $(a = k \cdot l,\ b = m \cdot l)$; rod material – steel; $[\sigma] = 160$ MPa ; $E = 2 \cdot 10^5$ MPa.

Plan of solving the task:

1. Complete the calculation model.
2. Draw the diagram of lineary forces.
3. From the strength condition, determine the diameters of the rod (bar) in all segments. Round off the obtained values to a size multiple of 2 or 5. Make a sketch of the rod (bar).
4. On each segment, calculate working (actual) normal stresses by the module σ_t and draw the diagram of working (actual) normal stresses.
5. Determine the lineary displacements of certain steps and the whole rod (bar).
6. Draw the diagram of the displacements distribution along the beam.

Table for task 1

Nr	F_1, kN	F_2, kN	F_3, kN	k	m
1	25	30	50	0,2	0,5
2	10	40	20	0,4	0,7
3	20	10	60	0,1	0,4
4	15	20	40	0,3	0,6
5	30	25	10	0,25	0,65
6	25	50	25	0,35	0,75
7	40	15	30	0,45	0,8
8	20	30	50	0,15	0,45
9	50	20	40	0,2	0,8
0	60	10	20	0,4	0,8

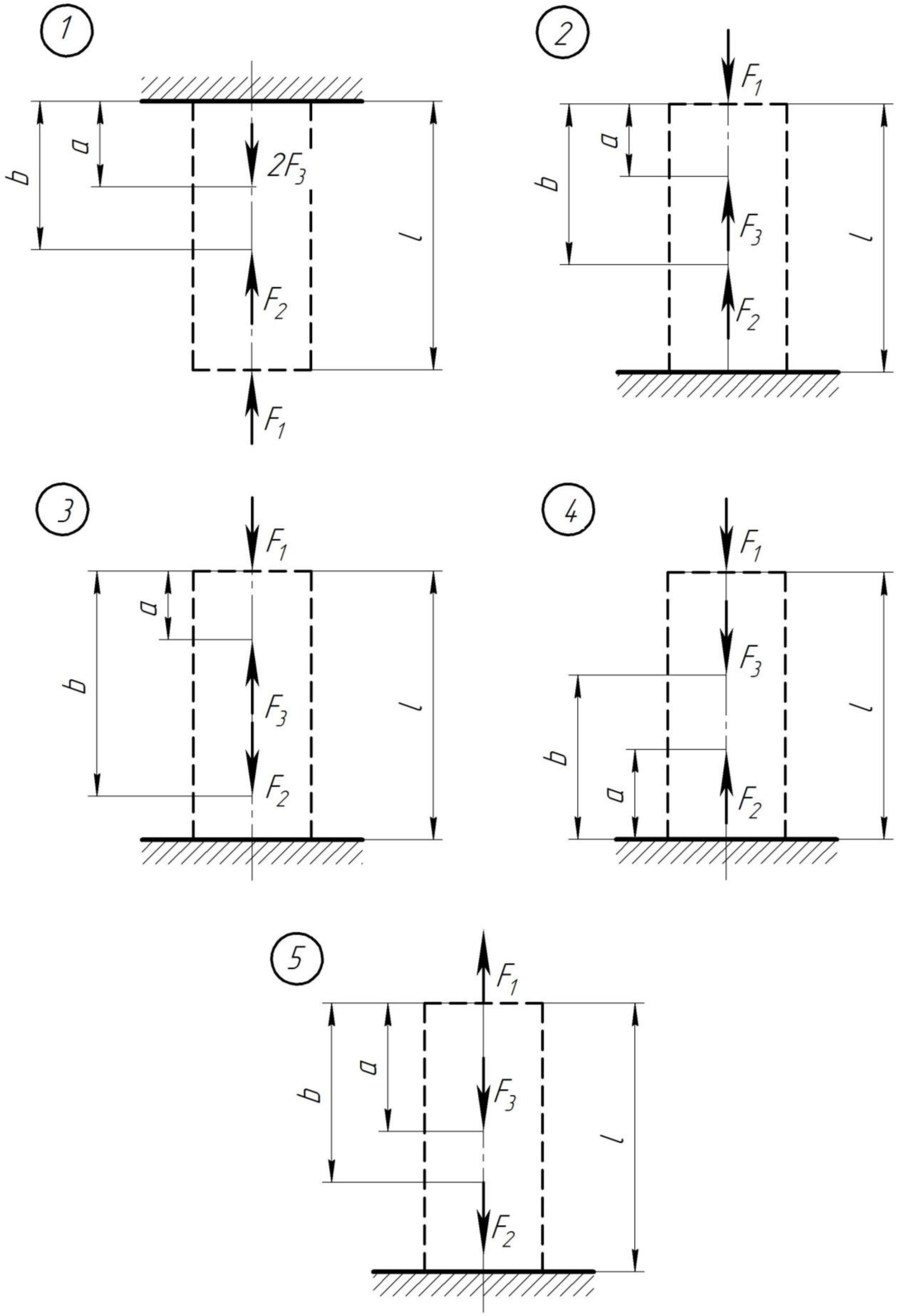


Figure for task 1

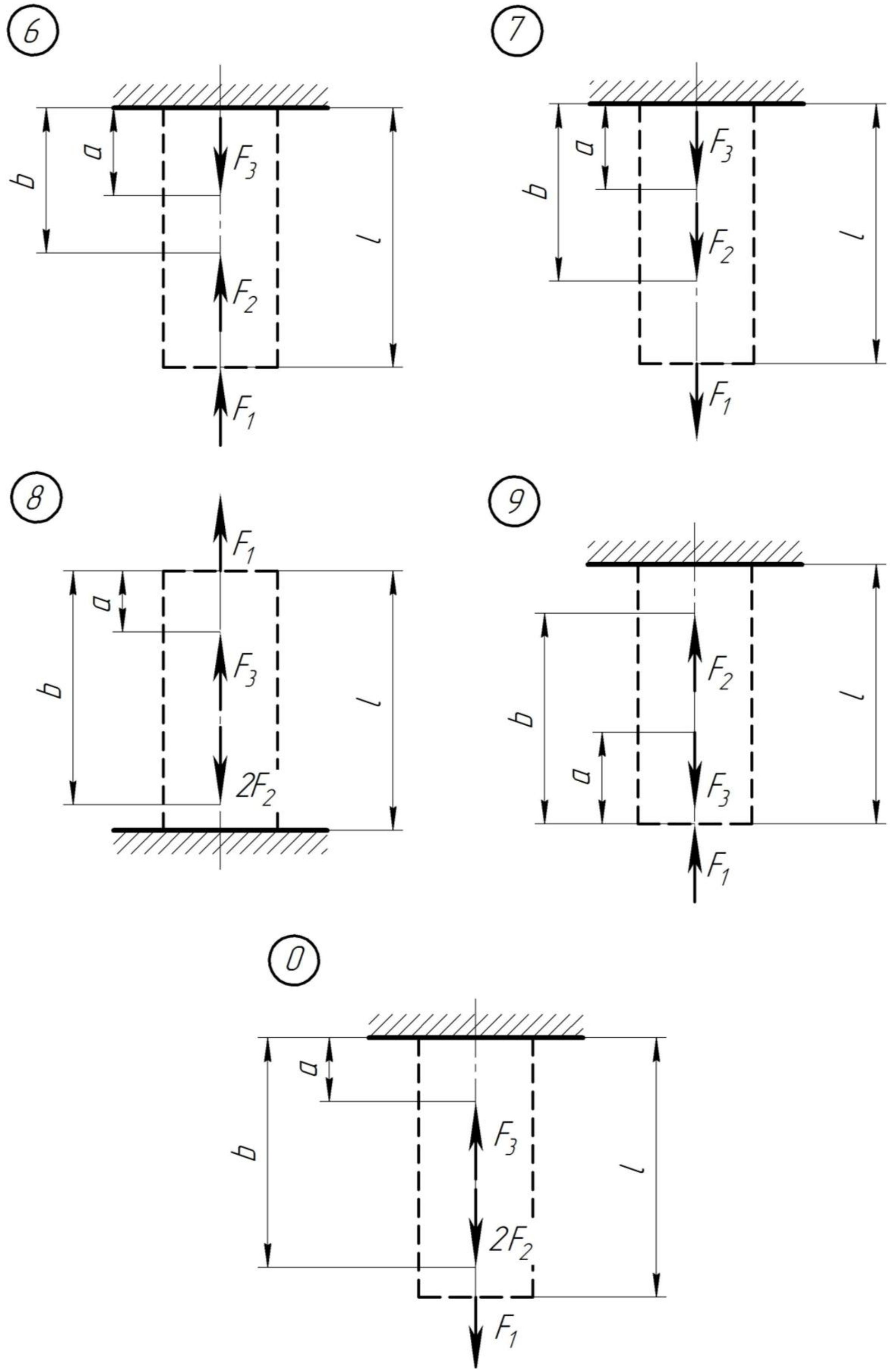


Figure for task 1 (contunied)

Example of solving the task 1
Strength calculation and displacement determination under tension and compression

For given straight steel rod (Fig. 2.5 *a*), determine the dimensions of the cross-section at all sections, provided that the cross-section is a circle; make the rod (bar) sketch; draw the diagram of the working (actual) normal stresses and longitudinal displacements of the rod, if $l = 8\ \text{m}$ $(a = k \cdot l,\ b = m \cdot l)$; rod (bar) material – steel; $[\sigma] = 160\ \text{MPa}$; $E = 2 \cdot 10^5$ MPa.

Solution

Divide the rod (bar) into three parts (Fig. 2.5*a*). For each part we determine the values of longitudinal (normal) forces:

$$N_1 = F_1 = 10\ \text{kN};$$

$$N_2 = F_1 + F_2 = 10 + 20 = 30\ \text{kN};$$

$$N_3 = F_1 + F_2 - 2F_3 = 10 + 20 - 2 \cdot 40 = -50\ \text{kN}.$$

Draw the diagram of longitudinal forces, N (Fig. 2.5 *b*).

From the condition of tensile-compressive strength $\sigma = N/A \le [\sigma]$ determine the required cross-section areas of the rod (bar) at each section

$$A_1 \ge \frac{|N_1|}{[\sigma]} = \frac{10 \cdot 10^{-3}}{160} = 0{,}625 \cdot 10^{-4}\ \text{m}^2;$$

$$A_2 \ge \frac{|N_2|}{[\sigma]} = \frac{30 \cdot 10^{-3}}{160} = 1{,}875 \cdot 10^{-4}\ \text{m}^2;$$

$$A_3 \ge \frac{|N_3|}{[\sigma]} = \frac{50 \cdot 10^{-3}}{160} = 3{,}125 \cdot 10^{-4}\ \text{m}^2.$$

The rod diameters determine by formula

$$A_i = \pi \cdot d_i^2 / 4, \quad \text{where} \quad i = 1, 2, 3, 4,$$

whence

$$d_i \ge \sqrt{4 A_i / \pi}\ .$$

Substituting data, obtain

$$d_1 \geq \sqrt{\frac{4 \cdot 62{,}5}{3{,}14}} = 8{,}92 \text{ mm};$$

$$d_2 \geq \sqrt{\frac{4 \cdot 187{,}5}{3{,}14}} = 15{,}45 \text{ mm};$$

$$d_3 \geq \sqrt{\frac{4 \cdot 312{,}5}{3{,}14}} = 19{,}95 \text{ mm}.$$

Round off the results: $d_{ac1} = 10$ mm, $d_{ac2} = 16$ mm, $d_{ac3} = 20$ mm. Then draw the sketch of the rod (bar) (Fig. 2.5 *c*).

Determine the actual cross-sectional areas of the rod (bar) at each section, taking into account the rounding of their diameters by the formula

$$A_{aci} = \pi \cdot d_{aci}^2 / 4.$$

Substituting data, obtain

$$A_{ac1} = 3{,}14 \cdot 10^2 / 4 = 78{,}5 \text{ mm}^2;$$

$$A_{ac2} = 3{,}14 \cdot 16^2 / 4 = 201 \text{ mm}^2;$$

$$A_{ac3} = 3{,}14 \cdot 20^2 / 4 = 314 \text{ mm}^2.$$

Then determine working (actual) normal stresses by the formula

$$\sigma_{aci} = |N_i| / A_{aci}.$$

Substituting data, obtain

$$\sigma_{ac1} = \frac{10 \cdot 10^{-3}}{78{,}5 \cdot 10^{-6}} = 127 \text{ MPa};$$

$$\sigma_{ac2} = \frac{30 \cdot 10^{-3}}{201 \cdot 10^{-6}} = 149 \text{ MPa};$$

$$\sigma_{ac3} = \frac{50 \cdot 10^{-3}}{314 \cdot 10^{-6}} = 159 \text{ MPa}.$$

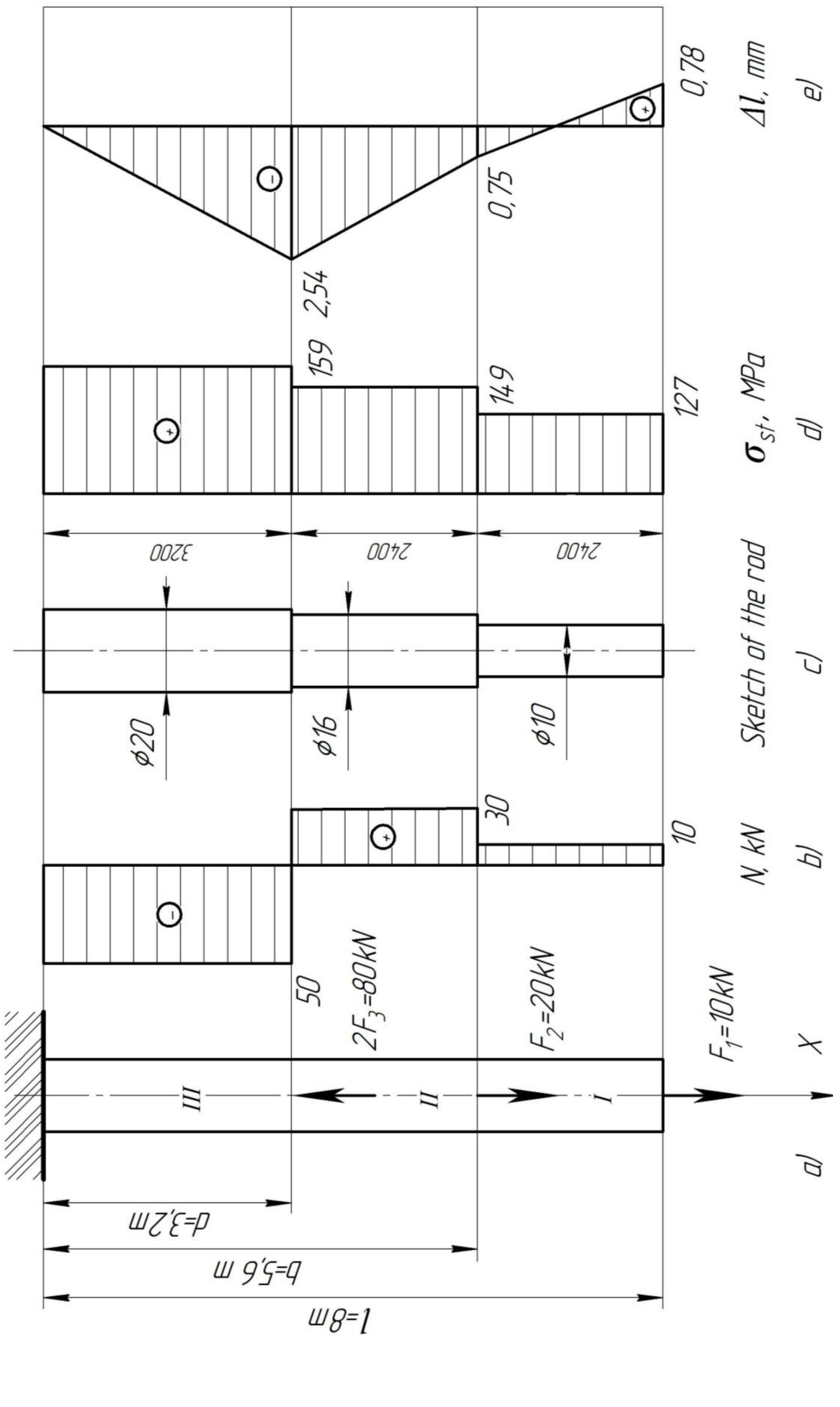

l=8m
b=5,6 m
d=3,2m
III
II
I
2F3=80kN
F2=20kN
F1=10kN
X
a)
50
30
10
N, kN
b)
ϕ20
ϕ16
ϕ10
3200
2400
2400
Sketch of the rod
c)
159
149
127
σst, MPa
d)
2,54
0,75
0,78
Δl, mm
e)

Figure 2.5

Draw the diagram of working (actual) normal stresses σ_{ac} by the module (Fig. 2.5 *d*).

The longitudinal (linear) deformations of each section of the rod (bar) are determined by the formula

$$\Delta l_i = \frac{N_i \cdot L_i}{E \cdot A_{aci}},$$

where L_i is the length of the rod (bar) section on which the longitudinal force acts.

In numerical form

$$\Delta l_1 = \frac{10 \cdot 2,4}{2 \cdot 10^8 \cdot 78,5 \cdot 10^{-6}} = 1,53 \cdot 10^{-3} \text{ m} = 1,53 \text{ mm};$$

$$\Delta l_2 = \frac{30 \cdot 2,4}{2 \cdot 10^8 \cdot 201 \cdot 10^{-6}} = 1,79 \cdot 10^{-3} \text{ m} = 1,79 \text{ mm};$$

$$\Delta l_3 = \frac{-50 \cdot 3,2}{2 \cdot 10^8 \cdot 314 \cdot 10^{-6}} = -2,54 \cdot 10^{-3} \text{ m} = -2,54 \text{ mm}.$$

Based on the obtained results, draw the diagram of linear displacements of the cross-sections (Fig. 2.5 *e*). The fixed end of the rod (bar) is taken as zero.

Task 2
Calculation of statically indeterminate rod (bar) system under tensile-compression

For the given rod (bar) system (Fig. for task 2, Table for task 2), to which force $F = 50$ kN is applied determine the diameters of the rods (bars) DE and KH, when the ratio of their areas $A_{DE} = k \cdot A_{KH}$ is known. Material of rods (bars) is steel St.3; $[\sigma] = 160$ MN/m^2, $a = 1$ m. The rod (bar) to which external force F is applied should be considered absolutely rigid.

Plan of solving the task:

1. Draw the scaled model of the rod (bar) system.
2. Determine the degree of static indeterminance of the system.
3. Considering the deformation of the system, make the auxiliary equations.
4. Determine the forces in the rods (bars).
5. Select the cross-section areas of the rods (bars).

Table for task 2

Nr	1	2	3	4	5	6	7	8	9	0
k	1	1,5	2	2,5	3	3,5	4	4,5	5	0,5

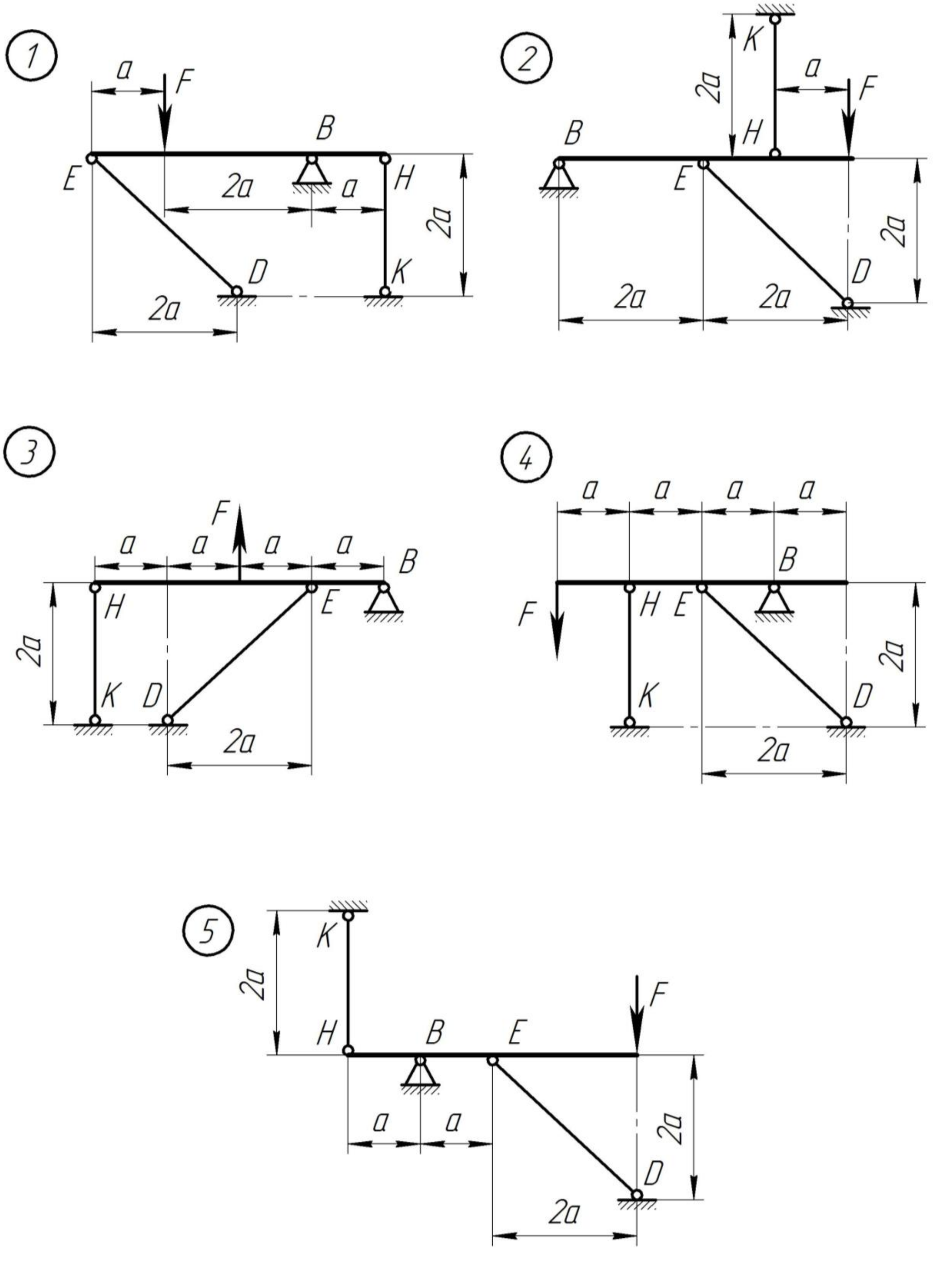
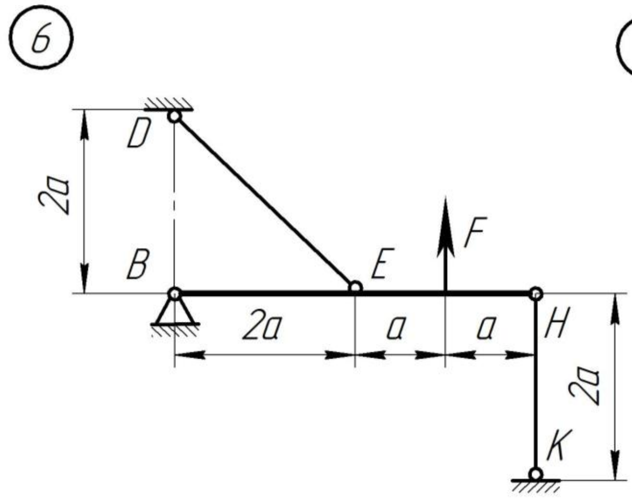
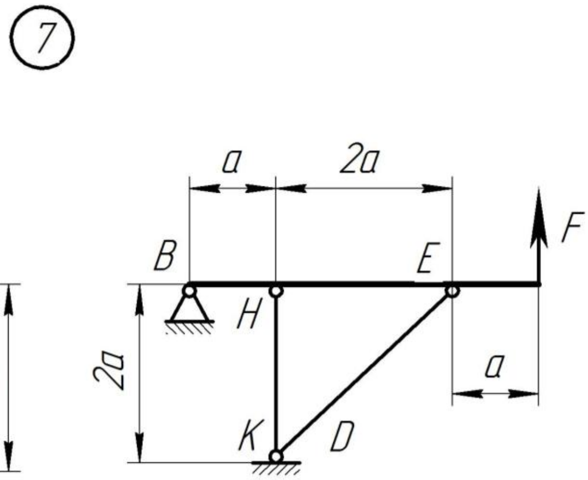
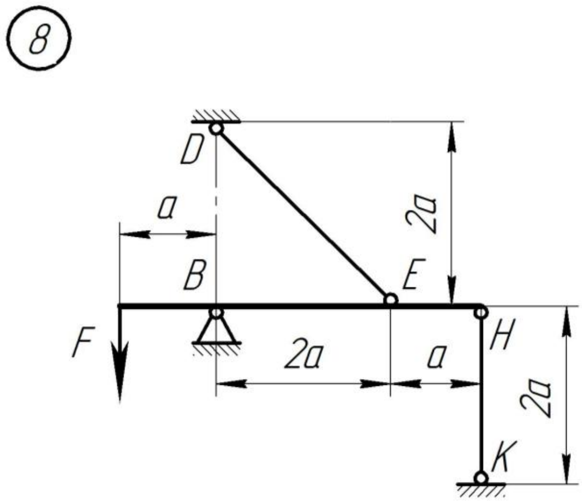
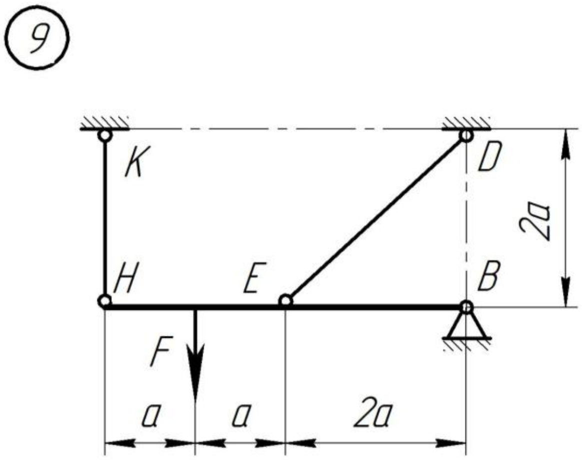


Figure for task 2









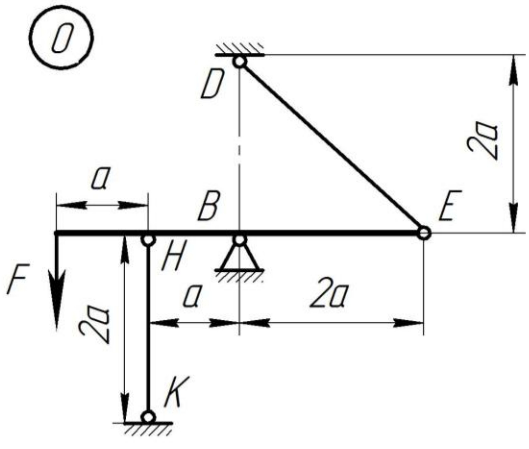


Figure for task 2 (contunied)

Example of solving the task 2
Calculation of statically indeterminate rod (bar) system under tensile-compression

For the given rod (bar) system (Fig. 2.6 *a*), to which force $F = 50$ kN is applied determine the diameters of the rods (bars) DE and KH, when the ratio of their areas $A_{DE} = k \cdot A_{KH}$, $k = 3$ is known. The rod (bar) to which external force F is applied should be considered absolutely rigid. Material of rods (bars) and bar (rod) is steel St.3; $[\sigma] = 160$ MPa ; $a = 1$ m.

Solution

When the system is loaded by force F, in rods (bars) DE and KH normal forces occur, in this case – compression forces. The cross-sectional area of the rods (bars) under compression is determined from the condition of tensile-compression strength

$$\sigma = \frac{N_i}{A_i} \le [\sigma], \qquad \text{whence} \qquad A_i \ge \frac{N_i}{[\sigma]}. \tag{1.1}$$

To determine the force in the rods (bars) DE and KH we derive the eqation of the bar equilibrium equilibrium (Fig. 2.6 *b*):

$$\sum X = 0; \qquad -N_{DE} \cdot \cos\alpha + B_X = 0; \tag{1.2}$$

$$\sum Y = 0; \qquad -F + N_{DE} \cdot \sin\alpha + B_Y - N_{KH} = 0; \tag{1.3}$$

$$\sum M_B = 0; \qquad F \cdot 3a - N_{DE} \cdot 2a \cdot \sin\alpha - N_{KH} \cdot a = 0, \tag{1.4}$$

where N_{DE}, N_{KH} are normal forces occurring in rods (bars) DE and KH;
B_X, B_Y are components of the reaction force of the support B.

There are four unknown forces and reactions (N_{DE}, N_{KH}, B_X, B_Y) in the system, and three equilibrium equations. Thus, the system is 4–3 = 1 time statically indeterminate.

We derive the additional equation, the equation of displacements compatibility (deformations).

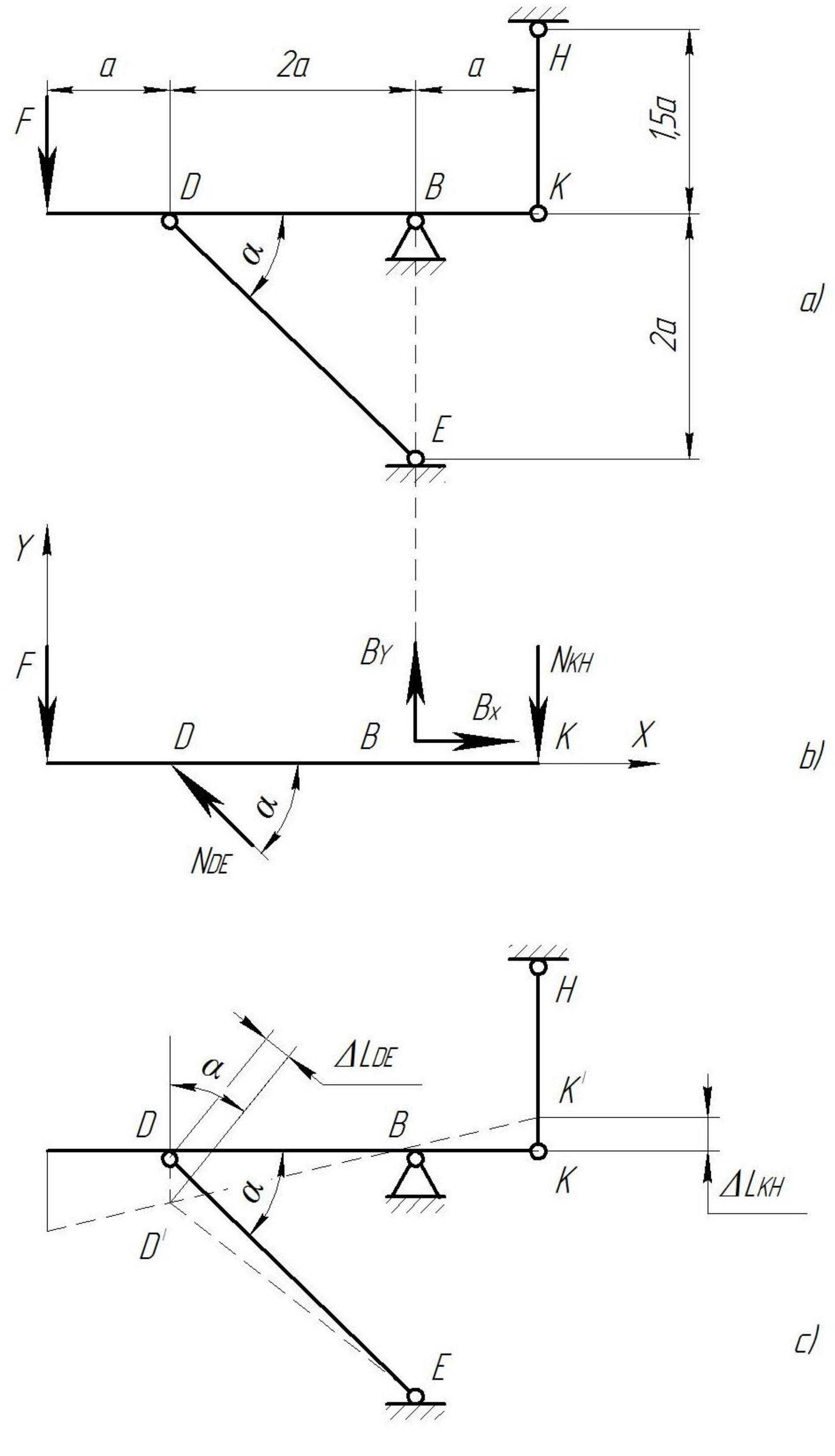

a
2a
a
H
1,5a
F
D
B
K
α
a)
2a
E
Y
F
BY
NKH
Bx
D
B
K
X
b)
α
NDE
H
ΔLDE
α
K'
D
B
K
α
ΔLKH
D'
c)
E

Figure 2.6

After applying the force F, the rods (bars) DE and KH deform and the system takes the position shown in Fig. 2.6 *c*. From the similarity of triangles DD_1B and KK_1B it follows that

$$\frac{DD_1}{KK_1} = \frac{DB}{KB} = \frac{2a}{a} = 2. \tag{1.5}$$

In this case

$$DD_1 = \frac{\Delta L_{DE}}{\sin\alpha}; \qquad KK_1 = \Delta L_{KH}, \tag{1.6}$$

where ΔL_{KH}, ΔL_{DE} are absolute deformations of rods (bars) KH and DE.

Rods deformations write by Hooke law in the following form:

$$\Delta L_{KH} = \frac{N_{KH}\cdot 1{,}5a}{E\cdot A_{KH}}; \qquad \Delta L_{DE} = \frac{N_{DE}\cdot 2a}{E\cdot A_{DE}\cdot\sin\alpha}.$$

Substituting values ΔL_{KH} and ΔL_{DE} into expressions (1.5) and (1.6), obtain

$$\frac{N_{DE}\cdot 2a\cdot E\cdot A_{KH}}{N_{KH}\cdot 1{,}5a\cdot E\cdot A_{DE}\cdot\sin^2\alpha} = 2.$$

Taking into account that $A_{DE} = 3A_{KH}$, we have

$$N_{DE} = 4{,}5N_{KH}\cdot\sin^2\alpha.$$

In this case α=45° (see Fig. 2.6*a*), then

$$N_{DE} = 4{,}5N_{KH}\cdot\sin^2 45^\circ = 2{,}25\,N_{KH}. \tag{1.7}$$

Solving equations (1.4) and (1.7), we determine unknown forces in the rods (bars)

$$\begin{cases} 3F - 2N_{DE}\cdot\sin 45^\circ - N_{KH} = 0; \\ N_{DE} = 2{,}25\,N_{KH}, \end{cases}$$

where

$$N_{KH} = \frac{3F}{4{,}5\sin 45^\circ + 1} = \frac{3 \cdot 50}{4{,}5 \cdot 0{,}7 + 1} = 35{,}9 \text{ kN},$$

respectively

$$N_{DE} = 2{,}25 \cdot 35{,}9 = 80{,}8 \text{ kN}.$$

Further equations (1.2) and (1.3) are not used in solving the task, since the unknown forces in the rods (bars) are defined, and according to the task statement it is not required to determine the reaction in support B.

The cross-section area of the rod (bar) KH is determined from the condition of tensile-compression strength (1.1).

$$A_{KH} \geq \frac{N_{KH}}{[\sigma]} = \frac{35{,}9 \cdot 10^{-3}}{160} = 2{,}24 \cdot 10^{-4} \text{ m}^2,$$

the cross-section area of rod (bar) DE determined from ratio

$$A_{DE} = 3A_{KH} = 3 \cdot 2{,}24 \cdot 10^{-4} = 6{,}72 \cdot 10^{-4} \text{ m}^2.$$

Estimate the strength of rod (bar) DE

$$\sigma_{DE} = \frac{N_{DE}}{A_{DE}} = \frac{80{,}8 \cdot 10^{-3}}{6{,}72 \cdot 10^{-4}} = 120 \text{ MPa} \leq [\sigma] = 160 \text{ MPa}.$$

The strength condition is ensured. Otherwise, the cross-section area of the rod (bar) DE should be determined from the strength condition and that of the rod (bar) KH from the ratio.

3. GEOMETRIC CHARACTERISTICS OF PLANE SECTIONS

Moments of inertia and center of gravity

The static moment of the plane figure area with respect to the axis lying in the same plane is the sum of the products of the areas of elementary planes at their distance from that axis.

The static moments of the section area of arbitrary shape (Fig. 3.1) are determined by the formulas

$$S_X = \int_A y dA \; ; \qquad S_Y = \int_A x dA \; ;$$

$$S_X = y_C \cdot A \; ; \qquad S_Y = x_C \cdot A \, ,$$

where x, y are coordinates (distances) that determine the position of the element area dA;

y_C, x_C are coordinates of the center of gravity of the section area;

A is a section area;

dA is an element of the area (elementary plane).

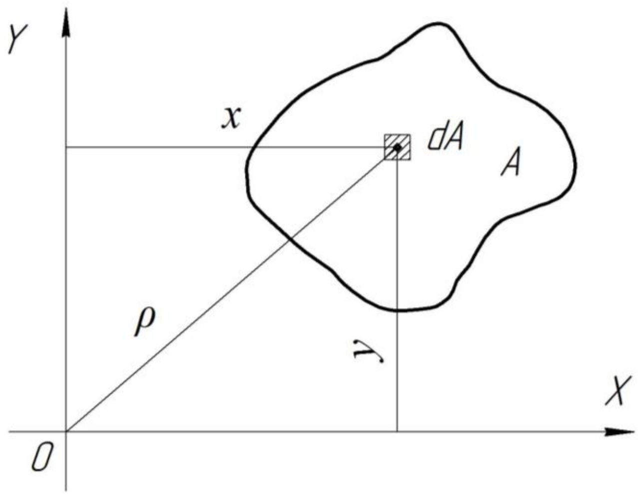


Figure 3.1

The static moment of the figure area relatively to axis lying in the same plane is equal to the product of the figure area at the distance from it to the center of gravity of that axis.

The static moment of figure area is the moment of the first order, its unit is m^3. It can be positive, negative and zero (relatively to the axis of figure symmetry or relatively to the central axis, that is, the axis passing through the center of gravity of the section).

The method of partitioning is used to determine the **center of gravity** of complex figures; the static moment of the area of the whole figure is defined as the algebraic sum of static moments of its individual parts. The coordinates of the center of gravity of the complex section is determined by the formulas

$$x_C = \frac{\sum S_Y}{\sum A}; \qquad y_C = \frac{\sum S_X}{\sum A},$$

where $\sum S_Y$, $\sum S_X$ are sums of static moments of separate areas;
$\sum A$ is a sum of separate areas.

The axes passing through the center of gravity of the section are called the **central axes.** The static moment of the area relatively to the central axis is zero.

The polar moment of inertia of the plane figure with respect to the pole lying in the same plane is the sum of the product of the areas of the elementary plane by the squares of their distances from the pole.

The polar moment of the section area of the arbitrary shape with respect to the pole O (see Fig. 3.1) is determined by the integral

$$I_P = \int_A \rho^2 dA,$$

where ρ is the distance from the center of the elementary plane (element of the area) dA to the axis perpendicular to the plane of the section through point O (pole),

$$\rho^2 = y^2 + x^2.$$

The axial moment of inertia of a plane figure with respect to the axis lying in the same plane is the sum products over the whole area by the elementary areas squared by their distance from that axis.

Axial moments of inertia of the section area of arbitrary shape (see Fig. 3.1) with respect to the axes OX and OY are determined by integrals

$$I_X = \int_A y^2 dA; \qquad I_Y = \int_A x^2 dA.$$

The polar and axial moments of inertia of the section are always positive and not equal to zero.

The dependence of axial and polar moments of inertia

$$I_P = \int_A \rho^2 dA = \int_A \left(y^2 + x^2\right) dA = I_X + I_Y.$$

Moments of figure inertia are moments of the second order, unit m^4.

The sum of the axial moments of inertia with respect to two mutually perpendicular axes is equal to the polar moment of inertia relatively to the point of intersection of these axes (the coordinate origin).

The dependence between the moments of inertia in parallel axes transfer

$$I_{X_1} = I_X + A \cdot a^2 .$$

The axial moment of inertia with respect to any axis X_1 is equal to the axial moment of inertia with respect to the central axis X , which is parallel to the axis X_1, plus the product of the area by the squared distance between the axes (a is the distance between the axes).

Main axes and main moments of inertia

Central axes are the axes that pass through the center of gravity of the plane figure.

Central moments of inertia of the plane figure (section) are moments of inertia relatively to the central axes.

If the axis of coordinates is rotated in its plane around the origin, the polar moment of inertia of the section will remain constant and the axial moments of inertia will change, and

$$I_X + I_Y = I_P = const \ .$$

If the sum of two variables remains constant, one of them decreases and the other increases. Therefore, at any position, one of the axial moments reaches the maximum and the other – the minimum values.

Main axes of inertia are axes in relation to which the axial moments of inertia of the section (plane figure) reach the maximum and minimum values.

The main moments of inertia of the section are the axial moments of inertia relatively to the principal axes.

The principal central axes are the main axes that pass through the center of gravity of the section (plane figure). If the figure has at least one axis of symmetry, then this axis will always be one of the main central axes.

The main central moments of inertia of the section (plane figure) are the moments of inertia with respect to the principal central axes.

In engineering calculations, **the main central moments of inertia** are important.

The moments of inertia of the sections are geometric characteristics that make it possible to compare the rigidity of the bars of the given material with their resistance to external forces.

Axial and polar moments of inertia gain only positive values.

The bar resistance to bending and torsion is also characterized by the moments of resistance of the sections: axial and polar.

The axial moments of intersection resistance are determined by the formulas

$$W_X = \frac{I_X}{|y_{max}|}; \qquad W_Y = \frac{I_Y}{|x_{max}|},$$

where y_{max}, x_{max} are the coordinates of the points of section are at maximum distance from the axes OX and OY.

The polar moment of intersection resistance, respectively

$$W_P = \frac{I_P}{\rho_{max}},$$

where ρ_{max} is the coordinate of the intersection point at maximum distance from the poles.

Polar moments of inertia and polar moments of resistance for cross-sections:

- circle (Fig. 3.2 *a*)

$$I_P = \pi \cdot D^4 / 32; \qquad W_P = \pi \cdot D^3 / 16;$$

- ring (Fig. 3.2 *b*)

$$I_P = \frac{\pi \cdot D^4}{32}\left(1 - \alpha^4\right); \qquad W_P = \frac{\pi \cdot D^3}{16}\left(1 - \alpha^4\right),$$ where $\alpha = d/D$.

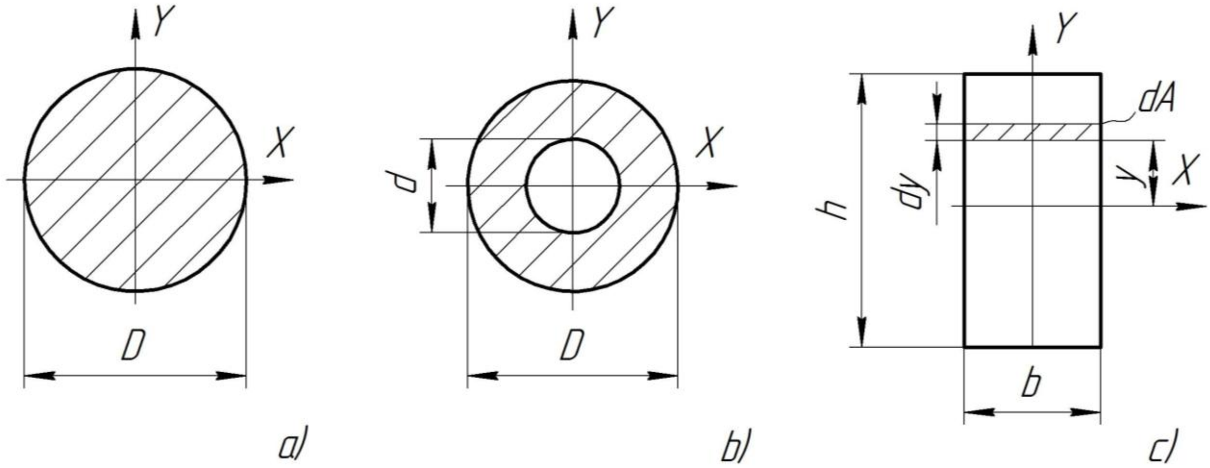


Figure 3.2

Axial moments of inertia and axial resistance moments for cross-sections:

- circle (see Fig. 3.2 *a*)

$$I_X = I_Y = I_0 = \pi \cdot D^4 / 64; \qquad W_X = W_Y = W_0 = \pi \cdot D^3 / 32;$$

- rectangle (Fig. 3.2 *c*)

$$I_X = \frac{b \cdot h^3}{12}; \qquad I_Y = \frac{h \cdot b^3}{12}; \qquad W_X = \frac{b \cdot h^2}{6}; \qquad W_Y = \frac{h \cdot b^2}{6}.$$

Task 3
Determination of axial moments of inertia of plane sections

For the given section (Fig. for task 3, Table for task 3) determine the position of the main central axes, the main central moments of inertia and the axial moments of resistance with respect to the main central axes.

Plan of solving the task:

1. Write out the data needed to solve the task from the assortment tables (*Annexs* 1, 2).
2. Determine the geometric characteristics of the strip (strips).
3. Draw a cross-section at a scale of 1 : 1 or 1 : 2. Mark all the dimensions used in the calculations in the drawing.
4. Choose a rational placement of auxiliary coordinate axes.
5. Determine the position of the center of gravity of the section.
6. Draw the main central axes parallel to the auxiliary axes and determine the values of the main central moments of inertia of the section.
7. Determine the axial moments of the section resistance relative to the main central axes.

Table for task 3

Nr	Geometrical characteristic (I-beam, U-beam)	Nr	Geometrical characteristic (I-beam, U-beam)
1	12	**6**	22
2	14	**7**	24
3	16	**8**	27
4	18	**9**	30
5	20	**0**	10

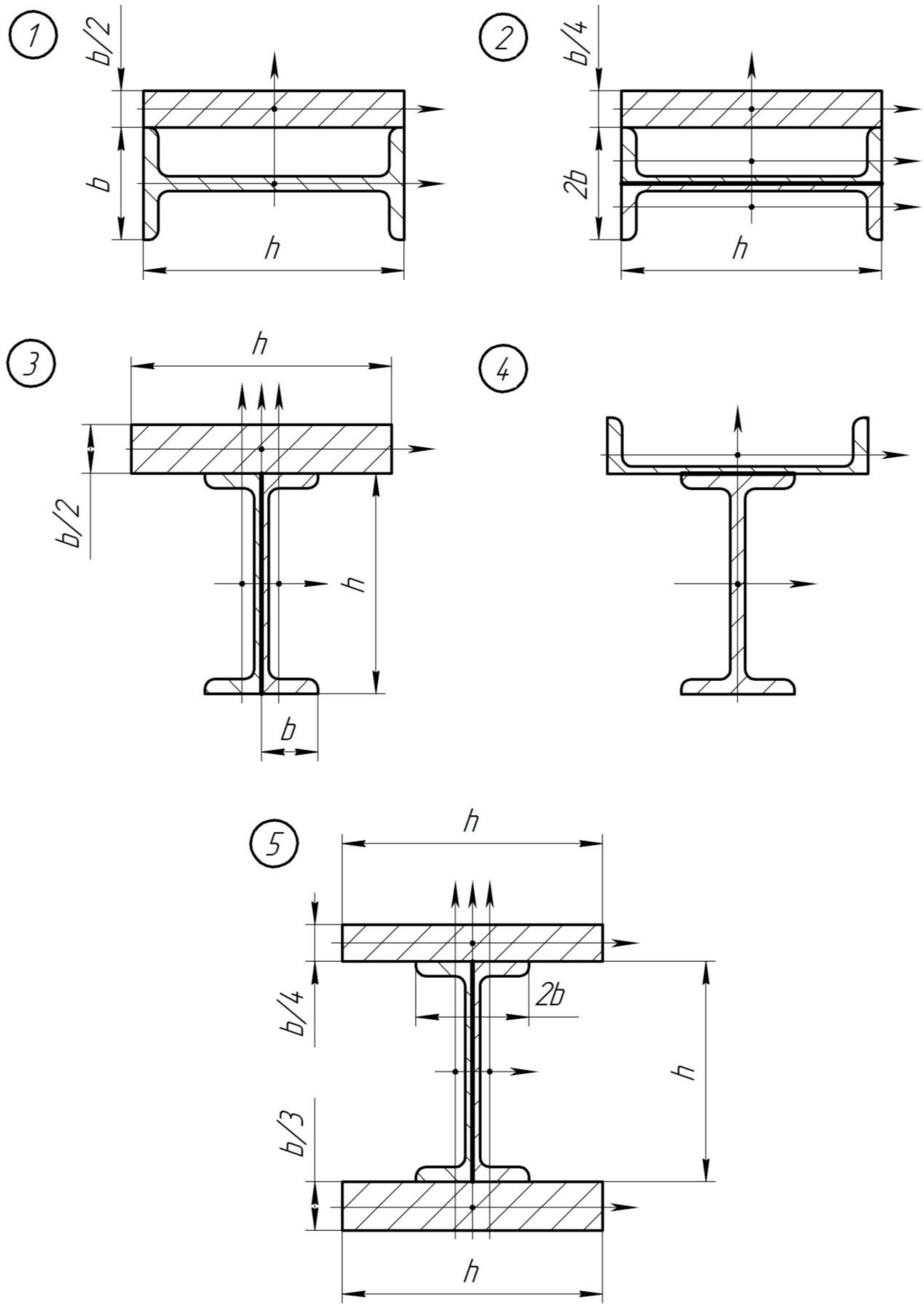


Figure for task 3

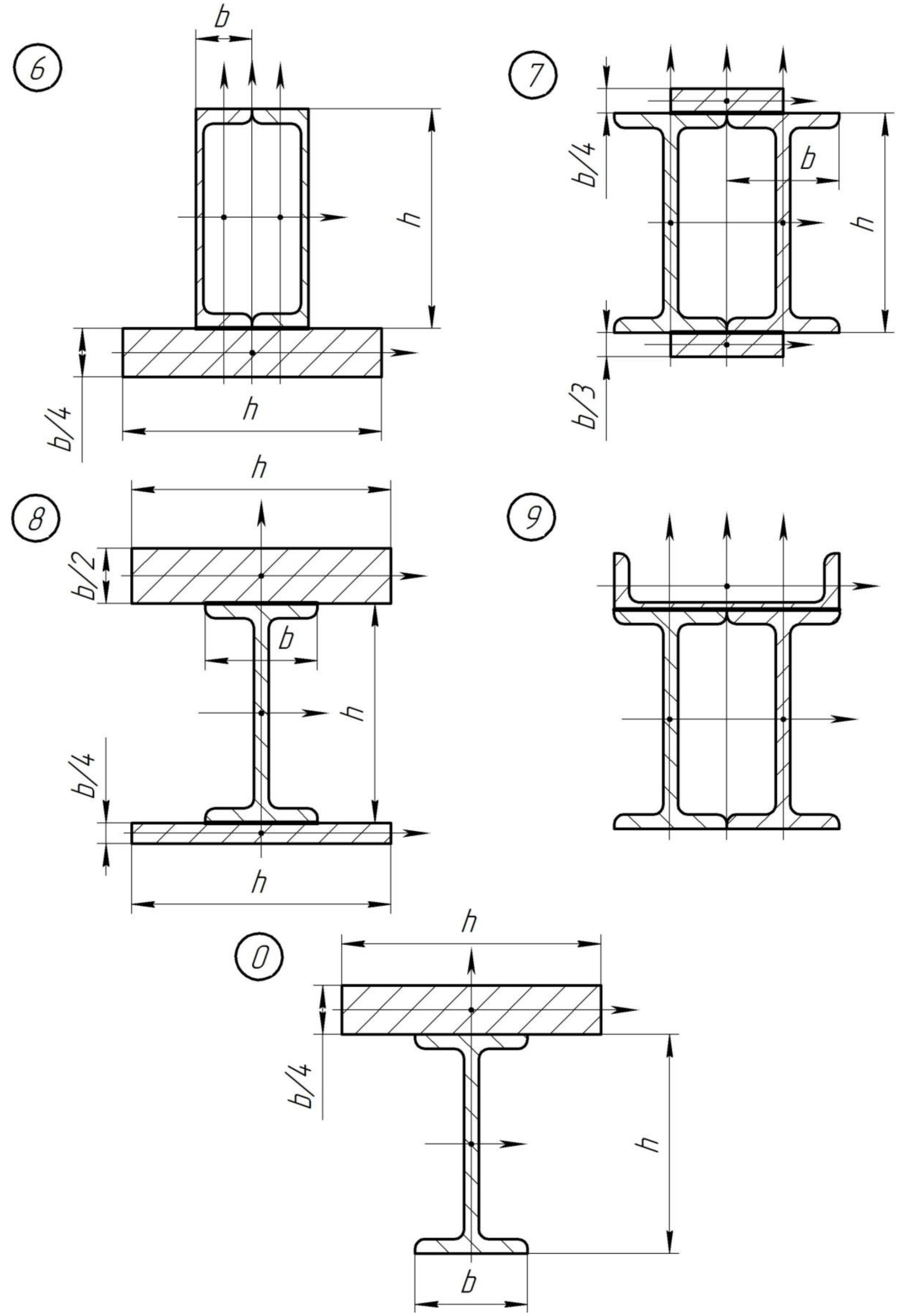


Figure for task 3 (contunied)

Example of solving the task 3
Determination of axial moments of inertia of plane sections

For the given section (Fig. 3.3) determine the position of the main central axes, the main central moments of inertia and the axial moments of resistance with respect to the main central axes, if I-beam is Nr 33.

Solution

The geometric characteristics of the specified rolling section are taken from the tables of assortment GOST 8239-89 (*Annex* 1). For I-beam Nr 33, indicate by 1.

$$h_1 = 330\ \text{mm};\ b_1 = 140\ \text{mm};\ d_1 = 7{,}0\ \text{mm};\ t_1 = 11{,}2\ \text{mm};\ A_1 = 53{,}8\ \text{cm}^2;$$

$$I_{X1} = 419\ \text{cm}^4;\ I_{Y1} = 9840\ \text{cm}^4.$$

Determine the geometric characteristics of the strip, indicate them by 2. The strip dimensions

$$b_2 = h_1/4 = 330/4 = 82{,}5\ \text{mm}; \qquad a_2 = h_1 = 330\ \text{mm}.$$

The cross-sectional area of the strip is

$$A_2 = b_2 \cdot a_2 = 8{,}25 \cdot 33 = 272\ \text{cm}^2.$$

Axial moments of strip inertia are

$$I_{X2} = \frac{a_2 \cdot b_2^3}{12} = \frac{33 \cdot 8{,}25^3}{12} = 1544\ \text{cm}^4;$$

$$I_{Y2} = \frac{b_2 \cdot a_2^3}{12} = \frac{8{,}25 \cdot 33^3}{12} = 24706\ \text{cm}^4.$$

Draw the section at scale (see Fig. 3.3).

The coordinates of the center of gravity of the section in the coordinate system $X_1 Y_1$ are:

$x_C = 0$, as axis Y is the axis of symmetry;

$$y_C = \frac{\sum S_X}{\sum A} = \frac{A_1 \cdot y_{C1} + A_2 \cdot y_{C2}}{A_1 + A_2},$$

where y_{C1} is the distance (coordinate) from the center of gravity of the area of the first figure of section (I-beam) to the axis X_1, $y_{C1} = 0$;

y_{C2} is the distance (coordinate) from the center of gravity of the area of the second figure of section (strip) to the axis X_1

$$y_{C2} = \frac{140}{2} + \frac{82,5}{2} = 111,25 \text{ mm}.$$

Substitute the value and obtain

$$y_C = \frac{0 + 272 \cdot 111,25}{53,8 + 272} = 92,9 \text{ mm}.$$

Draw the principal central axes $X\ Y$ through point C (see Fig. 3.3).

Determine the axial moments of inertia with respect to the principal central axes, i.e. the main central moments of inertia of the given section

$$I_Y = I_{Y1} + I_{Y2} = 9840 + 24706 \approx 34600 \text{ cm}^4;$$

$$I_X = I_{X1} + A_1 \cdot (-9,29)^2 + I_{X2} + A_2 \cdot (1,835)^2 =$$

$$= 419 + 53,8 \cdot (-9,29)^2 + 1544 + 272 \cdot 1,835^2 \approx 5980 \text{ cm}^4.$$

Determine the axial moments of resistance relatively to the principal central axes

$$W_X = \frac{I_X}{y_{max}} = \frac{5980}{16,29} = 367 \text{ cm}^3;$$

$$W_Y = \frac{I_Y}{x_{max}} = \frac{34600}{16,5} = 2097 \text{ cm}^3;$$

where x_{max}, y_{max} are the coordinates of points of the given section, maximum distant from the axes X and Y (see Fig. 3.3):

$$y_{max} = 9,29 + 7 = 16,29 \text{ cm}; \qquad x_{max} = 16,5 \text{ cm}.$$

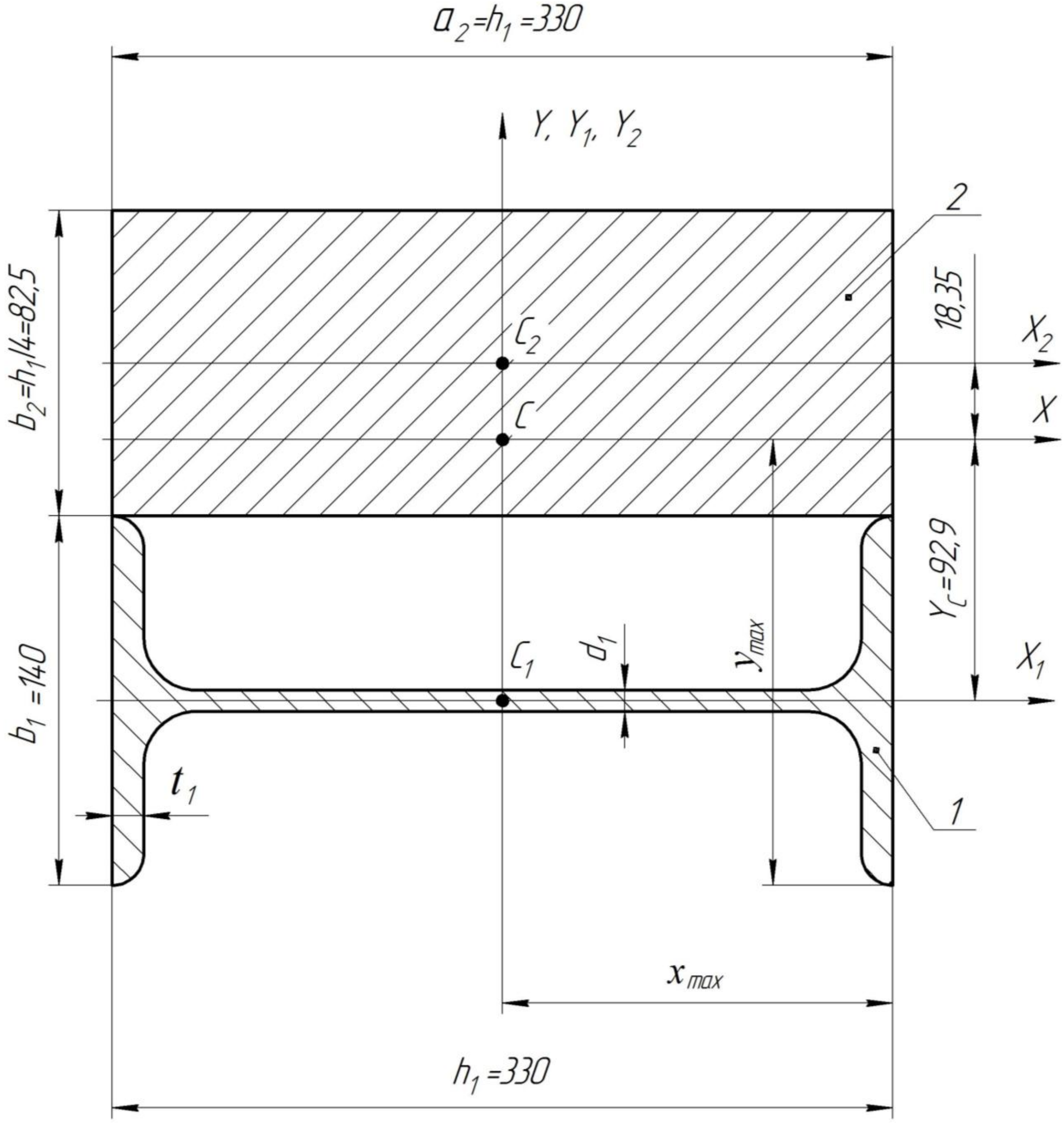

a2=h1=330
Y, Y1, Y2
2
b2=h1/4=82,5
18,35
C2
X2
C
X
Yc=92,9
ymax
d1
C1
X1
b1=140
t1
1
xmax
h1=330

Figure 3.3

4. SHEAR. TORSION

Shear stress, strain and Hooke's law

Shear is a type of deformation in which at any cross-section of the bar only shear (cutting) force Q acts (Fig. 4.1 *a*). The shear deformation resulting in material fracture is shear.

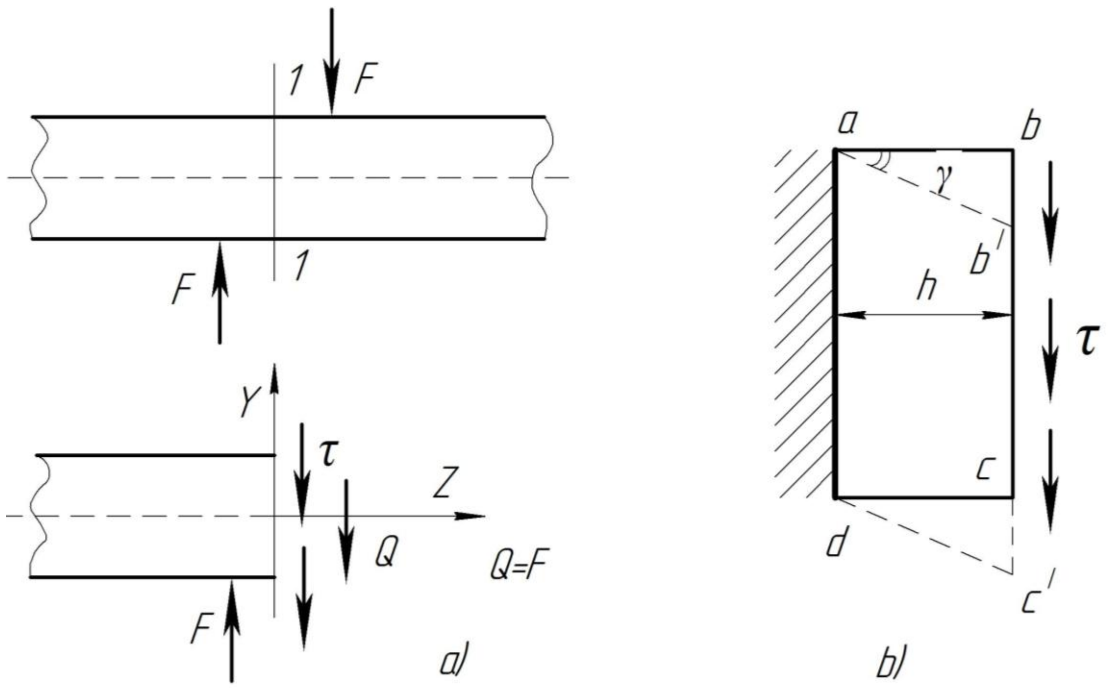


Figure 4.1

We assume that tangent stresses occuring at the cross-section of the bar under shear is $\tau = const$, then the shearing force is

$$Q = A \cdot \tau , \quad \text{i.e. stress} \quad \tau = Q / A .$$

Condition of shearing strength

$$\tau = \frac{Q}{A} \le [\tau]_{ss} ,$$

where $[\tau]_{ss}$ is the allowable shear stress, $[\tau]_{ss} = (0{,}25 \ldots 0{,}35)\, \sigma_{ye}$.

The shear deformation is determined by shear angle γ . Absolute bar shear (Fig. 4.1 *b*) $- bb'$, cc' .

Hooke's shear law

$$\tau = G \cdot \gamma ,$$

where G is the shear modulus or modulus of elasticity of the second type, characterizing the material rigidity.

The dependence between the elastic characteristics of the plastic material (steels) E , G , μ is

$$G = \frac{E}{2\,(1 + \mu)} .$$

Torsion

Torsion is a type of deformation in which ***only torque moment*** M_{TR} occurs at any cross-section of the bar.

Torsional deformations occur when a pair of forces M is applied to the straight bar in planes perpendicular to the axis (Fig. 4.2). The moments of these pairs are called ***rotating*** (if the bar rotates), they are indicated T, and ***twisting*** (if the bar does not rotate), they are indicated M.

The circular cross-section bar which operates for torsional deformation is called the **shaft**. The shafts of engines and machine tools or other metal structures are affected by torsion. The rods with the cross-sections of other shapes also operate for torsion.

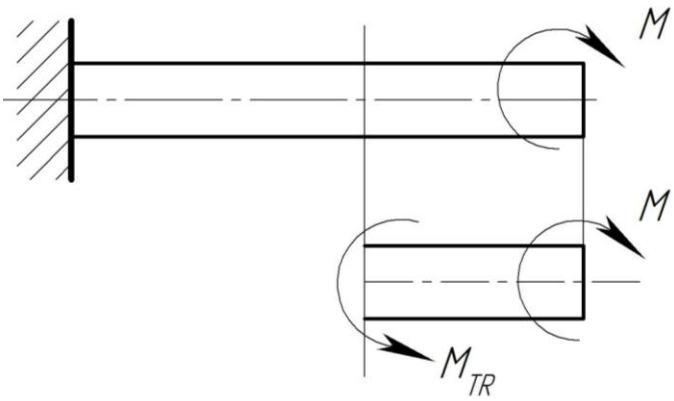


Figure 4.2

The torque M_{TR} at any section of the shaft cross-section is equal to the algebraic sum of the external twisting moments applied to the bar on the right or left of the section.

To calculate the bar for tensile strength, as well as for tensile (compression) it is necessary to determine the **dangerous section**. If the dimensions of the cross-section at bar length are constant, the sections at which torques are maximum are dangerous. **The torque diagram** is the graph showing the law of torque change along the bar length. It is drawn the same way as the diagram of longitudinal forces.

Under bar torsion, only tangential stresses occur in its cross sections. For the circular rod (shaft), the tangent stresses are determined by the formula

$$\tau = \frac{M_{TR}}{I_P} \cdot \rho,$$

where ρ is the distance from the center (pole) of the round section to the point at which the tangent stresses are determined (Fig. 4.3 *a*).

The diagram of the tangential (shear) stress distribution by the height of the cross-section is shown in Fig. 4.3 *b*. The shear stresses vary along the radius of section by linear law.

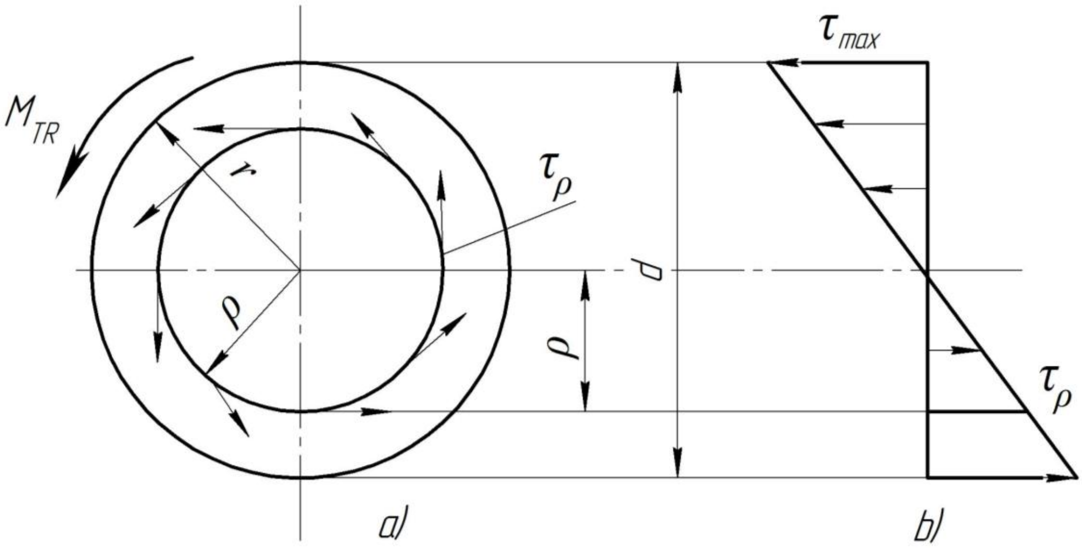


Figure 4.3

The tangential (shear) stresses are zero at the center of the section, $\rho = 0$ and reach the maximum value at the points of the contour, $\rho = d/2$. At the intermediate points of section, the tangent stresses depend linearly on the distance ρ (see Fig. 4.3 *b*). As

$$\frac{I_P}{(d/2)} = W_P, \qquad \text{then} \qquad \tau_{\max} = \frac{M_{TR}}{W_P}.$$

Condition of tensile strength. The strength of the shaft is ensured when the maximum tangential (shear) stress does not exceed the allowable one

$$\tau_{\max} = \frac{M_{TR}}{W_P} \le [\tau],$$

where $[\tau]$ is the allowable shear stress, determined depending on the allowable tensile stress $[\sigma]_P$:

for steels $\qquad [\tau] = (0{,}55 \ldots 0{,}6)[\sigma]_P$;

for cast iron $\qquad [\tau] = (1 \ldots 1{,}2)[\sigma]_P$.

Three types of tasks are solved by the torsional strength of the shaft.

Choosing the cross-section (design calculation), that is, determining its required sizes based on the polar moment of resistance

$$W_P \geq M_{TR} / [\tau], \text{ for round section } d = \sqrt[3]{16\, W_P / \pi}\ .$$

Validating calculation (testing calculation) is reduced to the comparison of actual (real) and allowable shear stresses by the formula

$$\tau_{\max} = \frac{M_{TR}}{W_P} \leq [\tau].$$

Determination of maximum torque

$$[M_{TR}] \leq [\tau] \cdot W_P\ .$$

Torsional deformation is characterized by the rotation of the cross-sections of the shaft relativly to each other by certain angle φ – **the twist angle**. For a shaft of constant rigidity $G \cdot I_P$ of length l with constant value of torque M_{TR} the twist angle (full twisting angle) is determined by the formulas:

$$\varphi = \frac{M_{TR} \cdot l}{G \cdot I_P}\ [\text{rad}]; \qquad \varphi = \frac{M_{TR} \cdot l}{G \cdot I_P} \cdot \frac{180^\circ}{\pi}\ [\text{degree}].$$

These relations are called ***Hooke's shear law***. For the cylindrical bar having several sections that differ in cross-section size, torque value, material, the full twist angle is equal to the algebraic sum of the twist angles of the separate sections $\varphi = \sum \varphi_i$.

The full twist angle of the shaft does not completely characterize the deformation of the torsion, since it depends on the length of the shaft. The rigidity of the shaft is estimated by the **relative twist angle**, which is determined by the formulas

$$\theta = \varphi / l; \qquad \theta = \frac{M_{TR}}{G \cdot I_P} \left[\frac{\text{rad}}{\text{m}}\right]; \qquad \theta = \frac{M_{TR}}{G \cdot I_P} \cdot \frac{180^\circ}{\pi} \left[\frac{\text{degree}}{\text{m}}\right].$$

Condition of rigidity of the shaft at rotation. The rigidity of the shaft is sufficient when the maximum relative twist angle does not exceed its allowable value

$$\theta = \frac{M_{TR}}{G \cdot I_P} \cdot \frac{180^\circ}{\pi} \leq [\theta],$$

where $[\theta]$ is the allowable angle of the shaft rotation.

Using rigidity conditions (as well as strength conditions), three types of structural calculations: **design, validation and determination of allowable load** are carried out.

Task 4
Shaft calculation for torsion

On the shaft (Fig. for task 4, Table for task 4) 5 pulleys are mounted, which transmit powers P_1, P_2, P_3, P_4, P_0. From the condition of torsional strength determine the diameters of individual sections of the shaft. Check shaft for rigidity at allowable angle of rotation $[\theta] = 2$ deg/m. Shaft rotation frequency ω; distance $a = 0{,}4$ m; material – steel 45; $[\tau] = 60$ MPa; $G = 8 \cdot 10^4$ MPa.

Plan of solving the task:

1. Determine the power on the pulley P_0 neglecting the friction in the bearings.
2. Find the torques transmitted by each pulley.
3. Determine the torques M_{TR} on each segment of the shaft. Draw the diagram of torques.
4. From the condition of torsional strength, determine the diameters of the shaft in its certain segments. Round off stepped shaft the obtained values to a size multiple of 2 or 5.
5. Draw the sketch of the (indicating the diameters and lengths of individual sections).
6. Determine the values of the torsion angles on the certain segments and draw a diagram of the torsion angles for the whole shaft, taking one of the ends of the shaft or the section where it P_0 acts, as a fixed section.
7. Check the shaft for rigidity.

Table for task 4

Nr	P_1, kW	P_2, kW	P_3, kW	P_4, kW	ω, rad/s
1	11	12	13	14	10
2	12	13	14	11	20
3	13	14	11	12	30
4	14	13	12	11	40
5	11	12	13	14	50
6	12	13	14	11	60
7	13	14	11	12	70
8	14	11	12	13	80
9	16	17	13	15	90
0	15	16	14	13	100

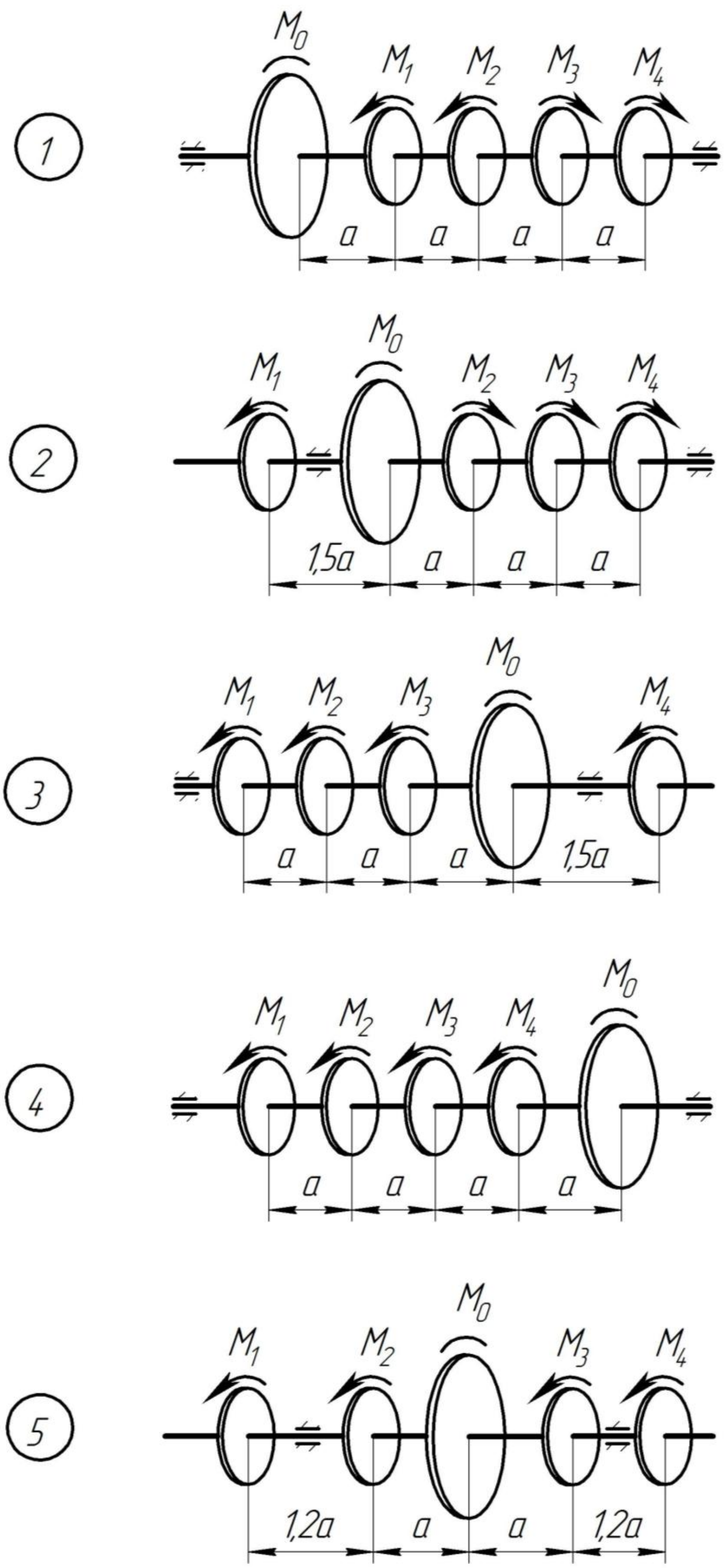


Figure for task 4

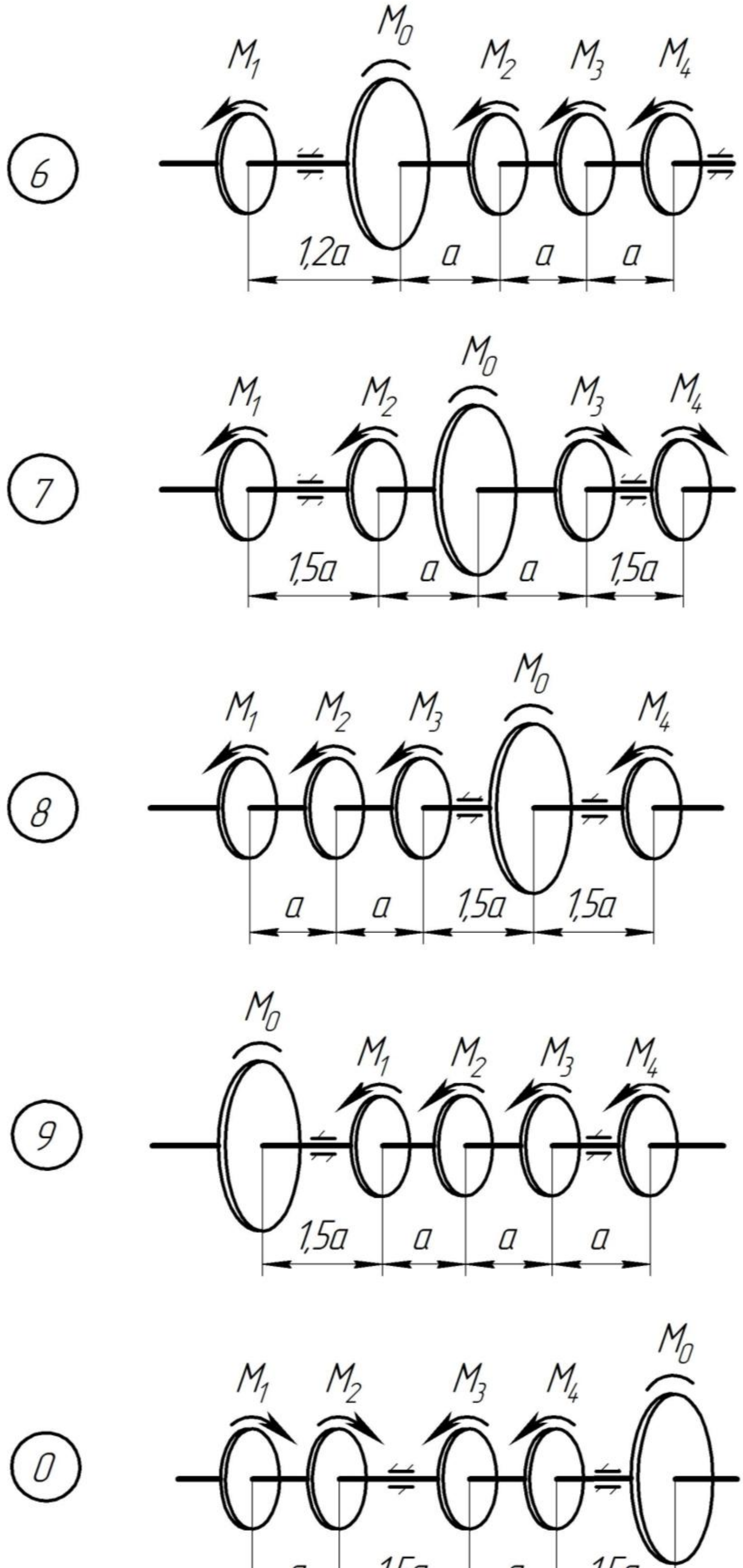


Figure for task 4 (contunied)

Example of solving the task 4
Shaft calculation for torsion

On the shaft (Fig. 4.4 *a*) 5 pulleys are mounted, which transmit powers $P_1 = 10$ kW; $P_2 = 50$ kW; $P_3 = 48$ kW; $P_4 = 80$ kW. From the condition of torsional strength, determine the diameters of individual sections of the shaft. Check shaft for rigidity at allowable angle of rotation $[\theta] = 2$ deg/m. Shaft rotation frequency $\omega = 40\ \text{s}^{-1}$; distance $a = 0{,}4$ m; material – steel 45; $[\tau] = 50$ MPa; $G = 8 \cdot 10^4$ MPa.

Solution

The value of power P_0 is determined on the basis equation of the power balance, written taking into account the direction of action of the concentrated moments (friction in the supports is neglected),

$$-P_0 + P_1 + P_2 + P_3 - P_4 = 0,$$

where

$$P_0 = P_1 + P_2 + P_3 - P_4 = 10 + 50 + 48 - 80 = 28 \text{ kW}.$$

The twisting moments M_i transmitted by each pulley are determined by the formula

$$M_i = \frac{P_i}{\omega}, \qquad \text{where} \quad i=0, 1, 2, 3, 4.$$

Substituting the value, obtain

$$M_0 = \frac{28}{40} = 0{,}7 \text{ kNm}; \quad M_1 = \frac{10}{40} = 0{,}25 \text{ kNm}; \quad M_2 = \frac{50}{40} = 1{,}25 \text{ kNm};$$

$$M_3 = \frac{48}{40} = 1{,}2 \text{ kNm}; \quad M_4 = \frac{80}{40} = 2{,}0 \text{ kNm}.$$

Torques M_{TRi} $(i = 1, 2, 3, 4)$ at each section of the shaft are determined considering the left and right sections (Fig. 4.4 *b*):

$$M_{TR1} = -M_0 = -0{,}7 \text{ kNm};$$

$$M_{TR2} = -M_0 + M_1 = -0{,}7 + 0{,}25 = -0{,}45 \text{ kNm};$$

$$M_{TR3} = M_4 = 2{,}0 \text{ kNm};$$

$$M_{TR4} = M_4 - M_3 = 2{,}0 - 1{,}2 = 0{,}8 \text{ kNm}.$$

Based on the obtained values, draw torque diagram (Fig. 4.4 *c*).

From the tensile strength condition $/\tau_{\max} = M_{TR}/W_P \le [\tau]$ taking into account that the moment of resistance of the round cross-section $W_P = \pi \cdot d^3/16 \approx 0{,}2d^3$, we determine the diameter of the shaft at each section by the formula

$$d_i \ge \sqrt[3]{\frac{M_{TRi}}{0{,}2\,[\tau]}}.$$

Substituting the values of torques, obtain

$$d_1 \ge \sqrt[3]{\frac{0{,}7}{0{,}2 \cdot 50 \cdot 10^3}} = 41{,}0 \cdot 10^{-3} \text{ m};$$

$$d_2 \ge \sqrt[3]{\frac{0{,}45}{0{,}2 \cdot 50 \cdot 10^3}} = 35{,}4 \cdot 10^{-3} \text{ m};$$

$$d_3 \ge \sqrt[3]{\frac{0{,}8}{0{,}2 \cdot 50 \cdot 10^3}} = 42{,}8 \cdot 10^{-3} \text{ m};$$

$$d_4 \ge \sqrt[3]{\frac{2{,}0}{0{,}2 \cdot 50 \cdot 10^3}} = 58{,}1 \cdot 10^{-3} \text{ m}.$$

Accept the diameters of the shaft sections

$$d_{1d} = 42 \text{ mm}; \; d_{2d} = 36 \text{ mm}; \; d_{3d} = 44 \text{ mm}; \; d_{4d} = 58 \text{ mm}.$$

According to the obtained values, draw a sketch of the shaft (Fig. 4.4 *d*).

The angles of twisting of individual sections of the shaft are determined by the formula

$$\varphi_i = \frac{M_{TRi} \cdot l_i}{G \cdot I_{Pi}}; \quad i=1,\ 2,\ 3,\ 4,$$

where l_i is the length of the shaft section;

G is shear modulus, $G = 8{,}1 \cdot 10^4$ MPa ;

I_{Pi} is the polar moment of inertia of the cross-section of the shaft

$$I_{Pi} = \frac{\pi \cdot d_{id}^4}{32} \cong 0{,}1 d_{id}^4 .$$

Substituting the value, obtain

$$\varphi_1 = \frac{-0{,}7 \cdot 0{,}4}{8{,}1 \cdot 10^7 \cdot 0{,}1 \cdot (0{,}042)^4} = -1{,}10 \cdot 10^{-2} \text{ rad};$$

$$\varphi_2 = \frac{-0{,}45 \cdot 0{,}6}{8{,}1 \cdot 10^7 \cdot 0{,}1 \cdot (0{,}036)^4} = -1{,}98 \cdot 10^{-2} \text{ rad};$$

$$\varphi_3 = \frac{0{,}8 \cdot 0{,}4}{8{,}1 \cdot 10^7 \cdot 0{,}1 \cdot (0{,}044)^4} = 1{,}10 \cdot 10^{-2} \text{ rad};$$

$$\varphi_4 = \frac{2{,}0 \cdot 0{,}5}{8{,}1 \cdot 10^7 \cdot 0{,}1 \cdot (0{,}058)^4} = 1{,}09 \cdot 10^{-2} \text{ rad}.$$

Determine the angles of the shaft cross-sections twist B, C, D, E relatively to the section A

$$\varphi_{BA} = \varphi_1 = -1{,}10 \cdot 10^{-2} \text{ rad};$$

$$\varphi_{CA} = \varphi_{BA} + \varphi_2 = -(1{,}10 + 1{,}98) \cdot 10^{-2} = -3{,}08 \cdot 10^{-2} \text{ rad};$$

$$\varphi_{DA} = \varphi_{CA} + \varphi_3 = (-3{,}08 + 1{,}10) \cdot 10^{-2} = -1{,}98 \cdot 10^{-2} \text{ rad};$$

$$\varphi_{EA} = \varphi_{DA} + \varphi_4 = (-1{,}98 + 1{,}09) \cdot 10^{-2} = -0{,}89 \cdot 10^{-2} \text{ rad}.$$

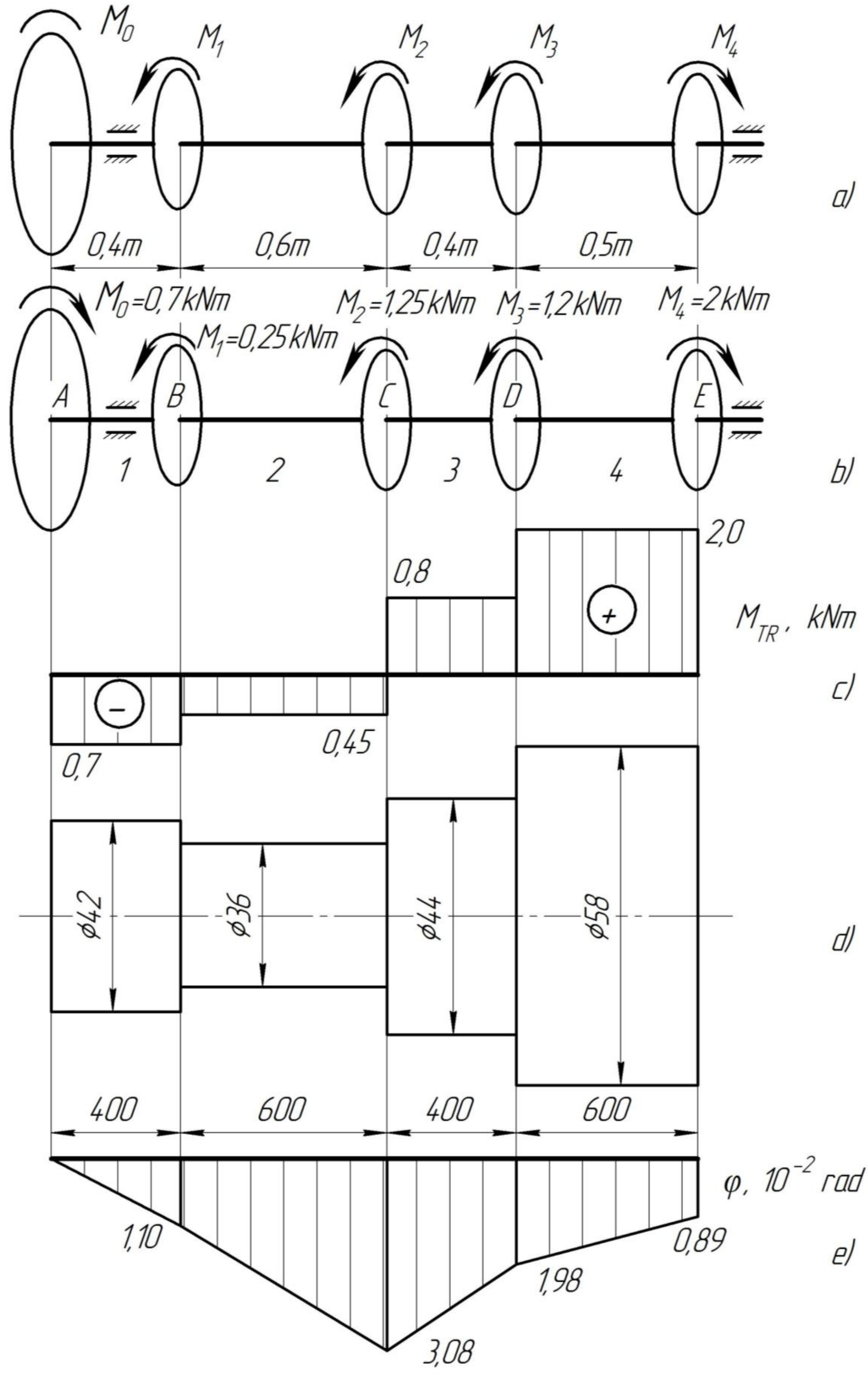

M0
M1
M2
M3
M4
a)
0,4m
0,6m
0,4m
0,5m
M0=0,7kNm
M1=0,25kNm
M2=1,25kNm
M3=1,2kNm
M4=2kNm
A
B
C
D
E
1
2
3
4
b)
2,0
0,8
+
MTR, kNm
c)
−
0,7
0,45
ϕ42
ϕ36
ϕ44
ϕ58
d)
400
600
400
600
φ, 10−2 rad
1,10
0,89
e)
1,98
3,08

Figure 4.4

Draw the diagram of twist angles of the shaft (Fig. 4.4 *e*), accepting section A for the beginning of reference.

Test of the shaft rigidity is carried out under the rigidity condition

$$\theta_{\max} \le [\theta],$$

where $\theta_{\max}$ is the maximum relative twist angle of the shaft.

Determine the relative twist angles on each section of the shaft by formula

$$\theta_i = \frac{\varphi_i}{l_i}, \text{ where } i = 1,\ 2,\ 3,\ 4.$$

Substituting the value, obtain

$$\theta_1 = \frac{1{,}10 \cdot 10^{-2}}{0{,}4} = 2{,}75 \cdot 10^{-2}\ \frac{\text{rad}}{\text{m}};$$

$$\theta_2 = \frac{1{,}98 \cdot 10^{-2}}{0{,}6} = 3{,}30 \cdot 10^{-2}\ \frac{\text{rad}}{\text{m}};$$

$$\theta_3 = \frac{1{,}10 \cdot 10^{-2}}{0{,}4} = 2{,}75 \cdot 10^{-2}\ \frac{\text{rad}}{\text{m}};$$

$$\theta_4 = \frac{1{,}09 \cdot 10^{-2}}{0{,}5} = 2{,}1 \cdot 10^{-2}\ \frac{\text{rad}}{\text{m}}.$$

Obtain

$$\theta_{\max} = \theta_2 \cdot \frac{180^\circ}{\pi} = 3{,}30 \cdot 10^{-2} \cdot \frac{180^\circ}{3{,}14} = 1{,}89\ \frac{\text{rad}}{\text{m}} < [\theta] = 2\ \frac{\text{rad}}{\text{m}}.$$

Hence, the rigidity condition is ensured.

5. COMPLEX STRESSED STATE

Through any point of the deformed massive body, it is possible to draw many differently oriented cutting planes (platforms). **The set of normal and tangential (shear) stresses occurring on planes crossing the given point characterize the stressed state of the body at that point.**

Normal stresses σ are considered to be positive if they stretch the material of the element. Tangential (shear) stresses are positive when they form a pair of forces relatively to the center of the element that tends to rotate it clockwise (Fig. 5.1).

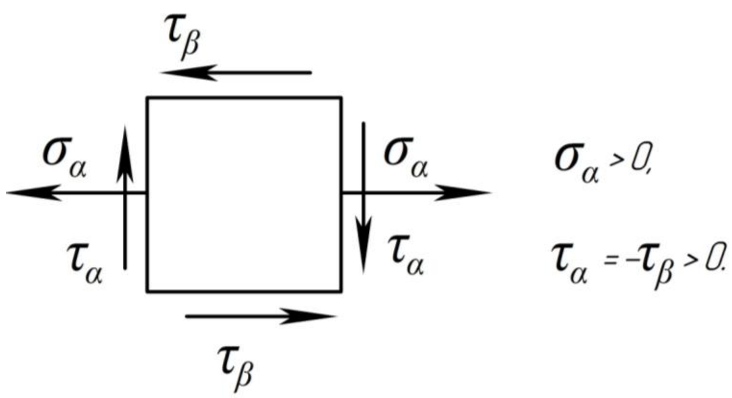


Figure 5.1

The tangential (shear) stresses at two mutually perpendicular planes are equal but opposite in sign. (Law of paired relationship of tangent stresses).

Three mutually perpendicular planes can be drawn through each point of the body at which the tangential (shear) stresses are zero. Such planes are called the **main planes**, and the stresses acting on them are the **main stresses.** They are indicated $\sigma_1, \sigma_2, \sigma_3$, besides $\sigma_1 \geq \sigma_2 \geq \sigma_3$. The main stresses at the given point in massive body reach extreme values for the given stress state.

There are three types of stress state (Fig. 5.2-5.4).

1. Linear

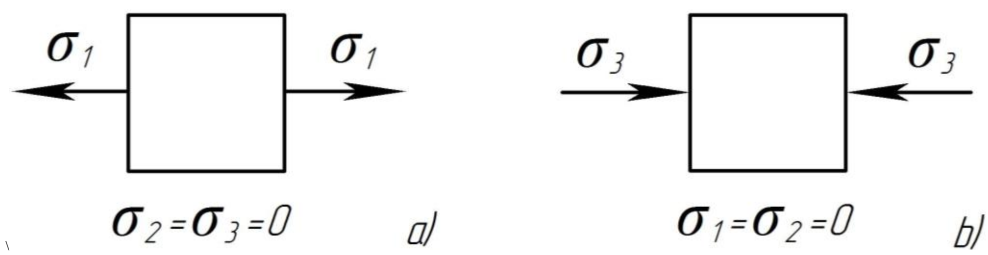


Figure 5.2

2. Plane

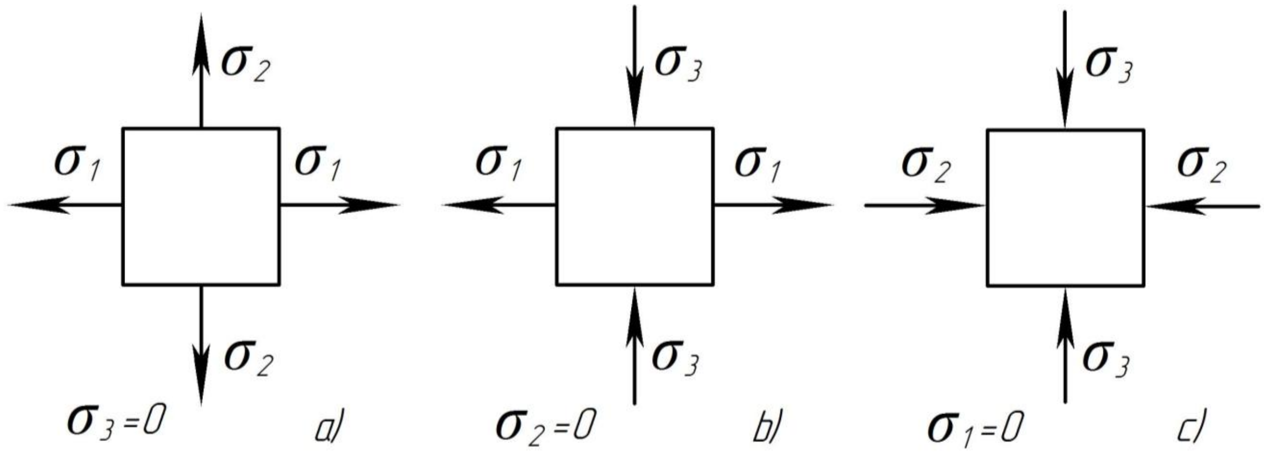


Figure 5.3

3. Volume

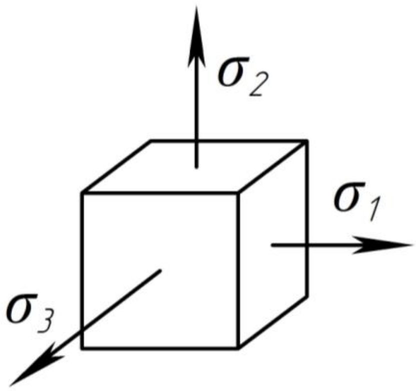


Figure 5.4

Then we consider the linear and flat stressed states.

Most of the tasks of complex stressed state are to determine the principal stresses by the known normal and tangent stresses at the planes.

The principal stresses are the extreme (maximum and minimum) stresses at which the strength of the structural material can be evaluated.

In the general case, for plane stressed state normal σ_α, σ_β and tangential (shear) $\tau_\alpha = -\tau_\beta$ stresses acting on mutually perpendicular sites are known. We assume that $\sigma_\alpha \geq \sigma_\beta$. Therefore, the calculation scheme can be taken as shown in Fig. 5.5.

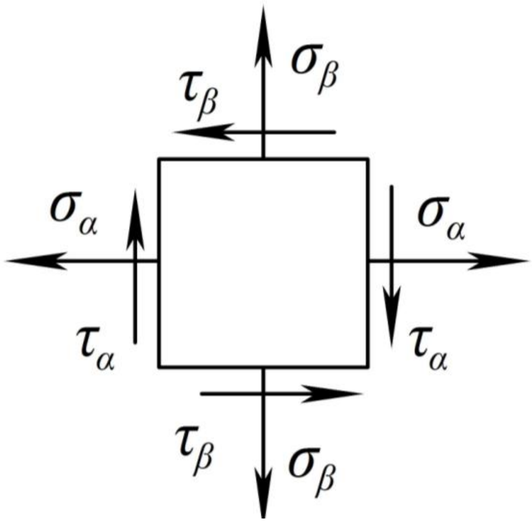


Figure 5.5

The value and direction of the principal stresses are determined by the formulas

$$\sigma_{\substack{max \\ min}} = \frac{1}{2}\left[\sigma_{\alpha} + \sigma_{\beta} \pm \sqrt{\left(\sigma_{\alpha} - \sigma_{\beta}\right)^{2} + 4\tau_{\alpha}^{2}}\right];$$

$$tg\,2a = -\frac{2\tau_{\alpha}}{\sigma_{\alpha} - \sigma_{\beta}},$$

where σ_{max} is **maximum main stress**, $\sigma_{max} = \sigma_1$ (Fig. 5.3 *a*, *b*), $\sigma_{max} = \sigma_2$ (Fig. 5.3 *c*);

σ_{min} is **minimum main stress**, $\sigma_{min} = \sigma_2$ (Fig. 5.3 *a*), $\sigma_{min} = \sigma_3$ (Fig. 5.3 *b*, *c*);

α is the angle to which the vector σ_{α} must be rotated in order to determine the direction of greater main stress (if the angle is positive, then you need to turn counterclockwise).

The values of the main stresses and their directions can be determined graphically using the **Mohr's circle**. This method is described in the example of solving of task 5.

Task 5
Analysis of stressed state

For a given element (Fig. for task 5, Table for task 5) determine: the position of the main planes (graphically and analytically), value and direction of the main stresses, linear deformations in the direction of all main stresses, relative change in volume, specific potential deformation energy. Check the element for strength according to the strength theories appropriate for the given materials.

Plan of solving the task:

1. Determine the values and directions of σ_α, σ_β, τ_α, τ_β (indeces V and H on the model stand for vertical and horizontal, replace them by α and β according to the value and with symbol σ), draw the given element.

2. Determine the values and direction of the main stresses graphically. In the middle of the given element draw the main element limited by the main planes.

3. Determine the value $\tau_{\max}$ and on the same figure, draw the position of the plane where $\tau_{\max}$ acts.

4. Validate the obtained results analytically.

5. Determine the relative deformations in the directions of all three main stresses.

6. Determine the relative volume change and specific potential deformation energy.

7. Determine the calculated stress according to one of the theories of strength relevant to the given material (at the student's choice) and compare it with the allowable, taking the margin of safety $n_T = 1{,}5$; $n_M = 2{,}5$.

Table for task 5

Nr	σ_V, $\mathrm{MN/m^2}$	σ_H, $\mathrm{MN/m^2}$	τ, $\mathrm{MN/m^2}$	Material	
				Cast iron	Steel
1	100	50	10	Ci 12-28	St. 1
2	0	60	20	Ci 15-32	St. 2
3	20	0	30	Ci 18-36	St. 3
4	30	80	40	Ci 21-40	St. 4
5	40	90	50	Ci 24-44	St. 5
6	50	0	25	Ci 28-48	St. 6
7	60	10	15	Ci 32-52	St. 1
8	0	20	45	Ci 35-56	St. 2
9	80	30	35	Ci 38-60	St. 3
0	90	40	10	Ci 18-36	St. 4

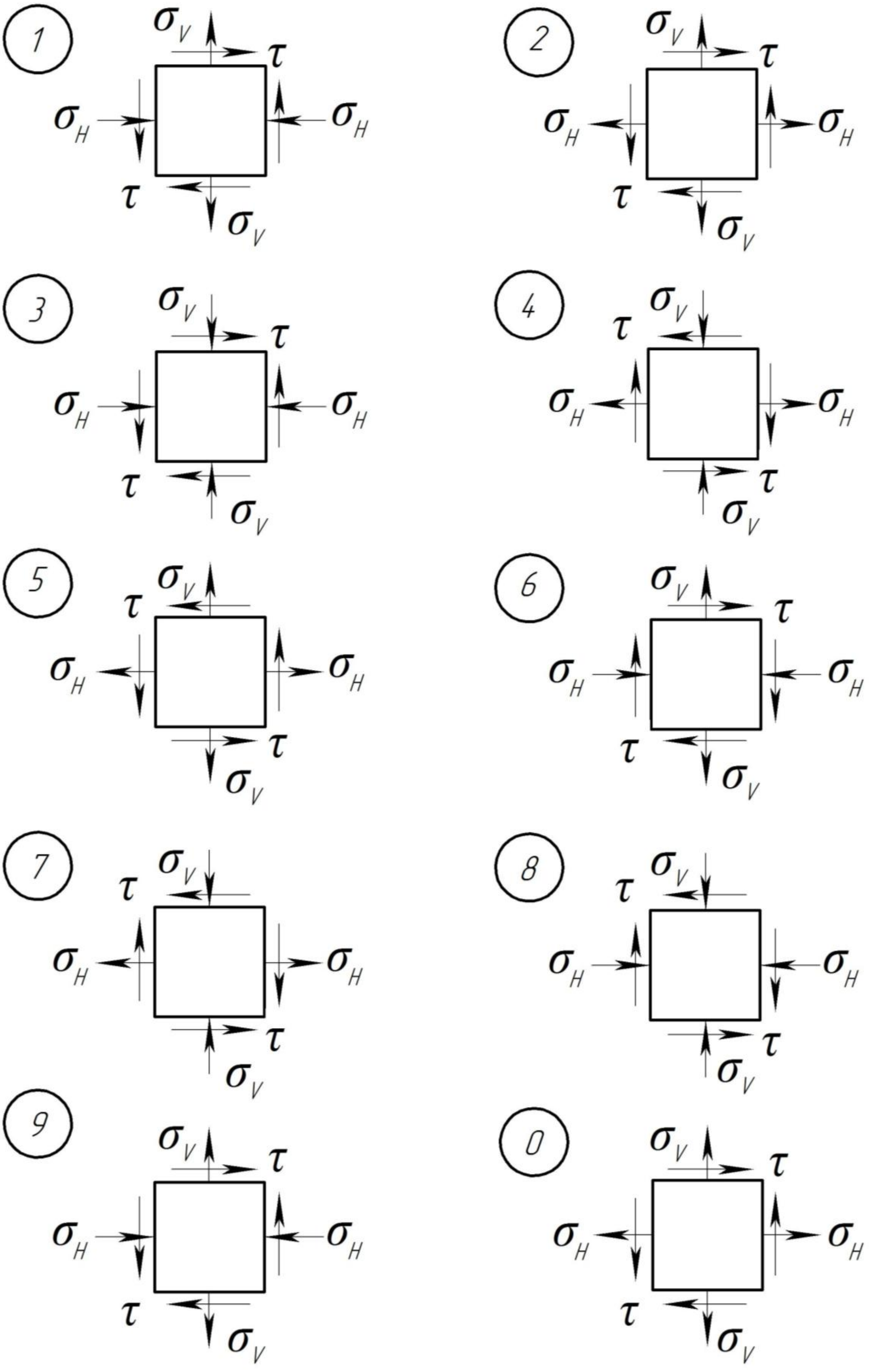


Figure for task 5

Example of solving the task 5
Analysis of plane stressed state

$\sigma_V = 0$; $\sigma_H = 60$ MPa act on the element shown in Fig. 5.6. Complete the following:

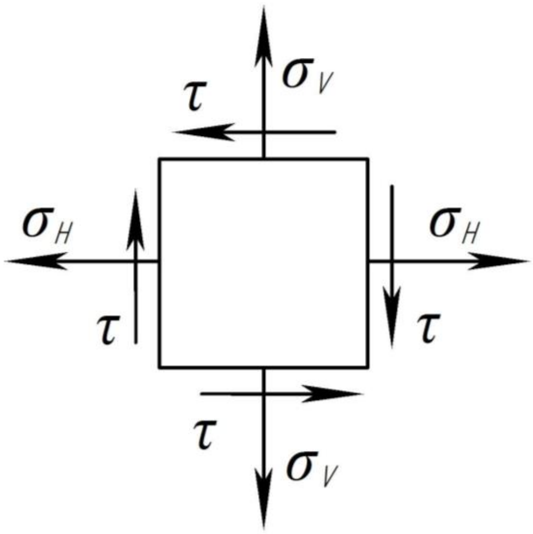


Figure 5.6

Determine the values and directions σ_α, σ_β, τ_α, τ_β (indices *C* and *D* in the diagram mean vertical and horizontal, replace them with α and β in accordance with the magnitude and sign σ), draw the element.

Determine graphically the values and direction of the main stresses. In the middle of the given element, draw the main element, restricted by the main planes.

Determine the value $\tau_{\max}$ and in the same figure to show the position of the plane on which $\tau_{\max}$ acts.

Analyze the obtained results analytically.

Determine the relative deformations in the directions of all three main stresses.

Determine the relative change in volume and the specific potential deformation energy.

Determine the calculation stresses according to one of the corresponding given material of strength theories for materials steel St.3 and cast iron Ci 18-36. Compare their values with the allowable stresses by taking the strength coefficients $n_T = 1{,}5$; $n_M = 2{,}5$.

Solution

Replace the indices of stresses acting on the element according to the data and calculation scheme (Fig. 5.7 *a*)

$$\sigma_\alpha = \sigma_H = 60 \text{ MPa}; \quad \sigma_\beta = \sigma_V = 0; \quad \sigma_\alpha > \sigma_\beta .$$

On the plane α the tangent stresses are $\tau_\alpha = 100$ MPa. According to the law of paired relationship of tangent stresses

$$\tau_\alpha = -\tau_\beta = 100 \text{ MPa} .$$

Draw out the rectangular coordinate system σ, τ. Axis σ is parallel to the greater normal stress σ_α (Fig. 5.7 *b*). In this coordinate system, we define points that correspond to the stresses on the planes α and β, these are points D_α and D_β. Since these points reflect the stresses acting on two mutually perpendicular planes, the segment $D_\alpha D_\beta$ is the diameter of the stress circle. The point of intersection of this diameter with the axis σ forms the center of the circle – point C. Points A and B at which the circle crosses axis σ $(\tau = 0)$ determine the values of the main normal stresses:

$$\sigma_1 = \overline{OA} = 135 \text{ MPa}; \quad \sigma_3 = \overline{OB} = -75 \text{ MPa} .$$

The stress direction σ_1 is determined by a vector BD'_α, $\sigma_\beta = 0$; $\sigma_2 = 0$.

The angle between normal stresses σ_α and σ_1 and is $\alpha = -37^\circ$. Minus sign indicates that it is set off from the axis σ clockwise direction.

The maximum tangential (shear) stress $\tau_{\max}$ is equal to the radius of Mohr's circle

$$\tau_{\max} = CT = 105 \text{ MPa} .$$

The angle between stress σ_α and greater main stress is determined by the formula

$$tg\, 2\alpha = -\frac{2\tau_\alpha}{\sigma_\alpha - \sigma_\beta} = -\frac{2 \cdot 100}{60 - 0} = -3{,}33 ;$$

$$\alpha = -\frac{1}{2} arctg\, (3{,}33) = -37^\circ .$$

The maximum tangential (shear) stress is equal

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{135 - (-75)}{2} = 105 \text{ MPa} .$$

The vectors of stresses σ_1, σ_3, τ_{max} and the planes on which they act are shown in Fig. 5.7 *a*.

The relative deformations in the direction of the main stresses for the steel element are determined by the formulas:

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)] = \frac{1}{2 \cdot 10^5} \cdot [135 - 0{,}3 \cdot (-75)] = 78{,}8 \cdot 10^{-5};$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \mu(\sigma_1 + \sigma_3)] = \frac{1}{2 \cdot 10^5} \cdot [0 - 0{,}3 \cdot (135 - 75)] = -9{,}00 \cdot 10^{-5};$$

$$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2)] = \frac{1}{2 \cdot 10^5} \cdot [-75 - 0{,}3 \cdot 135] = -57{,}8 \cdot 10^{-5}.$$

Determine the relative change in volume

$$\theta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = (78{,}8 - 9{,}00 - 57{,}8) \cdot 10^{-5} = 12 \cdot 10^{-5}.$$

Determine the specific potential energy of deformation of the steel element

$$U = \frac{1}{2}(\sigma_1 \cdot \varepsilon_1 + \sigma_2 \cdot \varepsilon_2 + \sigma_3 \cdot \varepsilon_3) =$$

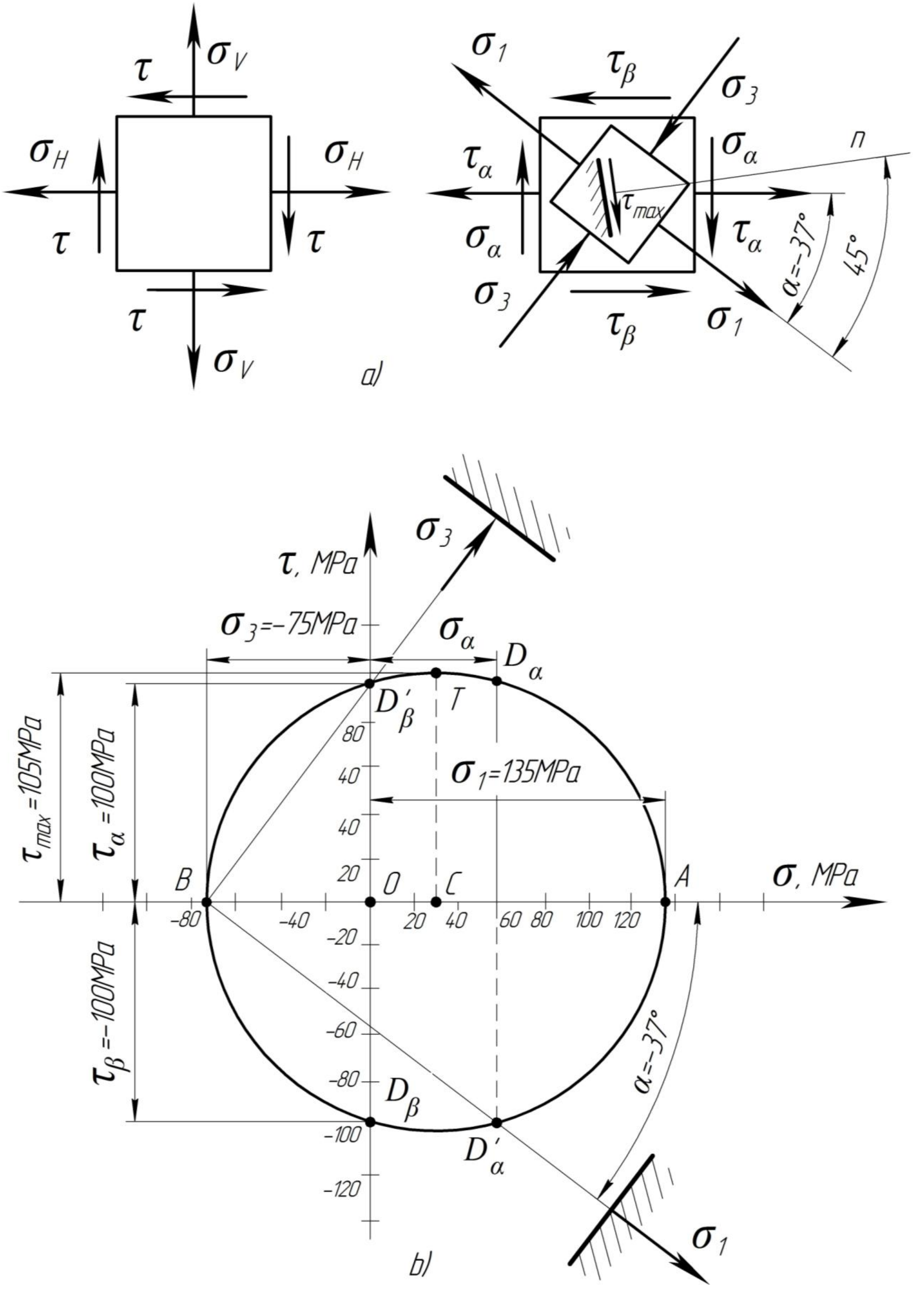
$$= \frac{1}{2}(78{,}8 \cdot 135 + 0 + 57{,}8 \cdot 75) \cdot 10^{-5} = 74{,}9 \cdot 10^{-3} \text{ MNm}/\text{m}^3 .$$

Determine the allowable stresses:

a) for steel St.3

$$[\sigma] = \frac{\sigma_T}{n_T} = \frac{220}{1{,}5} = 147 \text{ MPa} ,$$

where σ_T is the yield strength for steel Art.3; $\sigma_T = 220$ MPa ;



σ_V
τ
σ_H
σ_H
τ
τ
τ
σ_V
a)
σ_1
τ_β
σ_3
σ_α
n
τ_α
τ_max
σ_α
τ_α
α=-37°
45°
σ_3
τ_β
σ_1
σ_3
τ, MPa
σ_3=-75MPa
σ_α
D_α
D'_β
T
τ_max=105MPa
τ_α=100MPa
σ_1=135MPa
B
O
C
A
σ, MPa
-80
-40
20
40
60
80
100
120
τ_β=-100MPa
D_β
D'_α
α=-37°
σ_1
b)

Figure 5.7

b) for cast iron Ci 18-36:

$$[\sigma]_P = \frac{\sigma_{MP}}{n_M} = \frac{180}{2,5} = 72 \text{ MPa} ;$$

$$[\sigma]_C = \frac{\sigma_{MC}}{n_M} = \frac{700}{2,5} = 280 \text{ MPa} ,$$

where σ_{MP} , σ_{MC} are the tensile and compression strengths for brittle material, for Ci 18-36 $\sigma_{MP} = 180 \text{ MPa}$, $\sigma_{MC} = 700 \text{ MPa}$.

For steel St.3, which is a plastic material, the strength test can be performed according to the third or fourth theory of strength. According to the third theory of strength

$$\sigma_{P3} = \sigma_1 - \sigma_3 = 135 - (-75) = 210 \text{ MPa} ;$$

$$\sigma_{P3} = 210 \text{ MPa} > [\sigma] = 147 \text{ MPa} .$$

The strength condition is not ensured.

For Ci 18-36 cast iron, which is brittle material, we apply the theory of Mohr strength, since the investigated stress state of the material is between simple tension and simple compression

$$\sigma_P = \sigma_1 - \nu \cdot \sigma_3 = 135 - 0,257 \cdot (-75) = 154 \text{ MPa};$$

$$\sigma_P = 154 \text{ MPa} > [\sigma]_P = 72 \text{ MPa} ,$$

where

$$\nu = \frac{[\sigma]_P}{[\sigma]_C} = \frac{72}{280} = 0,257 .$$

The strength condition for cast iron is also not ensured.

6. STRAIGHT TRANSVERSE BENDING

Straight transverse bending. Internal force factors. Sign convention of bending

Bending is the bar resistance state in which bending or change of the curvature of its axis occurs. The bar that works in bending is called the ***beam***.

Many structural elements work for bending: axes of railway cars, shafts, overlapping panels, span bridges, crane arrows, flat car springs, etc.

Plane or **straight bending** is the case of bending in which the beam axis is curved in the direction of external forces and loads, i.e. in the same plane with external forces.

Straight transverse bending is a type of deformation in which the **shear (cutting) force** Q and **bending moment** M_{BN} occur in the cross-sections of the beam (Fig. 6.1 *a*). If the shear force does not occur, then it is the **pure bending** (Fig. 6.1 *b*).

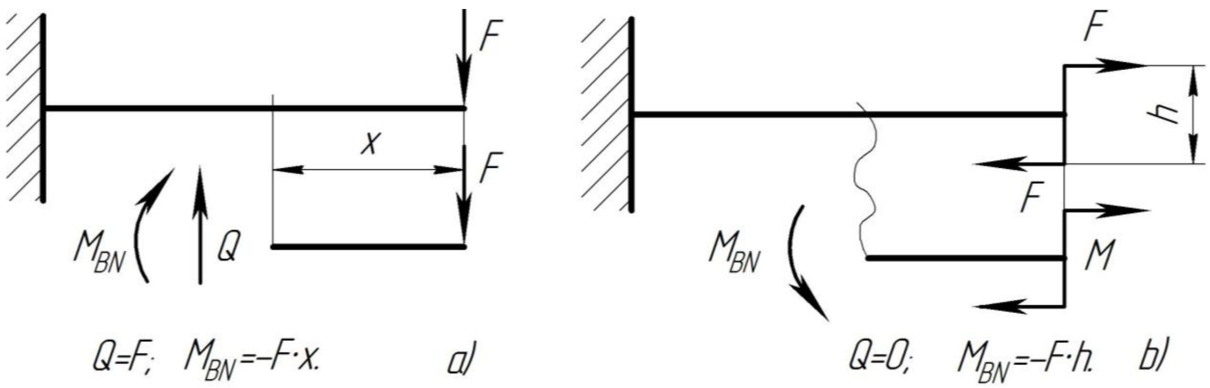


Figure 6.1

Shear (***cutting***) ***force*** at any cross-section of the beam is equal to the algebraic sum of the projections of all external forces acting on the right or left of the section on the axis perpendicular to the axis of the beam, i.e.

$$Q = \sum F_{iY} .$$

Bending moment at any cross-section of the beam is equal to the – algebraic sum of the moments of all external forces acting to the right or left of the section relatively to the center of gravity of the section.

$$M_{BN} = \sum M\left(F_i\right).$$

For the beam in equilibrium under the action of plane system of forces perpendicular to the axis (i.e. the system of parallel forces), the algebraic sum of all external forces is zero. Therefore, ***the sum of the external forces acting on the beam to the left of the section is numerically equal to the sum of the forces acting on the beam to the right of the intersection.***

Statics signs rules are unsuitable for determining the signs of the cross-section force Q and bending moment M_{BN} .

Sign rule of bending can be represented graphically – shear (cutting) force (Fig. 6.2) and bending moment (Fig. 6.3).

If the sum of external forces acting to the left of the section gives the equilibrium pointing upwards, then the ***shear* (*cutting*) *force*** in the section is considered to be ***positive***. Conversely: for the part of the beam to the right of the section, the signs of the lateral force will be opposite (see Fig. 6.2). Or the lateral forces are positive if they tend to rotate the beam element clockwise.

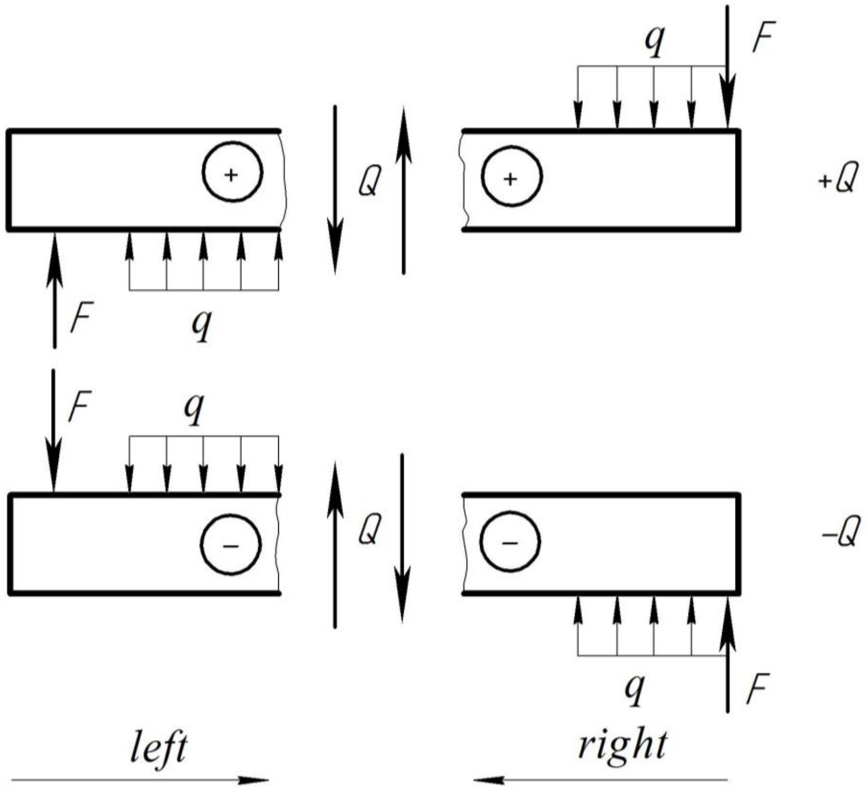


Figure 6.2

If the external load tends to bend the beam by the convexity downwards (the lower fibers are stretched), the ***bending moment*** in the section is considered ***positive*** and vice versa (see Fig. 6.3).

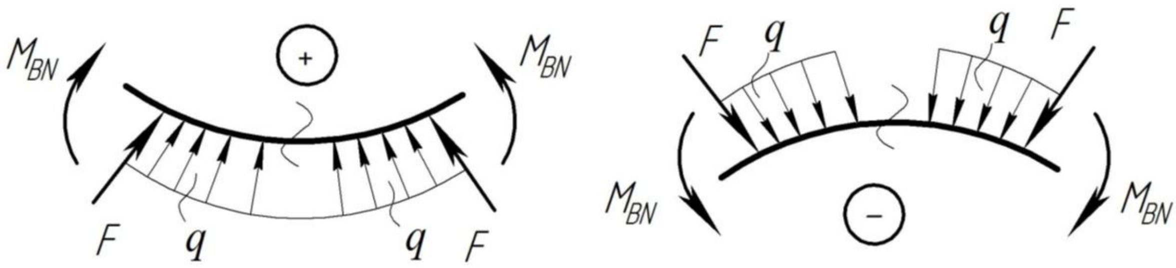


Figure 6.3

Using these rules, one should ***imagine the section of the beam rigidly fixed***, and the links rejected and replaced by reactions.

Statics sign rules are used to determine support reactions; to determine the signs of bending moment and shear (cutting) force the rules of strength of materials are applied.

All forces, active and reactive are the beam loads.

Simplified representation of the real support elements, that is, their schematization, which is used to construct the calculation schemes of beams in plane bending state, makes it possible to distinguish three main types of **supports**: ***hinged-movable***, ***hinged-fixed*** and ***clamping*** (rigid fastening). Each of them is model ed in the form of rods (bars) (Fig. 6.4).

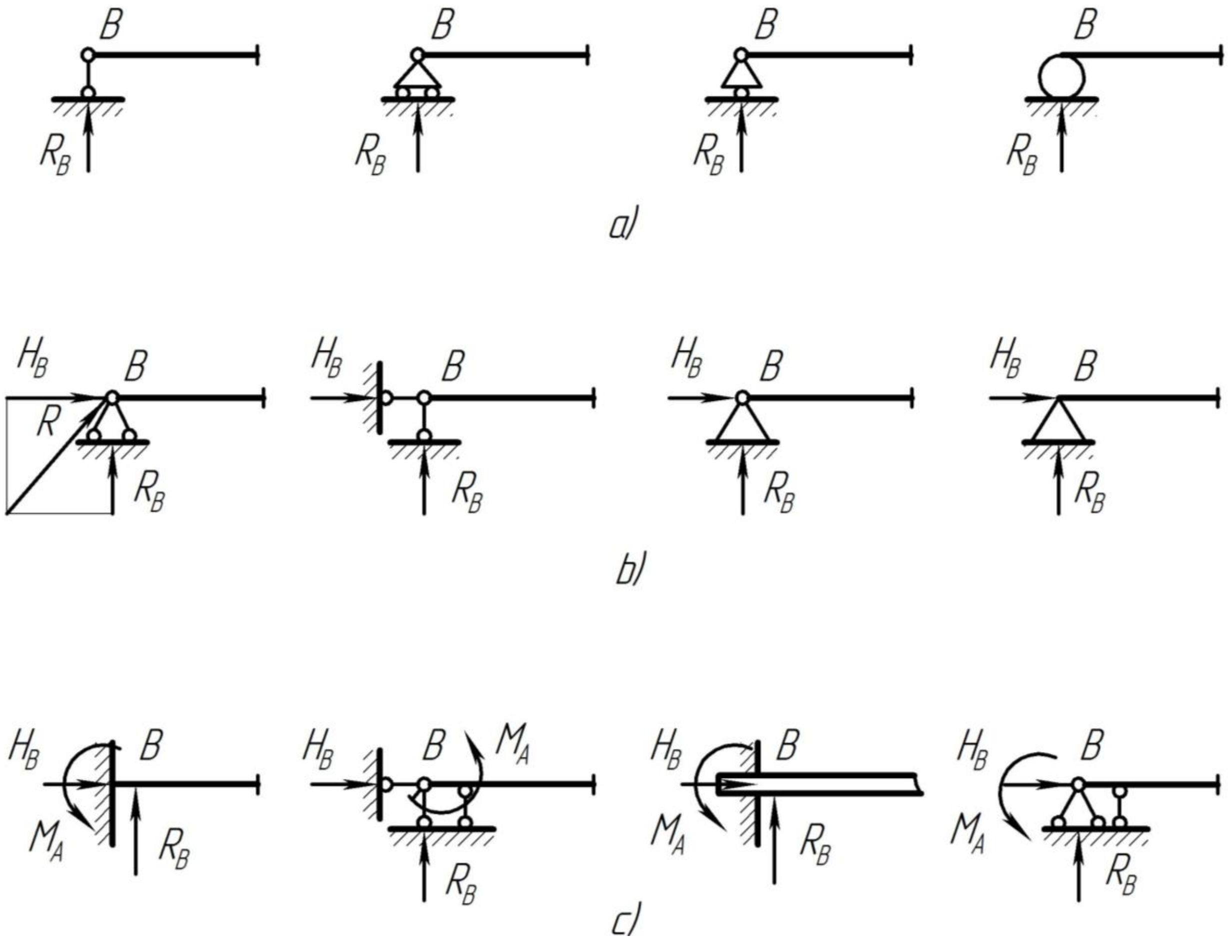


Figure 6.4. Schemes of beam supports:
a) hinged-movable; *b*) hinged-fixed; *c*) clamping

Differential dependencies at straight transverse bending

There are differential dependences between bending moment, shear (cutting) force, and intensity of the distributed load, on which the **Zhuravsky theorem** is based: ***the shear* (*cutting*) *force is equal to the first derivative from the bending moment by the abscissa of the beam section.***

Differential dependences between force factors under bending

$$\frac{dM(x)}{dx} = Q(x); \qquad \frac{d^2M(x)}{dx^2} = \frac{dQ(x)}{dx} = q(x).$$

The second derivative of bending moment or the first derivative from shear (cutting) force along abscissa of the intersection of the beam is equal to the intensity of the distributed load.

Diagrams of shear (cutting) forces and bending moment

To illustrate the distribution of the shear (cutting) forces and bending moments along the beam axis, the diagrams allowing to determine the ***possible dangerous section of the beam,*** to determine the value of shear (cutting) force and bending moment at this section are drawn. There are two methods of drawing the diagrams for shear (cutting) forces and bending moments.

The first method. Analytical expressions of shear forces and bending moments for each segment as the function of the current coordinate x of the cross-section are recorded

$$Q = f_1(x), \qquad M_{BN} = f_2(x).$$

Then the diarams are drawn according to the obtained results.

The second method. Diagrams are drawn according to the characteristic points and values of shear (cutting) forces and bending moments at the sections boundaries. Using this method, in most cases you can omit the addition of shear (cutting) forces and bending moments equations.

The construction of the diagrams of internal force factors under bending will be shown in the examples.

<u>Example 1</u> (Fig. 6.5).

Determine the support beam reactions (Fig. 6.5 *a*)

$$\sum M_C = 0; \qquad R_B(a+b) - F \cdot b = 0,$$

from which $R_B = \dfrac{F \cdot b}{a+b}$;

$$\sum M_B = 0; \qquad \sum M_B = R_C(a+b) - F \cdot a = 0,$$

from which $R_C = \dfrac{F \cdot a}{a+b}$.

Vlidate the correctness of the support beam reactions determination

$$\sum Y = R_B - F + R_C = \frac{F \cdot b}{a+b} - F + \frac{F \cdot a}{a+b} = 0 .$$

The support beam reactions are determined correctly.

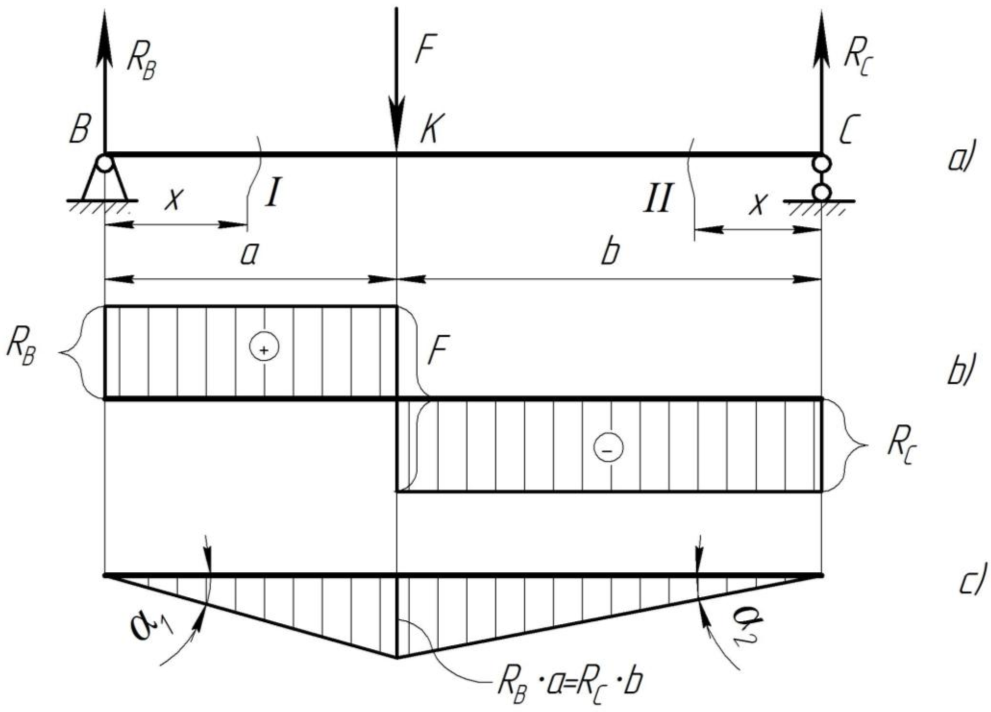


Figure 6.5

Use the first method of diagram drawing. Divide the beam into sections. For each section down functions $Q(x)$, $M_{BN}(x)$, as well as the boundaries within which these functions are true (Fig. 6.5 *a*).

Section *I*, $0 \le x \le a$ (left side):

$$Q(x) = R_B = \frac{F \cdot b}{a+b}; \qquad Q(0) = Q(a) = \frac{F \cdot b}{a+b};$$

$$M_{BN}(x) = R_B \cdot x = \frac{F \cdot b}{a+b} \cdot x; \qquad M_{BN}(0) = 0; \qquad M_{BN}(a) = \frac{F \cdot b \cdot a}{a+b}.$$

Section *II*, $0 \le x \le b$ (right side):

$$Q(x) = -R_C = -\frac{F \cdot a}{a+b}; \qquad Q(0) = Q(b) = -\frac{F \cdot a}{a+b};$$

$$M_{BN}(x) = R_C \cdot x = \frac{F \cdot a}{a+b} \cdot x; \qquad M_{BN}(0) = 0; \qquad M_{BN}(b) = \frac{F \cdot b \cdot a}{a+b}.$$

Based on the obtained results, draw the diagrams Q and M_{BN} (Fig. 6.5 *b, c*).

From the diagram M_{BN} (see Fig. 6.5 *c*) determine the dangerous section, i.e. the section in which the maximum bending moment acts – it is section K

$$M_{BN\ \max} = \frac{F \cdot b \cdot a}{a + b}.$$

Determine the values of the shear (cutting) forces using differential dependencies (for validation):

- on the first section

$$\frac{dM_{BN}(x)}{dx} = tg\,\alpha_1 = Q_1 = \frac{M_{BN\ \max}}{a} = R_B;$$

- on the second section

$$\frac{dM_{BN}(x)}{dx} = -tg\,\alpha_2 = Q_2 = -\frac{M_{BN\ \max}}{b} = -R_C.$$

Example 2 (Fig. 6.6). Use the second method of the diagram Q, M_{BN} construction.

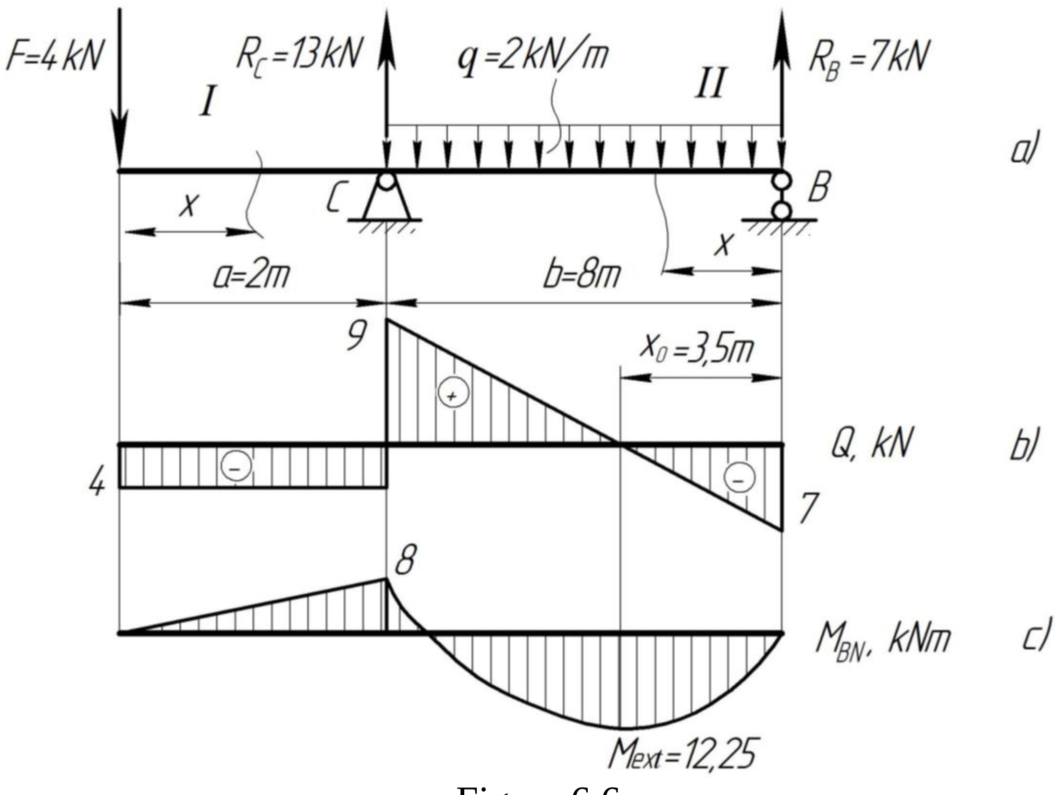


Figure 6.6

Analytically determine the value of extreme moment, for this case it is dangerous beam section.

Coordinate $x_0 = R_B / q = 7/2 = 3{,}5$ m.

$M_{BN}(x_0) = R_B \cdot x_0 - q \cdot x_0^2 / 2 = 7 \cdot 3{,}5 - 2 \cdot 3{,}5^2 / 2 = 12{,}25$ kNm.

I. Control of the correctness of drawing the diagrams of shear (cutting) forces and bending moments according to the calculation scheme of the beam

For shear (cutting) force diagram:

1. In the beam segment loaded by evenly distributed load q, the shear (cutting) force diagram is drawn by straight line inclined to the axis of the beam.

2. In the beam segment free from q, the shear (cutting) force diagram is drawn by straight line parallel to the beam axis.

3. Under the intersection of the beam where concentrated force is applied, there is a jump on the shear forces (cutting) diagram, which is equal to the magnitude of applied force.

4. At the section, where the concentrated pair of forces (concentrated moment) is applied, the diagram of shear (cutting) forces does not change its value.

For bending moment diagram:

1. In the beam segment loaded by evenly distributed load q, the diagram of bending moment is represented by quadratic parabola.

2. In the beam segment free from q, the diagram of bending moment is drawn as a straight line inclined to the axis of the beam.

3. The bending moment reaches extreme values at the sections where the shear forces are zero.

4. Under the section of the beam, where concentrated pair of forces (concentrated moment) is applied, there is a jump in the diagram of bending moments, which is equal to the magnitude of the concentrated moment.

5. On the beam segment where the shear (cutting) force is zero, the beam undergoes pure bending, the diagram of bending moments is straight line parallel to the axis of the beam.

II. Verification of the diagram of bending moments using bending differential (by diagram Q).

The cross-sectional diagram verification using dependency

$$\frac{dM_{BN}(x)}{dx} = Q(x),$$

is carried out taking into account that the diagram Q is graphical representation of the derivative of the bending moment M_{BN} :

1. The bending moment function $M_{BN}(x)$ increases when the derivative of the function, i.e. $Q(x)$ is positive.

2. The bending moment function $M_{BN}(x)$ decreases when the derivative of the function, i.e. $Q(x)$, is negative.

3. The bending moment function $M_{BN}(x)$ reaches extreme value at the point where its derivative $Q(x)$ is zero. The function at this section must be investigated for extremum.

Bending stress. Strength calculation

The bending moment is the destructive internal force factor in direct transverse (shear) bending. From the action of the bending moment in the cross-section of the beam the normal stresses occur. They are determined by the formula

$$\sigma = \frac{M_{BN}}{I_X} \cdot y,$$

where y is the distance (coordinate) from axis X (neutral axis) to points of the cross-section in which the normal stress is determined (Fig. 6.7 *a*).

Analyzing this formula, we obtain the diagram of the normal stresses distribution by the section height (Fig. 6.7*b*).

Maximum normal stresses and **bending strength condition under normal stresses**

$$\sigma_{\max} = \frac{M_{BN.\max}}{I_X} \cdot y_{\max} = \frac{M_{BN.\max}}{W_X} \le [\sigma],$$

where $M_{BN.\max}$ is the maximum bending moment, determined from the diagram M_{BN};

W_X is axial moment of cross-section resistance (see topic 3).

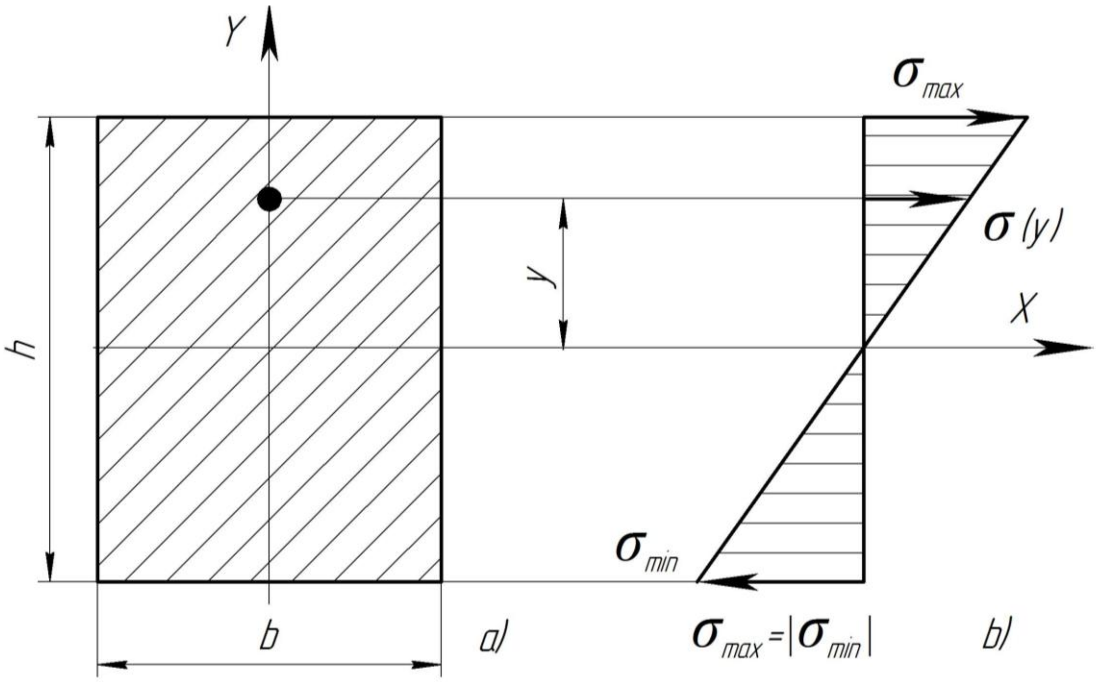


Figure 6.7

In the cross-sections of the beam under transverse bending deformation, not only normal but also tangential stresses occur as the result of shear (cutting) force Q action, which cause the shear deformation. According to the law of paired relationship, the same tangential stresses occur in the longitudinal sections parallel to the neutral layer. The presence of tangential (shear) stresses in the longitudinal sections under shear bending is confirmed by the occurence of longitudinal cracks in the wooden beams.

The **values of tangential (shear) stresses** are determined by **D.I. Zhuravsky formula**

$$\tau = \frac{Q_y \cdot S_X(y)}{b(y) \cdot I_X},$$

where τ is tangential (shear) stress at the considered point of cross-section;
Q_y is the absolute value of the shear (cutting) force in the considered section;
$S_X(y)$ is the absolute value of the static moment of section, cut off at the level of the point under consideration;
$b(y)$ is the width of the beam section at the level where tangential stresses (shear) are determined;
I_X is the moment of inertia of the entire section with respect to its central axis X.

The absolute value of the static moment of the section part cut off at the level of the considered point is determined by the formula

$$S_X(y) = A_{sh} \cdot y^*,$$

here A_{sh} is the area of the cut off part of the section;
y^* is the distance (coordinate) of the center of gravity C_B of the cut off part area relatively to the central axis.

For the cross-section, the values Q_y and I_X are constant values. Depending on the shape of the cross-section of the bar, the width $b(y)$ of the section may be variable (in the presence of the cross-section the function $b(y)$ is known). In contrast to Q_y and I_X, the value of the static moment $S_X(y)$ of the cut off part of the section has variable value that depends on y^*.

For the rectangular beam with sides b and h (Fig. 6.8 *a*)

- the area of the cut off section at the level of the layer under consideration of fibers $m\ n$

$$A_{sh} = b\left(\frac{h}{2} - y\right);$$

- the static moment of this area (i.e. at $m\ n$ level)

$$S_{adce} = b\left(\frac{h}{2} - y\right) \cdot \left[y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right] = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right).$$

The axial moment of inertia of rectangular cross-section

$$I_X = \frac{b \cdot h^3}{12}.$$

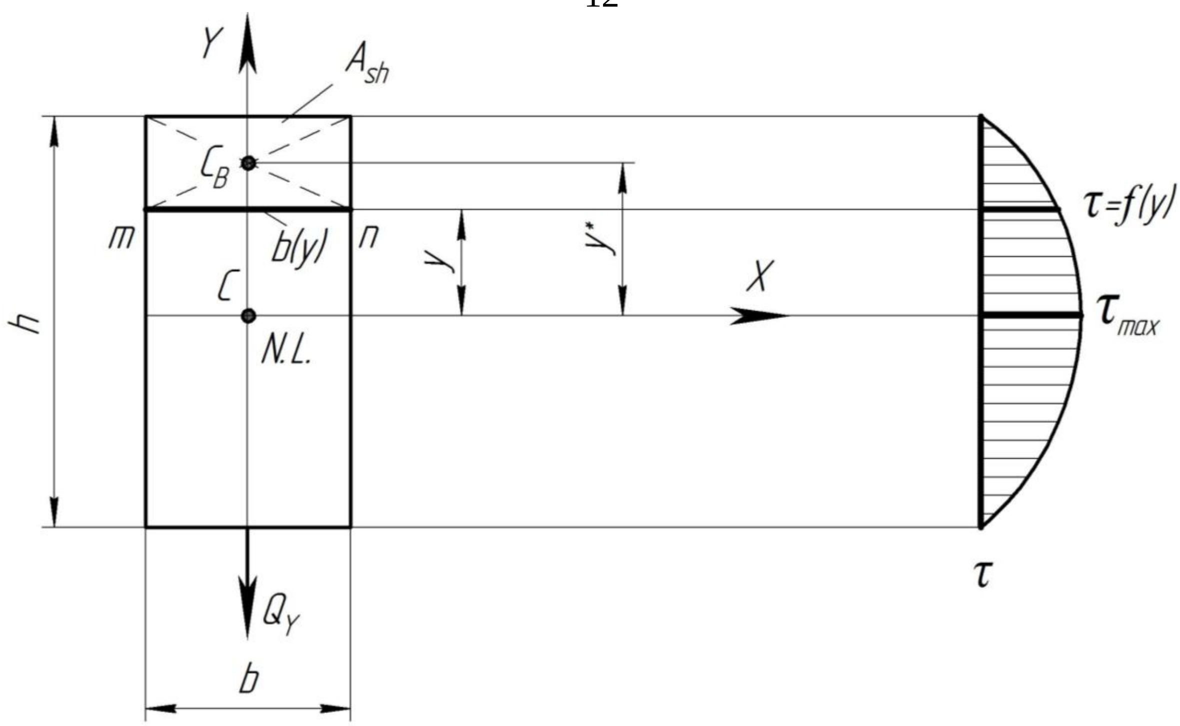


Figure 6.8

Define the law of distribution of tangential (shear) stresses for the rectangular cross-section beam. Do this for the fiber layer at mn level (see Fig. 6.8*a*)

$$\tau = \frac{Q \cdot S_{sh}}{b \cdot I_X} = \frac{Q \cdot (b/2) \cdot \left(h^2/4 - y^2\right)}{b \cdot b \cdot h^3/12} = \frac{6Q\left(h^2/4 - y^2\right)}{b \cdot h^3};$$

when $y = \pm h/2$, then $\tau = 0$;

when $y = 0$, then $\tau = \tau_{\max} = \dfrac{3Q}{2b \cdot h} = \dfrac{3Q}{2A} = \dfrac{3}{2}\tau$,

where τ are tangential (shear) stresses.

The ***diagram of tangential*** **(*shear) stresses*** at the height of rectangular section is indicated by ***quadratic parabola*** (Fig. 6.8 *b*). That is, in the upper and lower layers of fibers the tangential (shear) stresses are zero, and in the fibers of the neutral layer they reach the maximum value.

Thus, the ***tangential*** **(*shear*)** ***stresses*** in the beams ***correspond to shear deformation,*** and as the result plane cross-sections in direct transverse bending do not remain plane, as in pure bending, but distorted.

Most bending beams are calculated only under normal stresses. ***Three types of beams are verified for tangential stresses:***

1) wooden beams, because wood is not good for chipping;

2) narrow beams, for example, I-beams, since the maximum tangential (shear) stresses are inversely proportional to the width of the neutral layer;

3) short beams, because with relatively small bending moment and normal stresses such beams can produce considerable shear forces and tangent stresses.

Strength conditions under bending according to shear stresses

$$\tau_{\max} = \frac{Q_{y\,\max} \cdot S_X(y)_{\max}}{b(y) \cdot I_X} \le [\tau],$$

where $Q_{\max}$ is the maximum shear (cutting) force, determined from the diagram Q;

$S_X(y)_{\max}$ is the maximum static moment of the cross-sectional area crossing;

$[\tau]$ is allowable shear stress, $[\tau] \approx 0{,}5\,[\sigma]$.

Task 6
Drawing the diagrams of shear (cutting) force and bending moment for cantilever beam

For given cantilever beam (Fig. for task 6, Table for task 6) draw the diagram of shear (cutting) forces and bending moments, if $a = 3$ m .

Plan of solving the task:

1. Write down the functions of shear (cutting) forces and bending moments cantilever sections.
2. Diagram shear (cutting) forces and bending moments.

Table for task 6

Nr	q, kN/m	F, kN	M, kNm	b, m	c, m
1	2	$1{,}5\ q \cdot a$	$0{,}5\ q \cdot a^2$	$\frac{2}{3} a$	$\frac{1}{3} a$
2	3	$q \cdot a$	$q \cdot a^2$	$\frac{1}{3} a$	$\frac{2}{3} a$
3	6	$0{,}5\ q \cdot a$	$1{,}2\ q \cdot a^2$	$\frac{1}{3} a$	$\frac{1}{3} a$
4	2	$q \cdot a$	$q \cdot a^2$	$\frac{2}{3} a$	$\frac{2}{3} a$
5	4	$2\ q \cdot a$	$1{,}5\ q \cdot a^2$	$\frac{2}{3} a$	$\frac{1}{3} a$
6	6	$q \cdot a$	$q \cdot a^2$	$\frac{1}{3} a$	$\frac{2}{3} a$
7	2	$0{,}5\ q \cdot a$	$1{,}5\ q \cdot a^2$	$\frac{1}{3} a$	$\frac{1}{3} a$
8	5	$1{,}5\ q \cdot a$	$q \cdot a^2$	$\frac{2}{3} a$	$\frac{2}{3} a$
9	6	$q \cdot a$	$q \cdot a^2$	$\frac{2}{3} a$	$\frac{1}{3} a$
0	3	$2\ q \cdot a$	$0{,}2\ q \cdot a^2$	$\frac{1}{3} a$	$\frac{2}{3} a$

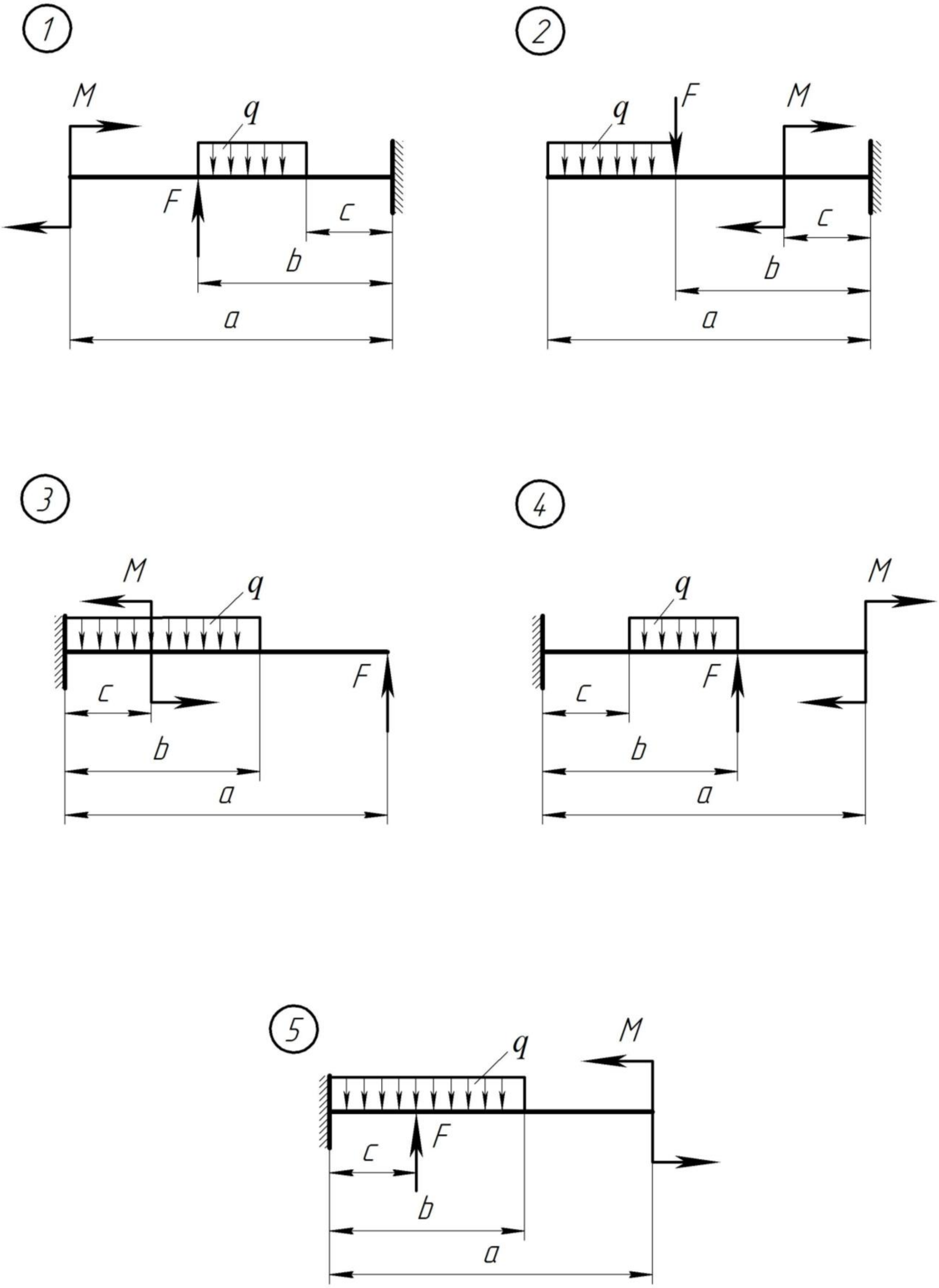


Figure for task 6

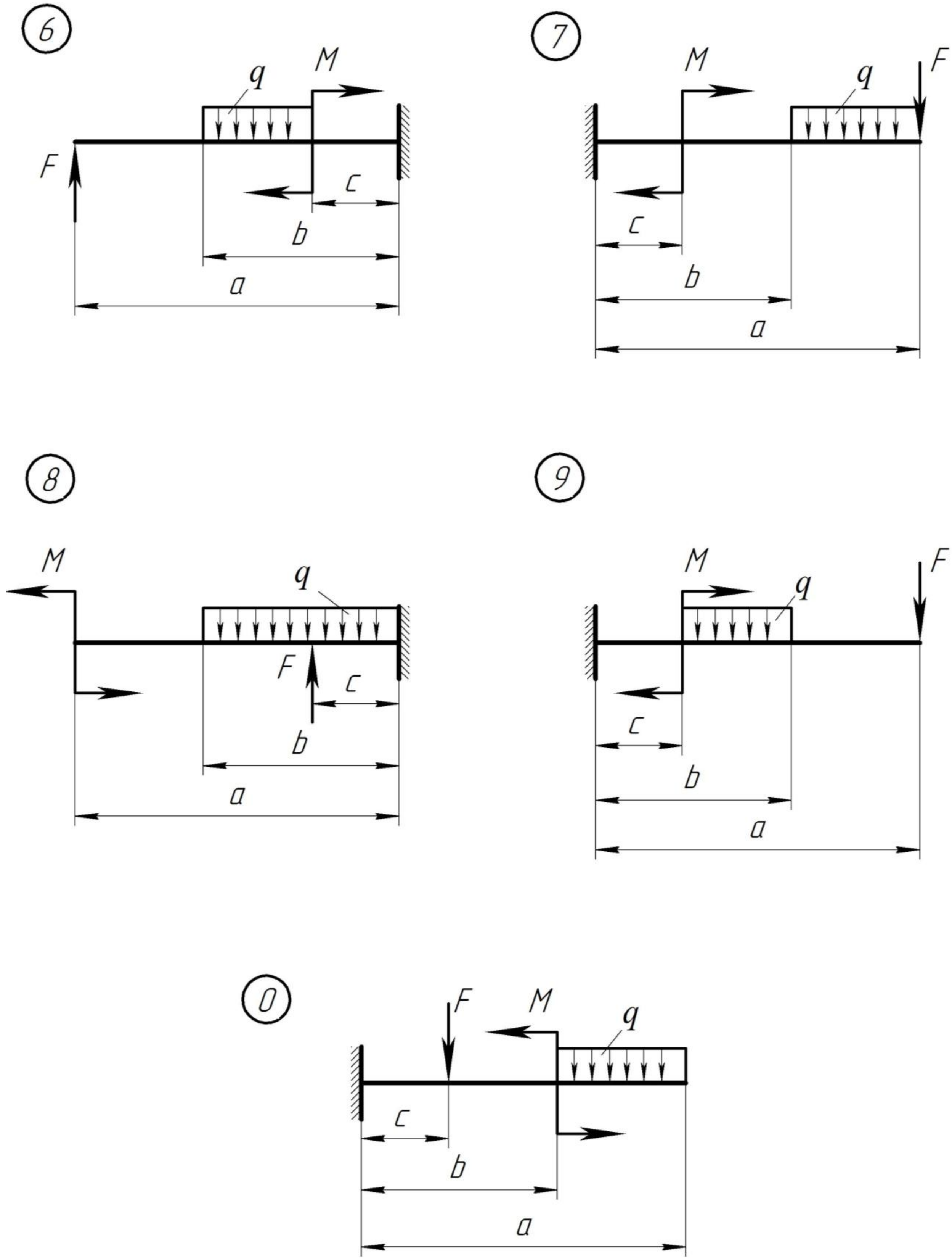


Figure for task 6 (contunied)

Example of solving the task 6
Drawing the diagrams of shear (cutting) force and bending moment for cantilever beam

For given cantilever beam (Fig. 6.9 *a*) diagram shear (cutting) forces and bending moment.

Solution

Divide the beam into three sections. The boundaries of the sections are the sections where the concentrated forces and bending moments are applied and the sections where the distributed load begins and ends.

In direct transverse bending, these internal force factors, such as bending moment $M_{BN}(x)$ and shear (cutting) force $Q(x)$, occur in the cross-sections. To determine them, we use the method of sections.

At each segment of the beam (for the cantilever beam we consider the segments from the free end) make imaginative sections, reject the left part of the beam and consider the equilibrium of the right one. The forces of interaction of beam parts are replaced by internal forces $Q(x)$ and $M_{BN}(x)$ (Fig. 6.9*b*). We define them as functions of the current coordinate x based on equilibrium equations $\sum Y = 0$; $\sum M_0 = 0$, using the sign rule.

Determine the lateral forces and bending moments at each segment of the cantilever beam

Section *I*, $0 \le x \le 1{,}3$ m (right side):

$$Q(x) = -F = -19 \text{ kN};$$

$$M_{BN}(x) = F \cdot x;$$

$$M_{BN}(0) = 0;$$

$$M_{BN}(1{,}3) = 19 \cdot 1{,}3 = 24{,}7 \text{ kNm} .$$

Section *II*, $1{,}3 \text{ m} \le x \le 3{,}2$ m (right side):

$$Q(x) = -F + q \cdot (x - 1{,}3);$$

$$Q(1{,}3) = -19 \text{ kN};$$

$$Q(3{,}2) = -19 + 13 \cdot (3{,}2 - 1{,}3) = 5{,}7 \text{ kN};$$

$$M_{BN}(x) = F \cdot x - q \cdot \frac{(x - 1{,}3)^2}{2};$$

$$M_{BN}(1{,}3) = 19 \cdot 1{,}3 = 24{,}7 \text{ kNm};$$

$$M_{BN}(3{,}2) = 19 \cdot 3{,}2 - 13 \cdot \frac{(3{,}2-1{,}3)^2}{2} = 37{,}3 \text{ kNm}.$$

Evaluate function $M_{BN}(x)$ for extremum

$$\frac{dM(x)}{dx} = Q(x) = F - q \cdot (x - 1{,}3) = 0;$$

$$x = \frac{F + 1{,}3 \cdot q}{q} = \frac{19 + 1{,}3 \cdot 13}{13} = 2{,}76 \text{ m};$$

then

$$M_{BN}(2{,}76) = M_{BN\,\max} = 19 \cdot 2{,}76 - 13 \cdot \frac{(2{,}76 - 1{,}3)^2}{2} = 38{,}6 \text{ kNm}.$$

It should be noted that the necessary condition for the extremum of function $M_{BN}(x)$ at the segment is zero value on this segment $Q(x)$.

Section *III*, $3{,}2 \text{ m} \le x \le 4{,}8 \text{ m}$ (right side):

$$Q(x) = -F + q \cdot (x - 1{,}3);$$

$$Q(3{,}2) = -19 + 13 \cdot (3{,}2 - 1{,}3) = 5{,}7 \text{ kN};$$

$$Q(4{,}8) = -19 + 13 \cdot (4{,}8 - 1{,}3) = 26{,}5 \text{ kN};$$

$$M_{BN}(x) = F \cdot x - q \cdot \frac{(x-1{,}3)^2}{2} - M;$$

$$M_{BN}(3{,}2) = 19 \cdot 3{,}2 - 13 \cdot \frac{(3{,}2-1{,}3)^2}{2} - 16 = 21{,}3 \text{ kNm};$$

$$M_{BN}(4{,}8) = 19 \cdot 4{,}8 - 13 \cdot \frac{(4{,}8-1{,}3)^2}{2} - 16 = -4{,}43 \text{ kNm}.$$

Based on the obtained results, draw the diagrams Q and M_{BN} (Fig. 6.9 *c*, *d*). The diagram of bending moments is draw from the side of stretched fibers, that is, the positive values of bending moments are placed down from the axis and the negative ones are up.

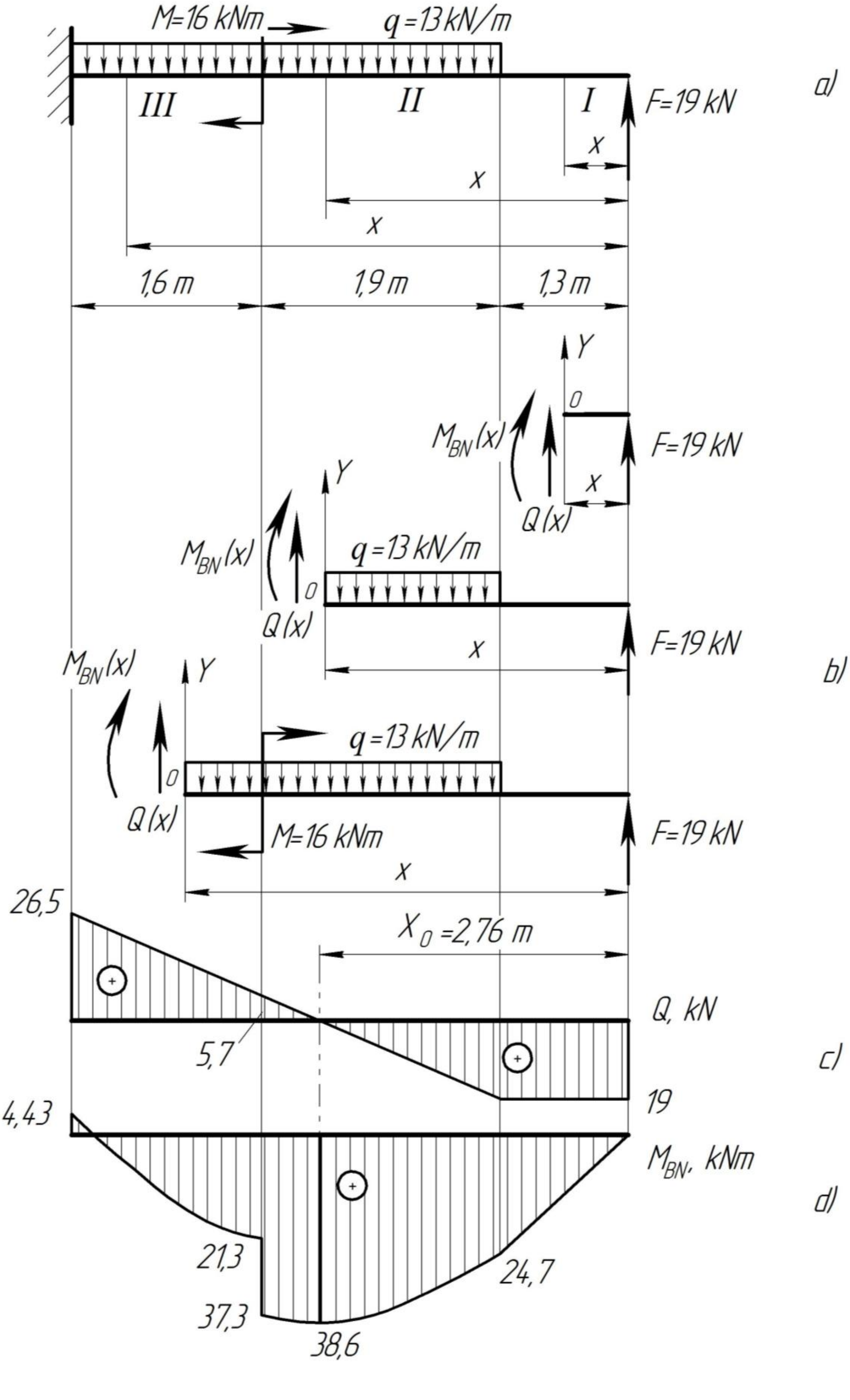

M=16 kNm
q=13 kN/m
a)
III
II
I
F=19 kN
x
x
x
1,6 m
1,9 m
1,3 m
Y
0
M_BN(x)
F=19 kN
x
Q(x)
Y
M_BN(x)
q=13 kN/m
0
Q(x)
F=19 kN
x
M_BN(x)
Y
b)
q=13 kN/m
0
Q(x)
M=16 kNm
F=19 kN
x
26,5
X_0 =2,76 m
Q, kN
5,7
c)
19
4,43
M_BN, kNm
d)
21,3
24,7
37,3
38,6

Figure 6.9

Task 7

Diagraming of shear (cutting) force and bending moment for simply supported beam

For the given steel beam (Fig. for task 7, Table for task 7) diagram shear (cutting) forces and bending moments.

Plan of solving the task:

1. Determine the support reactions, write down the functions of shear (cutting) forces and bending moments on the beam sections.
2. Diagram shear (cutting) forces and bending moments.

Table for task 7

Nr	q, kN/m	F, kN	M, kNm	a, m	Nr	q, kN/m	F, kN	M, kNm	a, m
1	4	6	8	3	**6**	3	3	7	3
2	5	7	9	2	**7**	2	2	4	5
3	3	3	5	4	**8**	3	4	5	4
4	2	4	6	5	**9**	5	8	7	3
5	3	5	4	4	**0**	4	9	8	2

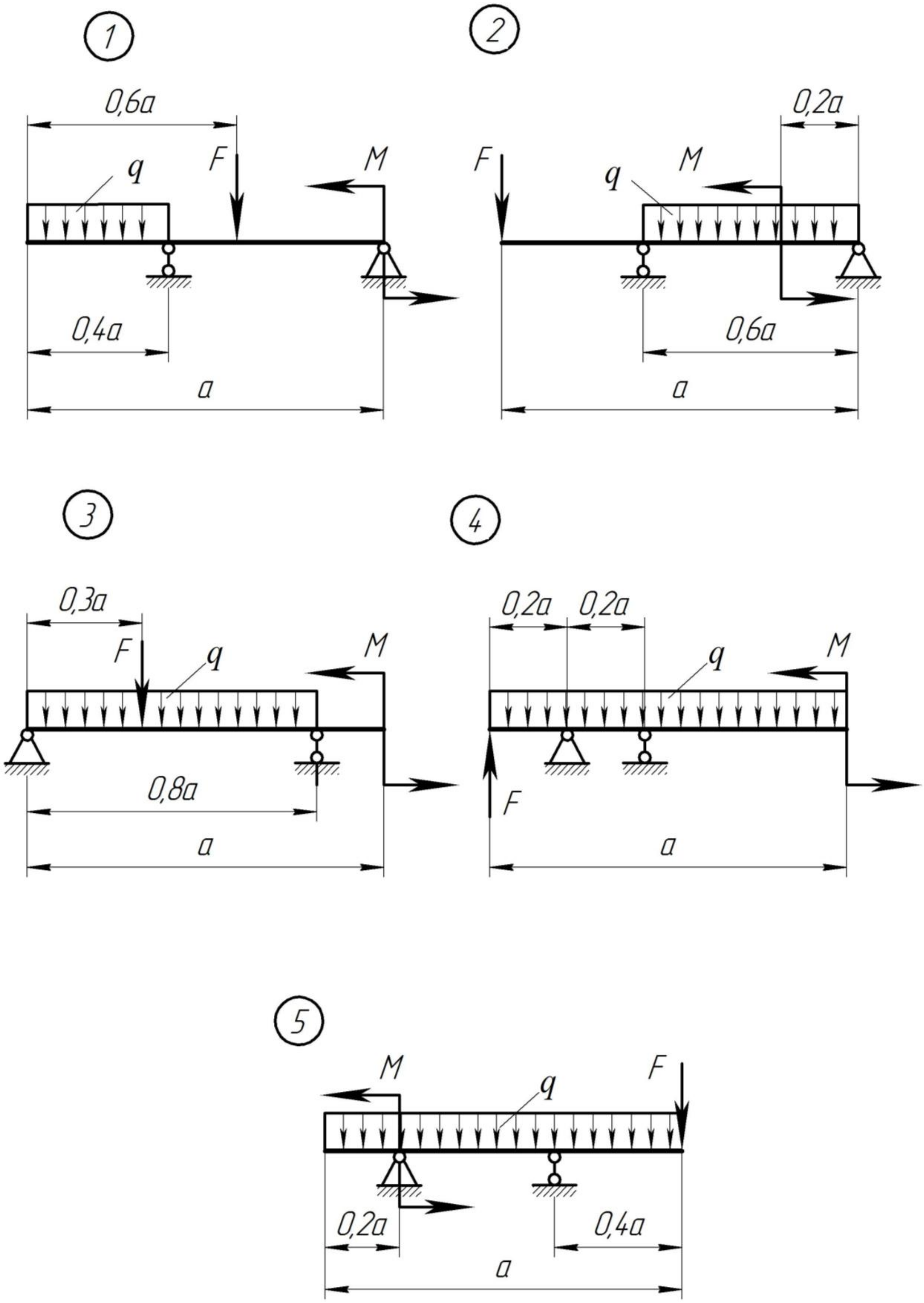


Figure for task 7

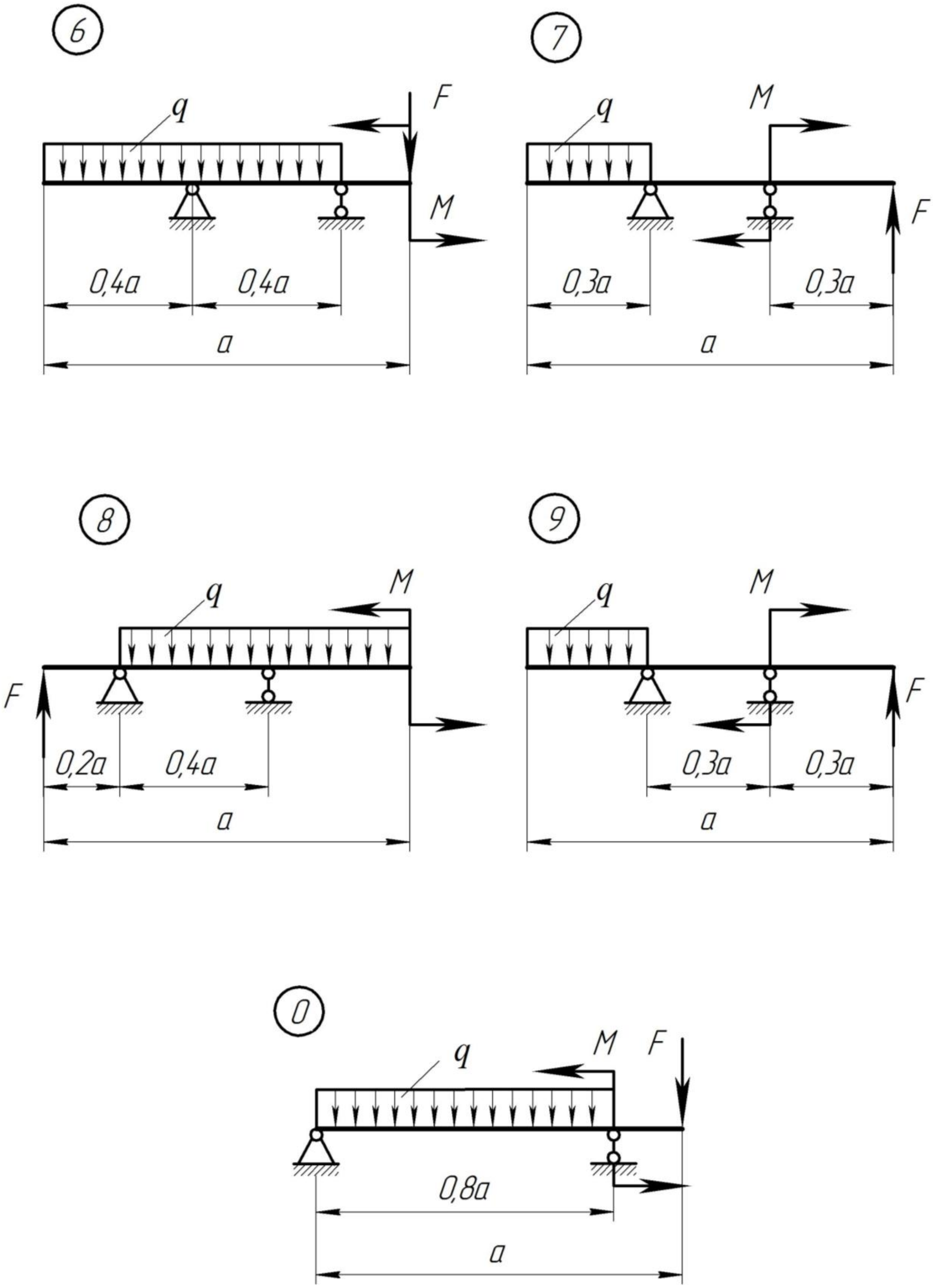


Figure for task 7 (contunied)

Task 8
Strength calculation under the bending of beams

For the given steel beam (Fig. for task 8, Table for task 8), diagram the shear (cutting) forces and bending moments for strength under normal stresses; choose I-beam, round and rectangular cross-section (putting for a rectangular cross-section the relation of height to width $h/b = 2$) sections of the beams and compare their weight. For I-beam, conduct the strength test by shear (cutting) stresses and complete test by the main stresses. Material of the beams is steel St.3; $[\sigma] = 160$ MPa.

Plan of solving the task:

1. Determine the support reactions, write down the functions of shear (cutting) forces and bending moments in the segments of the beam.
2. Diagram the shear (cutting) forces and bending moment. Determine the cross-section in which the maximum bending moment and the maximum cross-section force act.
3. Choose the dimensions of the sections (I-beam, round, rectangular) from the condition of strength under normal stresses.
4. Compare the weight of the beams, taking the weight of I-beam as 100%.
5. Test the strength of the selected I-beam for shear (cutting) stresses.
6. Determine the section in which the maximum main stresses occur. Conduct the complete strength test of I-beam at the dangerous point of this section.

Table for task 8

Nr	q, kN/m	F, kN	M, kNm	a, m	Nr	q, kN/m	F, kN	M, kNm	a, m
1	2	3	9	8	**6**	2	3,5	5,5	10
2	3	4	5	7	**7**	3	4,5	7,5	8
3	4	5	8	9	**8**	2	2,5	4,5	9
4	1	2	4	10	**9**	5	6,5	9,5	8
5	4	6	7	7	**0**	4	5,5	8,5	6

Task 9
Calculation for strength and determining displacements during the bending of beams

For the given beam (Fig. for task 8, Table for task 8) choose the I-beam. Determine the deflection of the beam in the section A by the method of initial parameters. Verify the obtained result by the Mohr method.

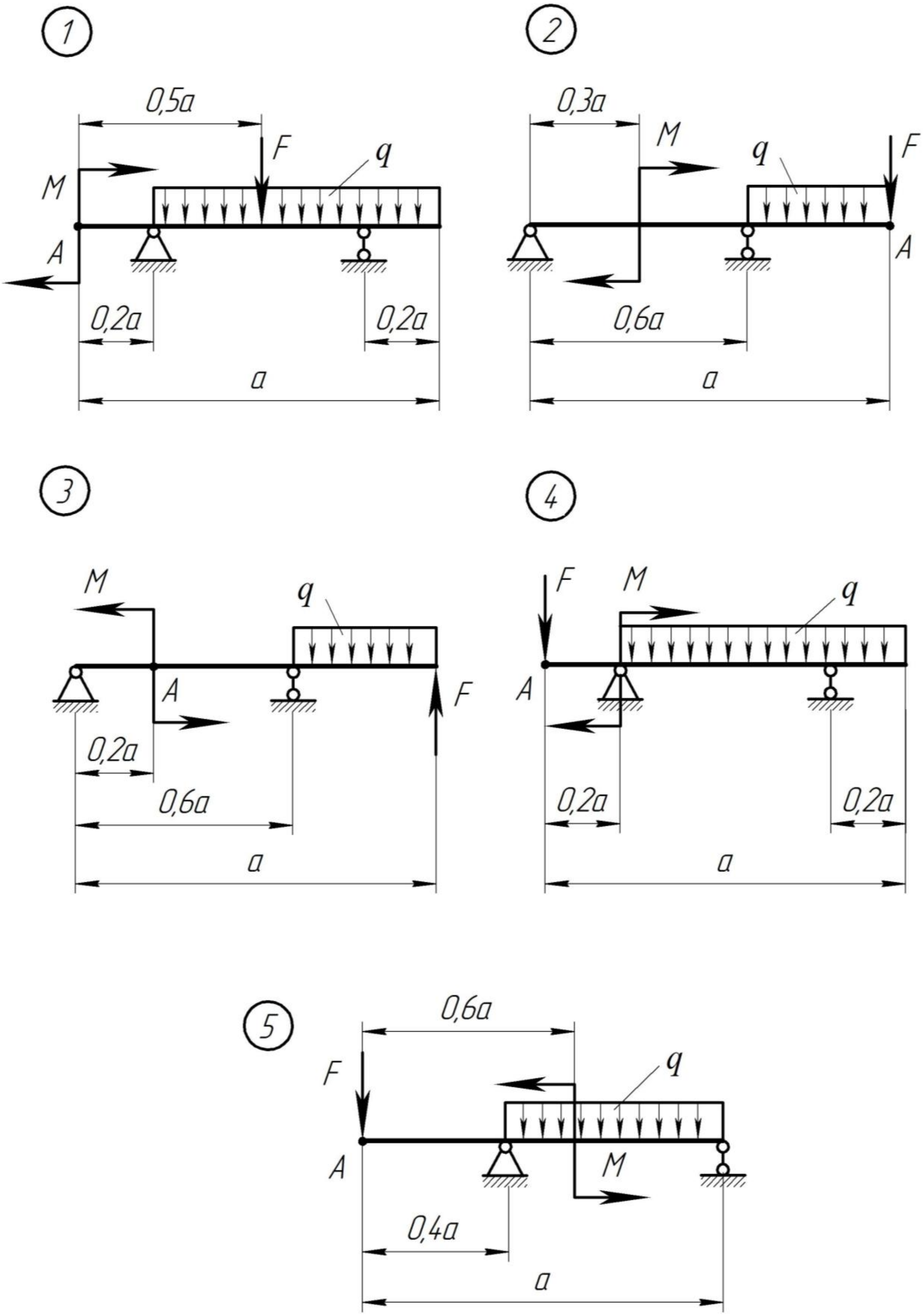


Figure for task 8

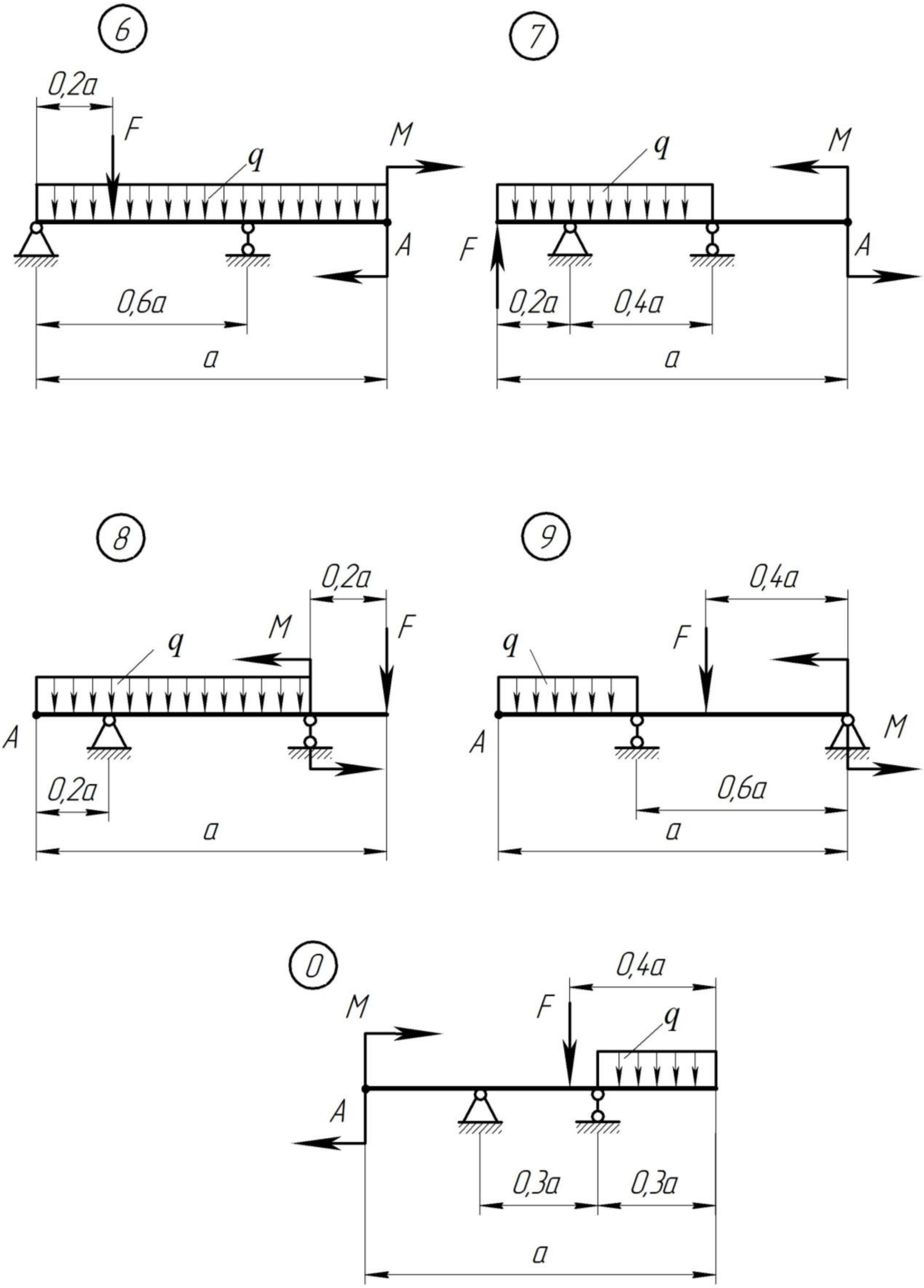


Figure for task 8 (contunied)

Example of solving the task 7 and 8
Diagraming of shear (cutting) force and bending moment for simply supported beam.
Strength calculation under the bending of beams

For the given steel beam (Fig. 6.10 *a*), diagram shear (cutting) forces and bending moments for strength under normal stresses; choose the I-beam, round and rectangular (putting for a rectangular cross-section the relation of height to width $h/b = 2$) sections of the beams and compare their weight. For I-beam, conduct the strength test by shear (cutting) stresses and complete test by the main stresses. Material of the beams is steel St.3; $[\sigma] = 160$ MPa.

Solution

Using the static equilibrium for the given beam scheme (Fig. 6.10 *b*), determine the vertical components of the forces reactions:

$$\sum M_B = 0; \qquad -M + A_Y \cdot 1{,}5 + q \cdot 1{,}5 \cdot 0{,}75 - F \cdot 2{,}0 = 0;$$

$$A_Y = \frac{16 - 20 \cdot 1{,}5 \cdot 0{,}75 + 15 \cdot 2{,}0}{1{,}5} = 15{,}7 \text{ kN};$$

$$\sum M_A = 0; \qquad -M - q \cdot 1{,}5 \cdot 0{,}75 + B_Y \cdot 1{,}5 - F \cdot 3{,}5 = 0;$$

$$B_Y = \frac{16 + 20 \cdot 1{,}5 \cdot 0{,}75 + 15 \cdot 3{,}5}{1{,}5} = 60{,}7 \text{ kN}.$$

Verification:

$$\sum Y = -A_Y - q \cdot 1{,}5 + B_Y - F = -15{,}7 - 20 \cdot 1{,}5 + 60{,}7 - 15 = 0.$$

Conclusion: the resistance responses are determined correctly.

Divide the beam into three sections. For each section write the functions of lateral force $Q(x)$ and bending moment $M_{BN}(x)$.

Section *I*, $0 \le x \le 1{,}6$ m (left side):

$$Q(x) = 0; \qquad\qquad M_{BN}(x) = M;$$

$$M_{BN}(0) = M_{BN}(1{,}6) = 16 \text{ kNm}.$$

Section *II*, $1{,}6 \text{ m} \le x \le 3{,}1 \text{ m}$ (left side):

$$Q(x) = -A_Y - q \cdot (x - 1{,}6);$$

$$Q(1{,}6) = -15{,}7 \text{ kN};$$

$$Q(3{,}1) = -15{,}7 - 20 \cdot (3{,}1 - 1{,}6) = -47{,}5 \text{ kN};$$

$$M_{BN}(x) = M - A_Y \cdot (x - 1{,}6) - q \cdot \frac{(x - 1{,}6)^2}{2};$$

$$M_{BN}(1{,}6) = 16 \text{ kNm};$$

$$M_{BN}(3{,}1) = 16 - 15{,}7 \cdot (3{,}1 - 1{,}6) - 10 \cdot (3{,}1 - 1{,}6)^2 = -30 \text{ kNm}.$$

Section *III*, $0 \le x \le 2{,}0 \text{ m}$ (right side):

$$Q(x) = F = 15 \text{ kN}; \qquad Q(0) = Q(2{,}0) = 15 \text{ kNm};$$

$$M_{BN}(x) = -F \cdot x; \qquad M_{BN}(0) = 0;$$

$$M_{BN}(2{,}0) = -15 \cdot 2{,}0 = -30 \text{ kNm}.$$

Draw the diagrams Q and M_{BN} (Fig. 6.10 *c*, *d*).

From the condition of bending strength by normal stresses

$$\sigma_{\max} = \frac{M_{BN\ \max}}{W_0} \le [\sigma],$$

determine the required axial moment of section resistance

$$W_0 \ge \frac{M_{BN\ \max}}{[\sigma]} = \frac{30 \cdot 10^{-3}}{160} = 188 \cdot 10^{-6} \text{ m}^3,$$

where $M_{BN\ \max}$ is the maximum bending moment acting on the beam, $M_{BN\ \max} = 30 \text{ kNm}$.

For the given beam, choose the following cross-sections:
a) I-beam Nr 20a (GOST 8239-56)

$$W_b = 203 \cdot 10^{-6} \text{ m}^3; \quad A_b = 28{,}8 \cdot 10^{-4} \text{ m}^2;$$

b) rectangular cross-section

$$W_{rc} = \frac{b \cdot h^2}{6} = \frac{b \cdot (2b)^2}{6} = \frac{2}{3} b^3;$$

$$b \geq \sqrt[3]{3 \cdot 188 \cdot 10^{-6} / 2} = 65{,}6 \cdot 10^{-3} \text{ m};$$

take $b = 70$ mm, then $h = 2 \cdot 70 = 140$ mm,
respectively $A_{rc} = b \cdot h = 70 \cdot 140 \cdot 10^{-6} = 98 \cdot 10^{-4} \text{ m}^2$;

c) round cross-section

$$W_{rn} = \pi \cdot d^3 / 32;$$

$$d \geq \sqrt[3]{32 \cdot 188 \cdot 10^{-6} / 3{,}14} = 124 \cdot 10^{-3} \text{ m};$$

take $d = 125$ mm, then

$$A_{rn} = \pi \cdot d^2 / 4 = 3{,}14 \cdot 125^2 \cdot 10^{-6} / 4 = 123 \cdot 10^{-4} \text{ m}^2.$$

Compare the beams weight

$$Q_b : Q_{rc} : Q_{rn} = A_b : A_{rc} : A_{rn} = 28{,}9 : 98 : 123 = 1 : 3{,}39 : 4{,}26.$$

Test the strength of I-beam.

Taking into account that the dimensions of cross-section of the I-beam were determined only by the condition of strength at normal stresses, it is necessary to test the strength of the beam by tangential (shear) and principal stresses.

From the assortment tables for I-beam Nr 20a according to GOST 8239-72 we take the necessary data for the calculation:

$$h_b = 200 \text{ mm}; \quad b_b = 110 \text{ mm}; \quad d_b = 5{,}2 \text{ mm}; \quad t_b = 8{,}6 \text{ mm};$$

$$I_X = 2030 \text{ cm}^4; \quad S_X = 114 \text{ cm}^3.$$

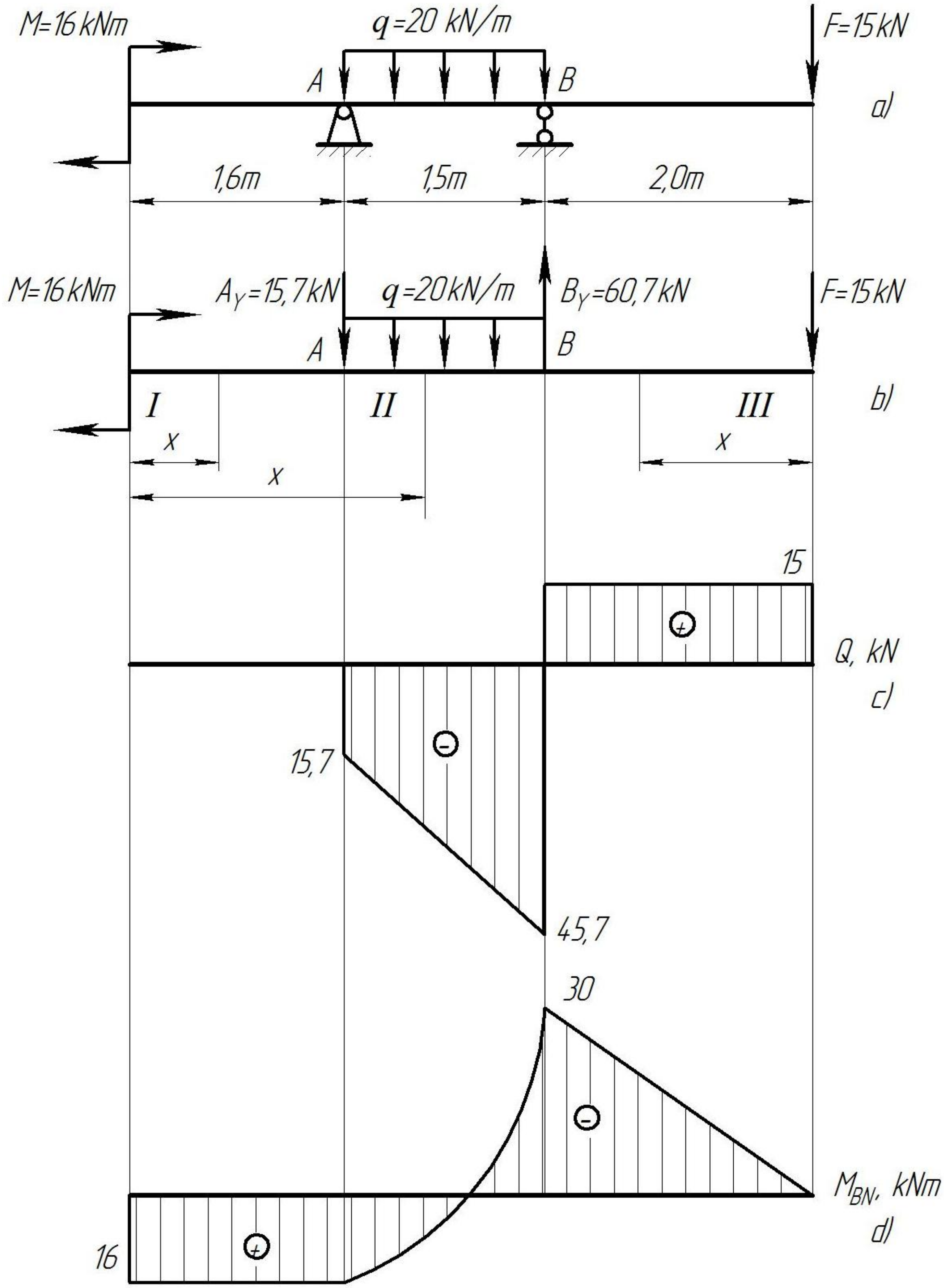

M=16 kNm
q=20 kN/m
F=15kN
A
B
a)
1,6m
1,5m
2,0m
M=16 kNm
A_Y=15,7kN
q=20kN/m
B_Y=60,7kN
F=15kN
A
B
I
II
III
b)
x
x
x
15
Q, kN
c)
15,7
45,7
30
M_BN, kNm
d)
16

Figure 6.10

Draw the simplified section of I-beam (Fig. 6.11 *a*).

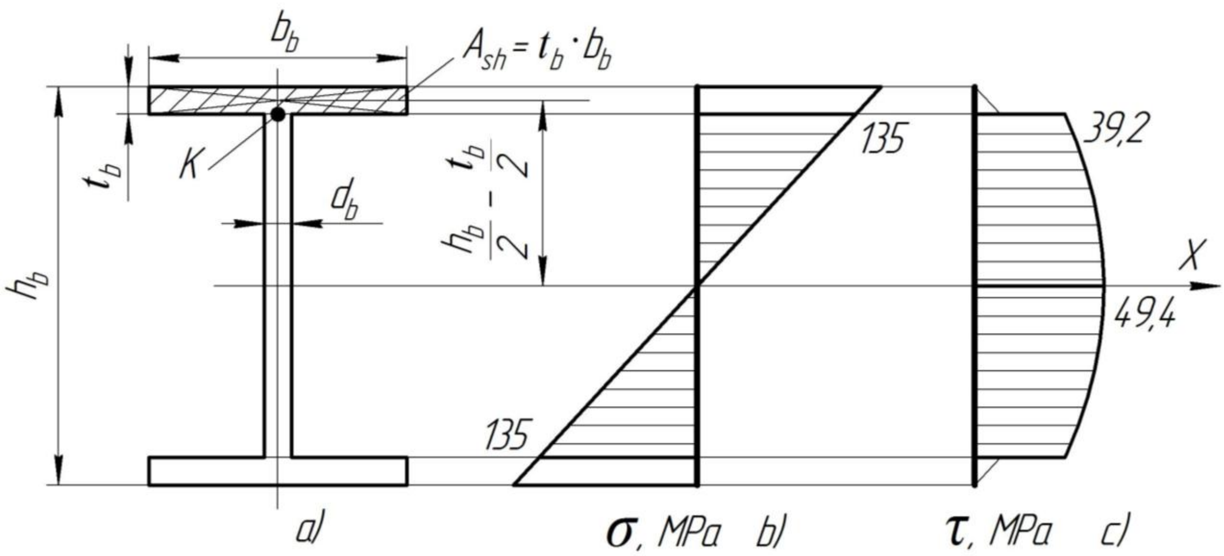


Figure 6.11

The dangerous section while testing for tensile strength is the section where the shear (cutting) force has maximum value $Q_{max} = 45,7$ kN (section at point B, see Fig. 6.10 *c*).

The maximum tangential (shear) stresses occur at the section points that coincide with the neutral axis (axis X, see Fig. 6.11).

Tensile strength condition

$$\tau_{max} = \frac{Q_{max} \cdot S_X}{d_b \cdot I_X} \le [\tau],$$

where $[\tau]$ is allowable shear stress,

$$[\tau] \approx 0,5\,[\sigma] = 0,5 \cdot 160 = 80 \text{ MPa} .$$

Substituting the values, obtain

$$\tau_{max} = \frac{45,7 \cdot 10^{-3} \cdot 114 \cdot 10^{-6}}{5,2 \cdot 10^{-3} \cdot 2030 \cdot 10^{-8}} = 49,4 \text{ MPa} < [\tau] = 80 \text{ MPa} .$$

The dangerous section while testing for main stresses is the section where bending moment and shear (cutting) force acquire maximum values or are close to them (the section point B, $M_{BN\,max} = 30$ kNm , $Q_{max} = 45,7$ kN , see Fig. 6.10 *c*, *d*).

Determine the normal and tangential (shear) stress in the dangerous section of the I-beam (point K, Fig. 6.11 *a*):

$$\sigma = \frac{M_{BN\,\max}}{I_X} \cdot \left(\frac{h_b}{2} - t_b \right) = \frac{30 \cdot 10^{-3}}{2030 \cdot 10^{-8}} \cdot \left(\frac{200}{2} - 8{,}6 \right) \cdot 10^{-3} = 135 \text{ MPa} \;;$$

$$\tau = \frac{Q \cdot S_{X\,sh}}{d_b \cdot I_X} = \frac{45{,}7 \cdot 10^{-3} \cdot 90{,}5 \cdot 10^{-6}}{5{,}2 \cdot 10^{-3} \cdot 2030 \cdot 10^{-8}} = 39{,}2 \text{ MPa} \;,$$

where $S_{X\,sh}$ is the static moment of the section area of the I-beam shelf relatively to axis X, is determined by the formula

$$S_{X\,sh} = A_{sh} \cdot \left(\frac{h_b}{2} - \frac{t_b}{2} \right) = \frac{b_b \cdot t_b \cdot (h_b - t_b)}{2} .$$

Substituting the data, obtain

$$S_{X\,sh} = \frac{110 \cdot 8{,}6 \cdot (200 - 8{,}6) \cdot 10^{-9}}{2} = 90{,}5 \cdot 10^{-6} \text{ m}^3 .$$

Here A_{sh} is the area of I-beam shelf, $A_{sh} = t_b \cdot b_b$.

Draw the diagrams of normal and tangent stresses for the I-beam section (Fig. 6.11 *a*, *b*, *c*).

Determine the calculated stress by the third theory of strength and test the strength of the material by the main stress:

$$\sigma_{R3} = \sqrt{\sigma^2 + 4\tau^2} = \sqrt{135^2 + 4 \cdot 39{,}2^2} = 156 \text{ MPa} \;;$$

$$\sigma_{R3} = 156 \text{ MPa} < [\sigma] = 160 \text{ MPa} .$$

Therefore, the strength of the beam by the main and tangential (shear) stresses is ensured.

7. DETERMINATION OF DISPLACEMENTS UNDER BENDING

Rods that undergo bending deformation have to be not only of adequate strength but also of sufficient rigidity. Under the action of external loads, the displacements of their sections must not exceed the allowable values established by the norms.

To perform the calculation for rigidity in practice, it is necessary to be able to calculate the corresponding displacements of the sections. Consider the movement of sections when bending the cantilever beam (Fig. 7.1).

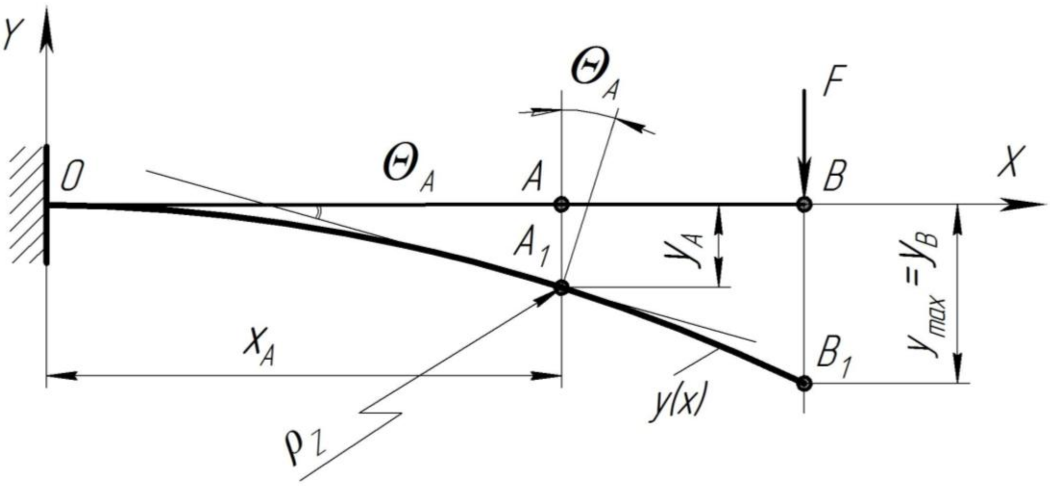


Figure 7.1

In Fig. 7.1 the symbols are used: OB – undeformed axis; OB_1 – bent axis of the beam; Θ_A – slope of the elastic curve A; y_A – linear displacement of the section; ρ_Z – curvature of the beam axis.

With a plane transverse bend, the bent axis of the beam lies in the force plane and coincides with the main plane. The curved axis of the beam is referred to as the **curved axis** or **elastic line**.

Position of the cross-section of the beam under bending is determined by two displacements (Fig. 7.1):

1. **Linear displacement** $y_A = y(x_A)$ ***of the gravity center of the section in the direction perpendicular to the undeformed axis of the beam, which is referred to as*** **deflection**.

2. **Angular displacement** $\Theta_A = \Theta(x_A)$ **is a slope of the elastic curve *around the neutral axis of the section relative to its initial position*.**

It is considered that the length of the curved longitudinal axis belonging to the neutral layer does not change when the beam is curved.

Deflections slopes of elastic curve are related by differential dependence

$$\Theta(x) = \frac{dy(x)}{dx} = y'(x). \tag{7.1}$$

Differential equation of the bent axis of the beam

There is such an analytical relationship between the curvature $1/\rho$ of the bent axis of the beam (elastic line), bending moment M_{BN} that determines this curvature, and the beam rigidity during bending EI_0

$$\frac{1}{\rho(x)} = \frac{M_{BN}(x)}{E \cdot I_0}, \tag{7.2}$$

where $\rho(x)$ is the curvature radius of the elastic line of the beam in the plane at distance x from the coordinates origin;

$M_{BN}(x)$ is the bending moment at the same cross-section of the beam.

Curvature of a plane curve (known from the course of higher mathematics) is described by the dependence

$$\frac{1}{\rho(x)} = \pm \frac{y''(x)}{\left[1 + y'(x)^2\right]^{3/2}}. \tag{7.3}$$

By equating the right-hand sides of relations (7.2) and (7.3), **the exact differential equation of the bent axis of the beam** is obtained.

$$E \cdot I_0 \frac{y''(x)}{\left[1 + y'(x)^2\right]^{3/2}} = \pm M_{BN}(x). \tag{7.4}$$

Given that the slopes of the elastic curve are small, the value $y'(x)^2$ compared to the unit can be neglected. Then, from expression (7.4), when choosing the direction of the upward axis Y, get the approximate differential **equation of the elastic line of the beam**

$$E \cdot I_0 \cdot y''(x) \approx M_{BN}(x). \tag{7.5}$$

By integrating it twice or once, it is possible to determine the linear $y(x)$ ***and angular*** $\Theta(x)$ ***displacements of the beam sections under any load conditions.***

There are several methods for determining displacements in direct transverse bending. Consider some of them.

Method of direct integration of differential equation of the bent axis of the beam

The method is based on the approximate differential equation of the bent axis of the beam

$$E \cdot I_0 \cdot y''(x) = M_{BN}(x), \tag{7.6}$$

where $E \cdot I$ is the rigidity of cross-section of a beam under bending;
E is the modulus of elasticity of the material from which the beam is made;
I_0 is the axial inertia moment of the cross-section of the beam, $I_0 = I$;
$M_{BN}(x)$ is the function of bending moment from external loading, acting on this section of the beam, hereinafter $M_{BN}(x) = M(x)$.

To obtain the function of the curved axis of the beam $y = f(x)$, integrate equation (7.6)

$$E \cdot I \cdot y'(x) = \int M(x)dx + C; \tag{7.7}$$

$$E \cdot I \cdot y(x) = \int dx \int M(x)dx + C \cdot x + D. \tag{7.8}$$

Therefore, the **equation of the curved axis of the beam** is

$$y(x) = \frac{1}{E \cdot I} \int dx \int M(x)dx + C \cdot x + D. \tag{7.9}$$

Equations (7.7)-(7.8) include constant integrations C and D, which are determined from the boundary conditions, i.e. the conditions of fixing the beam supports, the deflections and slopes of the elastic curve which are known.

Method of initial parameters

Method of initial parameters makes it possible to write **only one equation of deflections or slopes of the elastic curve, which is suitable for all sections of the beam.** This equation is called **the universal equation of the elastic line,** which takes into account all types of loads: concentrated force F, concentrated moment M, distributed load $q(x)$.

Method of initial parameters is obtained as a result of unification of the method of direct integration of the beam bent axis by equating the constant integrations at the boundaries of the sections. This method is a universal technique for determining displacements during bending.

For a prismatic beam (Fig. 7.2) with the selected coordinate system $X\ Y$ and different types of load, **the equation of the elastic line** can be written as

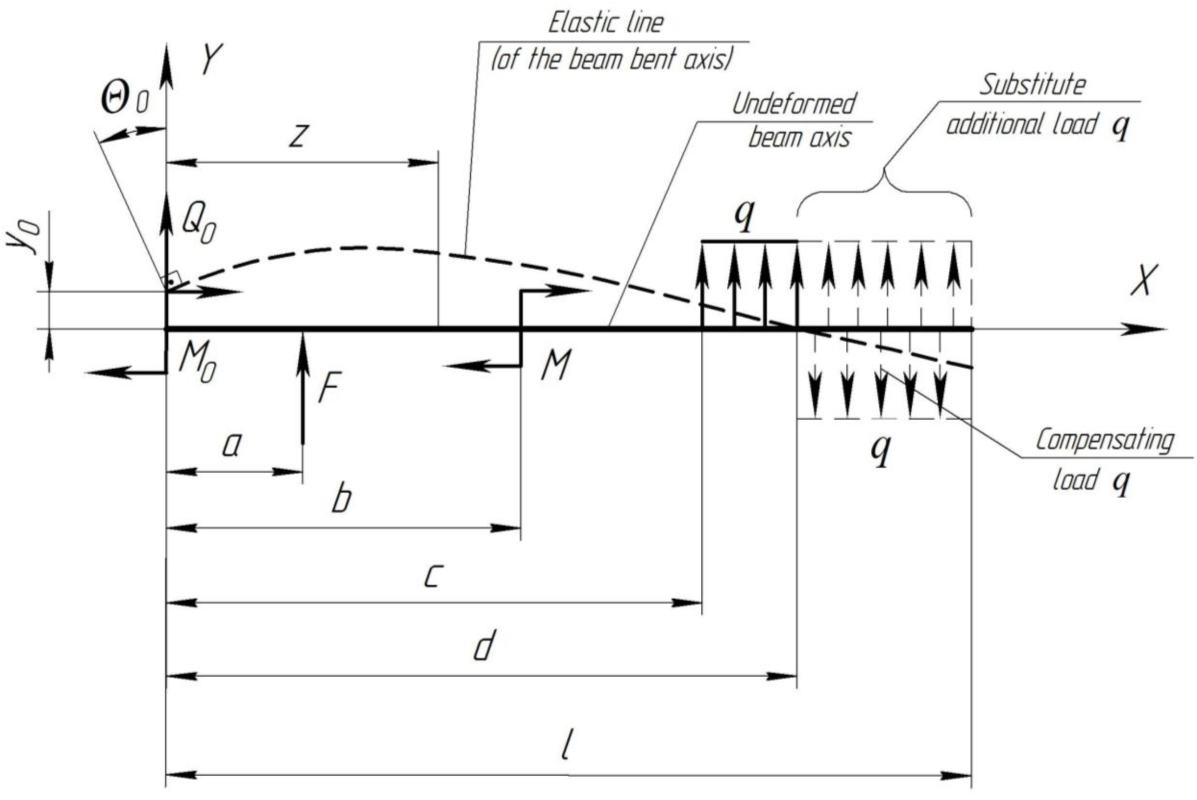


Figure 7.2

$$y(x) = y_0 + \Theta_0 \cdot x + \frac{1}{E \cdot I}\left(\frac{M_0 \cdot x^2}{2} + \frac{Q_0 \cdot x^3}{6} + \frac{F\,(x-a)^3}{6} + \right.$$
$$\left. + \frac{M\,(x-b)^2}{2} + \frac{q\,(x-c)^4}{24} - \frac{q\,(x-d)^4}{24} \right), \qquad (7.10)$$

where y_0, Θ_0, M_0, Q_0 are initial parameters, respectively: deflection, slope of the elastic curve, bending moment and shearing force at the coordinate origin;

- a is a distance from the coordinate origin to the section at which concentrated force F is applied;
- b is a distance from the coordinate origin to the section at which concentrated moment M is applied;
- c is a distance from the coordinate origin to the section at which the load q starts to act;
- d is a distance from the coordinate origin to the section at which the load q finishes its action.

Deflection y_0 and ***slope of the elastic curve*** Θ_0 are geometric initial parameters; ***bending moment*** M_0 and ***shear*** **(*cutting*)** ***force*** Q_0 in the crossing, which coincides with the coordinate origin are static initial parameters.

Deflection y_0 and slope of the elastic curve Θ_0 of the initial (right-hand final) beam section are determined from the conditions of beam fixation, bending moment M_0 and shear (cutting) force Q_0 are found from diagrams M_{BN} and Q.

If the simply supported beam is considered, y_0 and Θ_0 are determined from the conditions that deflections on the supports equal zero. If the cantilever beam is considered, these parameters are determined from the conditions that deflection and slope of the elastic curve in the clamp equal zero.

Initial parameters y_0, Θ_0, M_0, Q_0 can be positive, negative, or equal zero.

The signs of terms in the equation are determined by the signs of the corresponding external force factors. The rules of signs are the same as those adopted for shear (cutting) forces and bending moments.

The equation for determining slopes of the elastic curve of the prismatic beam (see Fig. 7.2) is

$$\Theta(x) = \Theta_0 + \frac{1}{E \cdot I}\left(M_0 + \frac{Q_0 \cdot x^2}{2} + \frac{F\,(x-a)^2}{2} + \frac{M\,(x-b)}{1} + \right.$$
$$\left. + \frac{q\,(x-c)^3}{6} - \frac{q\,(x-d)^3}{6} \right). \tag{7.11}$$

When compiling the equation of the elastic line of the beam, such rules should be followed:

1. The coordinate origin is chosen at the leftmost point of the beam and kept it common to all segments.

2. Only those loads that are applied to the left of the considered section are substituted into the equation.

3. If the distributed load $q(x)$ breaks on one of the sections of the beam, it is conventionally continued to the right end of the beam, while introducing a compensatory load of the same intensity, but in the opposite direction.

The Mohr method

The Mohr method is based on the principle of conservation of energy, i.e. the equality of work from external loads and the potential energy of deformation.

Displacement Δ (deflection y or slope of the elastic curve Θ) **is determined by Mohr integral** which spans all the length of the beam

$$\Delta(y,\ \Theta) = \sum \int_{l} \frac{M\,(x) \cdot \overline{M}\,(x)}{E \cdot I}\, dx\ , \tag{7.12}$$

where $M\,(x)$ are functions of bending moments from the external loads for the given (loaded) beam;

$\overline{M}\,(x)$ are functions of bending moments from a singular load for the redundant (auxiliary) — unloaded — beam.

Redundant (auxiliary) beam is a given beam without external loads.

Physical outline of the Mohr integral: the displacement of a random section of the beam is the work of a singular force, which is spent for displacement of its application point from a given load.

Sequence for determining displacements (deflections or slopes of the elastic curve) using the Mohr integral:

1. Compile the equations of bending moments $M\,(x)$ from the given load.

2. Having eliminated given loads from the system (beam), apply a force (pair of forces) equal one (singular force or singular moment) at that beam section, where the displacements are determined and in the direction of this displacement.

3. Compile the equations of bending moments $\overline{M}\,(x)$ from this singular force (pair of forces).

4. Calculate the integral sum (7.12) from the product of both moments divided by rigidity of the section.

Graphic-analytical solution of the Mohr integral

It is reasonable to calculate the Mohr integral (7.12) by graphic-analytical method.

Outline: ***the definite integral of the product of two functions, one of which is linear and the other arbitrary, is equal to the product of the area of the graph of an arbitrary function and the ordinate of the graph of the linear function taken under its center of gravity.***

Graphic-analytical method of solving the Mohr integral can be used when one diagram is traced with straight lines. This condition is satisfied for structures that consist of straight bars (elements), because the diagrams from the singular loads are always rectilinear.

General formula for determining displacements under bending

$$\Delta(y,\Theta)=\sum\frac{1}{E\cdot I}\int_l M(x)\cdot\overline{M}(x)dx=\sum_{i=1}^{n}\frac{\omega_i\cdot\overline{M}_{Ci}}{E\cdot I},\qquad(7.13)$$

where ω_i is the area of diagram (Fig. 7.3 *a*) of bending moments $M(F)$ from the external loads of the *i*-segment of a beam;

$\overline{M}_{Ci}$ is the ordinate of the linear diagram (Fig. 7.3 *b*) of bending moments from a singular load $\overline{M}_C$ of the *i*-section of the beam located under the gravity centre of nonlinear diagram

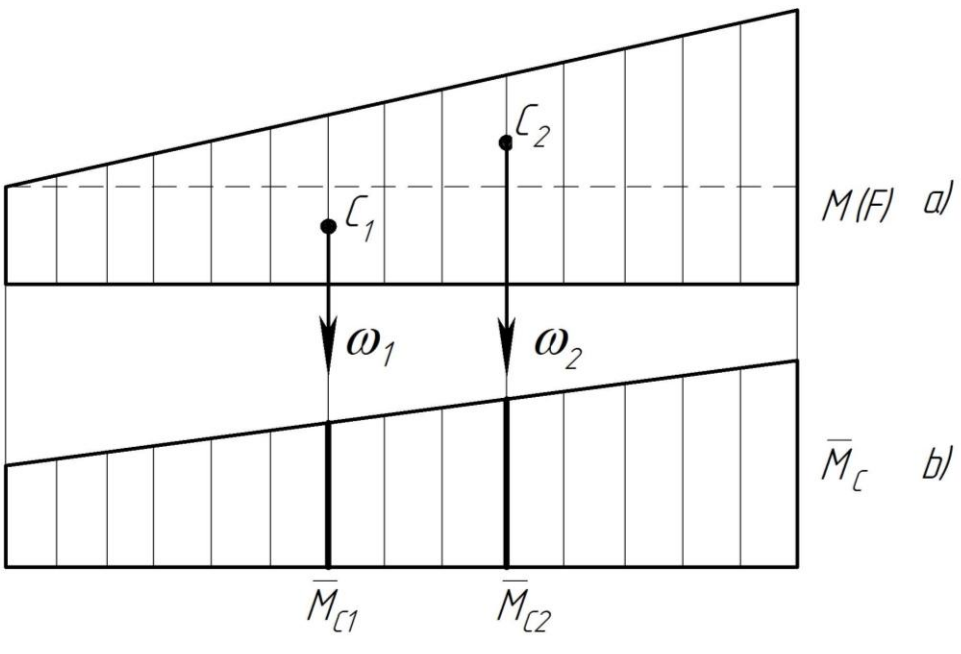


Figure 7.3

Graphic-analytical method of solving the Mohr integral is referred to as the ***method of multiplication of diagrams***. Herewith the diagram $M(F)$ is named the ***load*** one and diagram $\overline{M}_C$ is a ***singular***.

When using this method, the following should be considered:

1. Number of terms $n\left(\omega_i \cdot \overline{M}_{Ci}\right)$ has to be not less than the number of the Mohr integral sums.
2. If diagrams $M(F)$ (of external loads) and $\overline{M}_{Ci}$ (of singular loads) are of opposite sign (are on different sides of zero line), the result of diagram multiplication has the sign minus.
3. If the equation of bending moments is a polynomial, it is reasonable to draw the load diagram in layered form, i.e. to draw separate diagrams from external loads, each of which corresponds to one of the terms. Such diagrams are drawn by approaching the breaking point of a single diagram from both sides of the beam.
4. Diagrams drawn for use of the graphical-analytical method of the Mohr integral calculation are not hatched.

Measurements of singular diagrams of bending moments are the units of length.

The values of the diagrams areas and the coordinates of their gravity centre, which can be used to determine the displacements, are given in *Annex* 6.

Example of solving the task (cantilever beam)

By graphic-analytical solution of the Mohr integral, determine deflections and slopes of the elastic curve of sections A and B of cantilever beam shown in Fig. 7.4, provided that $E \cdot I = const$.

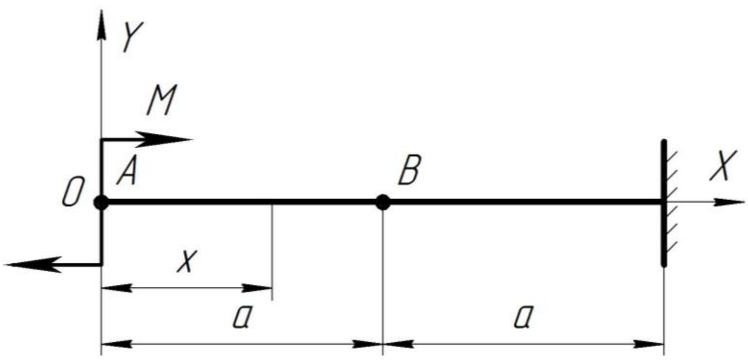


Figure 7.4

Solution

For a given cantilever beam (Fig. 7.5 *a*) draw a diagram of bending moments (Fig. 7.5 *b*) from external loads $M(F)$. To determine the deflection of the section A, in the redundant (auxiliary) beam (Fig. 7.5 *c*) apply a singular force at the same section and draw a singular diagram of bending moments $\overline{M}_1$ (Fig. 7.5 *d*).

Deflection of the section A determine by multiplying diagrams $M(F)$ and $\overline{M}_1$

$$y_A = \frac{1}{E \cdot I}\left(-\omega_1 \cdot \overline{M}_{1C1}\right) = -\frac{2M \cdot a^2}{E \cdot I},$$

where $\omega_1 = M \cdot 2a$; $\overline{M}_{1C1} = a$.

To determine the deflection of the beam at section B, it is reasonable to repeat diagram $M(F)$ again (Fig. 7.5 *e*). In the redundant (auxiliary) beam at section B, it is necessary to apply a singular force (Fig. 7.5 *f*) and draw a singular diagram of bending moments $\overline{M}_2$ (Fig. 7.5 *g*). Then the deflection of section B is

$$y_B = \frac{1}{E \cdot I} \cdot \left(-\omega_2 \cdot \overline{M}_{2C2} + \omega_3 \cdot \overline{M}_{2C3}\right) = -\frac{M \cdot a^2}{2E \cdot I},$$

where $\omega_2 = M \cdot a$; $\omega_3 = M \cdot a$; $\overline{M}_{2C2} = -a/2$; $\overline{M}_{2C3} = 0$.

To determine the slopes of the elastic curves of sections *A* and *B*, it is necessary to apply singular bending moments in the given sections of redundant (auxiliary) beams (Fig. 7.5 *h*, *j*) and draw the diagrams of bending moments (Fig. 7.5 *i*, *k*).

Slope of the elastic curve of section A is determined by multiplication of the diagram from exteral forces $M(F)$ (see Fig. 7.5 *b*) and singular moments $\overline{\overline{M}}_3$ (see Fig. 7.5 *i*)

$$\Theta_A = \frac{1}{E \cdot I}\left(\omega_1 \cdot \overline{M}_{3C1}\right) = \frac{2M \cdot a}{E \cdot I},$$

where $\overline{\overline{M}}_{3C1} = 1$.

Slope of the elastic curve of section B is determined by multiplication of the diagram from exteral forces $M(F)$ (see Fig. 7.5 *b*) and singular moments $\overline{\overline{M}}_4$ (see Fig. 7.5 *k*)

$$\Theta_B = \frac{1}{E \cdot I}\left(\omega_3 \cdot \overline{\overline{M}}_{4C3} + \omega_2 \cdot \overline{\overline{M}}_{4C2}\right) = \frac{M \cdot a}{E \cdot I},$$

where $\overline{\overline{M}}_{4C3} = 0$; $\overline{\overline{M}}_{4C2} = 1$.

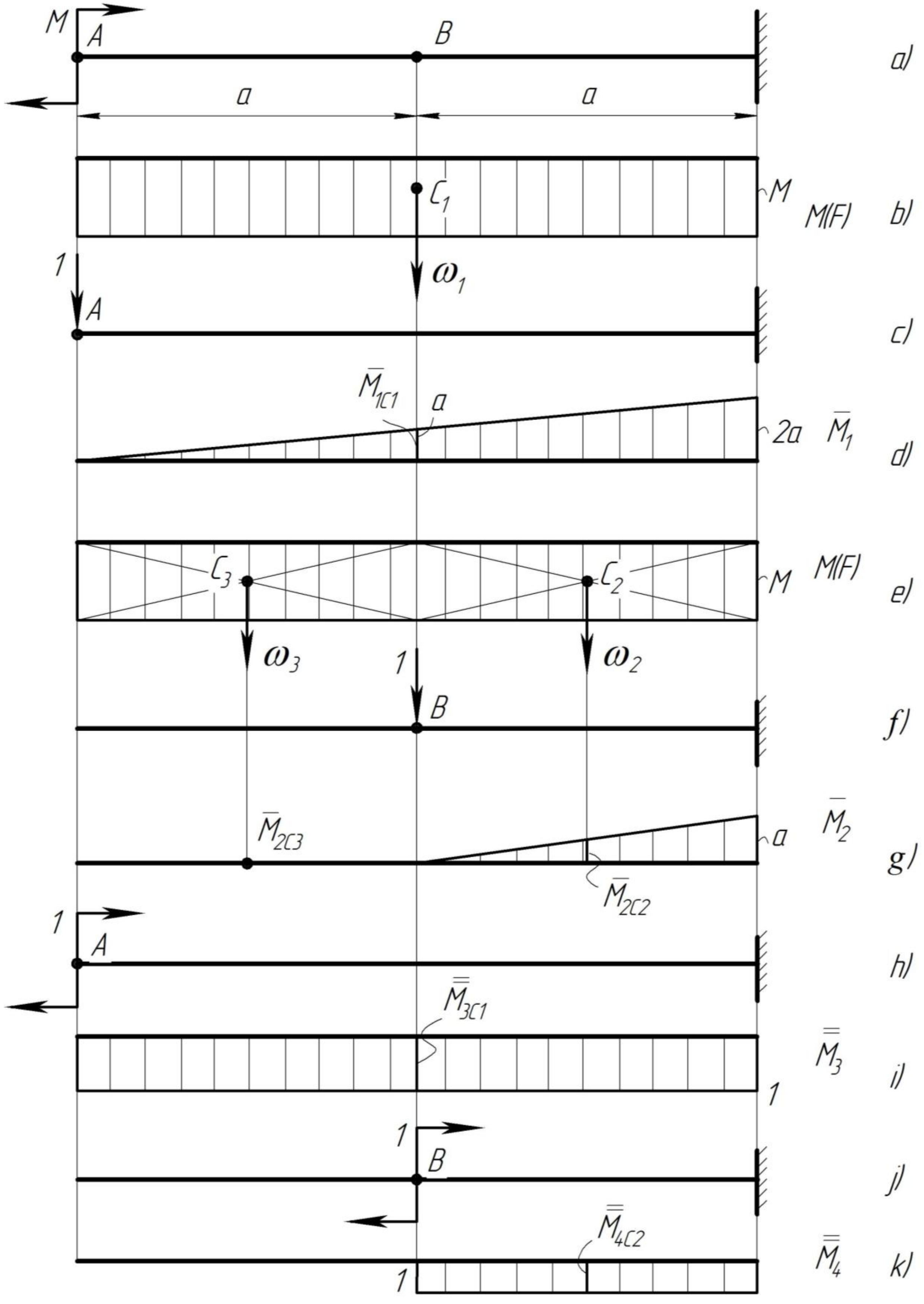

M
A
B
a
a
a)
C1
M
M(F)
b)
1
ω1
A
c)
M̄1C1
a
2a
M̄1
d)
C3
C2
M
M(F)
e)
ω3
1
ω2
B
f)
M̄2C3
a
M̄2
g)
M̄2C2
1
A
h)
M̿3C1
M̿3
i)
1
1
B
j)
M̿4C2
M̿4
k)
1

Figure 7.5

Example of solving the task (simply supported beam)

By graphical-analytical solution of the Mohr integral, determine the deflections in the sections K and D and the slope of the elastic curve of the beam on two supports, shown in Fig. 7.6 *a*, provided that $E \cdot I = const$.

Solution

From the equilibrium equations, determine the support reactions (see Fig. 7.6*a*):

$$\sum M_B = 0; \qquad -R_A \cdot 3a + F \cdot 2a = 0; \qquad R_A = \frac{2}{3}F;$$

$$\sum M_A = 0; \qquad R_B \cdot 3a - F \cdot a = 0; \qquad R_B = \frac{1}{3}F.$$

Verification $\quad \sum Y = R_A - F + R_B = \frac{2}{3}F - F + \frac{1}{3}F = 0.$

Draw a diagram of bending moments $M(F)$ (Fig. 7.6 *b*) from external loads.

To determine the deflection of the section K in the redundant (auxiliary) system (Fig. 7.6 *c*), apply a singular force in the section K. Determine the support reactions:

$$\sum M_B = 0; \qquad -\overline{R}_A \cdot 3a + 1 \cdot 2a = 0; \qquad \overline{R}_A = \frac{2}{3};$$

$$\sum M_A = 0; \qquad \overline{R}_B \cdot 3a - 1 \cdot a = 0; \qquad \overline{R}_B = \frac{1}{3}.$$

Verification $\quad \sum Y = \overline{R}_A - 1 + \overline{R}_B = \frac{2}{3} - 1 + \frac{1}{3} = 0.$

Draw a diagram of bending moments $\overline{M}_1$ (Fig. 7.6 *d*) from a singular force.

Determine deflection of section K by multiplying diagrams $M(F)$ and $\overline{M}_1$

$$y_K = \frac{1}{E \cdot I} \cdot \left(\omega_1 \cdot \overline{M}_{1C1} + \omega_2 \cdot \overline{M}_{2C2}\right),$$

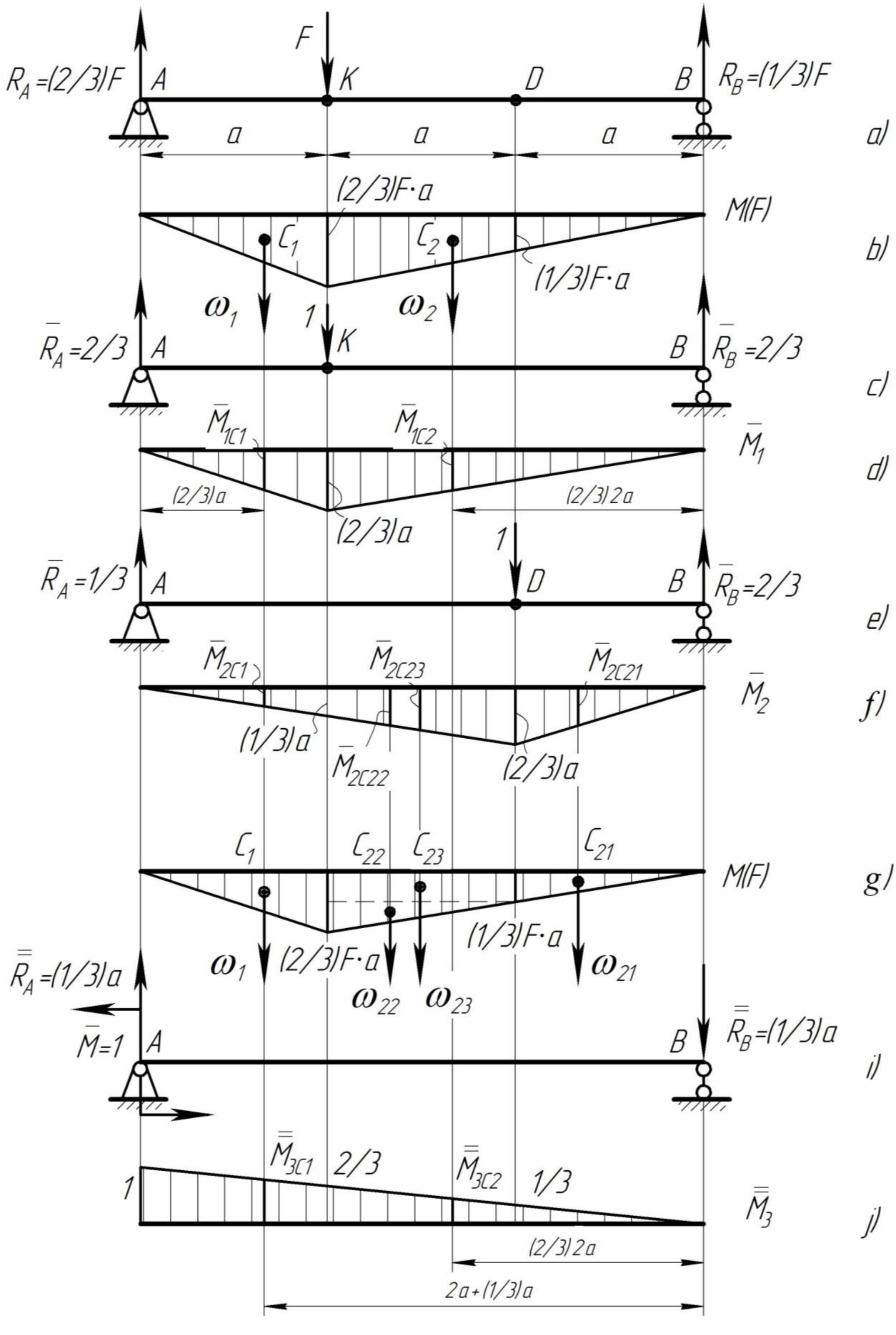

F
$R_A=(2/3)F$
A
K
D
B
$R_B=(1/3)F$
a
a
a
a)
$(2/3)F\cdot a$
M(F)
C_1
C_2
b)
$(1/3)F\cdot a$
ω_1
1
ω_2
$\bar{R}_A=2/3$
A
K
B
$\bar{R}_B=2/3$
c)
$\bar{M}_{1C1}$
$\bar{M}_{1C2}$
$\bar{M}_1$
d)
(2/3)a
(2/3)2a
(2/3)a
1
$\bar{R}_A=1/3$
A
D
B
$\bar{R}_B=2/3$
e)
$\bar{M}_{2C1}$
$\bar{M}_{2C23}$
$\bar{M}_{2C21}$
$\bar{M}_2$
f)
(1/3)a
$\bar{M}_{2C22}$
(2/3)a
C_1
C_{22}
C_{23}
C_{21}
M(F)
g)
$(1/3)F\cdot a$
ω_1
$(2/3)F\cdot a$
ω_{22}
ω_{23}
ω_{21}
$\bar{\bar{R}}_A=(1/3)a$
$\bar{M}=1$
A
B
$\bar{\bar{R}}_B=(1/3)a$
i)
$\bar{\bar{M}}_{3C1}$
2/3
$\bar{\bar{M}}_{3C2}$
1/3
1
$\bar{\bar{M}}_3$
j)
(2/3)2a
2a+(1/3)a

Figure 7.6

where

$$\omega_1 = \frac{1}{2} \cdot \frac{2}{3} F \cdot a \cdot a = \frac{F \cdot a^2}{3};$$

$$\omega_2 = \frac{1}{2} \cdot \frac{2}{3} F \cdot a \cdot 2a = \frac{2 F \cdot a^2}{3};$$

$$\overline{M}_{1C1} = \overline{R}_A \cdot \frac{2}{3} a = \frac{2}{3} \cdot \frac{2}{3} a = \frac{4}{9} a;$$

$$\overline{M}_{1C2} = \overline{R}_B \cdot \frac{2}{3} 2a = \frac{1}{3} \cdot \frac{4}{3} a = \frac{4}{9} a.$$

Substituting the data, obtain

$$y_K = \frac{1}{E \cdot I} \left(\frac{F \cdot a^2}{3} \cdot \frac{4}{9} a + \frac{2F \cdot a^2}{3} \cdot \frac{4}{9} a \right) = \frac{12\ F \cdot a^3}{E \cdot I}.$$

To determine the deflection of section *D*, do the similar operations, i.e. apply a singular force in section *D* of the redundant (auxiliary) beam (Fig. 7.6 *e*) and draw the diagram $\overline{M}_2$ (Fig. 7.6 *f*). For convenience and clarity, place the diagram from external forces $M(F)$ under the diagram $\overline{M}_2$ (Fig. 7.6 *g*).

Determining the deflection of section *D* is complicated by the increase of terms $\omega_i \cdot \overline{M}_{Ci}$. This is due to the fact that we have three areas of integration by the Mohr method, and also divide the middle shape (trapezoid) into two shapes — triangle and rectangle (Fig. 7.6 *g*).

Deflection of the section D determine by formula

$$y_D = \frac{1}{E \cdot I} \left(\omega_1 \cdot \overline{M}_{2C1} + \omega_{21} \cdot \overline{M}_{2C21} + \omega_{22} \cdot \overline{M}_{2C22} + \omega_{23} \cdot \overline{M}_{2C23} \right),$$

where $\omega_1 = \frac{1}{2} \cdot \frac{2}{3} F \cdot a \cdot a = \frac{F \cdot a^2}{3};$ $\omega_{21} = \frac{1}{2} \cdot \frac{1}{3} F \cdot a \cdot a = \frac{F \cdot a^2}{6};$

$\omega_{22} = \frac{1}{2} \cdot \frac{1}{3} F \cdot a \cdot a = \frac{F \cdot a^2}{6};$ $\omega_{23} = \frac{1}{3} F \cdot a \cdot a = \frac{F \cdot a^2}{3};$

$$\overline{M}_{2C1}=\frac{2}{3}\cdot\frac{1}{3}a=\frac{2}{9}a;\qquad \overline{M}_{2C21}=\frac{2}{3}\cdot\frac{2}{3}a=\frac{4}{9}a;$$

$$\overline{M}_{2C22}=\frac{1}{3}\cdot\left(a+\frac{1}{3}a\right)=\frac{4}{9}a;\qquad \overline{M}_{2C23}=\frac{1}{3}\cdot\left(a+\frac{1}{2}a\right)=\frac{1}{2}a.$$

Substituting data, obtain

$$y_D=\frac{1}{E\cdot I}\left(\frac{F\cdot a^2}{3}\cdot\frac{2}{9}a+\frac{F\cdot a^2}{6}\cdot\frac{4}{9}a+\frac{Fa^2}{6}\cdot\frac{4}{9}a+\frac{F\cdot a^2}{3}\cdot\frac{1}{2}a\right)=\frac{23\,F\cdot a^3}{54\,E\cdot I}.$$

To determine a slope of the elastic curve of section A in the redundant (auxiliary) beam (Fig. 7.6 *i*), apply $\overline{M}=1$ in section *A*, find supporting reactions and draw the diagram of bending moments from a singular load $\overline{\overline{M}}$ (Fig. 7.6 *j*).

Slope of the elastic curve of section *A* detrmine by multiplying diagram $M(F)$ (Fig. 7.6 *b*) by $\overline{\overline{M}}_3$ (Fig. 7.6 *j*)

$$\Theta_A=\frac{1}{E\cdot I}\left(-\omega_1\cdot\overline{\overline{M}}_{3C1}-\omega_2\cdot\overline{\overline{M}}_{3C2}\right),$$

where $\overline{\overline{M}}_{3C1}=\frac{1}{3a}\left(2a+\frac{1}{3}a\right)=\frac{7}{9};$

$$\overline{\overline{M}}_{3C2}=\frac{1}{3a}\left(\frac{2}{3}\cdot 2a\right)=\frac{4}{9}.$$

Substituting the data, obtain

$$\Theta_A=-\frac{1}{E\cdot I}\left(\frac{F\cdot a^2}{3}\cdot\frac{7}{9}+\frac{2F\cdot a^2}{3}\cdot\frac{4}{9}\right)=-\frac{15\,F\cdot a^2}{27\,E\cdot I}.$$

Sign minus means that the slope of the elastic curve of section *A* occurs in the direction opposite to the action of singular moment.

Example of solving the task 9 by the method of initial parameters

For the given beam (Fig. 7.7), determine deflections of sections C and D and the slope of the elastic curve of section A by the method of initial parameters, when $F = 10$ kN , $M = 40$ kNm , $q = 20$ kN / m , $E \cdot I = const$.

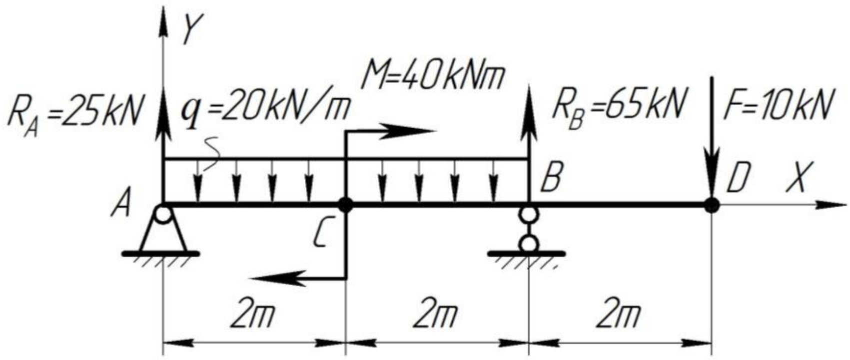


Figure 7.7

Solution

From the equilibrium condition write down:

$$\sum M_B = 0; \qquad -R_A \cdot 4 + q \cdot 4 \cdot 2 - M - F \cdot 2 = 0;$$

$$\sum M_A = 0; \qquad -F \cdot 6 + R_B \cdot 4 - M - q \cdot 4 \cdot 2 = 0.$$

Determine the support reactions:

$$R_A = \frac{q \cdot 8 - M - F \cdot 2}{4} = \frac{20 \cdot 8 - 40 - 10 \cdot 2}{4} = 25 \text{ kN};$$

$$R_B = \frac{F \cdot 6 + M + q \cdot 8}{4} = \frac{10 \cdot 6 + 40 + 20 \cdot 8}{4} = 65 \text{ kN}.$$

Compile the validation equation

$$\sum Y = R_A - q \cdot 4 + R_B - F = 25 - 20 \cdot 4 + 65 - 10 = 0,$$

that is support reactions are determined correctly.

Choose the coordinate origin on the left-most support A. Write down the general equation of the bent axis of the beam by the method of initial parameters

$$E \cdot I \cdot y(x) = E \cdot I \cdot y_0 + E \cdot I \cdot \Theta_0 \cdot x + \frac{R_A \cdot x^3}{6} + \frac{M\,(x-2)^2}{2} + \\ + \frac{R_B\,(x-4)^3}{6} - \frac{q \cdot x^4}{24} + \frac{q\,(x-4)^4}{24}. \qquad (7.14)$$

Find the initial parameters y_0 and Θ_0 from the conditions:

$$E \cdot I \cdot y(0) = 0\,; \qquad (7.15)$$

$$E \cdot I \cdot y(4) = 0\,;$$

or

$$\begin{cases} E \cdot I \cdot y_0 = 0; \\ E \cdot I \cdot y_0 + E \cdot I \cdot \Theta_0 \cdot 4 + \dfrac{R_A \cdot 4^3}{6} - \dfrac{q \cdot 4^4}{24} + \dfrac{M \cdot (4-2)^2}{2} = 0. \end{cases} \qquad (7.16)$$

From the system (7.16) obtain:

$$E \cdot I \cdot y_0 = 0\,;$$

$$E \cdot I \cdot \Theta_0 = \frac{1}{4}\left(-\frac{25 \cdot 4^3}{6} + \frac{20 \cdot 4^4}{24} - \frac{40 \cdot 2^2}{2}\right) = -33{,}3.$$

On substituting the initial parameters, write down the equation of the bent axis of a beam

$$E \cdot I \cdot y\,(x) = -33{,}3 \cdot x + \frac{R_A \cdot x^3}{6} - \frac{q \cdot x^4}{24} + \frac{M \cdot (x-2)^2}{2} + \frac{R_B \cdot (x-4)^3}{6} + \frac{q \cdot (x-4)^4}{24}.$$

Find deflections of the beam in sections C and D.

Section C, $x_C = 2$ m:

$$E \cdot I \cdot y_C\,(2) = -33{,}3 \cdot 2 + \frac{25 \cdot 2^3}{24} - \frac{20 \cdot 2^4}{24} = -46{,}7;$$

from which

$$y_C\,(2) = -\frac{46{,}7}{E \cdot I}.$$

Section D, $x_D = 6$ m:

$$E \cdot I \cdot y_D\,(6) = -33{,}3 \cdot 6 + \frac{25 \cdot 6^3}{6} - \frac{20 \cdot 6^4}{24} + \frac{40 \cdot 4^2}{2} + \frac{65 \cdot 2^3}{6} + \frac{20 \cdot 2^4}{24} = 40{,}0;$$

from which

$$y_D\,(6) = \frac{40{,}0}{E \cdot I}.$$

Slope of the elastic curve of section A

$$E \cdot I \cdot \Theta_0 = E \cdot I \cdot \Theta_A = -33{,}3\,;$$

from which

$$\Theta_A = -\frac{33{,}3}{E \cdot I}.$$

Example of solving the task 9 by Mohr method

Let us solve the task (see Fig. 7.7) by Mohr method. Given: $q = 20$ kN / m; $M = 40$ kNm ; $F = 10$ kN ; $R_A = 25$ kN ; $R_B = 65$ kN ; $E \cdot I = const$. Calculation model is shown in Fig. 7.8 *a*.

Solution

Determine the deflection of section C . In the redundant (auxiliary) beam at point *C* (Fig. 7.8 *b*) apply a singular force. Determine the support reactions. In the given case (symmetric application of force)

$$\overline{R}_A = \overline{R}_B = \frac{1}{2} = 0{,}5.$$

Determine deflection of section C by Mohr method using the formula

$$EIy_C = \int_l M(x) \cdot \overline{M}(x)\, dx . \qquad (7.17)$$

Write down the expressions $M(x)$ and $\overline{M}(x)$ on the segments of the beam:

Segment *I*; $0 \le x \le 2$ m (left side)

$$M(x) = 25x - 10x^2;$$
$$\overline{M}(x) = 0{,}5x.$$

Segment *II*; $2 \text{ m} \le x \le 4$ m (left side)

$$M(x) = 25x - 10x^2 + 40;$$
$$\overline{M}(x) = 0{,}5x - 1(x-2).$$

Segment *III*; $0 \le x \le 2$ m (right side)

$$M(x) = -10x;$$
$$\overline{M}(x) = 0.$$

Substitute expressions $M(x)$ and $\overline{M}(x)$ into Mohr integral (7.17) and integrate

$$E \cdot I \cdot y_C = \int_0^2 (25x - 10x^2) \cdot 0{,}5x dx + \int_2^4 \left(25x - 10x^2 + 40\right) \cdot \left(0{,}5x - (x-2)\right) dx + 0 =$$

$$= \int_0^2 \left(12{,}5x^2 - 5x^3\right) dx + \int_2^4 \left(50x - 20x^2 + 80 - 12{,}5x^2 + 5x^3 - 20x\right) dx =$$

$$= \frac{12{,}5 \cdot x^3}{3}\Bigg|_0^2 - \frac{5 \cdot x^4}{4}\Bigg|_0^2 + \frac{30 \cdot x^2}{2}\Bigg|_2^4 - \frac{32{,}5 \cdot x^3}{3}\Bigg|_2^4 + \frac{5 \cdot x^4}{4}\Bigg|_2^4 + \frac{80 \cdot x}{1}\Bigg|_2^4 = 46{,}7;$$

from which the deflection of point C is

$$y_C = \frac{46,7}{E \cdot I}.$$

Sign plus means that the deflection coincides with the direction of a singular force action.

To determine the deflection of section D in redundant (auxiliary) beam (Fig. 7.8 *c*), apply a singular force in the same section. Determine the support reactions:

$$\sum M_A = 0; \quad \overline{\overline{R}}_B \cdot 4 - 1 \cdot 6 = 0; \quad \overline{\overline{R}}_B = 1,5;$$

$$\sum M_B = 0; \quad \overline{\overline{R}}_A \cdot 4 - 1 \cdot 2 = 0; \quad \overline{\overline{R}}_A = 0,5.$$

Write down the expressions of bending moments from the singular load on the segments of the:

segment *I*; $0 \le x \le 2$ m (left side)

$$\overline{\overline{M}}(x) = -0,5x;$$

segment *II*; $2 \text{ m} \le x \le 4$ m (left side)

$$\overline{\overline{M}}(x) = -0,5x;$$

segment *III*; $0 \le x \le 2$ m (right side)

$$\overline{\overline{M}}(x) = -1\,x.$$

Substitute expressions $M(x)$ and $\overline{\overline{M}}(x)$ in the Mohr integral (7.17) and integrate

$$EIy_D = \int_0^2 \left(25x - 10x^2\right) \cdot (-0,5x)dx + \int_2^4 \left(25x - 10x^2 + 40\right) \cdot (-0,5x)dx + \int_0^2 (-10x) \cdot (-1x)dx =$$

$$= \int_0^2 \left(-12,5 \cdot x^2 - 5x^3\right)dx + \int_2^4 \left(-12,5x^2 + 5x^3 - 20x\right)dx + \int_0^2 10x^2 dx =$$

$$= \frac{12,5 \cdot x^3}{3}\Bigg|_0^2 + \frac{5 \cdot x^4}{4}\Bigg|_0^2 - \frac{12,5 \cdot x^3}{2}\Bigg|_2^4 + \frac{5 \cdot x^4}{4}\Bigg|_2^4 - \frac{20 \cdot x^2}{2}\Bigg|_2^4 + \frac{10 \cdot x^3}{3}\Bigg|_0^2 = -40,$$

from which the deflection of point D is

$$y_D = -\frac{40}{E \cdot I}.$$

Sign minus means that the deflection of point D does not coincide with the direction of a singular force action (see. Fig. 7.8 *c*).

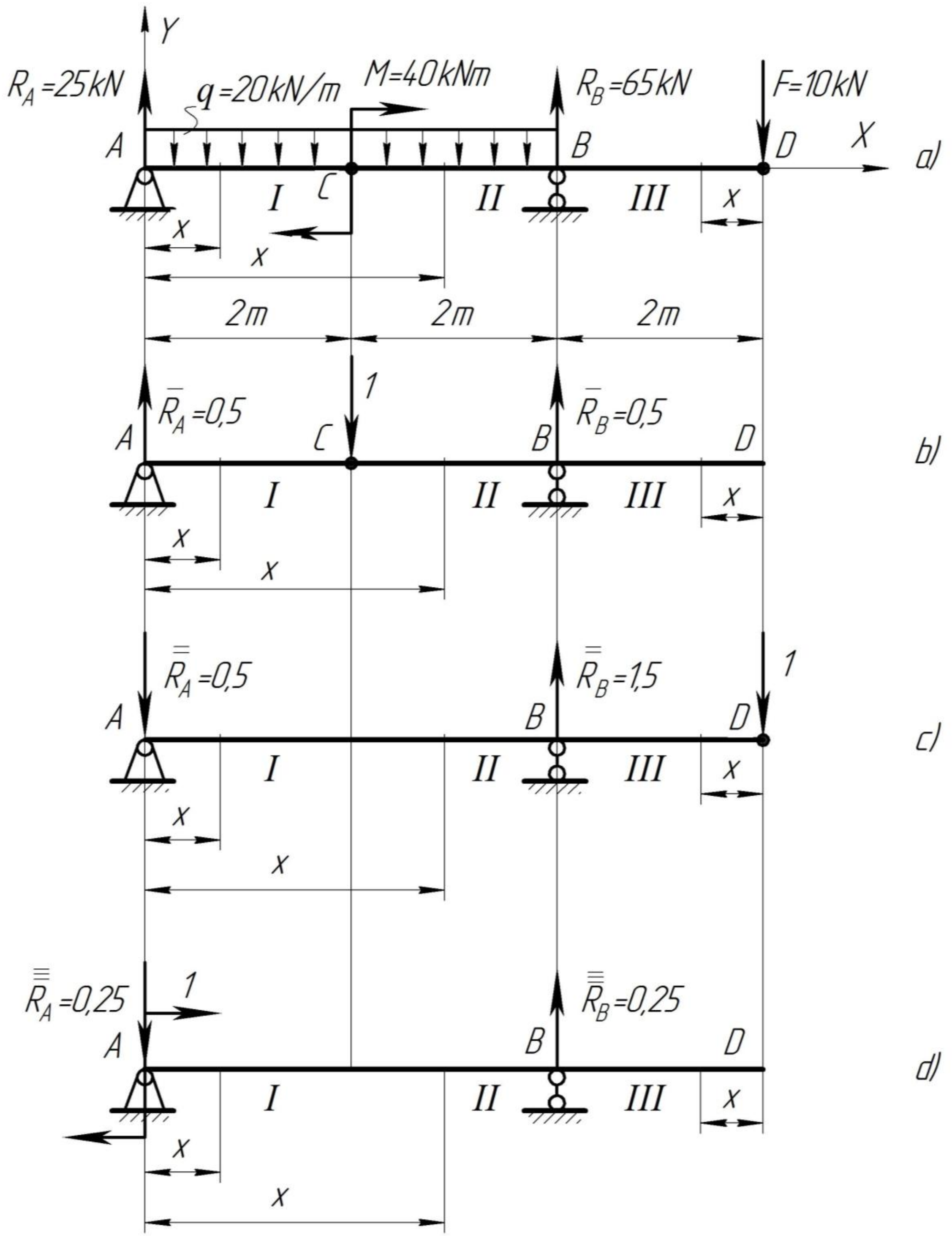


Figure 7.8

To determine the slope of the elastic curve A, in the redundant (auxiliary) beam (Fig. 7.8 *d*), apply a single moment.

Find the support reactions from the action of a single moment

$$\overset{\equiv}{R}_A = \overset{\equiv}{R}_B = \frac{1}{4} = 0{,}25 .$$

Expressions of bending moments for the beam (Fig. 7.8 *d*):
segment *I*; $0 \le x \le 2$ m (left side)

$$\overline{\overline{\overline{M}}}(x) = 1 - 0{,}25\,x\,;$$

segment *II*; $2\text{ m} \le x \le 4$ m (left side)

$$\overline{\overline{\overline{M}}}(x) = 1 - 0{,}25\,x\,;$$

segment *III*; $0 \le x \le 2$ m (right side)

$$\overline{\overline{\overline{M}}}(x) = 0\,.$$

Determine the slope of the elastic curve A by the Mohr method using the formula

$$E \cdot I \cdot \Theta_A = \int_l M(x) \cdot \overline{\overline{\overline{M}}}(x)\,dx\,. \qquad (7.18)$$

Substituting the data, obtain

$$E \cdot I \cdot \Theta_A = \int_0^2 \left(25\,x - 10\,x^2\right) \cdot (1 - 0{,}25\,x)\,dx +$$

$$+ \int_2^4 \left(25\,x - 10\,x^2 + 40\right) \cdot (1 - 0{,}25\,x)\,dx + 0 =$$

$$= \int_0^2 \left(25\,x - 10\,x^2 - 6{,}25\,x^2 + 2{,}5 \cdot x^3\right) dx +$$

$$+ \int_2^4 \left(25\,x - 10\,x^2 + 40 - 6{,}25\,x^2 + 2{,}5\,x^3 - 10\,x\right) dx =$$

$$= \left.\frac{25 \cdot x^2}{2}\right|_0^2 - \left.\frac{16{,}25 \cdot x^3}{3}\right|_0^2 + \left.\frac{2{,}5 \cdot x^4}{4}\right|_0^2 + \left.\frac{40 \cdot x}{1}\right|_2^4 + \left.\frac{15 \cdot x^2}{2}\right|_2^4 -$$

$$- \left.\frac{16{,}25 \cdot x^3}{3}\right|_2^4 + \left.\frac{2{,}5 \cdot x^4}{4}\right|_2^4 = 33{,}4;$$

from which the slope of the elastic curve A is

$$\Theta_A = \frac{33{,}4}{E \cdot I}\,.$$

8. STATICALLY INDETERMINATE SYSTEMS

General concept

Statically indeterminate systems are systems in which the reactions of junctions and internal forces are impossible to determine by the equilibrium equations only.

Such systems (constructions) are the most spread as they are more reliable and rigid in comparison with statically determinate ones.

Statically determinate (isostatic) beam or frame can be transformed into statically indeterminate (hyperstatic) by setting extra (excessive from the point if view of the system's equilibrium) support. Advantages of such system: the loading over it can be increased without changing the crossing of the beam; when one of the supports in isostatic system is damaged, it is turned into mechanism whilst hyperstatic system remains unmoveable, capable to take loads, in other words, it is safer. In many cases statically indeterminate systems are the only possible variant of construction.

Advantages of statically indeterminate systems are: decrease of elastic displacements; increase of stiffness and stability of the system elements; significant decrease of the working stresses at their crossings; economical efficiency as having the same size of crossings, they can carry more load; when losing some excessive relations they remain immovable and geometrically unchanged; have higher reliability and connectedness of elements during work; capable to redistribute the load between elements if some of them damage or weaken (in case of setting down of one or several supports).

Drawbacks: there occur the temperature stresses as well as assembly ones in case if their size changes in relation to designed dimensions.

Peculiarities: the supports reactions and internal forces in the elements depend on stiffness of diametrical crossing of the rod system; it is impossible to provide the equal safety margine, i.e. one elements can be underloaded, and the others overloaded which requires them to be optimally designed.

Main methods of evaluating the systems indeterminence

Since there are more unknown forces than the equilibrium equations, static indeterminance of the system can be evaluated only with redundant (auxiliary) equations. These equations have to show the peculiarities of geometric relations put over the rod system. Such equations are composed by figuring out and drawing the picture of displacement of the construction elements sections during its deformation and that is why they are defined as ***displacement*** **(*deformation*)** ***compatability equations.***

Metods of calculation of statically indeterminate systems are classified according to whaich is taken as an unknown value. If displacements are considered as unknown, the calculation method is called ***the displacement method;*** and if the forces are unknown, the method of their calculation is ***the force method.*** If partially forces and partially displacemetnts are unknown, the method of calculation is ***mixed.***

The displacement method, in which the linear and angular displacement of rigid nodes of the pin system are taken as unknowns in the static equations, appeared in 1880, the force method is known since 1807.

In strength of materials the force method is used more frequently.

Force method

Calculation of statically indeterminate system begins with its analysis. It is necessary for determining the degree of static indeterminance. **The degree of static indeterminance** equals the number of redundant junctions removing of which turns the indeterminate system into determinate one (main), geometrically changeable. The term redundant (auxiliary) junction is ment as excessive junction, not as unnecessary junction.

In Fig. 8.1 *a* there is a statically indeterminate beam.

Over this beam, four junctions are placed X_1, X_2, X_3, X_4. For the plane force system, only **three static equations** can be formed, so this beam is $4-3=1$ times statically indeterminate. As a redundant (auxiliary) junction X_1 is taken, hence the beam (Fig. 8.1 *b*) is statically determinate (the main).

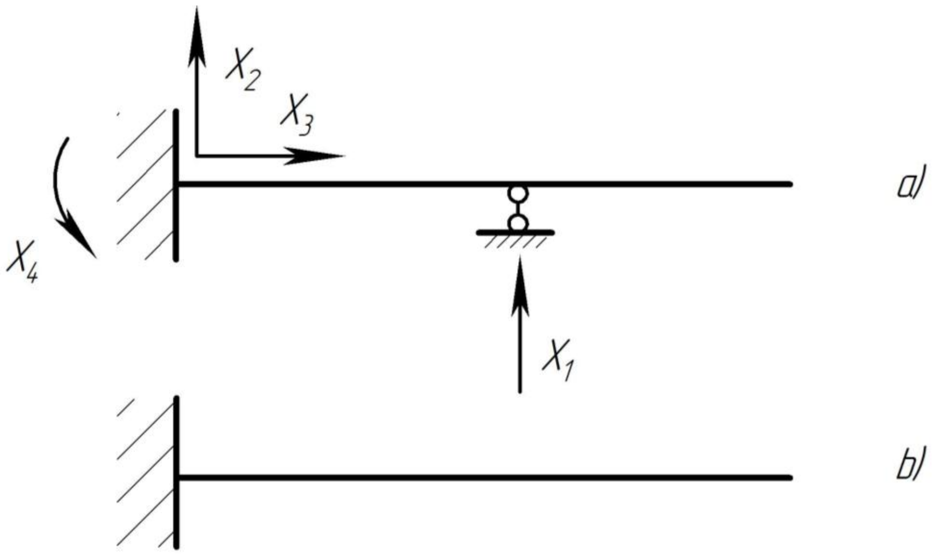


Figure 8.1

In Fig. 8.2 *a* the plane frame is drawn. This system is $5-3=2$ times statically indeterminate.

Having removed redundant (auxiliary) junctions X_1 and X_2, we transform statically indeterminate system into statically determinate geometrically unchangeable one (Fig. 8.2 *b*).

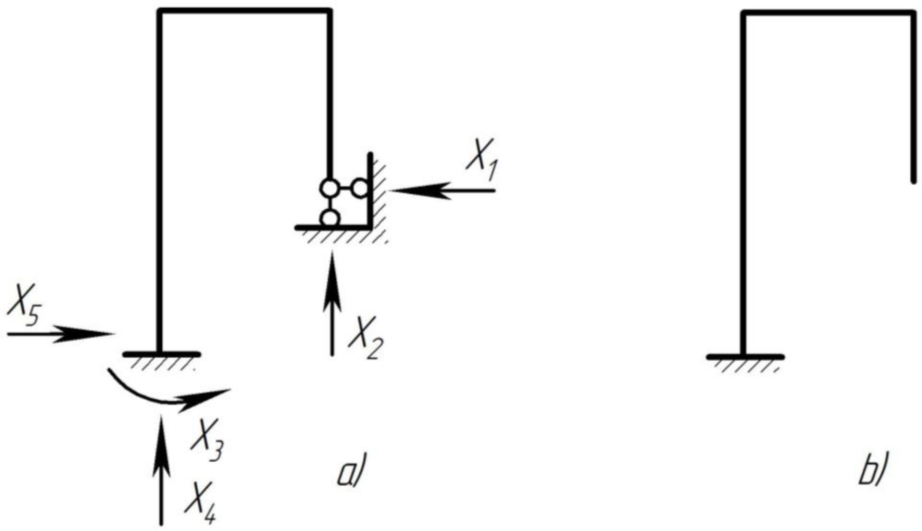


Figure 8.2

The main system is statically determinate geometrically unchangeable system made of statically indetermitate one is defined.

In Fig. 8.1 *b* the main system is drawn.

The principle of the independence of force action makes the basis of **the force method.**

The order of calculation of statically indeterminate systems using the force method:

1. ***Determining the degree of static indeterminance of the system*** (see Fig. 8.1, 8.2)

2. ***Choosing the main system*** by remowing the redundant junctions. The main system has to be statically determinate, unmovable and geometrically ungchangeable after applying the load as well. For every given system, a few auxiliary can be chosen so it is reasonable to take the optimal system which significantly simplifies the further calculations.

3. ***Formation of the equivalent system***. Artificial changes in given statically indeterminate system during transition to the main system have to be compensated by introduction of corresponding unknown generalized forces that are applied instead of the removed junctions. In those sections where the linear displacements are impossible, the concentrated forces are applied, and where the angular displacements are unallowable the moments are introduced. These unknown for present forces are indicated as X_i, where i is the number of unknown redundant force. In other words, by substituting of removed redundant junctions with the force X_i and applying external load, the equivalent system is formed. During transition to it, the force scheme of the given hyperstatic system as well as its deformation scheme have to be kept, i.e. the equivalent system has to deform in the same way as the given hyperstatic one. These demands can be formulated as so called conditions of continuity or ***strain compatability***.

4. ***Writing down the equation of strain compatability*** (continuity of displacements) ***for the equivalent system*** taking ino account the condition that displacements which are impossible in the given system have to be impossible in the equivalent system under stated active load and unknown redundant forces (reactions) as well.

In the extended geometric form this equation can be written down as ***the condition of zero equality of displacements by the direction of any removed junction***

$$\Delta_i = X_1 \cdot \delta_{i1} + X_2 \cdot \delta_{i2} + ... + X_{in} \cdot \delta_{in} + \Delta_i(F, q, M) = 0 ; \quad (8.1)$$

where X_1, X_2, X_n are values of reactions of redundant unknowns;

$\delta_{i1}, \delta_{i2}, \delta_{in}$ are displacements in the main system by the direction of redundant unknown *i* from the singular loads that are applied in the main system in the directions X_1, X_2, X_n;

$\Delta_i(F, q, M)$ is displacement in the main system by the direction of redundant unknown *i* from the external loads.

With introducing the concept of singular effort being applied instead of unknown $X_i = 1$ and the term singular displacement occurring in the direction of *i*-force from *n*-singular force for *n*-times statically indeterminate scheme, the equation (8.1) is written down as the system of canonic (the simplest) equations:

$$\cdot\begin{cases} X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} + \cdots + X_n \cdot \delta_{1n} + \Delta_1(F) = 0; \\ X_2 \cdot \delta_{21} + X_2 \cdot \delta_{22} + \cdots + X_n \cdot \delta_{2n} + \Delta_2(F) = 0; \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ X_1 \cdot \delta_{n1} + X_2 \cdot \delta_{n2} + \cdots + X_n \cdot \delta_{nn} + \Delta_n(F) = 0. \end{cases} \quad (8.2)$$

Singular displacements are the coefficients in canonic equations and ***absolute displacements are absolute terms***. During displacements, the first index shows in the direction of which force the displacement of the section with this force is applied, the second index indicates the force that caused this displacement. Singular displacements with the same indeces (δ_{11}) are denoted as ***the main coefficients*** of the system of canonic equations, and with different indeces are (δ_{12}) — ***secondary***, while $\delta_{12} = \delta_{21}$.

Every single canonic equation of the system (8.2) shows that displacement of the section where the redundant junction is remowed is impossible in the direction of this junction reaction under the stated load and all unknown forces.

The total number of displacement compatability equations equals the number of unknown forces (removed redundant junctions), in other words, the degree of static indeterminance of the stated system.

5. ***Calculation of coefficients and absolute terms of canonic equations.*** It is reasonable to work out these displacements by formulas of energy method (Mohr integral).

While determining δ_{ij} and Δ_{iF} expressions for rigidity $E \cdot I$ elements of the system, it is advisable to solve them in general (not numeric) form in order to simplify the canonic equations and make the calculations shorter.

To establish the absolute terms of the system of equations (8.2), i.e. complete displacements Δ_{iF}, the diagrams (epures) caused by the external forces action have to be drawn. It is better to draw these diagrams (epures) from each force separately. Multiplying these real diagrams (epures) by appropreate singular ones, the values of displacements Δ_{iF} are determined.

6. ***Determining unknown forces*** from the system of canonic equations.

7. ***Calculations of strength, rigidity and stability*** can be made similarly to the way it is done in case of statically deretminant systems. ***Determination of total bending moments and other internal force factors in the sections is carried basing on the principle of the action independence using the classical method of sections or by the method of drawing appropreate diagrams (epures).***

While determining real displacements of single sections of the system, the singular action has to be applied to the main system; draw the diagram of bending moments of this force and multiply it by the resultant diagram of external load. On order not to divide the resultat diagram into simple segments, singular diagram can be multiplied by single real diagrams from the action of each force and the results can be added. ***Displacement of characteristic crossings*** (fixations on supports) ***are determined to test the correctness of all previous calculations of statically indeterminate system.***

Metod of minimum potential energy of deformation

While considering statically indeterminate frame constructions, taking into account additivity (continuity) of the function of potential energy of deformation, the expression of full potential energy of construction deformation can be written down

$$U = U_M + U_K + U_Q + U_N , \tag{8.3}$$

where U_M is potential energy of bending strain of the frame elements,

$$U_M = \sum \int \frac{[M(x)]^2}{2E \cdot I_0} dx ;$$

U_K is potential energy of tortion strain,

$$U_K = \sum \int \frac{[K(x)]^2}{2G \cdot I_P} dx ;$$

U_Q is potential energy of shearing strain,

$$U_Q = \sum \int \frac{[Q(x)]^2}{2G \cdot A} dx ;$$

U_N **is** potential energy of tensile (compressive) strain,

$$U_N = \sum \int \frac{[N(x)]^2}{2E \cdot A} dx ,$$

here E and G are elasticity and creep module correspondingly;
I is an axial moment of cross-section inertia;
I_P is a polar moment of cross-section inertia;
A is a cross-section area;
$M(x)$ is a functions of bending moment;
$K(x)$ is a functions of tortion moment;
$Q(x)$ is a functions of cross-cut forces;
$N(x)$ is a functions of tensile (compessing) forces.

In these formulas integration is made along the elements of frame (beam).

Formula (8.3) and its components are **the main expressions of potential energy of deformation during evaluation of static indeterminance of any system.**

Using Castigliano theorem $\partial U / \partial X_i = 0$, the system of equations is formed and the values of redundant unknowns are calculated.

While calculating frame constructions from the normal and cross-cut forces, potential energies are neglected and only potential energies from the bending moment and tortion are considered.

Task 10
Calculation of statically indeterminate frame

For the given statically indeterminate frame (Fig. for task 10, Table for task 10) evaluate static indeterminance using the force method and validate the obtained result by the method of minimum potential energy of deformation (MMPED). Draw the diagrams of scoss-cut and axial forces, bending moments. Carry out static assessment of any frame nod. Choose I-shaped section, when $[\sigma] = 160$ MPa ; $a = 1$ m; $q = 20$ kN/m ; $E \cdot I = const$. From two binders (1 and 2) leave the one from Table for task 10.

Plan of solving the task:

1. Choose the main system and draw the diagrams of bending moments from external and singular loads in the main system.
2. Write down the canonic equation of the force method.
3. Determine coefficient δ_{11} and absolute term $\Delta_1(F)$ of the canonic equation.
4. Solve the canonic equation.
5. Check the correctness of evaluation of static indeterminance by MMPED.
6. Write down the axial N, cross-cut (cutting) Q, and bending moments on the frame segments.
7. Draw diagrams N, Q, M for the equivalent system.
8. Carry out static assessment of any frame nod.
9. Determine the dangerous frame section and choose I-shaped section from the terms of strength with normal stresses that appear because of bending.

Table for task 10

Nr	F, kN	M, kNm	*Binder number*
1	$q \cdot a$	$F \cdot a$	1
2	$2\ q \cdot a$	$q \cdot a^2$	2
3	$3\ q \cdot a$	$F \cdot a$	1
4	$q \cdot a$	$q \cdot a^2$	2
5	$2\ q \cdot a$	$F \cdot a$	1
6	$3\ q \cdot a$	$q \cdot a^2$	2
7	$q \cdot a$	$F \cdot a$	1
8	$2\ q \cdot a$	$q \cdot a^2$	2
9	$3\ q \cdot a$	$F \cdot a$	1
0	$q \cdot a$	$q \cdot a^2$	2

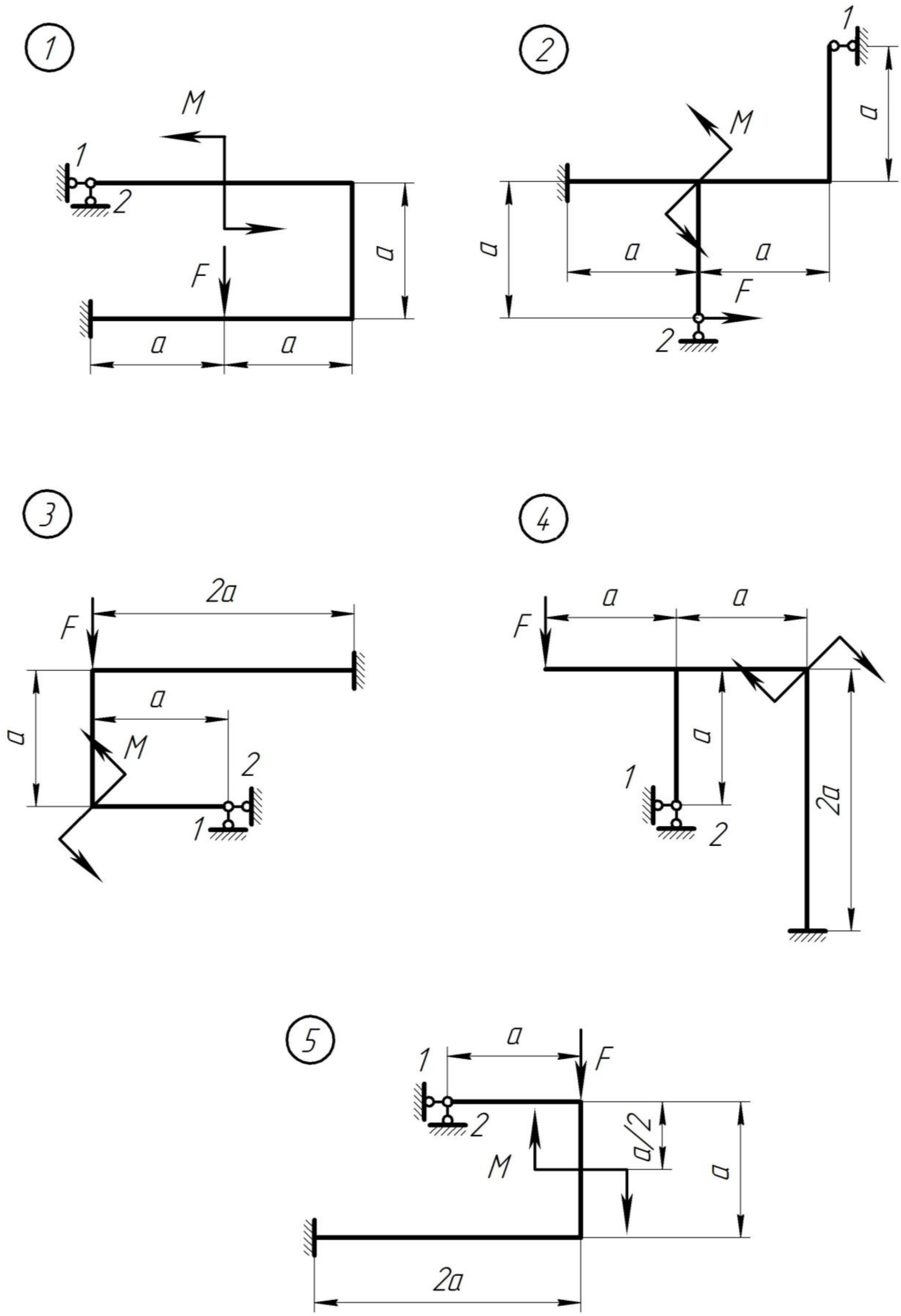
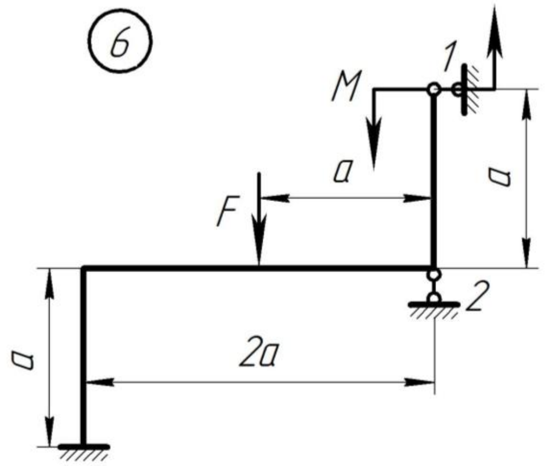
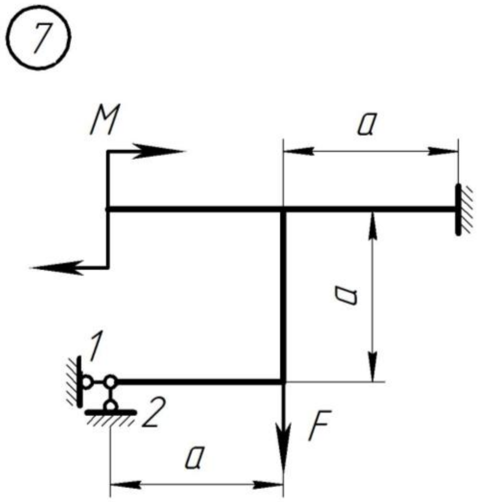
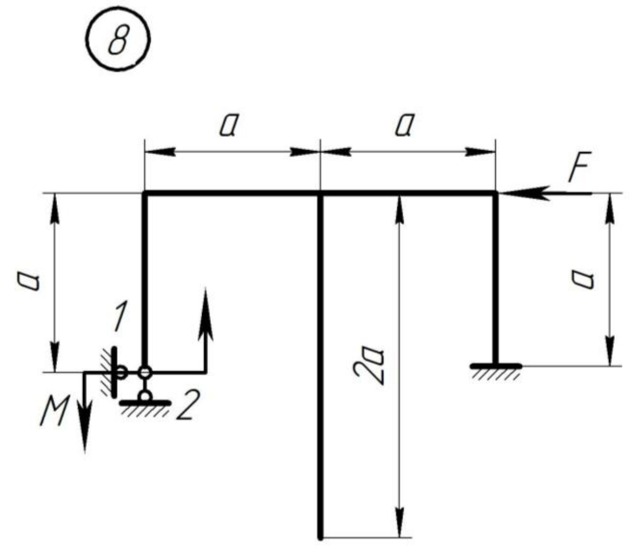
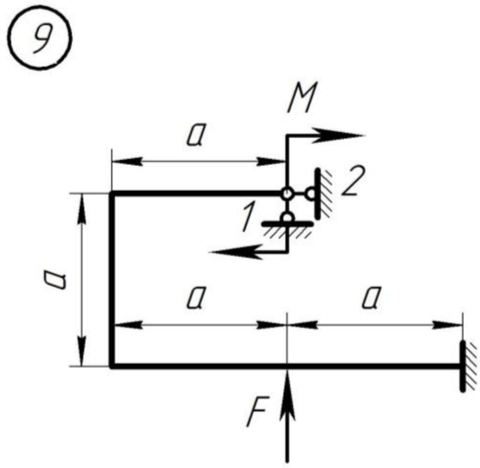


Figure for task 10









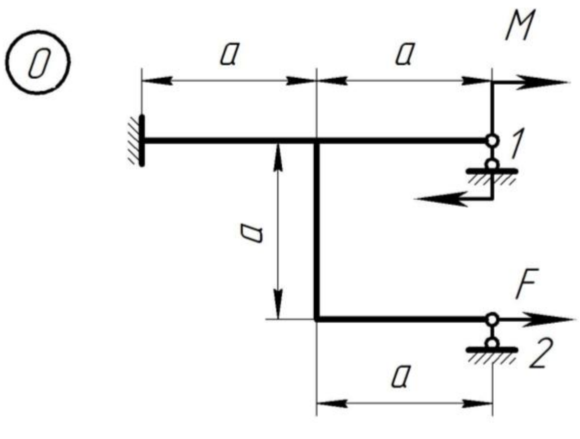


Figure for task 10 (contunied)

Example of solving the task 10 using the force method

For the given statically indeterminate frame (Fig. 8.3*a*) evaluate static indeterminance using the force method and validate the obtained result by the method of minimum potential energy of deformation (MMPED). Draw the diagrams of shear (cutting, cross-cut) and axial forces, bending moments. Carry out static evaluation of any frame nod. Choose the cross-cut, when $[\sigma] = 160$ MPa ; $a = 1$ m; $F = 40$ kN; $M = 60$ kNm; $E \cdot I = const$.

Solution

The given frame construction (see Fig. 8.3 *a*) is $4 - 3 = 1$ time statically indeterminate. Statical indeterminance is evaluated using the force method. X_1 is taken as excessive unknown. The main system is shown at Fig. 8.3 *b*.

Write down the canonic equation of the force method

$$X_1 \cdot \delta_{11} = -\Delta_1(F).$$

Displacement (coefficient) δ_{11} and the absolute term of equation $\Delta_1(F)$ are evaluated by grapho-analytical method using Mohr integral solution approach.

We load the main system with singular force (Fig. 8.3 *c*). Draw the diagram of bending moments from the singular force (Fig. 8.3 *d*). Work out

$$E \cdot I \cdot \delta_{11} = \omega_1 \cdot \overline{M}_{C1},$$

where $\omega_1 = \frac{1}{2} \cdot 2 \cdot 2 = 2;$ $\quad \overline{M}_{C1} = \frac{2}{3} \cdot 2 = \frac{4}{3}.$

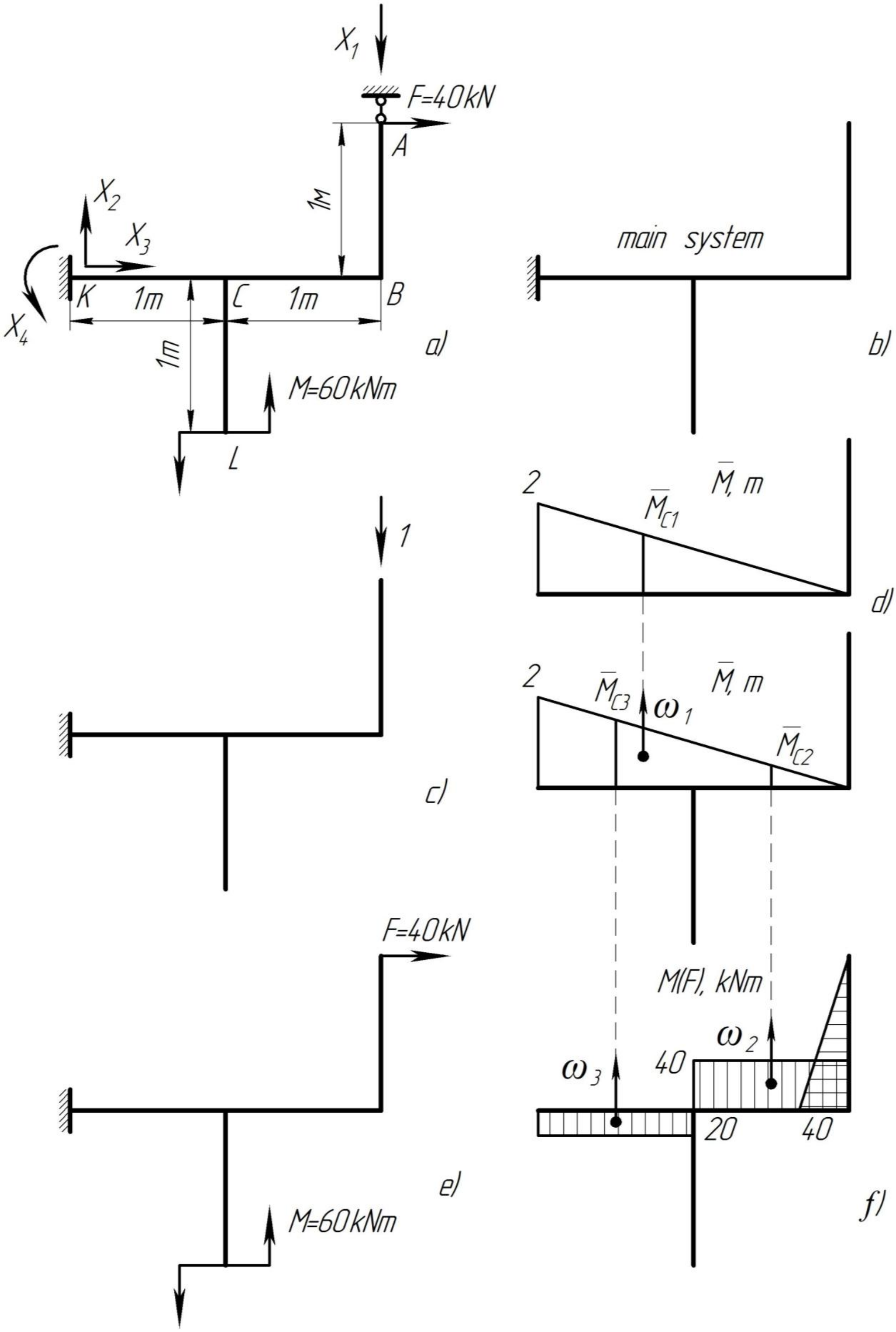
Then

$$E \cdot I \cdot \delta_{11} = 2 \cdot \frac{4}{3} = \frac{8}{3}.$$

To determine the absolute term of the equation $\Delta_1(F)$, apply the external load to the main system (Fig. 8.3 *e*). Draw the diagram of bending moments from the loads (Fig. 8.3 *f*). Write down the equation

$$E \cdot I \cdot \Delta_1(F) = \omega_2 \cdot \overline{M}_{C2} - \omega_3 \cdot \overline{M}_{C3},$$

where $\omega_2 = 40 \cdot 1 = 40;$ $\quad \omega_3 = 20 \cdot 1 = 20;$ $\quad \overline{M}_{C2} = \frac{1}{2} = 0{,}5;$ $\quad \overline{M}_{C3} = 1{,}5.$



X1
F=40kN
A
1м
X2
X3
X4
K
1m
C
1m
B
a)
1m
M=60kNm
L
main system
b)
1
c)
2
M̄, m
M̄C1
d)
2
M̄C3
ω1
M̄, m
M̄C2
F=40kN
M(F), kNm
ω2
40
ω3
20
40
e)
M=60kNm
f)

Figure 8.3

Then

$$E \cdot I \cdot \Delta_1(F) = 40 \cdot 0{,}5 - 20 \cdot 1{,}5 = -10 \,.$$

Solve the canonic equation using the force method

$$X_1 = -\frac{\Delta_1(F)}{\delta_{11}} = -\frac{(-10)\cdot 3}{8} = 3{,}75 \text{ kN} \,.$$

Sign plus means that the direction of reaction force X_1 is chosen correctly.

Draw the equivalent scheme (Fig. 8.3 *g*). **Equivalent scheme** is the main scheme loaded with external load and determined reaction forces, in other words, it is **the given initial scheme with determined reactions.**

Divide the frame into segments. Work out the values of internal forces factors for each of them. On the scheme (see Fig. 8.3 *g*) on the frame contour, there are signs plus for positive values of bending moments indicated. The bending moment is considered to be positive if it stretches the lower fibres. Write down the functions of axial N, cross-cut (shear, cutting) forces Q, and bending moments M_{BN} on the frame segments.

Segment AB, $0 \le x \le 1$ m:

$N(x) = -X_1 = -3{,}75$ kN; $\quad N_A = N_B = -3{,}75$ kN;

$Q(x) = F = 40$ kN; $\quad Q_A = Q_B = 40$ kN;

$M_{BN}(x) = -F \cdot x = -40x$; $\quad M_{BN\,A} = 0$; $\quad M_{BN\,B} = -40$ kNm.

Segment BC, $0 \le x \le 1$ m:

$N(x) = F = 40$ kN; $\quad N_B = N_C = 40$ kN;

$Q(x) = X_1 = 3{,}75$ kN; $\quad Q_B = Q_C = 3{,}75$ kN;

$M_{BN}(x) = -F \cdot 1 - X_1 \cdot x = -40 - 3{,}75x$;

$M_{BN\,B} = -40$ kNm; $\quad M_{BN\,C} = -43{,}75$ kNm.

Segment LC, $0 \le x \le 1$ m:

$N(x) = 0;$ $N_L = N_C = 0;$

$Q(x) = 0;$ $Q_L = Q_C = 0;$

$M_{BN}(x) = M = -60$ kNm; $M_{BN\ L} = M_{BN\ C} = -60$ kNm.

Segment CK, $1\text{ m} \le x \le 2$ m:

$N(x) = F = 40$ kN; $N_C = N_K = 40$ kN;

$Q(x) = X_1 = 3{,}75$ kN; $Q_C = Q_K = 3{,}75$ kN;

$M_{BN}(x) = -F_1 \cdot 1 - X_1 \cdot 1 - X_1 \cdot x + M = -40 - 3{,}75 - 3{,}75 \cdot x + 60 = 16{,}25 - 3{,}75x;$

$M_{BN\ C} = 16{,}25$ kNm; $M_{BN\ K} = 12{,}5$ kNm.

By the obtained results, draw the diagrams N, Q and M_{BN} (Fig. 8.3 *h*, *i*, *j*).

Validate the evaluation of static indeterminance.
Static test. Consider equilibrium of the nod C (Fig. 8.3 *k*):
Write down the equilibrium equation:

$$\sum F_{iX} = F - Q_A = 40 - 40 = 0;$$

$$\sum F_{iY} = -X_1 + N = -3{,}75 + 3{,}75 = 0;$$

$$\sum M_{iC} = 60 - 16{,}25 - 43{,}75 = 0.$$

From the diagram of bending moments determine (see Fig. 8.3 *j*)

$$M_{BN.\max} = 60 \text{ kNm}.$$

To select the frame section from the terms of bending strength at normal stress, determine the axial resisting moment of one I-beam

$$W_0 = \frac{M_{BN\ \max}}{2[\sigma]} = \frac{60 \cdot 10^{-3}}{2 \cdot 160} = 187{,}5 \cdot 10^{-6} \text{ m}^3,$$

take I-beam Nr 22a (standard GOST 8240-72, *Annex* 1), for which $W_0 = 192 \text{ cm}^3$.

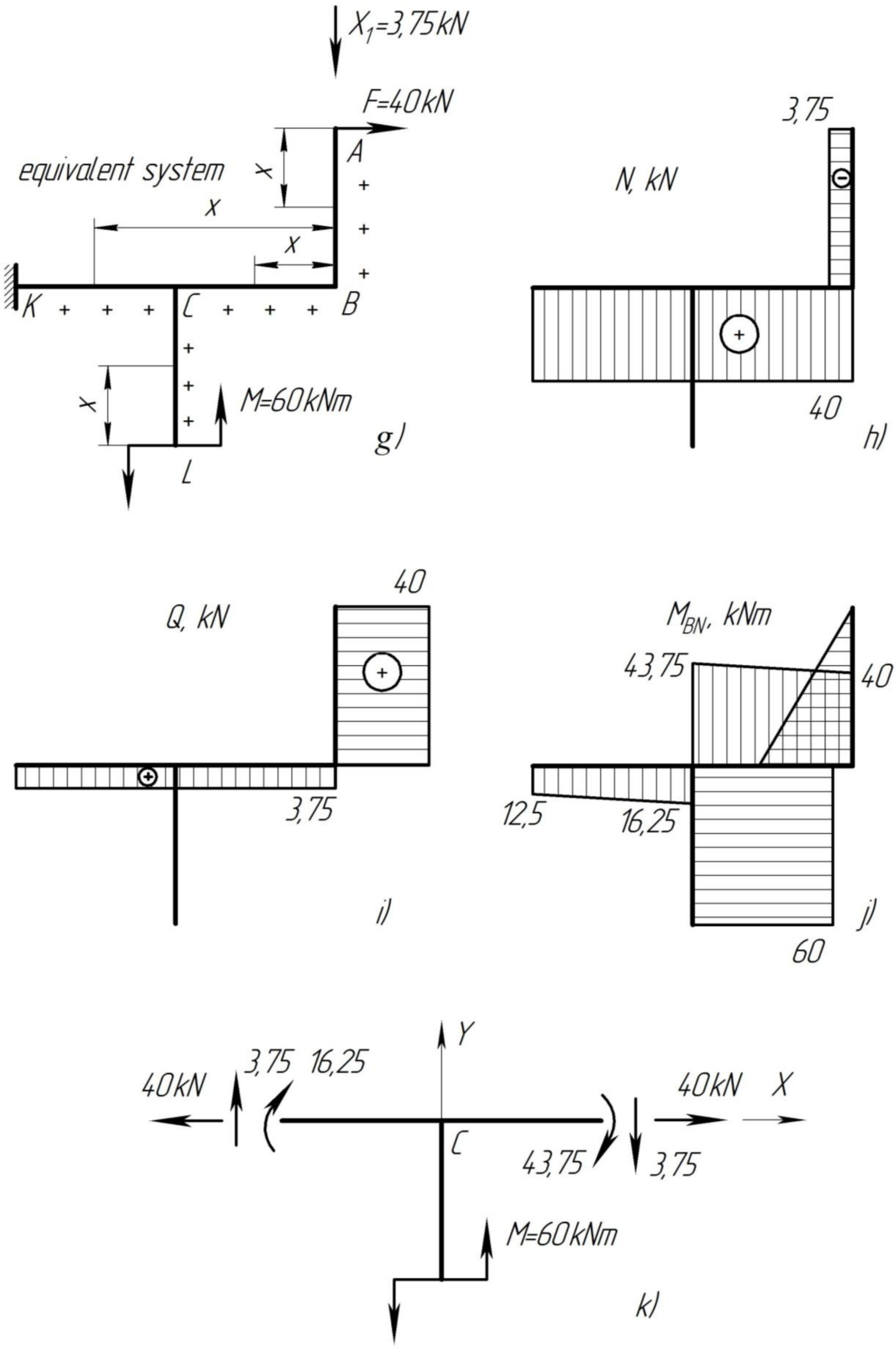


Figure 8.3 (continued)

Example of solving the task 10
by the metod of minimum potential energy of deformation

Evaluate the static indereminance of the frame construction (see Fig. 8.3 *a*) by the metod of minimum potential energy of deformation (MMPED).

Solution

Write down potential energies of bending deformation for each element of the frame.

Segment AB, $0 \le x \le 1$ m:

$$M_{BN}(x) = F \cdot x = 40 \cdot x;$$

$$U_1 = \int_0^1 \frac{(40 \cdot x)^2}{2\,E \cdot I}\,dx.$$

Segment BC, $0 \le x \le 1$ m:

$$M(x) = 40 \cdot 1 + X_1 \cdot x;$$

$$U_2 = \int_0^1 \frac{(40 + X_1 \cdot x)^2}{2\,E \cdot I}\,dx.$$

Segment LC, $0 \le x \le 1$ m:

$$M(x) = M = 60;$$

$$U_3 = \int_0^1 \frac{(60)^2}{2\,E \cdot I}\,dx.$$

Segment CK, $0 \le x \le 1$ m:

$$M(x) = 40 + X_1 \cdot 1 + X_1 \cdot x - 60 = -20 + X_1 + X_1 \cdot x;$$

$$U_4 = \int_0^1 \frac{(-20 + X_1 + X_1 \cdot x)^2}{2E \cdot I} dx.$$

Total potential energy of bending deformation

$$U = U_1 + U_2 + U_3 + U_4 = \int_0^1 \frac{(40x)^2}{2E \cdot I} dx + \int_0^1 \frac{(40 + X_1 \cdot x)^2}{2E \cdot I} dx +$$

$$+ \int_0^1 \frac{(60)^2}{2E \cdot I} dx + \int_0^1 \frac{(-20 + X_1 + X_1 \cdot x)^2}{2E \cdot I} dx.$$

From equation $\frac{\partial U}{\partial X_1} = 0$, work out the value of the reaction X_1:

$$\frac{\partial U}{\partial X_1} = \frac{1}{2E \cdot I} \left\{ 0 + \int_0^1 2(40 + X_1 \cdot x) \cdot x dx + 0 + \int_0^1 2(-20 + X_1 + X_1 \cdot x)(1 + x) dx \right\} =$$

$$= \frac{1}{2E \cdot I_0} \left\{ \left. \frac{80 \cdot x^2}{2} \right|_0^1 + \left. \frac{2X_1 \cdot x^3}{3} \right|_0^1 - \left. \frac{40 \cdot x}{1} \right|_0^1 + \left. \frac{2X_1 \cdot x}{1} \right|_0^1 + \left. \frac{2X_1 \cdot x^2}{2} \right|_0^1 - \left. \frac{40 \cdot x^2}{2} \right|_0^1 + \right.$$

$$\left. + \left. \frac{2X_1 \cdot x^2}{2} \right|_0^1 + \left. \frac{2X_1 \cdot x^3}{3} \right|_0^1 \right\} = 40 + \frac{2}{3} X_1 - 40 + 2X_1 + X_1 - 20 + X_1 + \frac{2}{3} X_1 =$$

$$= -20 + \frac{16}{3} X_1 = 0;$$

then

$$X_1 = \frac{20 \cdot 3}{16} = 3{,}75 \text{ kN},$$

that consists with the definitions of the forces method.

9. EVALUATION OF STRESSES AND DISPLACEMENTS AT OBLIQUE BENDING

Oblique bending is a complex type of deformation. It occurs when the plane of absolute bending moment action does not coincide with any of its main planes, i.e. planes drawn through the beam axis and the main axis of cross-cut inertia.

Consider the example of pure oblique bending. In a random cross-cut the force plane of bending pair M makes the angle α with the inertia axis Y (Fig. 9.1).

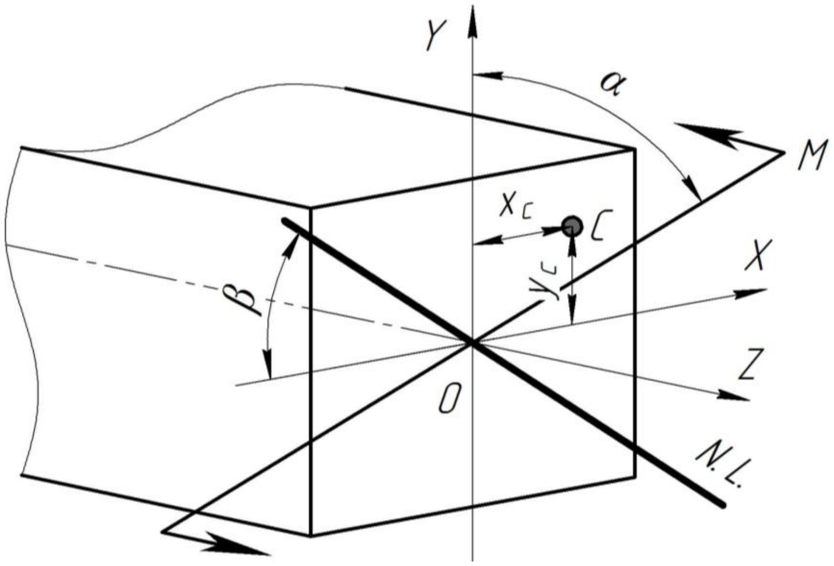


Figure 9.1

Oblique bending is considered as combination of two right bendings in the main planes XZ and YZ (Fig. 9.2). Axes X and Y are the main central crossing inertia axes, axis Z coincides with longitudinal axis of the beam.

Components M_X and M_Y of the general bending moment M that act in the main planes are calculated by formulas:

$$M_X = M \cdot \cos\alpha; \qquad M_Y = M \cdot \sin\alpha.$$

Normal stress at oblique bending at any cross-cut point, e.g. at point C with coordinates x_C and y_C (see Fig. 9.1), ***is found as algebraic sum of normal stresses*** from the components of the bending moment M_X and M_Y,

$$\sigma_{Z\ sum} = \sigma_Z(M_X) + \sigma_Z(M_Y) = -\left(\frac{M_X}{I_X} \cdot y_C + \frac{M_Y}{I_Y} \cdot x_C\right) \tag{9.1}$$

or

$$\sigma_{Z\ sum} = -M\left(\frac{y_C}{I_X}\cdot\cos\alpha + \frac{x_C}{I_Y}\cdot\sin\alpha\right). \tag{9.2}$$

Coordinate system XYZ is chosen in such a way that compression stresses act in the I-st quadrant.

The neutral (zero) **section line is a geometric place of the points where normal stresses equal zero.** This line must run through the weight centre of the cross-cut.

Equation of the neutral line at oblique bending

$$\frac{M_X}{I_X}\cdot y_0 + \frac{M_Y}{I_Y}\cdot x_0 = 0, \tag{9.3}$$

or

$$\frac{y_0}{I_X}\cdot\cos\alpha + \frac{x_0}{I_Y}\cdot\sin\alpha = 0, \tag{9.4}$$

where x_0, y_0 are coordinates of the points of the neutral crossing line (Fig. 9.3).

Since $x_0 = 0$, then $y_0 = 0$ as well. The position of such line is evaluated by the angle of its inclination to the axis X (Fig. 9.2).

$$tg\beta = \frac{y_0}{x_0} = -\frac{I_X}{I_Y}\cdot tg\alpha. \tag{9.5}$$

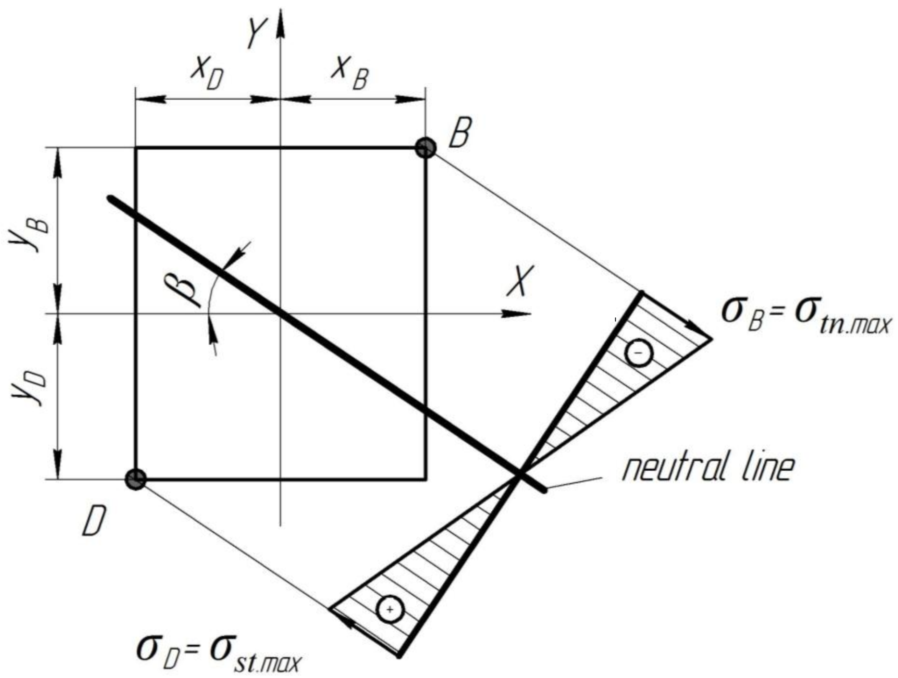


Figure 9.2

Stresses at oblique bending. Strength condition

Maximum normal stresses occuring at the points most remote from the neutral line, symetric crossing points, e.g., rectangle (see Fig. 9.2), points *B* and *D* are of the same size but with different signs. They are worked out by formula

$$\sigma_{\substack{\max\\ \min}} = \pm\left(\frac{M_X}{W_X} + \frac{M_Y}{W_Y}\right), \tag{9.6}$$

where M_X and M_Y are bending moments in relation to the main axes in the most loaded dangerous crossing section.

For the elastic materials, which cross-cuts have two symmetry axes **the strength condition by normal stresses at oblique bending is**

$$\sigma_{\max} = \frac{M_X}{W_X} + \frac{M_Y}{W_Y} \le [\sigma]\cdot \tag{9.7}$$

Shearing stresses at oblique bending are determined as a sum of shearing stresses τ_X, τ_Y obtained from the cross-cut forces Q_X, Q_Y

$$\tau = \sqrt{\tau_X^2 + \tau_Y^2}\,. \tag{9.8}$$

The shearing stresses components τ_Y, τ_Z are calculated by D.I. Zhuravskyi formula

$$\tau_X = \frac{Q_X \cdot S_Y^{sh}}{b_1 \cdot I_Y}; \qquad \tau_Y = \frac{Q_Y \cdot S_X^{sh}}{b_2 \cdot I_X}. \tag{9.9}$$

Deformations at oblique bending

In general, for sections with different values of axial inertia moments, in other words, when $I_X \ne I_Y$ and $tg\beta \ne tg\alpha$, the neutral line is not perpendicular to the force line, but deviated in the direction to the axis of minimum moment of crossing inertia.

Since the direction of absolute bending f and the neutral line are always orthogonal (Fig. 9.3), the beam at the oblique bending bends not in the force plain, but in some other plain where the bending rigidity is less.

Oblique bending is brought to two plane ones. Using the principle of superposition, the displacements f_X and f_Y in each of main planes are determined.

The absolute bending of the beam f (see Fig. 9.3) at oblique bending is evaluated as a geometrical sum of bendings

$$f = \sqrt{f_X^2 + f_Y^2}. \qquad (9.10)$$

The direction of absolute bending is determined by angle

$$\gamma = arctg\left(\frac{f_X}{f_Y}\right),$$

equal with angle β.

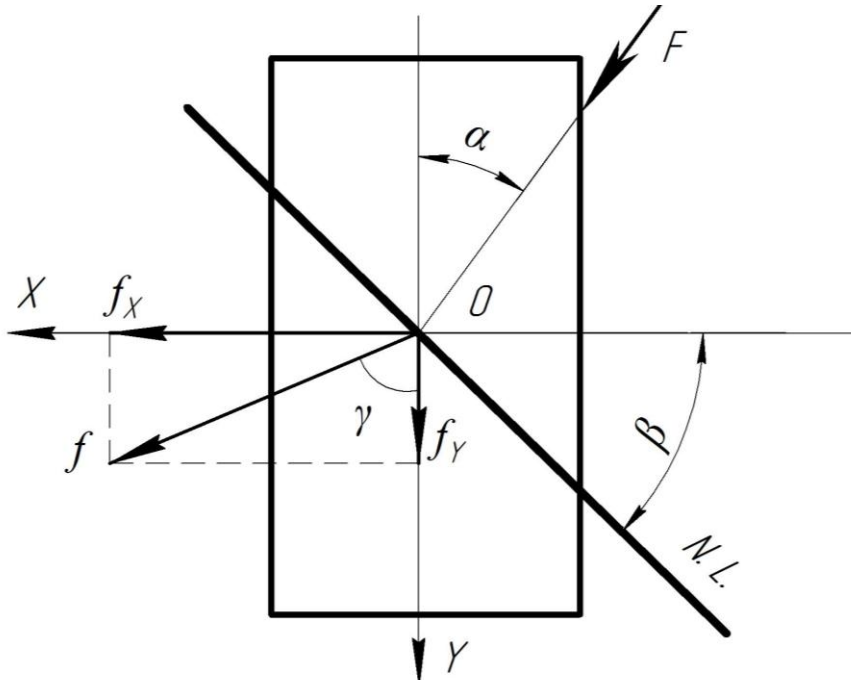


Figure 9.3

The oblique bending phenomenon is dangerous for cross-cut that are significantly different from the moments of inertia with respect to the major central crossing axes (e.g., the I-axis). Beams with such cross-cuts bend a little when bending in the plane of greatest rigidity; but even at slight angles of inclination of the external forces actions plane to the plane of greatest rigidity in beams, there is a significant deviation of the absolute bending line toward the least rigidity.

Task 11
Choosing the beam section at oblique bending deformation

For the given beam (Fig. for task 11, Table for task 11) choose a rectangular cross-cut, when relation of the beam height to its length is $h/b = 2$, and placing sides b and h parallel to axes X and Y most rationally. Evaluate the position of the neutral axis in the dangerous cross-cut of the beam. Draw the spatial diagram of distribution of normal stresses in the dangerous cross-cut. Determine the absolute displacement of the cross-cut pointed A at the figure, provived $a = 1$ m; material of the beam is steel St.3; $E = 2 \cdot 10^5$ MPa ; $[\sigma] = 160$ MPa .

Plan of solving the task:

1. Lay out given loads on axes X and Y. Write down the functions of shearing forces and bending moments in horizontal and vertical planes.

2. Draw the diagrams of shearing forces and bending moments in the horizontal and vertical planes.

3. Determine the dangerous cross-cut and its rational position in relation to the load.

4. Determine the cross-cut dimensions of the beam with the condition of strength under normal stresses.

5. Evaluate the position of the neutral line in the dangerous cross-cut of the beam and draw a spatial diagram of distribution of normal stresses in the crossing.

6. Determine horizontal, vertical and absolute bending deflections of the beam at crossing A.

Table for task 11

Nr	q, kN/m	F, kN	M, kNm	α° (degree)
1	5	12	12	0
2	6	10	10	90
3	8	8	8	180
4	10	6	6	270
5	12	5	5	360
6	5	12	12	0
7	6	10	10	90
8	8	8	8	180
9	10	6	6	270
0	12	5	5	360

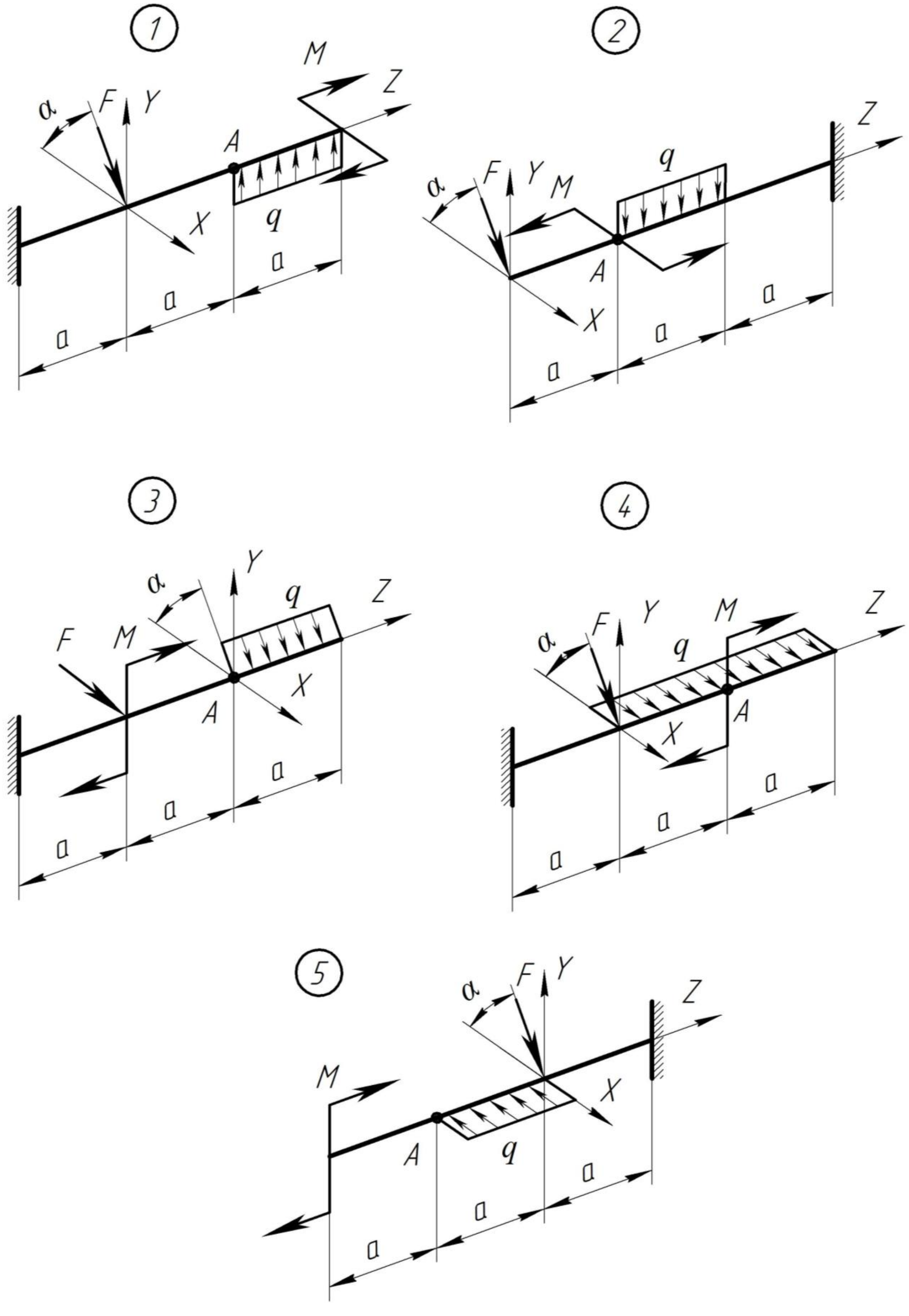


Figure for task 11

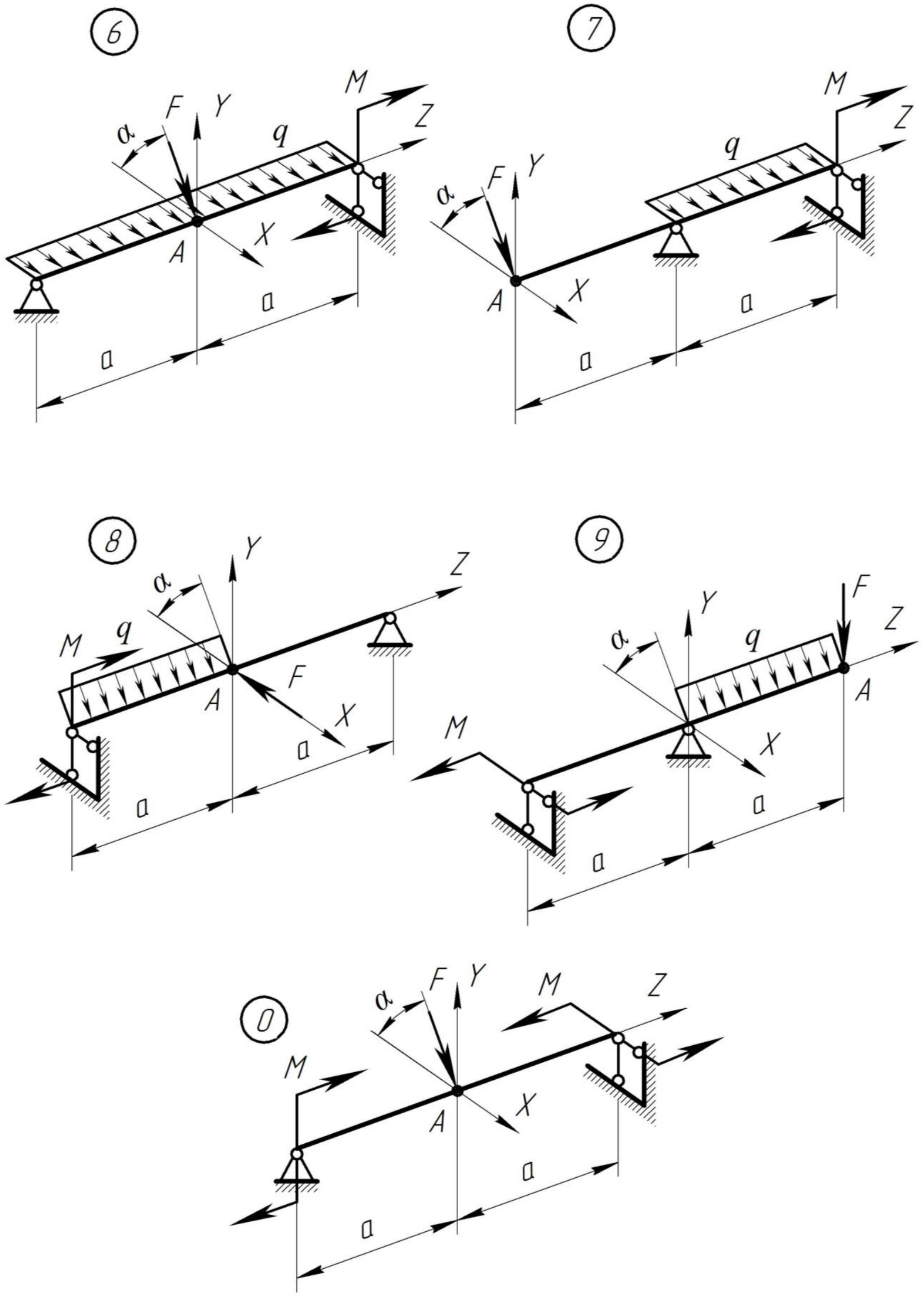


Figure for task 11 (continued)

Example of solving the task 11
Choosing the beam section at oblique bending deformation

For the given beam (Fig. 9.4 *a*) choose a rectangular cross-section, when relation of the beam height to its length is $h/b = 2$, and placing sides b and h parallel to axes X and Y most rationally. Evaluate the position of the neutral line in the dangerous cross-section of the beam. Draw the spatial diagram of distribution of normal stresses in the dangerous cross-section. Determine the absolute displacement of the cross-section pointed A at the figure, provived $a = 1$ m; material of the beam is steel St.3; $E = 2 \cdot 10^5$ MPa ; $[\sigma] = 160$ MPa .

Solution

Separate force F into vertical and horizontal components:

$$F_X = F \cdot \sin 45^\circ = 5 \cdot \sin 45^\circ = 3{,}54 \text{ kN};$$

$$F_Y = F \cdot \cos 45^\circ = 5 \cdot \cos 45^\circ = 3{,}54 \text{ kN}.$$

Load the beam in vertical (Fig. 9.4 *b*) and horizontal planes (Fig. 9.4 *e*). Draw the diagrams of shear (cutting) forces (Fig. 9.4 *c*, *f*) and bending moments (Fig. 9.4 *d*, *g*). These diagrams are drawn by characteristic points, values of shearing forces and bending moments on the segments bounds. Using this method, we do without making equations of shear (cutting) forces and bending moments. From the diagrams analysis, we find dangerous cross-section – this is cross-section B, where there are:

$$|M_X| = 12{,}08 \text{ kNm}; \qquad |M_Y| = 37{,}08 \text{ kNm}.$$

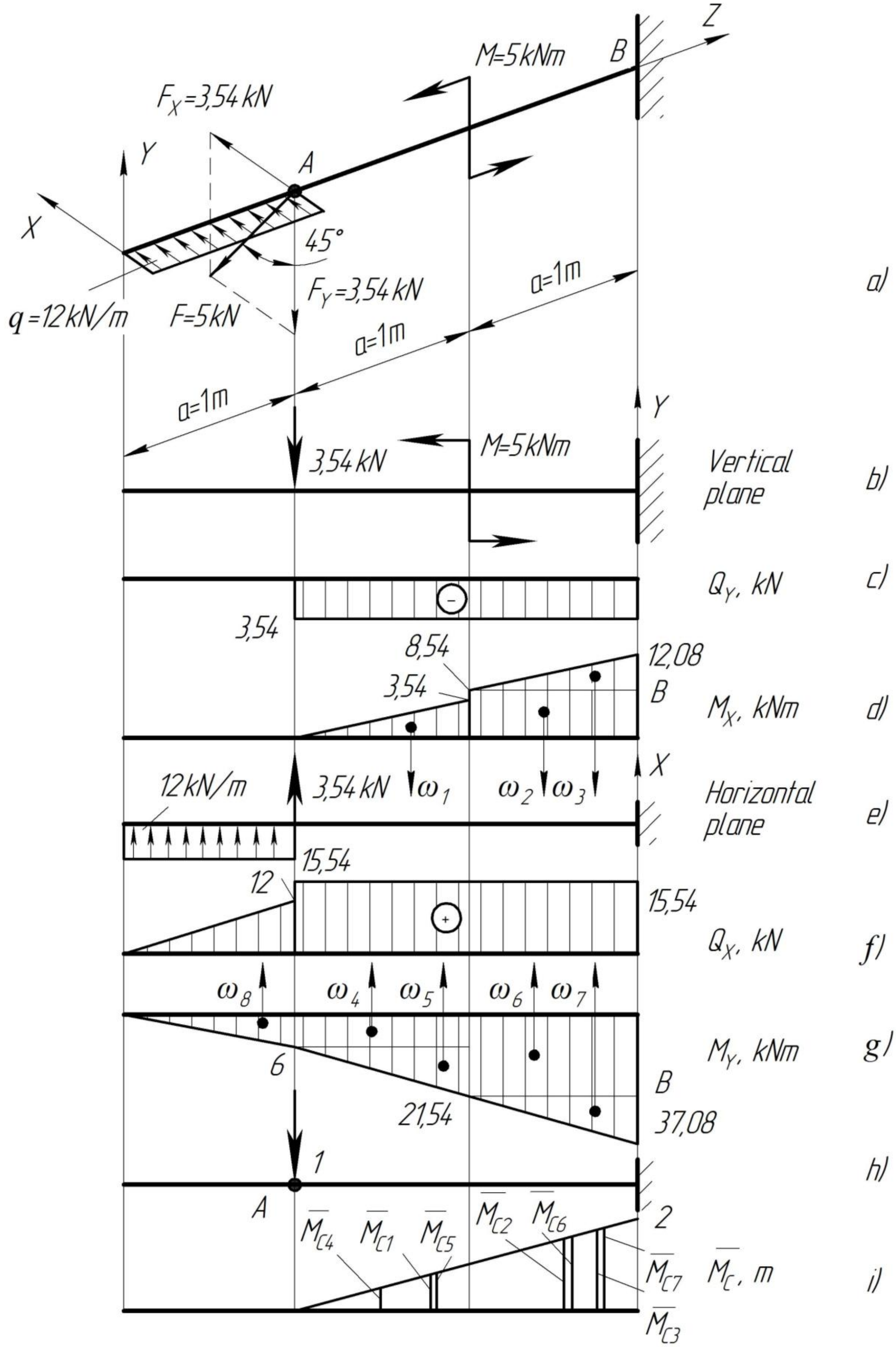
Rationally place the cross-section of the beam in relation to the external load (Fig. 9.5), using condition $|M_Y| > |M_X|$.

For the scheme (Fig. 9.5 *b*) the axial moments of the cross-section resistance are

$$W_X = \frac{b \cdot h^2}{6} = \frac{2}{3} b^3; \qquad W_Y = \frac{h \cdot b^2}{6} = \frac{1}{3} b^3.$$

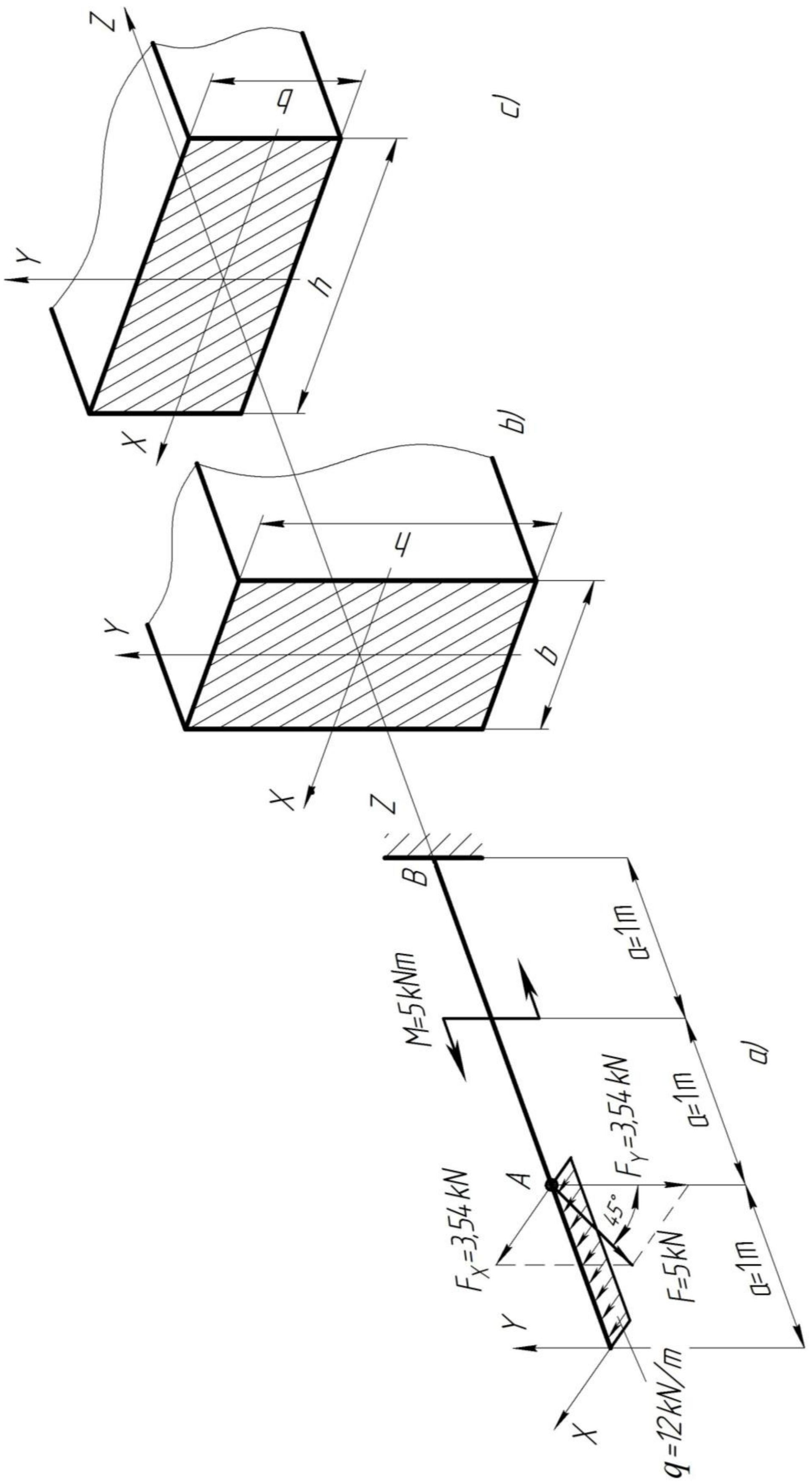
For the scheme (Fig. 9.5 *c*) the axial moments of the cross-section resistance are

$$W_X = \frac{h \cdot b^2}{6} = \frac{1}{3} b^3; \qquad W_Y = \frac{b \cdot h^2}{6} = \frac{2}{3} b^3.$$



Z
M=5kNm
B
F_X=3,54kN
Y
A
X
45°
F_Y=3,54kN
a=1m
q=12kN/m
F=5kN
a=1m
a)
a=1m
Y
3,54kN
M=5kNm
Vertical plane
b)
Q_Y, kN
c)
3,54
8,54
12,08
3,54
B
M_X, kNm
d)
12kN/m
3,54kN
ω_1
ω_2
ω_3
X
Horizontal plane
e)
15,54
12
15,54
Q_X, kN
f)
ω_8
ω_4
ω_5
ω_6
ω_7
M_Y, kNm
g)
6
B
21,54
37,08
1
h)
A
$\overline{M}_{C4}$
$\overline{M}_{C1}$
$\overline{M}_{C5}$
$\overline{M}_{C2}$
$\overline{M}_{C6}$
2
$\overline{M}_{C7}$
$\overline{M}_C$, m
i)
$\overline{M}_{C3}$

Figure 9.4



a)
b)
c)
M=5kNm
a=1m
a=1m
a=1m
F=5kN
F_X=3,54kN
F_Y=3,54kN
45°
q=12kN/m
A
B
X
Y
Z
b
h

Figure 9.5

From the condition of the rational placement of the beam cross-section in relation to axes X and Y and the dangerous cross-section (cross-section B), determined by the diagrams of bending moments (see Fig. 9.4 *d, g*), take the rational scheme drawn in Fig. 9.5 *c*, for which $W_Y > W_X$.

Write down the strength condition by normal stresses at oblique bending for the chosen cross-section (Fig. 9.5 *c*)

$$\sigma_{\max} = \frac{M_X}{W_X} + \frac{M_Y}{W_Y} \le [\sigma].$$

From the strength condition, determine the width of the rectangular cross-section

$$b = \sqrt[3]{\frac{3M_X + 1{,}5M_Y}{[\sigma]}} = \sqrt[3]{\frac{3 \cdot 12{,}08 + 1{,}5 \cdot 37{,}08}{160 \cdot 10^{-3}}} = 0{,}0831 \text{ m.}$$

Take $b = 85$ mm, correspondingly, the cross-cut height $h = 170$ mm.
Write down the equation of the neutral line in the dangerous beam cross-section

$$\frac{M_X}{I_X} \cdot y_0 + \frac{M_Y}{I_Y} \cdot x_0 = 0,$$

where

$$I_X = \frac{h \cdot b^3}{12} = \frac{17 \cdot 8{,}5^3}{12} \cdot 10^{-8} = 870 \cdot 10^{-8} \text{ m}^4;$$

$$I_Y = \frac{b \cdot h^3}{12} = \frac{8{,}5 \cdot 17^3}{12} \cdot 10^{-8} = 3480 \cdot 10^{-8} \text{ m}^4.$$

Substituting the values, obtain

$$\frac{12{,}08 \cdot 10^{-3}}{870 \cdot 10^{-8}} \cdot y_0 + \frac{37{,}08 \cdot 10^{-3}}{3480 \cdot 10^{-8}} \cdot x_0 = 0;$$

then

$$y_0 = -0{,}767\, x_0; \qquad tg\beta = \frac{y_0}{x_0} = -0{,}767; \qquad \beta = -37{,}5^\circ.$$

In Fig. 9.6 the rational placement of the beam cross-section relative to the load and the location of the neutral line are shown.

To draw the spatial diagram of stresses distribution on the contour of the dangerous cross-section, normal stresses in the junction points of this cross-section (see Fig. 9.6) are determined by formula

$$\sigma_i = -\left(\frac{M_X}{I_X} \cdot y_i + \frac{M_Y}{I_Y} \cdot x_i \right),$$

where x_i, y_i are coordinates of the junction section points:

$x_1 = 8{,}5 \cdot 10^{-2}$ m; $y_1 = 4{,}25 \cdot 10^{-2}$ m;

$x_2 = 8{,}5 \cdot 10^{-2}$ m; $y_2 = -4{,}25 \cdot 10^{-2}$ m;

$x_3 = -8{,}5 \cdot 10^{-2}$ m; $y_3 = -4{,}25 \cdot 10^{-2}$ m;

$x_4 = -8{,}5 \cdot 10^{-2}$ m; $y_4 = 4{,}25 \cdot 10^{-2}$ m.

Determine the stresses at the junction section points:

$$\sigma_1 = -\left(\frac{12{,}08 \cdot 10^{-3} \cdot 4{,}25 \cdot 10^{-2}}{870 \cdot 10^{-8}} + \frac{37{,}08 \cdot 10^{-3} \cdot 8{,}5 \cdot 10^{-2}}{3480 \cdot 10^{-8}} \right) = -150 \text{ MPa};$$

$$\sigma_2 = -(-59 + 91) = -32 \text{ MPa};$$

$$\sigma_3 = -(-59 - 91) = 150 \text{ MPa};$$

$$\sigma_4 = -(59 - 91) = 32 \text{ MPa}.$$

By the obtained results, draw the diagram of distribution of normal stresses at the cross-section (see Fig. 9.6).

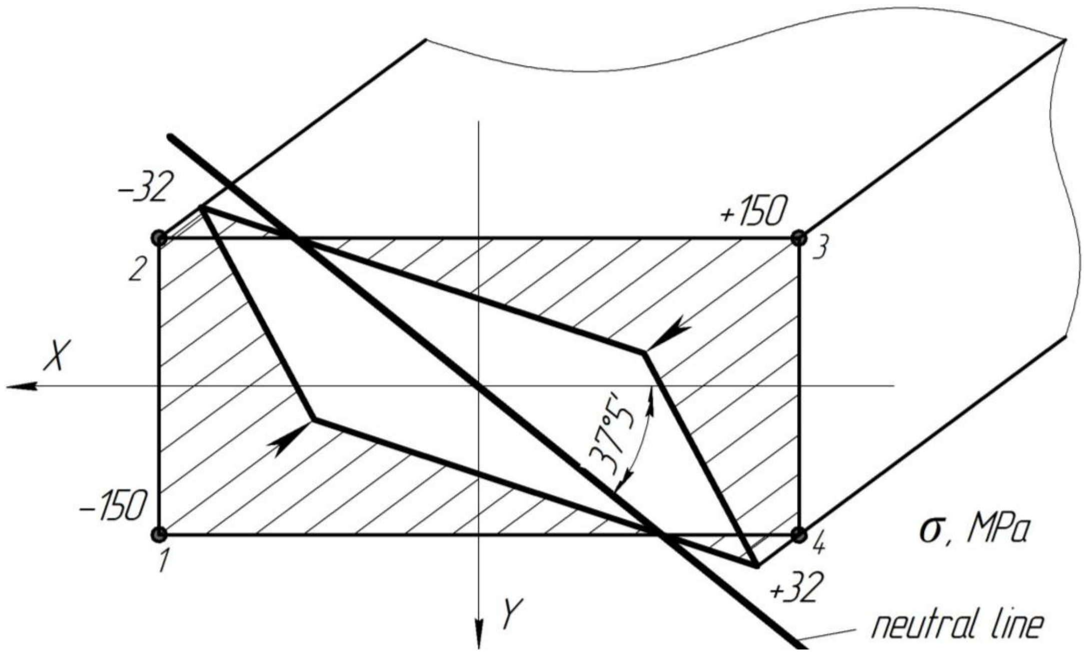


Figure 9.6

Determine deflexion of the beam in section A using graphic-analytical method of solving Mohr integral.

Vertical deflexion in point A determine by formula

$$f_A^{ver} = \sum_1^3 \frac{\omega_i \cdot \overline{M}_{Ci}}{E \cdot I_Y};$$

where ω_i is the area of the diagram of bending moments from external loads of the *i*-segment (Fig. 9.5 *d*);

$\overline{M}_{Ci}$ is the value of the bending moment from the singular load (force) (Fig. 9.5 *h*), that lies under the gravity (weight) center of *i*-diagram (Fig. 9.5 *i*).

Respectively,

$$f_A^{ver} = \frac{\omega_1 \cdot \overline{M}_{C1} + \omega_2 \cdot \overline{M}_{C2} + \omega_3 \cdot \overline{M}_{C2}}{E \cdot I_Y},$$

where $\omega_1 = -\frac{1}{2} \cdot 3{,}54 \cdot 1 = -1{,}77 \text{ kNm}^2$; $\quad \overline{M}_{C1} = -0{,}667 \text{ m}$;

$\omega_2 = -8{,}54 \cdot 1 = -8{,}54 \text{ kNm}^2$; $\quad \overline{M}_{C2} = -1{,}5 \text{ m}$;

$\omega_3 = -\frac{1}{2} \cdot (12{,}08 - 8{,}54) \cdot 1 = -1{,}77 \text{ kNm}^2$; $\quad \overline{M}_{C3} = -1{,}667 \text{ m}$.

Having substituted the values, obtain

$$f_A^{ver} = \frac{1{,}77 \cdot 0{,}667 + 8{,}54 \cdot 1{,}5 + 1{,}77 \cdot 1{,}667}{2 \cdot 10^5 \cdot 870 \cdot 10^{-8}} \cdot 10^{-3} = 0{,}0097 \text{ m} = 9{,}7 \text{ m}.$$

Horizontal deflection in point A work out by formula

$$f_A^{hor} = \sum_4^7 \frac{\omega_i \cdot \overline{M}_{Ci}}{E \cdot I_X};$$

or

$$f_A^{hor} = \frac{\omega_4 \cdot \overline{M}_{C_4} + \omega_5 \cdot \overline{M}_{C_5} + \omega_6 \cdot \overline{M}_{C_6} + \omega_7 \cdot \overline{M}_{C_7}}{E \cdot I_X};$$

where $\omega_4 = 6 \cdot 1 = 6 \text{ kNm}^2$; $\overline{M}_{C4} = -0{,}5 \text{ m}$;

$$\omega_5 = \frac{1}{2} \cdot (21{,}54 - 6) \cdot 1 = 7{,}77 \text{ kNm}^2; \qquad \overline{M}_{C5} = -0{,}667 \text{ m};$$

$$\omega_6 = 21{,}54 \cdot 1 = 21{,}54 \text{ kNm}^2; \qquad \overline{M}_{C6} = -1{,}5 \text{ m};$$

$$\omega_7 = \frac{1}{2} \cdot (37{,}08 - 21{,}54) \cdot 1 = 7{,}77 \text{ kNm}^2; \qquad \overline{M}_{C7} = -1{,}667 \text{ m}.$$

Substituting the values, obtain

$$f_A^{hor} = \frac{6 \cdot (-0{,}5) + 7{,}77(-0{,}667) + 21{,}54(-1{,}5) + 7{,}77(-1{,}667)}{2 \cdot 10^5 \cdot 3480 \cdot 10^{-8}} \cdot 10^{-3} = -7{,}7 \text{ mm.}$$

Absolute section of deflextion in point A equals

$$f_A = \sqrt{\left(f_A^{ver}\right)^2 + \left(f_A^{hor}\right)^2} = \sqrt{9{,}7^2 + (-7{,}7)^2} = 12{,}4 \text{ mm}.$$

10. JOINT ACTION OF BENDING WITH TORSION

Joint action of bending with torsion is a type of resistance to combined stress in which external forces acting on the beam cause the following internal force factors: ***torque, bending moments and shear*** **(*cutting*)** ***forces***.

Under the action of bending and torsion in the cross section of the beam there are normal stresses from the bending moment in two planes and tangential shear stresses from torsion and shear forces.

Most shafts (straight bars of round or annular cross-section) undergo simultaneous bending and torsion deformations.

When calculating the shafts, the torque and bending moments are taken into account. Shear (cutting) forces are not considered, as the corresponding to them tangential (shear) stresses are relatively small.

With the combined action of bending and torsion, the material element in the dangerous section is in a plane stress state (Fig. 10.1).

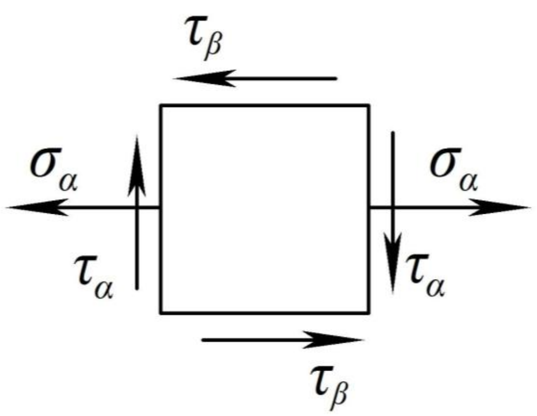


Figure 10.1

The maximum normal and tangential (shear) stresses for round shafts are determined by formula

$$|\sigma_\alpha| = \frac{M_{BN}}{W_0} = \frac{32\,M_{BN}}{\pi \cdot d^3};$$

$$|\tau_\alpha| = \frac{M_{TR}}{W_P} = \frac{16\,M_{TR}}{\pi \cdot d^3},$$

where $W_P = W_0/2$;

$\tau_\alpha = -\tau_\beta$, law of parity of tangential (shear) stresses.

Normal and tangential stresses reach the greatest value on a shaft surface.

To determine the bending moment, the bending of the shaft in two mutually perpendicular planes (vertical and horizontal) is considered. Diagrams of bending moments in two planes and total are drawn. The values of bending moments in the characteristic sections are reduced to the total (equivalent) by the formula

$$M_{BN} = \sqrt{M_Z^2 + M_Y^2}.$$

Dangerous sections of the shaft are determined by comparing the plots of the total bending moments and torque. Sections are dangerous, where M_{BN} and M_{TR} simultaneously reach the highest values.

Under the simultaneous action of normal and tangential (shear) stresses, the strength of the material is evaluated by one of the theories of strength.

Theories of strength are used for their intended purpose, i.e. the first and second theory are used for brittle materials, the third and fourth for the plastic ones; Mohr and Pisarenko-Lebedev theories are used for materials with different yield strengths during tension and compression.

The calculation of shaft strength at resistance to combined stress is carried out by the reduced (equivalent, calculation) ***moment*** M_R $\left(M_{eqv}\right)$. ***It is determined depending on the accepted theory of strength:***

– according to the third theory of strength (maximum tangential stresses)

$$M_R = \sqrt{M_{BN}^2 + M_{TR}^2};$$

– according to the third theory of strength (energetic)

$$M_R = \sqrt{M_{BN}^2 + 0{,}75\,M_{TR}^2}.$$

Condition of strength under the joint action of bending with torsion

$$\sigma_{eqv} = \frac{M_R}{W_0} = \frac{32\ M_R}{\pi \cdot d^3} \le [\sigma],$$

where σ_{eqv} is the equivalent (calculation) normal stress;

W_0 is the axial moment of the beam resistance section for the round cross-section $W_0 = \pi \cdot d^3 / 32$.

Task 12
Calculation of the shaft for bending with torsion

Steel transmission shaft (Fig. for task 12, Table for task 12) rotates at speed n rpm and transmits the power through two driven pulleys of belt transmission given in table 12. Diameters of the pulleys: $D_1 = 60$ cm, $D_2 = 40$ cm, $D_3 = 30$ cm; distance $a = 100$ cm; material is steel 45, $[\sigma] = 100$ MPa. Find the diameter of the shaft from the condition of strength.

Plan of solving the task:

1. Determine the power on the pulley (from the condition of power balance) where it is not specified.
2. Determine the torques on each pulley, the torques on the shaft sections and draw a diagram of the torques.
3. Determine the pressure transmitted by each pulley to the shaft, assuming that the tension of the leading branch of the belt is twice more than the tension of the driven, $T_i = 2t_i$.
4. Determine the values of the components of the pressure forces acting in the horizontal and vertical planes.
5. Draw the diagrams of bending moments in the horizontal and vertical planes.
6. Determine the total bending moments in the characteristic cross-sections of the shaft. Draw a diagram of the total bending moments.
7. Determine the calculation moment using the third theory of strength.
8. Determine the diameter of the shaft from the condition of strength.

Table for task 12

Nr	a_1° (degree)	a_2° (degree)	a_3° (degree)	n, rpm	P_1, kW	P_2, kW	P_3, kW
1	0	270	360	150	-	10	20
2	90	0	180	100	10	-	20
3	180	270	0	200	10	20	-
4	270	360	0	300	-	30	40
5	360	0	90	400	30	-	40
6	0	90	180	500	30	40	-
7	90	180	270	600	-	50	60
8	180	270	360	700	50	-	60
9	270	360	0	800	50	60	-
0	90	0	180	900	-	90	50

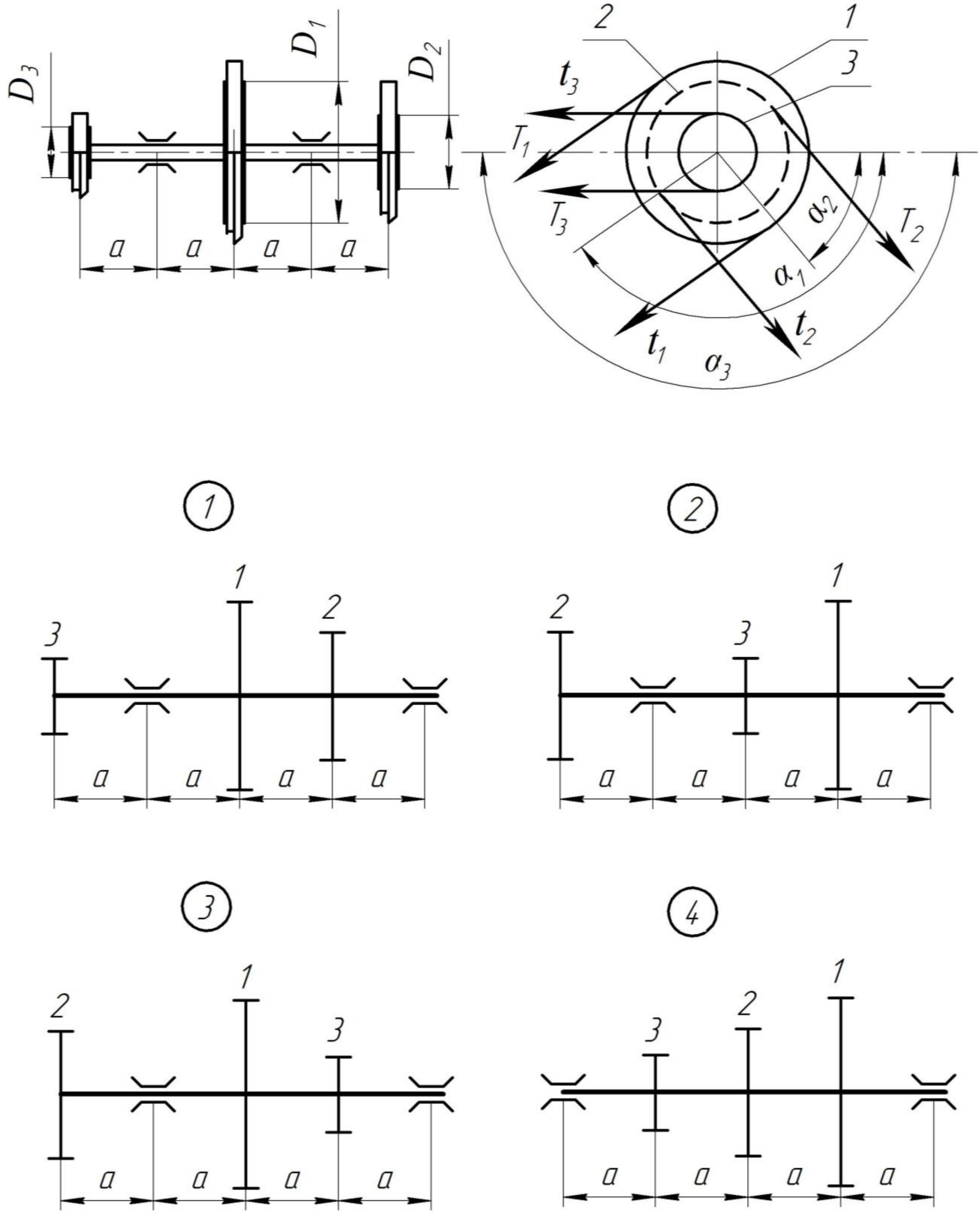


Figure for task 12

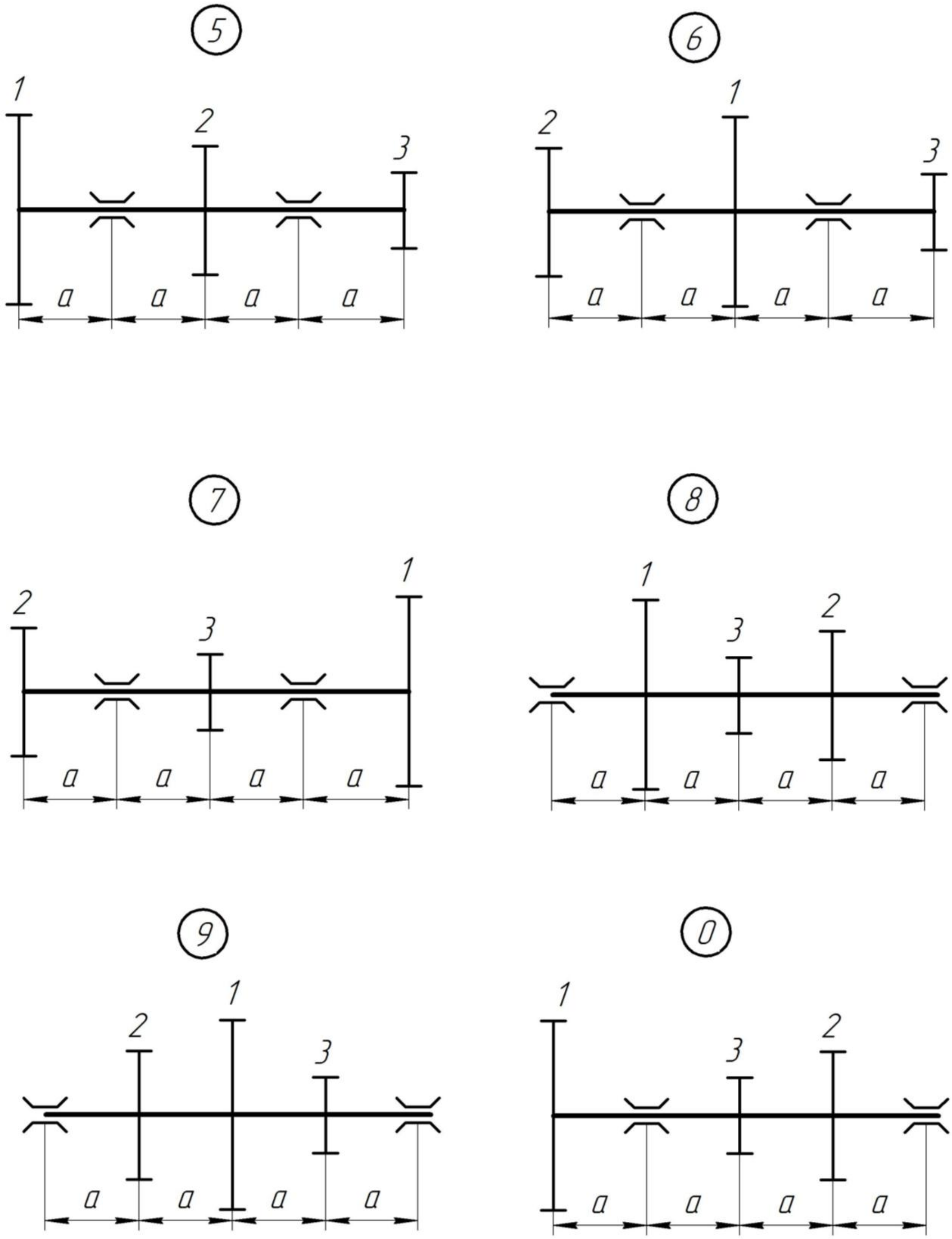


Figure for task 12 (continued)

Example of solving the task 12
Calculation of the shaft for bending with torsion

Steel transmission shaft rotates at speed $n = 300$ rpm and transmits the power through two driven pulleys of belt transmission $P_1 = 70$ kW and $P_3 = 40$ kW (Fig. 10.2 *a*). Diameters of the pulleys are: $D_1 = 60$ cm, $D_2 = 40$ cm, $D_3 = 30$ cm. Inclination angle of pulleys are $\alpha_1 = 30°$, $\alpha_2 = 240°$, $\alpha_3 = 180°$ (on Fig. 10.2 the angles are indicated from the axe Y), distance $a = 100$ cm. The material of the shaft is steel 45; $[\sigma] = 100$ MPa. Find the diameter of the shaft from the condition of strength.

Solution

From the balance of power determine the power on the pulley transmitted by the driven pulley

$$P_2 = P_1 + P_3 = 70 + 40 = 110 \text{ kW.}$$

The values of the moments transmitted by the pulleys determine by formula

$$M_i = P_i / \omega ,$$

where ω is the angular speed of the shaft, determine it by formula

$$\omega = \frac{\pi \cdot n}{30} = \frac{\pi \cdot 300}{30} = 31{,}4 \text{ s}^{-1} .$$

Torques on pulleys:

$$M_1 = 70/31{,}4 = 2{,}23 \text{ kNm} ;$$

$$M_2 = 110/31{,}4 = 3{,}50 \text{ kNm} ;$$

$$M_3 = 40/31{,}4 = 1{,}27 \text{ kNm} .$$

Using the sections method, draw a diagram of torques M_{TR} (Fig. 10.2 *b*).

Determine the tensile forces of the belt drives by the formula

$$t_i = 2M_i / D_i .$$

Respectively

$$t_1 = \frac{2 \cdot 2{,}23}{0{,}6} = 7{,}43 \text{ kN} ; \quad t_2 = \frac{2 \cdot 3{,}50}{0{,}4} = 17{,}5 \text{ kN} ; \quad t_3 = \frac{2 \cdot 1{,}27}{0{,}3} = 8{,}47 \text{ kN} .$$

The pressure force on the shaft at the pulley fits is determined by the formula

$$F_i = 3t_i .$$

That is

$$F_1 = 3 \cdot 7{,}43 = 22{,}3 \text{ kN};$$

$$F_2 = 3 \cdot 17{,}5 = 52{,}4 \text{ kN};$$

$$F_3 = 3 \cdot 8{,}47 = 25{,}4 \text{ kN}.$$

Resolve the pressure forces into vertical and horizontal components:

$$F_{1Z} = -F_1 \cdot \sin 30° = -22{,}3 \cdot \sin 30° = -11{,}2 \text{ kN};$$

$$F_{1Y} = F_1 \cdot \cos 30° = 22{,}3 \cdot \cos 30° = 19{,}3 \text{ kN};$$

$$F_{2Z} = F_2 \cdot \sin 60° = 52{,}4 \cdot \sin 60° = 45{,}4 \text{ kN};$$

$$F_{2Y} = -F_2 \cdot \cos 60° = -52{,}4 \cdot \cos 60° = -26{,}2 \text{ kN};$$

$$F_{3Z} = 0;$$

$$F_{3Y} = -F_3 = 25{,}4 \text{ kN}.$$

Consider the vertical plane (Fig. 10.2 *c*).

Vertical components of the reaction of supports A and B are determined from the equilibrium equations:

$$\begin{cases} M_B = 0; \qquad A_Z \cdot 4 - F_{2Z} \cdot 3 + F_{1Z} \cdot 2 = 0; \\ M_A = 0; \qquad -B_Z \cdot 4 - F_{1Z} \cdot 2 + F_{2Z} \cdot 1 = 0; \end{cases}$$

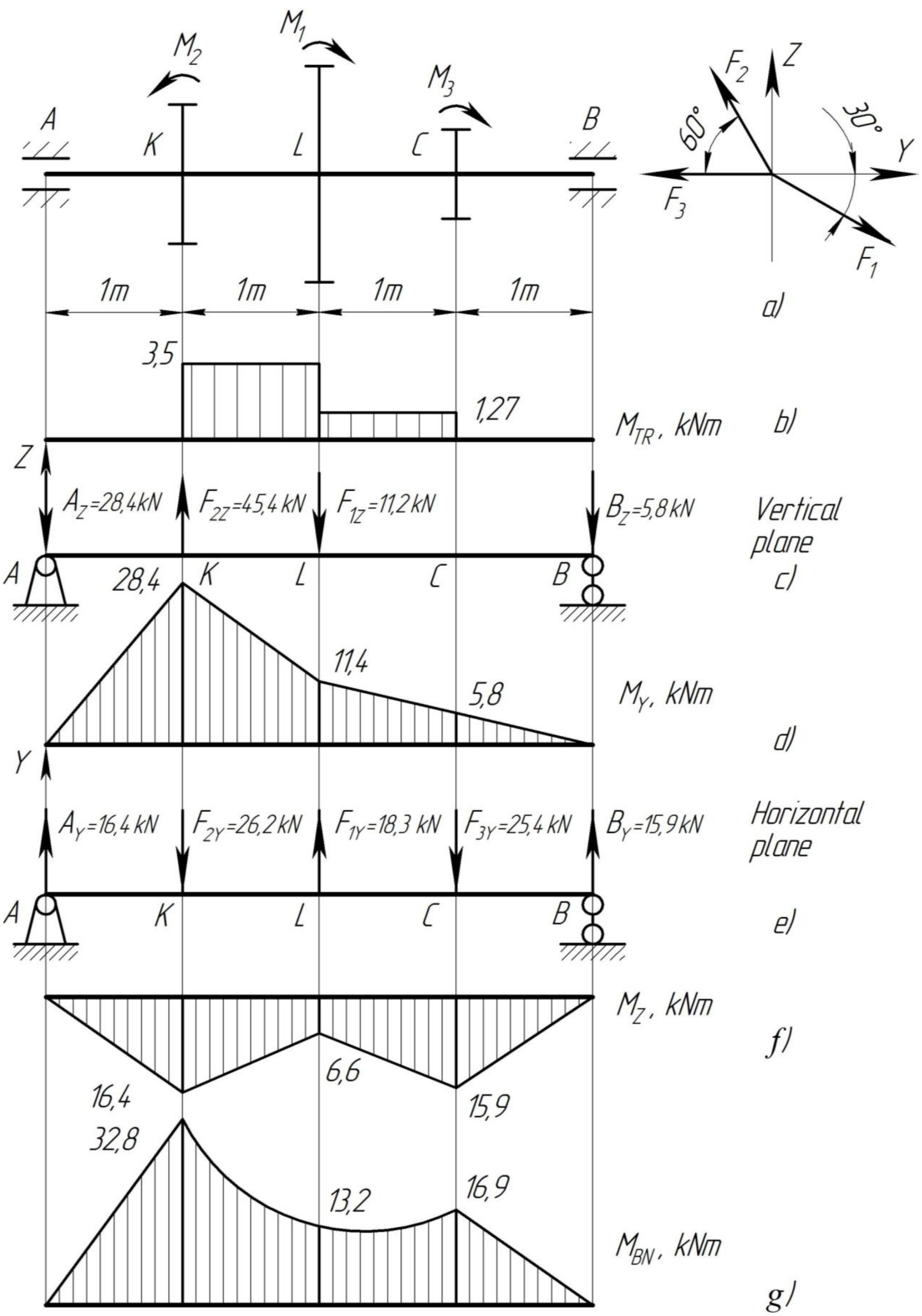
then

$$A_Z = \frac{45{,}4 \cdot 3 - 11{,}2 \cdot 2}{4} = 28{,}4 \text{ kN};$$

$$B_Z = \frac{45{,}4 \cdot 1 - 11{,}2 \cdot 2}{4} = 5{,}8 \text{ kN}.$$

Verification: $\sum Z = -A_Z + F_{2Z} - F_{1Z} - B_Z =$

$$= -28{,}4 + 45{,}4 - 11{,}2 - 5{,}8 = 45{,}4 - 45{,}4 = 0.$$



M1
M2
M3
A
K
L
C
B
Z
F2
60°
30°
Y
F3
F1
1m
1m
1m
1m
a)
3,5
1,27
MTR, kNm
b)
Z
AZ=28,4kN
F2Z=45,4kN
F1Z=11,2kN
BZ=5,8kN
Vertical plane
A
K
L
C
B
c)
28,4
11,4
5,8
MY, kNm
d)
Y
AY=16,4kN
F2Y=26,2kN
F1Y=18,3kN
F3Y=25,4kN
BY=15,9kN
Horizontal plane
A
K
L
C
B
e)
MZ, kNm
f)
6,6
16,4
15,9
32,8
13,2
16,9
MBN, kNm
g)

Figure 10.2

Determine the bending moments in the characteristic points of the vertical plane:

$$M_Y^A = M_Y^B = 0;$$

$$M_Y^K = -A_Z \cdot 1 = -28{,}4 \cdot 1 = -28{,}4 \text{ kNm};$$

$$M_Y^L = -A_Z \cdot 2 + F_{2Z} \cdot 1 = -28{,}4 \cdot 2 + 45{,}4 \cdot 1 = -11{,}4 \text{ kNm};$$

$$M_Y^C = -B_Z \cdot 1 = -5{,}8 \cdot 1 = -5{,}8 \text{ kNm}.$$

Draw the diagrams of bending moments in the vertical plane (Fig. 10.2 *d*).

Consider a horizontal plane (Fig. 10.2 *e*).

Determine the supporting reactions:

$$\sum M_B = 0; \quad -A_Y \cdot 4 + F_{2Y} \cdot 3 - F_{1Y} \cdot 2 + F_{3Y} \cdot 1 = 0; \quad A_Y = 16{,}4 \text{ kN};$$

$$\sum M_A = 0; \quad B_Y \cdot 4 - F_{3Y} \cdot 3 + F_{1Y} \cdot 2 - F_{2Y} \cdot 1 = 0; \quad B_Y = 15{,}9 \text{ kN}.$$

Verification: $\sum Y = A_Y - F_{2Y} + F_{1Y} - F_{3Y} + B_Y =$

$$= 16{,}4 - 26{,}2 + 19{,}3 - 25{,}4 + 15{,}9 = 51{,}6 - 51{,}6 = 0;$$

that is the supporting reactions are determined correctly.

Determine the bending moments in the characteristic points of the horizontal plane:

$$M_Z^A = M_Z^B = 0;$$

$$M_Z^K = A_Y \cdot 1 = 16{,}4 \cdot 1 = 16{,}4 \text{ kNm};$$

$$M_Z^L = A_Y \cdot 2 - F_{2Y} \cdot 1 = 16{,}4 \cdot 2 - 26{,}2 \cdot 1 = 6{,}6 \text{ kNm};$$

$$M_Z^C = B_Y \cdot 1 = 15{,}9 \cdot 1 = 15{,}9 \text{ kNm}.$$

Draw a diagram of bending moments in horizontal plane (Fig. 10.2 *f*).

Determine the total bending moments in the characteristic cross-sections of the shaft by formula

$$M_{BN} = \sqrt{M_Y^2 + M_Z^2}.$$

Find:

$$M_{BN}^{A} = M_{BN}^{B} = 0;$$

$$M_{BN}^{K} = \sqrt{28{,}4^2 + 16{,}4^2} = 32{,}8 \text{ kNm};$$

$$M_{BN}^{L} = \sqrt{11{,}4^2 + 6{,}6^2} = 13{,}2 \text{ kNm};$$

$$M_{BN}^{C} = \sqrt{5{,}8^2 + 15{,}9^2} = 16{,}9 \text{ kNm}.$$

Draw a diagram of the total bending moments (Fig. 10.2 *g*).

From the analysis of the diagrams M_{TR} (see Fig. 10.2 *b*) and M_{BN} (see Fig. 10.2 *g*) find the dangerous section; it is section K in which

$$M_{BN} = 32{,}8 \text{ kNm}; \qquad M_{TR} = 3{,}5 \text{ kNm}.$$

Determine the calculation moment using the third theory of strength.

$$M_R = \sqrt{M_{BN}^2 + M_{TR}^2} = \sqrt{32{,}8^2 + 3{,}5^2} = 33 \text{ kNm}.$$

From the condition of strength under joint action of bending and torsion

$$\sigma_{\max} = \frac{M_R}{W_0} = \frac{32\,M_R}{\pi \cdot d^3} \le [\sigma],$$

determine the diameter of the shaft

$$d \ge \sqrt[3]{\frac{32\,M_R}{\pi \cdot [\sigma]}} = \sqrt[3]{\frac{32 \cdot 33{,}0}{\pi \cdot 100 \cdot 10^3}} = 149 \cdot 10^{-3} \text{ m},$$

accept $d = 150$ mm.

11. STABILITY OF CENTRALLY-COMPRESSED RODS

The conditions of strength and rigidity are supplemented by the condition of stability, which provides preserving of the original form of equilibrium of the structure or its individual elements under the action of a given load.

Loads at which stability is lost are called **critical** and the corresponding states are called ***critical states***.

The danger of loss of stability arises for thin-walled structures such as flexible rods, long compressed rods, plates and shells.

The critical force is the largest value of the compressive force applied centrally, to which the rectilinear form of equilibrium of the rod is stable. The bend caused by the loss of stability of the rectilinear shaped rod is called ***the longitudinal bend***.

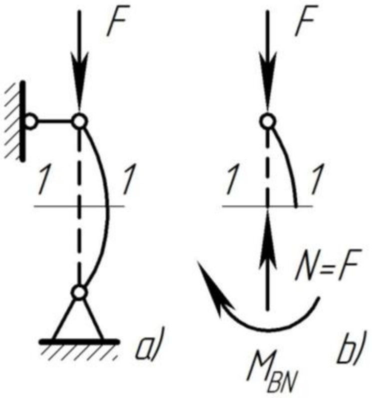


Figure 11.1

Due to the curvature of the axis in the cross-sections of the rod there are two internal force factors – the ***longitudinal force*** $N = F$ and bending moment M_{BN} (Fig. 11.1). Therefore, the curved rod undergoes both deformations of central compression and transverse bending.

Determination of critical loads is an important part of the calculation of structure and makes it possible to avoid loss of stability by introducing the appropriate stability margin coefficient.

$$n_{cm} = \frac{F_{CR}}{F}.$$

To ensure stability, it is necessary that the compressive force F acting on the rod is less than critical F_{CR}. The rod stability is sufficient if $n_{cm} > 1$. The value of the coefficient of stability depends on the purpose of the rod and its material. For steels $n_{cm} = 1{,}8 \ldots 3$; for cast iron $n_{cm} = 5 \ldots 5{,}5$; for wood $n_{cm} = 2{,}8 \ldots 3{,}2$.

The equilibrium of absolutely rigid solid can be stable, indifferent and unstable. It can be similarly referred to a deformed solid.

The long rod under the action of axial compressive load undergoes **three forms of equilibrium: *stable, indifferent, unstable*.**

The compressed rod is in the state of ***stable*** equilibrium (Fig. 11.2 *a*) if the compressive force F does not exceed the critical value F_{CR}. That is, if the rod is slightly bent by some transverse load and then when this load is removed, the rod will align again and take the initial position.

The equilibrium form of a compressed rod is ***indifferent*** (Fig. 11.2 *b*) if the compressive force reaches a certain value equal to the critical force. With a slight deviation from the initial position, under the action of shear force, the rod does not return back.

When the value of the compressive force exceeds the critical, the rectilinear form of equilibrium of the rod becomes ***unstable***, the rod loses its original shape (Fig. 11.2 *c*).

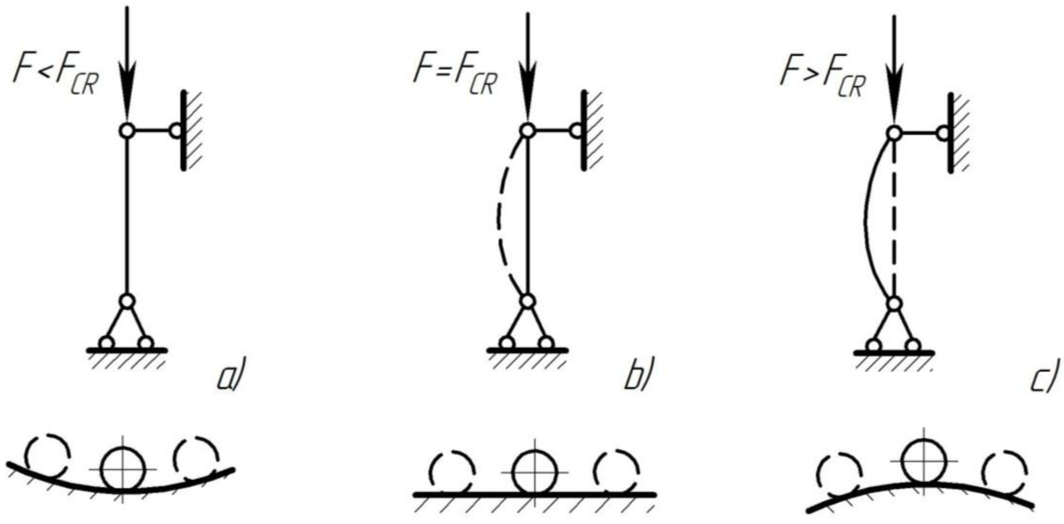


Figure 11.2

The loss of stability of the rod may occur even when the stress under the action of a critical force has not reached the limit of proportionality.

The smallest value of the compressive force at which the rod loses the ability to keep a rectilinear shape is called critical and is indicated F_{CR} .

The task of determining the magnitude of the critical force was first solved by the academician of the St. Petersburg Academy of Sciences Leonard Euler in 1744. **Euler's formula**

$$F_{CR} = \frac{\pi^2 E \cdot I_{\min}}{(\mu \cdot l)^2},$$

where E is the elasticity modulus of the first kind;

$I_{\min}$ is the minimum axial moment of inertia of the rod cross-section;

μ is the coefficient of reduction of length depends on the method of fixing the ends of the rod;

l is the length of the rod.

Ways of fixing the ends of the rod are shown in Fig. 11.3.

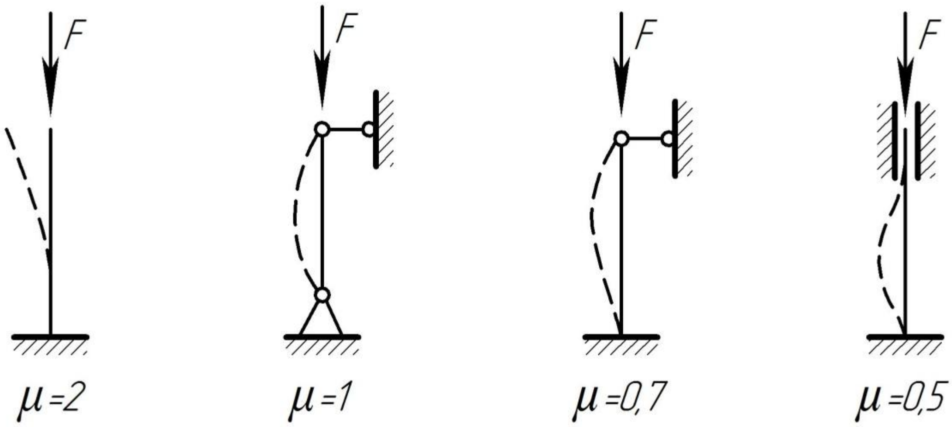


Figure 11.3

Use of Euler's formula. Yasinsky's formula

Euler's formula is obtained from the differential equation of the curved axis of the rod with hinged ends. The Euler's formula is derived basing on Hooke's law, which is valid until the stress in the material does not exceed the limit of proportionality.

$$\sigma_{CR} \le \sigma_{pr}; \qquad \sigma_{CR} = \frac{\pi^2 \cdot E \cdot I_{\min}}{(\mu \cdot l)^2 \cdot A} = \frac{\pi^2 \cdot E}{\lambda_{\max}^2} \le \sigma_{pr},$$

where A is the area of the rod cross-section;

$\lambda_{\max}$ is maximum flexibility of the rod; depends on the geometry of the rod, ways of fixation of its ends. It is determined by formula

$$\lambda_{\max} = \frac{\mu \cdot l}{i_{\min}},$$

where $i_{\min}$ is minimum radius of inertia of the rod cross-section, depends on geometric parameters. It is determined by formula

$$i_{\min} = \sqrt{I_{\min} / A}.$$

Euler's formula is used for flexibilities that are greater than the ultimate flexibility of the rod λ_0 which depends on the material of the rod and is determined by formula

$$\lambda_0 = \sqrt{\frac{\pi^2 \cdot E}{\sigma_{pr}}}.$$

Euler's formula is used when the flexibility of the rod is greater or equal to the ultimate flexibility of the material from which it is made

$$\lambda_{\max} \geq \lambda_0 .$$

As the example, the ultimate flexibility of the steel St.3 can be determined, for which $\sigma_{pr} = 200$ MPa, modulus of elasticity $E = 2 \cdot 10^5$ MPa.

$$\lambda_0 = \sqrt{3{,}14^2 \cdot 2 \cdot 10^5 \cdot 10^6 / \left(200 \cdot 10^6\right)} = 100 .$$

For low-carbon steel rods, Euler's formula is used when their flexibility $\lambda \geq 100$. Similarly the ultimate flexibility of other materials is determined. In particular, for cast iron $\lambda_0 = 80$; for wood $\lambda_0 = 110$.

If the flexibility of the rods is less than the ultimate one, in particular, for steels $\lambda = 40 \ldots 100$, **Yasinsky's** empirical **formula *is used to determine stresses***

$$\sigma_{CR} = a - b \cdot \lambda_{\max} ,$$

where a, b are coefficients that depend on the material of the rod. For steel St.3 these values are equal

$$a = 310 \text{ MPa} ; \qquad b = 1{,}14 \text{ MPa} .$$

If flexibility is $\lambda < 40$, rods can be calculated for strength under simple compression without taking into account the danger of the longitudinal bending, that is by formulas

$$\sigma = F/A ; \qquad [\sigma] = \sigma_{ye}/n_{cm} = \sigma_{CR}/n_{cm} .$$

The graph of the dependence of critical stresses on flexibility for rods made of low-carbon steel is shown in Fig. 11.4.

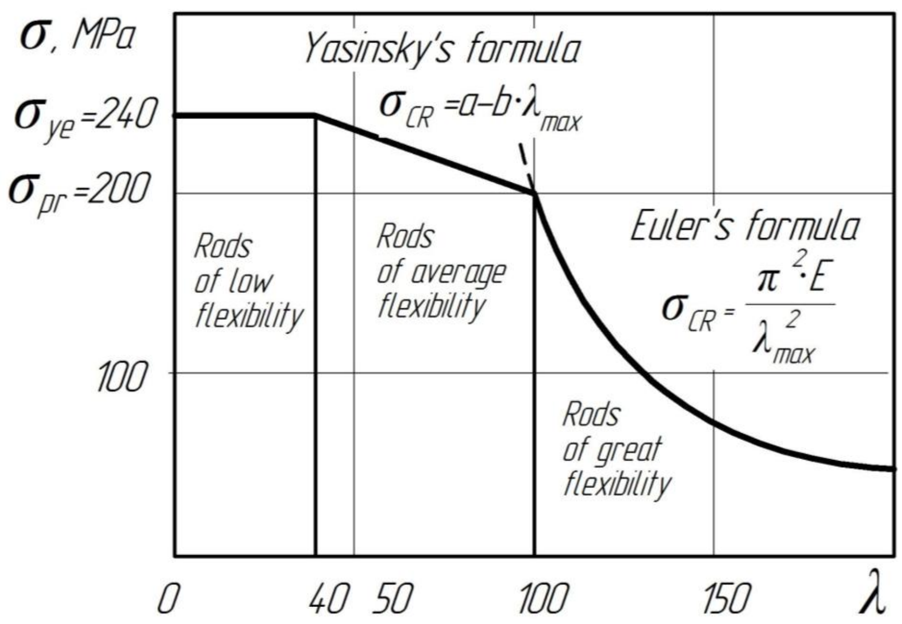


Figure 11.4

Note that:

1) at small values of λ the critical stress is equal to the yield strength $\sigma_{CR} = \sigma_{ye}$;

2) at average values of λ the critical stress is less than the yield strength but more than the proportionality limit $\sigma_{pr} < \sigma_{CR} < \sigma_{ye}$;

3) at large values of λ the critical stress is less than the proportionality limit $\sigma_{CR} < \sigma_{pr}$.

The value of the critical force that can be applied to the rod is obtained in the following sequence. Determine:

a) ultimate flexibility λ_0;

b) maximum actual flexibility of the rod $\lambda_{\max}$.

c) with λ_0 and $\lambda_{\max}$, to determine the critical stresses, use one of the following formulas:

- when $\lambda_{\max} \geq \lambda_0$ Euler's formula, $\sigma_{CR} = \pi^2 \cdot E / \lambda_{\max}^2$;
- when $\lambda_{cp} \leq \lambda_{\max} \leq \lambda_0$ Yasinsky's formula, $\sigma_{CR} = a - b \cdot \lambda_{\max}$;
- when $\lambda_{\max} < \lambda_{cp}$, formula for compression, $\sigma_{CR} = \sigma_{ye}$;

d) with σ_{CR} find $F_{CR} = \sigma_{CR} \cdot A$.

The allowable value of the force applied to the rod is defined as

$$[F] = F_{CR} / n_{cm}.$$

In calculations of stability, the critical stress is as destructive as the yield strength or strength limit in calculations of strength. Therefore, the concept of **allowable stability stress** $[\sigma_{ST}]$ is introduced, which is defined as part of the critical stress

$$[\sigma_{ST}] = \sigma_{CR} / n_{cm}.$$

The stability condition requires that the stress occuring during compression does not exceed the allowable stability stress

$$\sigma = \frac{F_{\max}}{A} \leq [\sigma_{ST}].$$

However, the calculation of the allowable stability stress is complicated by the fact that the critical stress depends not only on the properties of the material, but also on the flexibility of the rod. Therefore, the concept of the ***coefficient of reduction of the main allowable strength stress when calculating the stability*** is introduced

$$\varphi = \frac{[\sigma_{ST}]}{[\sigma]},$$

where $[\sigma]$ is the allowable strength stress under compression $[\sigma] = \sigma_{ye} / n$.

The coefficient φ for each material can be determined at any value of flexibility. Its value for certain materials is given in *Annex* 5.

Thus, **the condition of stability** is

$$\sigma = \frac{F_{\max}}{\varphi \cdot A} \leq [\sigma].$$

Three types of tasks are solved with stability condition.

Choosing the cross-section of the rod or **project calculation.**

This calculation is carried out by determining the cross-sectional area from the condition of stability

$$A \geq \frac{F}{\varphi \cdot [\sigma]}.$$

Determining of allowable load from the condition of stability is performed similarly to p.1, except for the last action, instead of which the allowable load is calculated.

Verification calculation. Stability test, i.e. compliance with the condition of stability. Perform in the following sequence:

- determine the minimum moment of inertia of the rod cross-section and the minimum radius of inertia (with the same fixation in the main planes);
- flexibility of the rod is calculated;
- choose the reduction factor of the main allowable stress φ;
- obtained data are substituted in a condition of stability and their performance is verified.

There is no single solution to this task, because the inequality includes two unknown quantities: the cross-sectional area A and the coefficient φ which depends on still undetermined cross-sectional dimensions, its shape and the length of the rod. The task is solved by the method of successive approximations with verification of intermediate results using the stability condition. In the first approximation, the random value of the reduction factor of the main allowable stress, approximately φ =0,5…0,6, is taken.

Determining the size of the cross-section of the rod during stability is complicated by the fact that it is not known in advance in which range the actual flexibility of the rod will occur, i.e. which of the formulas to use: Euler's, Yasinsky's or for simple compression.

Task 13
Calculation of stability of compressed rod

For the given rod (Fig. for task 13, Table for task 13) choose the elements of its cross-section from the condition of stability. Material of the elements is steel St.3; $[\sigma] = 160$ MPa ; $\lambda_0 = 100$. Elements of the rod are welded to each other.

Plan of solving the task:

1. Draw the given scheme, placing the elements (angles, channels or I-beams) under the rod.
2. Determine the plane of the minimum rigidity (the plane in which the axis is deformed when the force reaches a critical value).
3. Carry out the calculation of the rod stability in the plane of maximum flexibility, using the table of assortment for shaped rolling (*Annexes* 1, 2, 3, 4) and the table of coefficients of reduction of the main allowable stress (*Annex* 5).

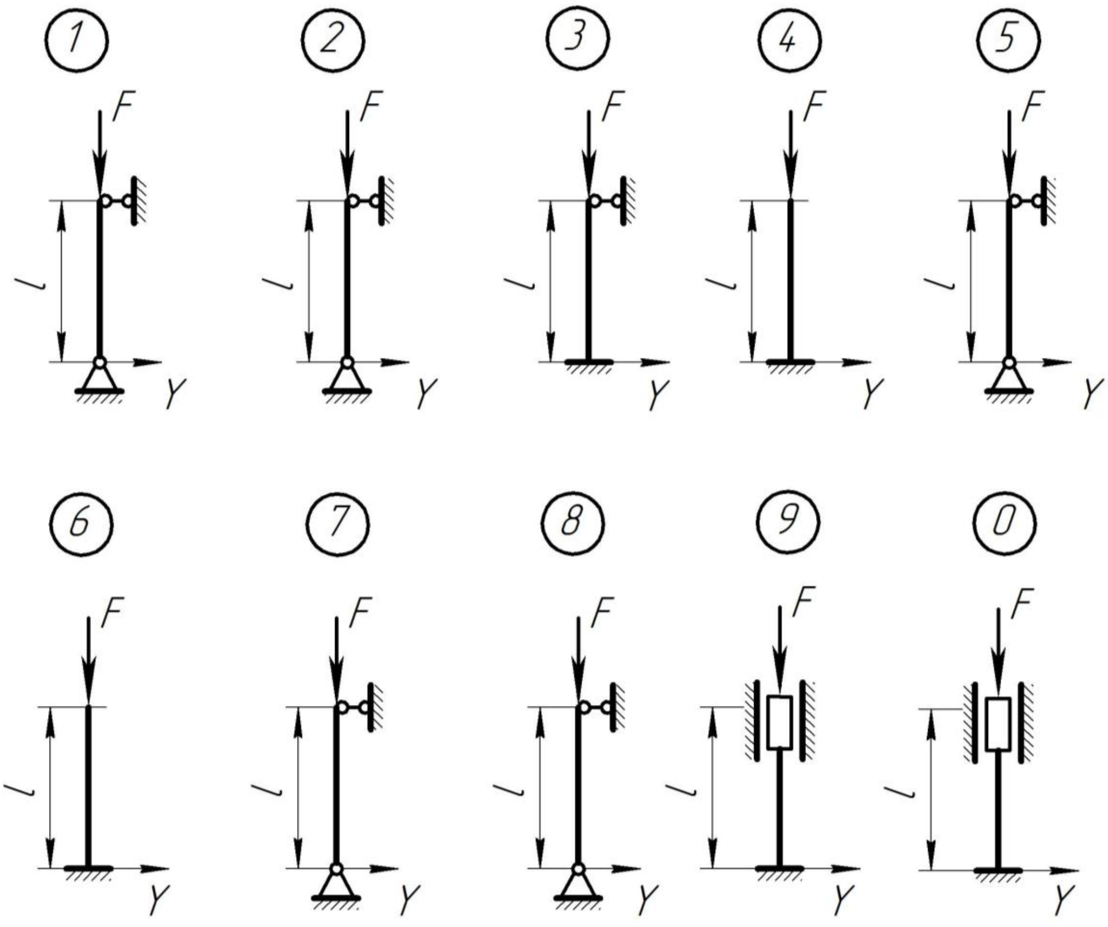
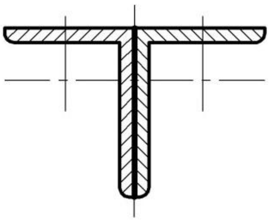
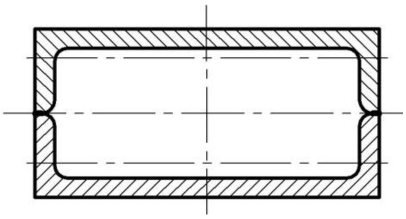
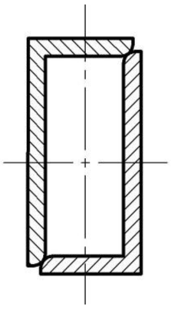
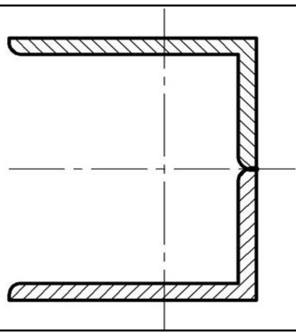
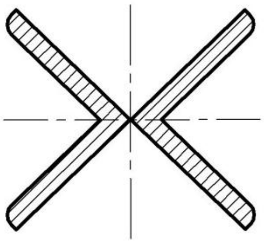
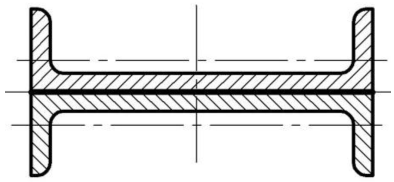
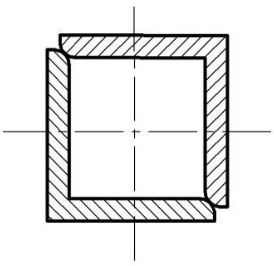
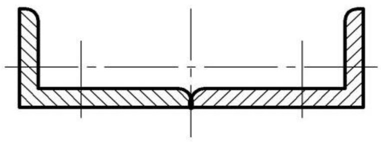
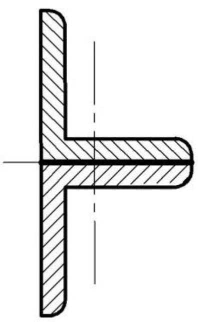
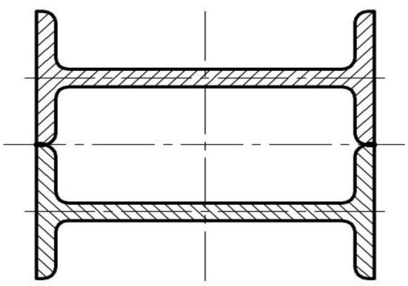


Figure for task 13

Table for task 13

Nr	F, kN	l, m	Cross-section	Nr	F, kN	l, m	Cross-section
1	130	2,5		**6**	130	2,0	
2	180	4,5		**7**	180	2,5	
3	100	1,5		**8**	100	4,0	
4	80	3,0		**9**	80	2,0	
5	90	3,5		**0**	90	6,0	

Example of solving the task 13
Calculation of stability of compressed rod

For the given rod (Fig. 11.5 *a*) choose the I-beam section. Material of the rod is steel St.3; $[\sigma] = 160$ MPa ; $\lambda_0 = 100$; $F = 200$ kN ; $l = 3$ m.

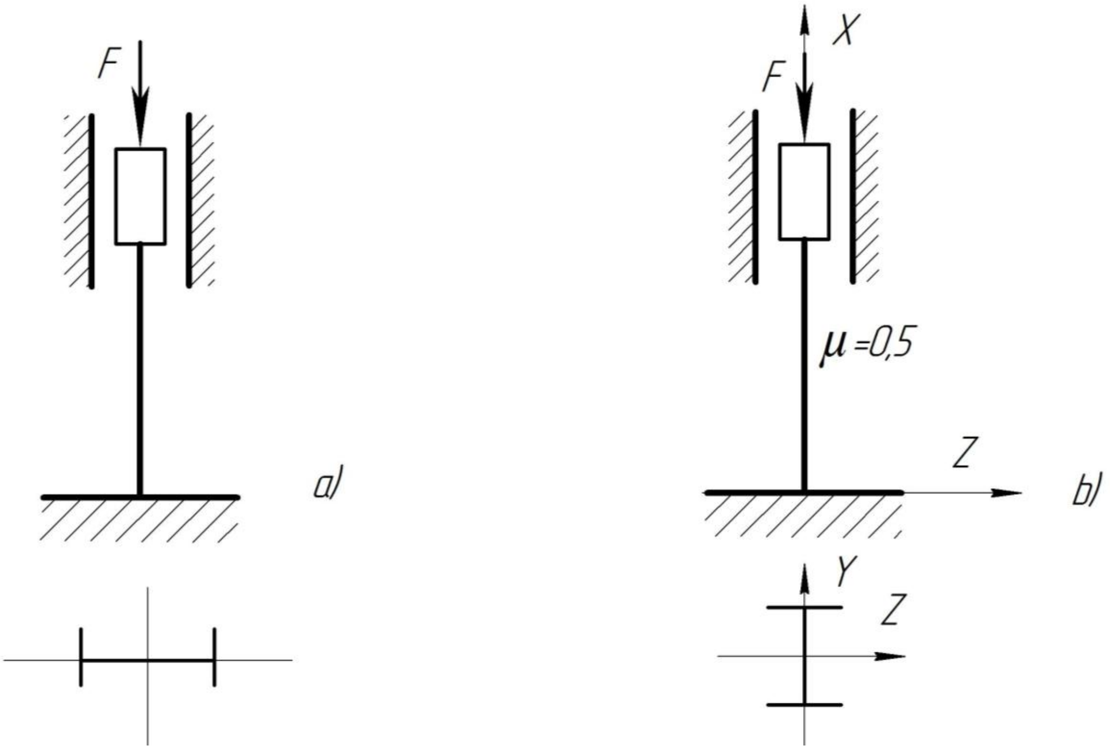


Figure 11.5

Solution

Place the cross-section under the rod (Fig. 11.5 *b*).

Calculation is carried out by the method of approximation.

The first approximation, take the value of the coefficient of longitudinal bending $\varphi_1 = 0{,}5$.

From the condition of stability determine the cross-sectional area

$$A_1 \geq \frac{F}{\varphi \cdot [\sigma]} = \frac{200}{0{,}5 \cdot 160 \cdot 10^3} = 25{,}0 \cdot 10^{-4} \text{ m}^2 = 25 \text{ cm}^2 .$$

From the standard GOST 8239-56 (*Annex* 1) choose I-beam Nr 10 for which: $A_{b1} = 25{,}4 \text{ cm}^2$, $i_{y1} = 2{,}12$ cm , $i_{z1} = 7{,}51$ cm .

Determine the flexibility of the rod in two planes:

$$\lambda_{Y1} = \frac{\mu \cdot l}{i_y} = \frac{0{,}5 \cdot 300}{2{,}12} \approx 71 ;$$

$$\lambda_{Z1} = \frac{\mu \cdot l}{i_z} = \frac{0{,}5 \cdot 300}{7{,}51} \approx 20 .$$

Since $\lambda_{Y1} > \lambda_{Z1}$, the calculation is made in the plane XOZ (see Fig. 11.5 *b*). Using the method of linear interpolation, we determine the specified value of the coefficient of longitudinal bending, which corresponds to the flexibility $\lambda_{Y1} = 71$. From the Table (*Annex* 5) write out:

when $\lambda = 70$; $\varphi = 0{,}81$; when $\lambda = 80$; $\varphi = 0{,}75$.

Determine the coefficients of reduction of the main allowable stress for flexibility $\lambda_{Y1} = 71$

$$\varphi_1^{/} = 0{,}81 - \frac{0{,}81 - 0{,}75}{10} \cdot 1 = 0{,}804 .$$

Determine the actual stress

$$\sigma_{s.ac1} = \frac{F}{A_{\partial в 1}} = \frac{200 \cdot 10^{-3}}{25{,}4 \cdot 10^{-4}} = 78{,}7 \text{ MPa} .$$

Determine the allowable stress

$$[\sigma]_{s.al1} = \varphi_1^{/} \cdot [\sigma] = 0{,}804 \cdot 160 = 129 \text{ MPa} .$$

Understress makes

$$\frac{\left|\sigma_{s.ac1} - [\sigma]_{s.al1}\right|}{[\sigma]_{s.al1}} \cdot 100\ \% = \frac{78{,}7 - 129}{129} \cdot 100\ \% = 38{,}7\ \%,$$ which is unacceptable.

The second approximation, we take

$$\varphi_2 = \frac{\varphi_1 + \varphi_1^{/}}{2} = \frac{0{,}5 + 0{,}804}{2} = 0{,}652 .$$

Then

$$A_2 \geq \frac{200}{0{,}652 \cdot 160 \cdot 10^3} = 19{,}2 \cdot 10^{-4} \text{ m}^2 = 19{,}2 \text{ cm}^2 .$$

Accept I-beam Nr 16 (*Annex* 1), for which $A_{b2} = 20{,}2 \text{ cm}^2$; $i_{y2} = 1{,}70$ cm; $i_{z2} = 6{,}57$ cm.

Determine the flexibility of the rod

$$\lambda_{Y2} = \frac{0{,}5 \cdot 300}{1{,}70} = 88 ,$$

which corresponds

$$\varphi_2^{/} = 0{,}75 - \frac{0{,}75 - 0{,}69}{10} \cdot 8 = 0{,}702 .$$

Actual stress

$$\sigma_{s.ac\,2} = \frac{200 \cdot 10^{-3}}{20,2 \cdot 10^{-4}} = 99 \text{ MPa} .$$

Allowable stress

$$[\sigma]_{s.al\,2} = 0,702 \cdot 160 = 112 \text{ MPa} .$$

Understressing makes

$$\frac{112 - 99}{112} \cdot 100 \% = 11,6 \%$$, which is more than 5%.

The third approximation, we take

$$\varphi_3 = \frac{\varphi_2 + \varphi_2^{/}}{2} = \frac{0,652 + 0,702}{2} = 0,677 .$$

Then

$$A_3 = \frac{200}{0,677 \cdot 160 \cdot 10^3} = 18,5 \cdot 10^{-4} \text{ m}^2 = 18,5 \text{ cm}^2 .$$

Take I-beam Nr 14 (*Annex* 1), for which $A_{b3} = 17,4 \text{ cm}^2$; $i_{y3} = 1,55 \text{ cm}$; $i_{z3} = 5,73$ cm.

Determine flexibility

$$\lambda_{Z3} = \frac{0,5 \cdot 300}{1,55} = 97 ,$$

that corresponds

$$\varphi_3^{/} = 0,69 - \frac{0,69 - 0,60}{10} \cdot 7 = 0,627 .$$

Actual stress

$$\sigma_{s.ac\,3} = \frac{200 \cdot 10^{-3}}{17,4 \cdot 10^{-4}} = 115 \text{ MPa} .$$

Allowable stresses

$$[\sigma]_{s.al\,3} = 0,627 \cdot 160 = 100 \text{ MPa} .$$

Overstressing makes $\frac{115 - 100}{100} \cdot 100 \% = 15 \%$, which is unacceptable.

Therefore, for this rod we take I-beam cross-section Nr 16, for which the understress is 11,6 %, because for the rod with I-beam cross-section Nr 14 overstressing is 15 %, which is unacceptable.

12. DYNAMIC LOADS. DETERMINING IMPACT STRESSES AND DISPLACEMENTS

Dynamic load ***is load which is partially or completely caused by the forces of inertia*** (at accelerated movement of parts, during their rotation and oscillation), as well as ***at instantaneous load and impact.*** The same structural elements and their material are deformed differently depending on how they are loaded: statically or dynamically.

The peculiarity of fracture under dynamic action of forces is that plastic materials, such as low-carbon steel, demonstrate brittle properties under instantaneous (impact) load, i.e. they are destroyed without significant residual deformations and at much lower deformation energy. ***Mechanical characteristic of material, which reflects its ability to resist impact loads, is called impact viscosity.*** Impact strength is characterized by the area of the stress diagram $\varepsilon - \sigma$ before failure (*see Chapter* 2). The modules of elasticity under dynamic loading are also different than under the static one. In strength of materials, approximate ***theory of impact is used***, taking into consideration that **Hooke's law is kept, the modules of elasticity are unchanged and there is no energy dissipation during impact**.

Operation of some machines (pressing, driving in piles, etc.) is accompanied by an impact load, for example, a load Q falling from a certain height h on a stationary elastic system. At the moment of impact, stresses and deformations reach the maximum values in a structure.

Impact load by a free-falling body

In the systems in which the load is falling there may occur different kinds of deformations: compression (Fig. 12.1 *a*), bending (Fig. 12.1 *b*, *c*), torsion (Fig. 12.1 *d*).

To obtain the formulas of strength and rigidity under such a load (in approximate form) the following assumptions are taken:

1. Acceleration and inertia force of the body causing the impact increase without changing the direction from zero to the final value.

2. Body under impact has only one degree of freedom.

3. Body that impacts is absolutely rigid and does not deform; impact is elastic, but the bodies after the impact displace together (without bounce).

4. Deformations of the body under the impact are elastic and Hooke's law is acceptable to it.

5. Mass of the elastic system is neglected in approximate calculations.

6. Energy dissipation during impact is neglected. Kinetic energy of the falling load is completely converted into potential energy of elastic system.

Let us consider the simplest case of the impact load of a vertical column by a free-falling perfectly rigid body (Fig. 12.1 *a*).

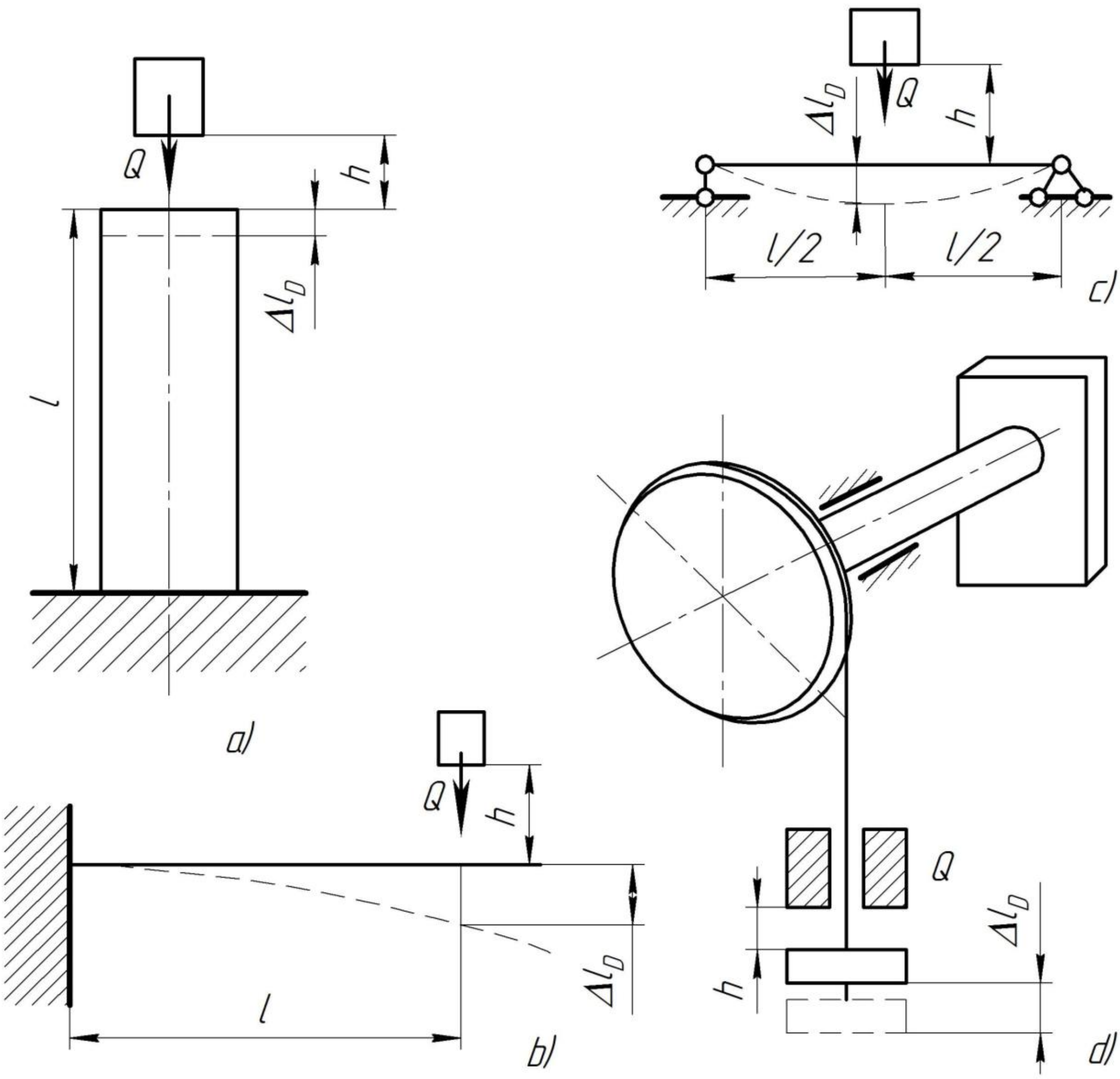


Figure 12.1

It is impossible to use the D'Alembert principle in this case, because the acceleration is unknown when the column itself is deformed. Using the law of conservation of energy, we make the equation of energy balance of the system falling body – structure for the moment of maximum displacement Δl_D

$$K_T = U_{col} , \qquad (12.1)$$

where K_T is kinematic energy of the falling body with taking into account its displacement (together with the dynamic shortening of the column by Δl_D) up to stop at the end of the maximum displacement of impact point

$$K_T = Q\left(h + \Delta l_D\right). \tag{12.2}$$

Potential energy of elastic deformation that will accumulate during such shortening of the column

$$U = \frac{1}{2} N_D \cdot \Delta l_D , \tag{12.3}$$

where N_D is maximum internal force during elastic deformation,
$N_D \neq Q$ is unknown parameter as well as Δl_D.

We write down the relations between these parameters, assuming that Hooke's law is satisfied:

$$\Delta l_D = \frac{N_D \cdot l}{E \cdot A}, \qquad \text{then } N_D = \frac{\Delta l_D \cdot E \cdot A}{l}.$$

The energy of deformation will be corresponding to (12.3)

$$U = \frac{\Delta l_D^2 \cdot E \cdot A}{2\,l}. \tag{12.4}$$

Due to the balance of energy (12.2) and (12.4) compare

$$Q\left(h + \Delta l_D\right) = \frac{\Delta l_D^2 \cdot E \cdot A}{2\,l}. \tag{12.5}$$

Write down the quadratic equation in the simplest way

$$\Delta l_D^2 - \frac{2\,Q \cdot l}{E \cdot A} \cdot \Delta l_D - \frac{2\,Q \cdot l \cdot h}{E \cdot A} = 0 . \tag{12.6}$$

Expression $\frac{Q \cdot l}{E \cdot A}$ is shortening of the column (according to Hooke’s law) under the static action of the weight Q, i.e. it is Δl_{ST} (displacement of the cross-section of the elastic system from the static action of the load).

Dynamic shortening can be obtained as

$$\Delta l_D = \Delta l_{ST} + \sqrt{\Delta l_{ST}^2 + 2\Delta l_{ST} \cdot h}\,, \tag{12.7}$$

or

$$\Delta l_D = \Delta l_{ST}\left(1 + \sqrt{1 + \frac{2\,h}{\Delta l_{ST}}}\right) = \Delta l_{ST} \cdot k_D . \tag{12.8}$$

The expression in brackets is considered to be the ***impact coefficient of a free-falling body.***

When $h = 0$, that is when the body falls from zero height (or under the so-called instantaneous application of force $k_D = 1 + \sqrt{1} = 2$.

In most cases elastic deformations are much less than h, therefore the coefficient of impact can be taken

$$k_D \approx \sqrt{\frac{2h}{\Delta l_{ST}}}. \qquad (12.9)$$

The larger the denominator, the smaller k_D is. This means that a more susceptible to deformation (less rigid) system is stronger under impacts and vibrations.

When obtaining the dependence for the coefficient of impact, the own mass of the deformed body is neglected. This is acceptable only for approximate calculations.

Similarly, determine the displacement and stress of the beam under the axial impact (Fig. 12.2).

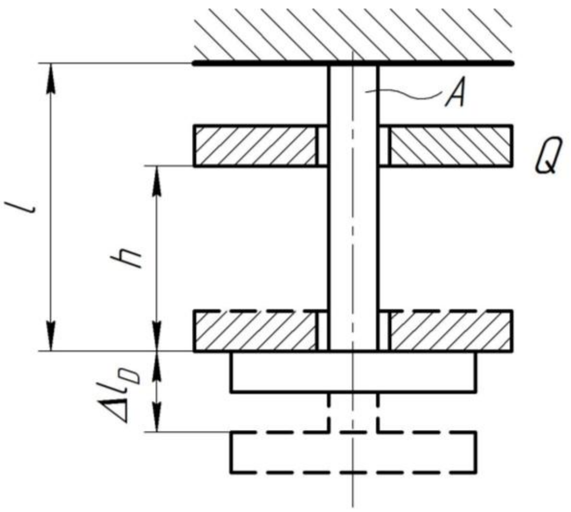


Figure 12.2

Based on the linear relationship between force and displacement, it can be written down that

$$\sigma_D = \sigma_{ST} \cdot k_D; \qquad \tau_D = \tau_{ST} \cdot k_D,$$

where σ_D, τ_D are dynamic normal and tangential (shear) stresses;

σ_{ST}, τ_{ST} are static normal and tangential (shear) stresses determined in the structural elements from the static action of the load.

Task 14
Determining maximum dynamic stresses and displacements under the impact

For the given elastic system (Fig. for the task 14, Table for the task 14) determine the maximum stresses under the impact that occur during falling of the load $Q = 100$ N from the height $h = 0{,}5$ m and the displacement value (see table for the task Nr 14) in the direction of the impact. The material of the elastic system is steel; $l = 2$ m; $d = 4$ cm.

Plan of solving the task:

1. Determine the types of deformation for which the structural elements work.
2. Draw the diagrams of internal force factors under static action of the load Q.
3. Determine the maximum static stresses in the structural elements.
4. Determine the static displacement in the given cross-section.
5. Determine the static displacement in the place of impact.
6. Determine the coefficient of impact (without considering the own mass of the elastic system).
7. Determine the maximum dynamic stresses.
8. Determine displacement during the impact in the given cross-section (f_A^{ver} або Θ_B).

Table for task 14

Nr	D, cm	Displacement
1	4,5	f_A^{ver}
2	4,0	Θ_B
3	5,0	f_A^{ver}
4	6,0	Θ_B
5	7,0	f_A^{ver}
6	8,0	Θ_B
7	9,0	f_A^{ver}
8	10,0	Θ_B
9	11,0	f_A^{ver}
0	12,0	Θ_B

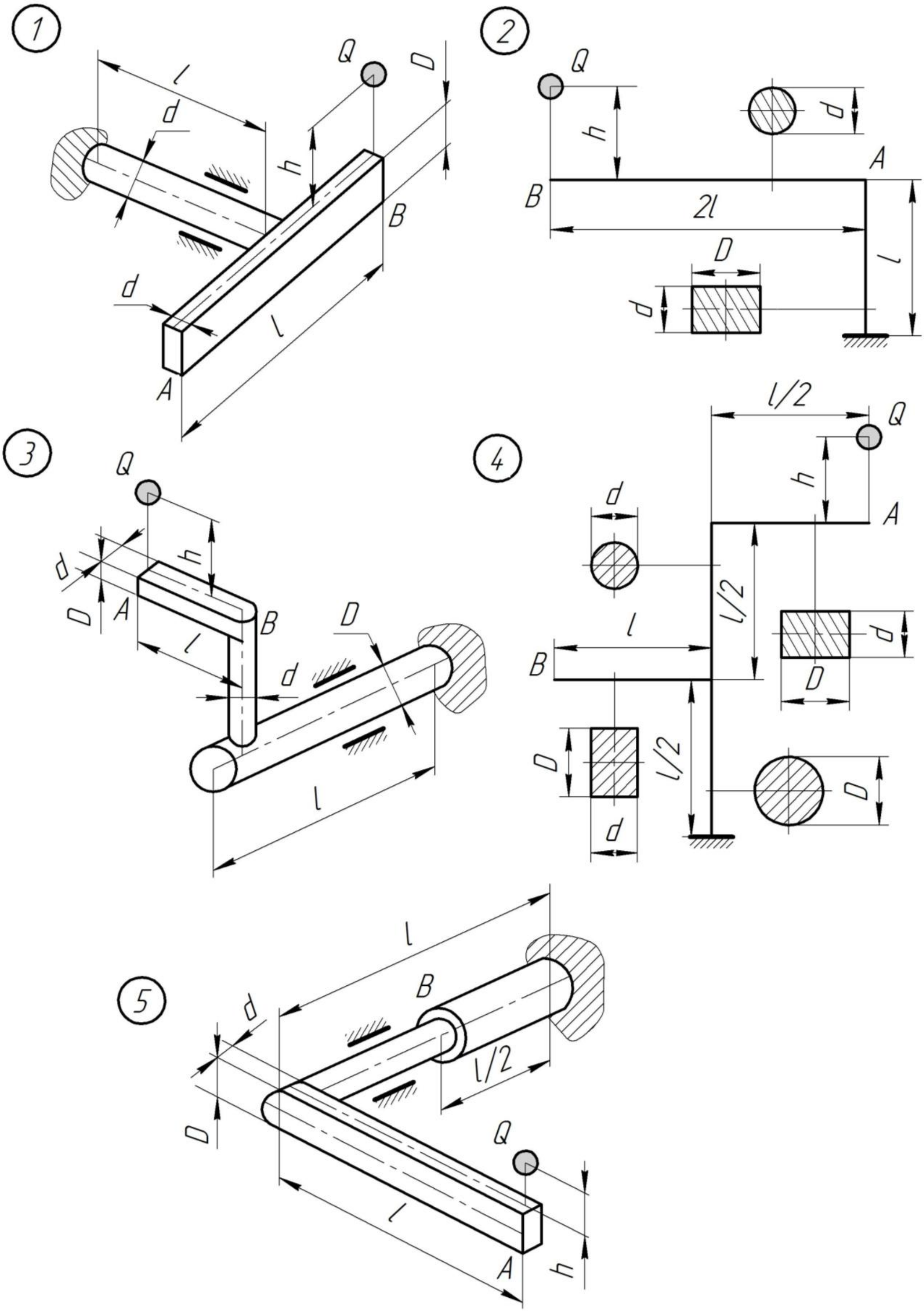


Figure for task 14

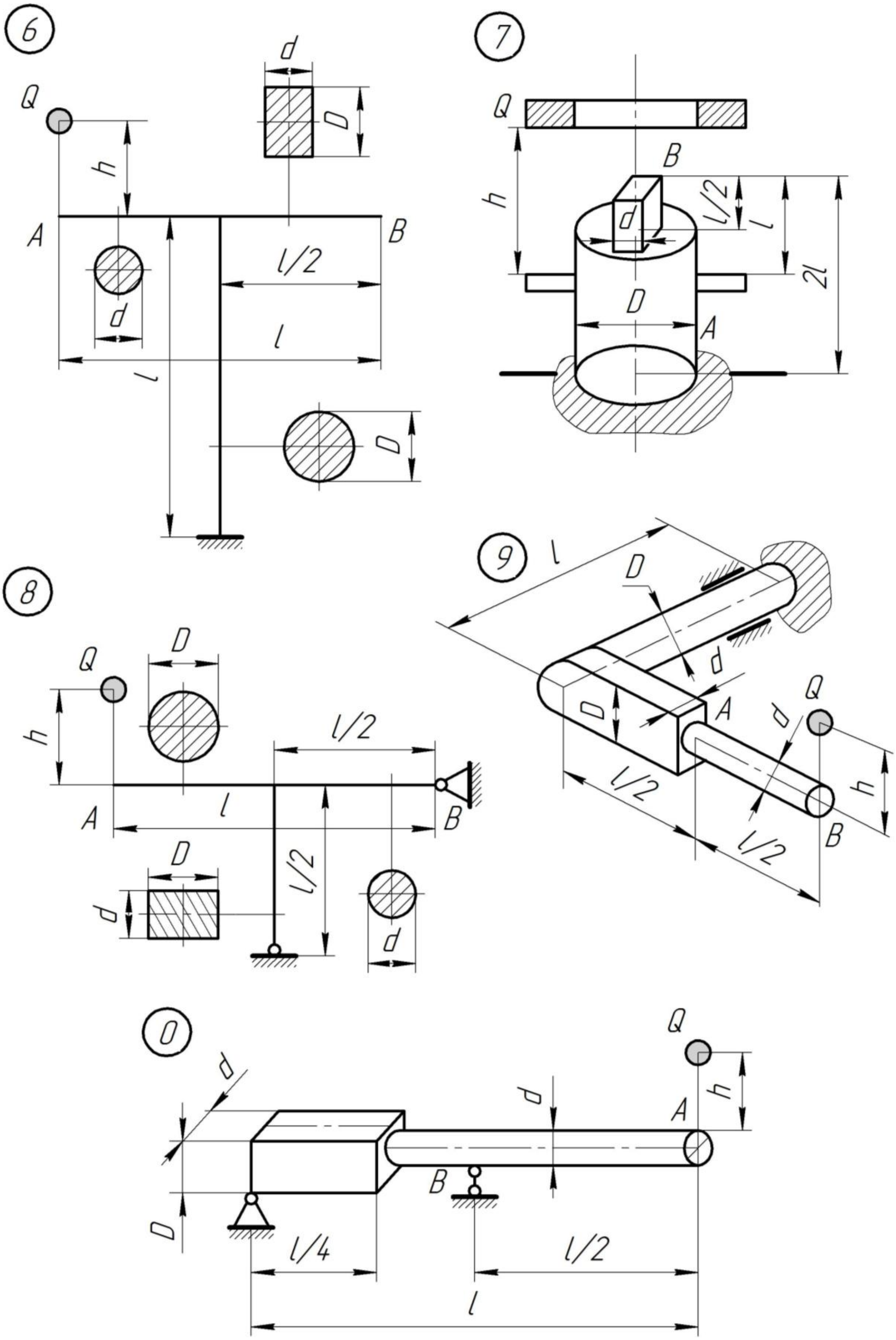


Figure for task 14 (continued)

Example of solving the task 14.1

For the given elastic system (Fig. 12.3 *a*) determine the maximum stresses that occur under the impact of the load $Q = 100$ N falling from the height $h = 0{,}5$ m and the value of displacement in the cross-section on which the load is falling. Given: $E = 2 \cdot 10^{11}$ Pa; $l = 2$ m; $d = 4$ cm; $D = 5$ cm.

Solution

The given rod system (Fig. 12.3 *b*) works for such kinds of deformation: segments AB, BC – symmetrical transverse bending.

Determine the maximum internal force factors under the static load $Q = 100$ N. Draw the diagram of bending moments (Fig. 12.3 *c*).

Segment AB, cross-section B: $M_{BN.B} = 100$ Nm.

Segment BC, $M_{BN.BC} = 100$ Nm.

Determine maximum static stresses.

Segment B:

$$\sigma_{ST.B} = \frac{M_{BN.B}}{W_{01}} = \frac{100}{6{,}28 \cdot 10^{-6}} = 15{,}9 \text{ MPa},$$

where

$$W_{01} = \frac{\pi \cdot d^3}{32} = \frac{\pi \cdot 4^3}{32} = 6{,}28 \text{ cm}^3 = 6{,}28 \cdot 10^{-6} \text{ m}^3.$$

Segment BC:

$$\sigma_{ST.BC} = \frac{M_{BN.BC}}{W_{02}} = \frac{100}{12{,}3 \cdot 10^{-6}} = 8{,}15 \text{ MPa},$$

where

$$W_{02} = \frac{\pi \cdot D^3}{32} = \frac{\pi \cdot 5^3}{32} = 12{,}3 \text{ cm}^3 = 12{,}3 \cdot 10^{-6} \text{ m}^3.$$

Determine the static displacement in the cross-section on which the load is falling by the graphoanalytical method of solving the Mohr integral (Fig. 12.3 *d*, *e*)

$$\Delta l_{ST} = \frac{1}{E \cdot I_{01}} \omega_1 \cdot M_{C1} + \frac{1}{E \cdot I_{02}} \omega_2 \cdot M_{C2} =$$

$$= \frac{1}{2 \cdot 10^{11}} \left(\frac{50 \cdot 0{,}67}{12{,}56} + \frac{200 \cdot 1}{30{,}6} \right) \cdot \frac{1}{10^{-8}} = 5{,}93 \cdot 10^{-3} \text{ m};$$

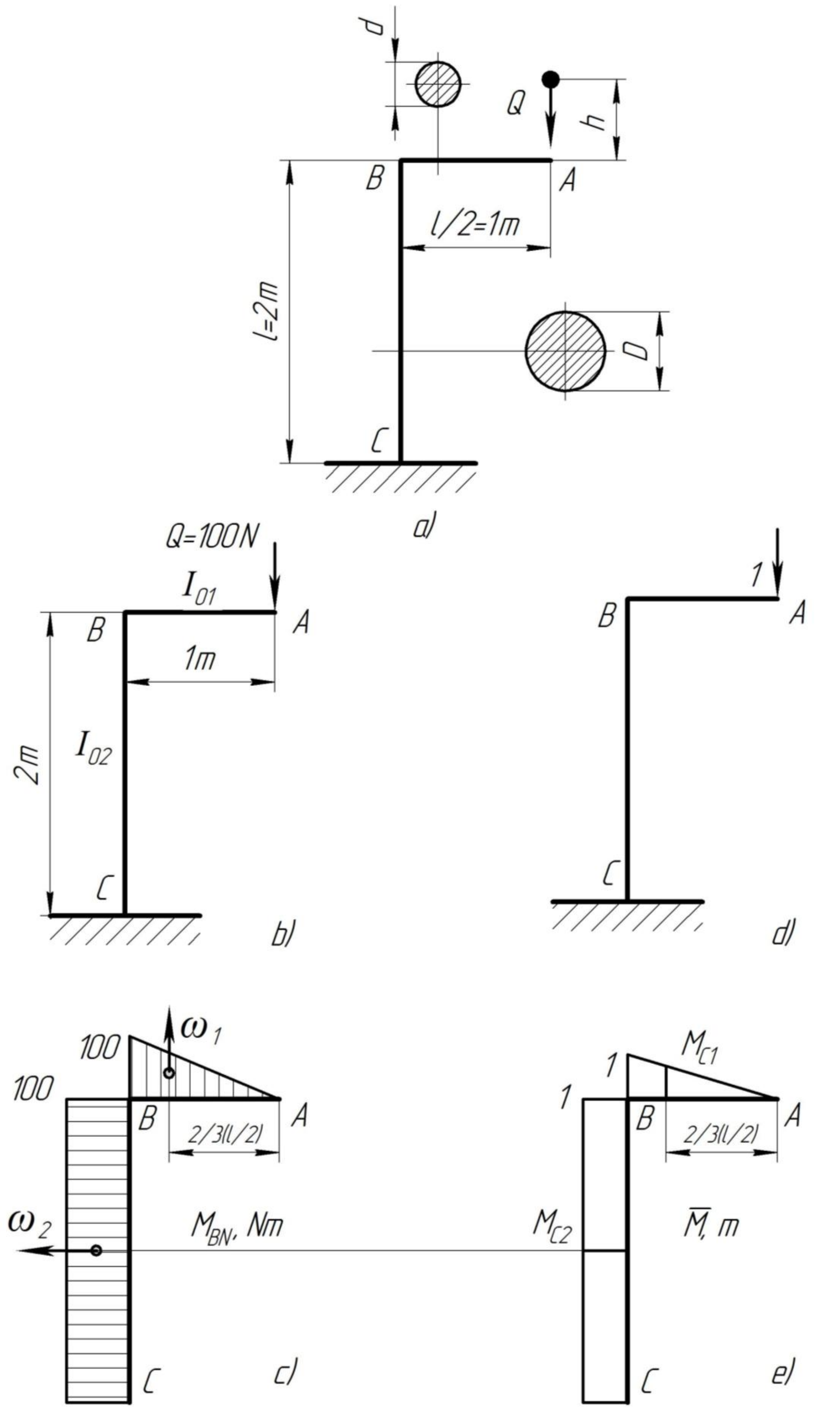

d
Q
h
B
A
l/2=1m
l=2m
D
C
a)
Q=100N
I01
B
A
1m
2m
I02
C
b)
1
B
A
C
d)
ω1
100
100
B
2/3(l/2)
A
ω2
MBN, Nm
C
c)
1
MC1
1
B
2/3(l/2)
A
MC2
M̅, m
C
e)

Figure 12.3

where $\omega_1 = \frac{1}{2} \cdot 100 \cdot 1 = 50$; $\omega_2 = 100 \cdot 2 = 200$;

$$M_{C1} = \frac{2}{3} \cdot 1 = 0{,}67; \qquad M_{C2} = 1;$$

$$I_{01} = \frac{\pi \cdot d^4}{64} = \frac{\pi \cdot 4^4}{64} = 12{,}56 \text{ cm}^4 = 12{,}56 \cdot 10^{-8} \text{ m}^4;$$

$$I_{02} = \frac{\pi \cdot D^4}{64} = \frac{\pi \cdot 5^4}{64} = 30{,}6 \text{ cm}^4 = 30{,}6 \cdot 10^{-8} \text{ m}^4.$$

Find the coefficient of impact

$$k_D = 1 + \sqrt{1 + \frac{2 \cdot h}{\Delta l_{ST}}} = 1 + \sqrt{1 + \frac{2 \cdot 0{,}5}{5{,}93 \cdot 10^{-3}}} = 14{,}0.$$

Determine dynamic stresses in the construction elements:
cross-section B,

$$\sigma_{D.B} = \sigma_{ST.B} \cdot k_D = 15{,}9 \cdot 14 = 223 \text{ MPa}.$$

segment BC,

$$\sigma_{D.BC} = 8{,}15 \cdot 14 = 114 \text{ MPa}.$$

Vertical bending of cross-section A during the impact

$$f_{D.A} = \Delta l_{ST.A} \cdot k_D = 5{,}93 \cdot 14 = 83 \text{ mm},$$

where $\Delta l_{ST.A} = \Delta l_{ST}$.

Example of solving the task 14.2

For the given elastic system (Fig. 12.4) determine the maximum stresses that occur under the impact of the load $Q = 100$ N falling from the height $h = 0{,}5$ m and the value of displacement in the cross-section B on which the load is falling. Given: $E = 2 \cdot 10^5$ MPa; $l = 2$ m; $d = 4$ cm; $D = 4$ cm; $G = 8 \cdot 10^4$ MPa.

Solution

The given rod system (Fig. 12.4 *a*) works for such kinds of deformation: segments BA and AC – bending; segment CK – torsion.

Determine the maximum internal force factors under the static load $Q = 100$ N (Fig. 12.4 *b*).

Segment BA, A; $M_{BN.A} = 100 \cdot 1 = 100$ Nm.

Segment AC, cross-section C; $M_{BN.C} = 100 \cdot 2 = 200$ Nm.

Segment CK, $M_{TR.C} = 200$ Nm.

Draw the diagrams of internal force factors (Fig. 12.4 *c*).

Determine the maximum static stresses.

Cross-section A:
$$\sigma_{ST.A} = \frac{M_{BN.A}}{W_{01}} = \frac{100}{6{,}28} = 15{,}9 \text{ MPa},$$

where
$$W_{01} = \frac{\pi \cdot d^3}{32} = \frac{\pi \cdot 4^3}{32} = 6{,}28 \text{ cm}^3 = 6{,}28 \cdot 10^{-6} \text{ m}^3.$$

Cross-section C:
$$\sigma_{ST.C} = \frac{M_{BN.C}}{W_{02}} = \frac{200}{10{,}7} = 18{,}7 \text{ MPa},$$

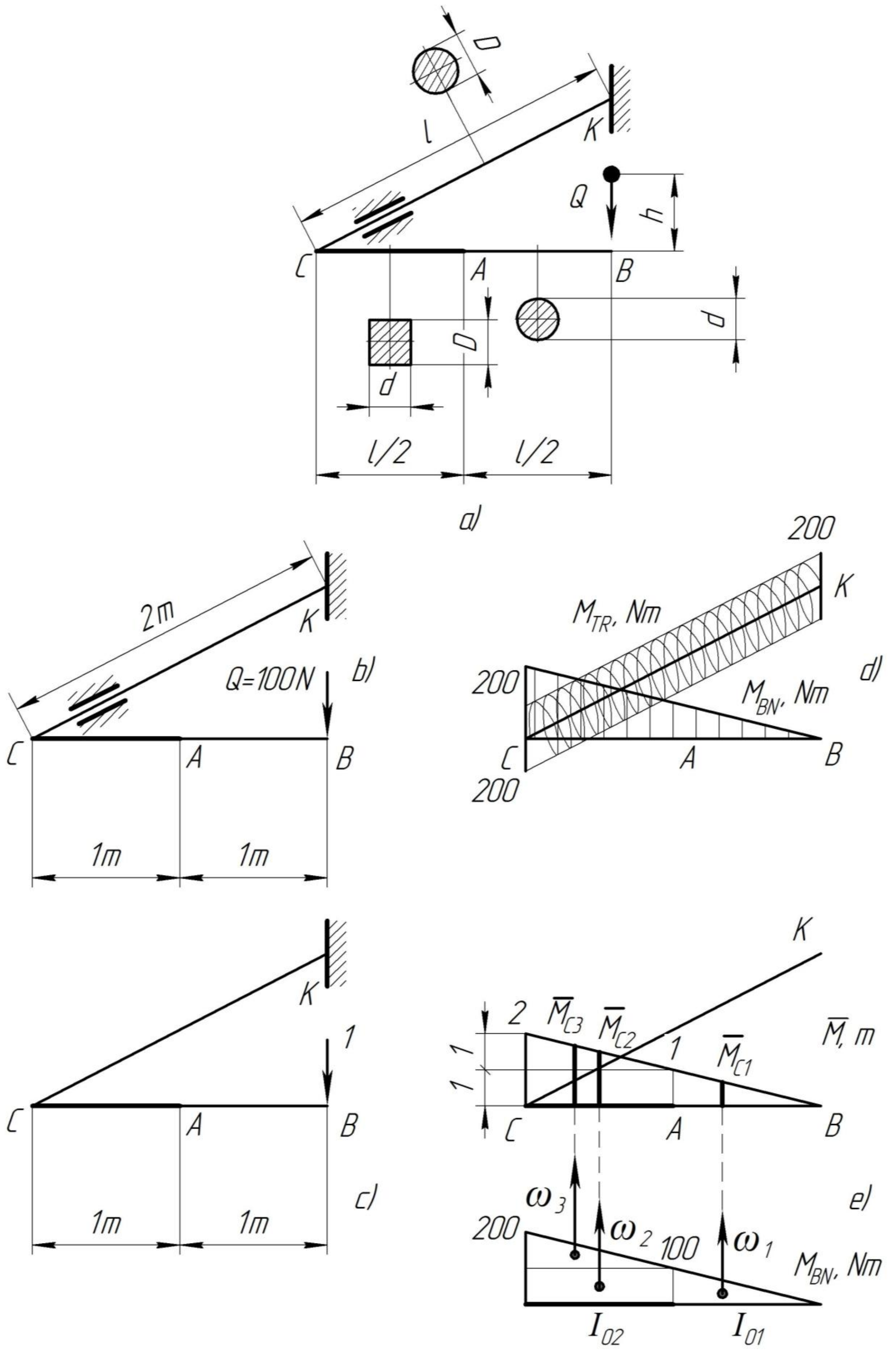
where
$$W_{02} = \frac{d \cdot D^2}{6} = \frac{4^3}{6} = 10{,}7 \text{ cm}^3 = 10{,}7 \cdot 10^{-6} \text{ m}^3.$$

Cross-section CK:
$$\tau_{ST.CK} = \frac{M_{TR}}{W_P} = \frac{200}{12{,}6} = 15{,}9 \text{ MPa},$$

where
$$W_P = \frac{\pi \cdot D^3}{16} = \frac{\pi \cdot 4^3}{16} = 12{,}6 \text{ cm}^3 = 12{,}6 \cdot 10^{-6} \text{ m}^3.$$

Determine the static displacement in the cross-section on which the load is falling
$$\Delta l_{ST} = \Delta l_{ST.TR} + \Delta l_{ST.BN} = 0{,}398 + 0{,}00679 = 0{,}0466 \text{ m},$$

where $\Delta l_{ST.TR}$ is displacement of the segment from the torsion strain of the segment CK
$$\Delta l_{ST.TR} = \varphi \cdot 2 = 0{,}0199 \cdot 2 = 0{,}0398 \text{ m};$$



D
l
K
Q
h
C
A
B
d
D
d
l/2
l/2
a)
2m
K
Q=100N
b)
C
A
B
1m
1m
200
K
M_TR, Nm
d)
200
M_BN, Nm
C
A
B
200
K
1
C
A
B
1m
1m
c)
K
2
M̄_C3
M̄_C2
1
M̄, m
M̄_C1
1
1
C
A
B
ω3
ω2
ω1
e)
200
100
M_BN, Nm
I_02
I_01

Figure 12.4

$$\varphi = \frac{M_{TR.CK} \cdot l_{CK}}{G \cdot I_P} = \frac{200 \cdot 2}{8 \cdot 10^{10} \cdot 25{,}1 \cdot 10^{-8}} = 0{,}0199 \text{ rad};$$

$$I_P = \frac{\pi \cdot D^4}{32} = \frac{\pi \cdot 4^4}{32} = 25{,}1 \text{ cm}^4 = 25{,}1 \cdot 10^{-8} \text{ m}^4;$$

$\Delta l_{ST.BN}$ is displacement of the section B under the bending deformation of BC segment, determined by the graphoanalytical method of solving the Mohr integral (Fig. 12.4 *d*)

$$\Delta l_{ST.BN} = \frac{\omega_1 \cdot M_{C1}}{E \cdot I_{01}} + \frac{1}{E \cdot I_{02}}\left(\omega_2 \cdot M_{C2} + \omega_3 \cdot M_{C3}\right) =$$

$$= \frac{50 \cdot 0{,}667}{2 \cdot 10^{11} \cdot 12{,}56 \cdot 10^{-8}} + \frac{1}{2 \cdot 10^{11} \cdot 21{,}3 \cdot 10^{-8}}\left(100 \cdot 1{,}5 + 50 \cdot 1{,}667\right) =$$

$$= 6{,}79 \cdot 10^{-3} \text{ m};$$

where $\omega_1 = 0{,}5 \cdot 100 = 50$; $M_{C1} = 0{,}667$;

$\omega_2 = 100 \cdot 1 = 100$; $M_{C2} = 1{,}5$;

$\omega_3 = 50$; $M_{C3} = 1 + 0{,}667 = 1{,}667$;

$$I_{01} = \frac{\pi \cdot d^4}{64} = \frac{\pi \cdot 4^4}{64} = 12{,}56 \text{ cm}^4 = 12{,}56 \cdot 10^{-8} \text{ m}^4;$$

$$I_{02} = \frac{d \cdot D^3}{12} = \frac{4^4}{12} = 21{,}3 \text{ cm}^4 = 21{,}3 \cdot 10^{-8} \text{ m}^4.$$

Determine the coefficient of impact

$$k_D = 1 + \sqrt{1 + \frac{2H}{\Delta l_{ST}}} = 1 + \sqrt{1 + \frac{2 \cdot 0{,}5}{0{,}0466}} = 5{,}74.$$

Determine the maximum dynamic stresses and displacements of the section B at the time of falling of the load:

$$\sigma_{D.A} = \sigma_{ST.A} \cdot k_D = 15{,}9 \cdot 5{,}74 = 91{,}3 \text{ MPa};$$

$$\sigma_{D.C} = \sigma_{ST.C} \cdot k_D = 18{,}7 \cdot 5{,}74 = 107 \text{ MPa};$$

$$\tau_{D.CK} = \tau_{ST.CK} \cdot k_D = 15{,}9 \cdot 5{,}74 = 91{,}3 \text{ MPa};$$

$$f_D = \delta_{ST} \cdot k_D = 0{,}0466 \cdot 5{,}74 = 0{,}268 \text{ m}.$$

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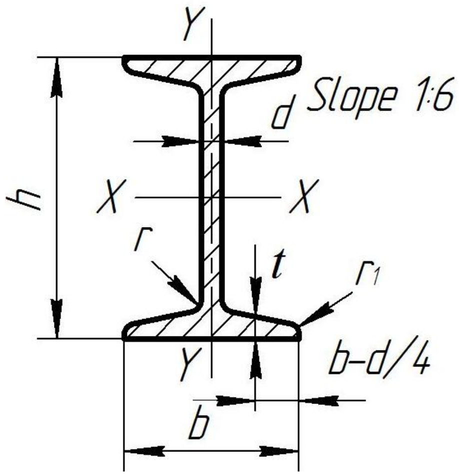
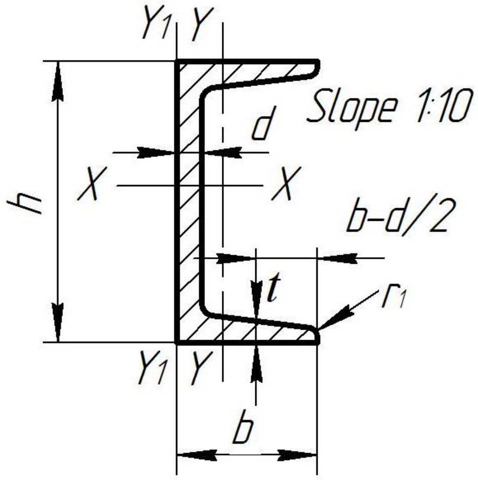
Annex 1



Rolled steel. **I-beams**. Assortment. ГОСТ 8239-89

Shape num-bers	Dimensios				Section area A, sm²	Reference values for axes						
						x – x				y – y		
	h	b	d	t		J_X	W_X	i_x	S_X	J_Y	W_Y	i_y
	mm					sm⁴ → sm^4	sm^3	sm	sm	sm^4	sm^3	sm
10	100	55	4,5	7,2	12,0	198	39,7	4,06	23,0	17,9	6,49	1,22
12	120	64	4,8	7,3	14,7	350	58,4	4,88	33,7	27,9	8,72	1,38
14	140	73	4,9	7,5	17,4	572	81,7	5,73	46,8	41,9	11,5	1,55
16	160	81	5,0	7,8	20,2	873	109	6,57	62,3	58,6	14,5	1,70
18	180	90	5,1	8,1	23,4	1290	143	7,42	81,4	82,6	18,4	1,88
20	200	100	5,2	8,4	26,8	1840	184	8,28	104	115	23,1	2,07
22	220	110	5,4	8,7	30,6	2550	232	9,13	131	157	28,6	2,27
24	240	115	5,6	9,5	34,8	3460	289	9,97	163	198	34,5	2,37
27	270	125	6.0	9,8	40,2	5010	371	11,2	210	260	41,5	2,54
30	300	135	6,5	10,2	46,5	7080	472	12,3	268	337	49,9	2,69
33	330	140	7,0	11,2	53,8	9840	597	13,5	339	419	59,9	2,79
36	360	145	7,5	12,3	61,9	13380	743	14,7	423	516	71,1	2,89
40	400	155	8,0	13,0	71,4	18930	947	16,3	540	665	85,9	3,05
45	450	160	6,6	14,2	83,0	27450	1220	18,2	699	807	101	3,12
50	500	170	9,5	15,2	97,3	39120	1560	20,1	899	1040	122	3,28
55	550	180	10,0	16,5	113	54810	1990	22,0	1150	1350	150	3,46
60	600	190	10,8	17,8	131	75010	2500	23,9	1440	1720	181	3,62

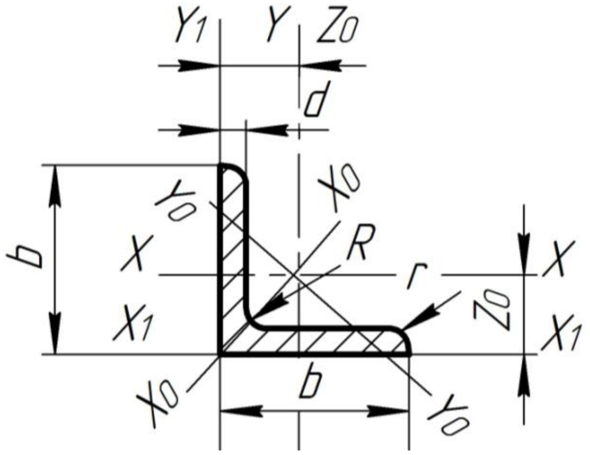
Annex 2



Rolled steel. **U-beam**. Assortment. ГОСТ 8240-89

Shape num-bers	Dimensios				Section area A, sm^2	Reference values for axes						
						$x-x$			$y-y$			z_0
	h	b	d	t		J_x	W_x	i_x	J_y	W_y	i_y	
	mm					sm^4	sm^3	sm	sm^4	sm^3	sm	sm
5	50	32	4,4	7,0	6,16	22,8	9,10	1,92	5,61	2,75	0,954	1,16
6,5	65	36	4,4	7,2	7,51	48,6	15,0	2,54	8,70	3,68	1,08	1,24
8	80	40	4,5	7,4	8,98	89,4	22,4	3,16	12,8	4,75	1,19	1,31
10	100	46	4,5	7,6	10,9	174	34,8	3,99	20,4	6,46	1,37	1,44
12	120	52	4,8	7,8	13,3	304	50,6	4,78	31,2	8,52	1,53	1,54
14	140	58	4,9	8,1	15,6	491	70,2	5,60	45,4	11,0	1,70	1,67
16	160	64	5,0	8,4	18,1	747	93,4	6,42	63,3	13,8	1,87	1,80
18	180	70	5,1	8,7	20,7	1090	121	7,24	86,0	17,0	2,04	1,94
20	200	76	5,2	9,0	23,4	1520	152	8,07	113	20,5	2,20	2,07
22	220	82	5,4	9,5	26,7	2110	192	8,89	151	25,1	2,37	2,21
24	240	90	5,6	10,0	30,6	2900	242	9,73	208	31,6	2,60	2,42
27	270	95	6,0	10,5	35,2	4160	308	10,9	262	37,3	2,73	2,47
30	300	100	6,5	11,0	40,5	5810	387	12,0	327	43,6	2,84	2,52
33	330	105	7,0	11,7	46,5	7980	484	13,1	410	51,8	2,97	2,59
36	360	110	7,5	12,6	53,4	10820	601	14,2	513	61,7	3,10	2,68
40	400	115	8,0	13,5	61,5	15220	761	15,7	642	73,4	3,23	2,75

Annex 3



Equal leg angle steel rolled steel. Assortment. ГОСТ 8509-86

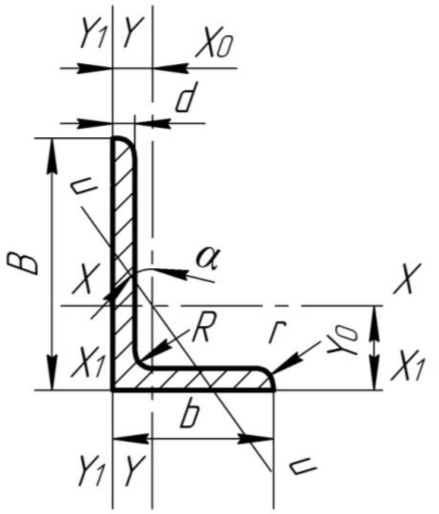
Shape numbers	Dimensios			Section area A, sm^2	Reference values for axes		
					$x-x$		z_0
	b	d	R		J_x	i_x	
	mm				sm^4	sm	sm
1	2	3	4	5	6	7	8
2	20{	3 4	3,5	1,13 1,46	0,40 0,50	0,59 0,58	0,60 0,64
2,5	25{	3 4	3,5	1,43 1,86	0,81 1,03	0,75 0,74	0,73 0,76
2,8	28	3	4	1,62	1,16	0,85	0,80
3,2	32{	3 4	4,5	1,86 2,43	1,77 2,26	0,97 0,96	0,89 0,94
3,6	36{	3 4	4,5	2,10 2,75	2,56 3,29	1,10 1,09	0,99 1,04
4	40{	3 4	5	2,35 3,08	3,55 4,58	1,23 1,22	1,09 1,13
4,5	45{	3 4 5	5	2,65 3,48 4,29	5,13 6,63 8,03	1,39 1,38 1,37	1,21 1,26 1,30
5	50{	3 4 5	5,5	2,96 3,89 4,80	7,11 9,21 11,2	1,55 1,54 1,53	1,33 1,38 1,42
5,6	56{	3,5 4 5	6	3,86 4,38 5,41	11,6 13,1 16,0	1,73 1,73 1,72	1,50 1,52 1,57
6,3	63{	4 5 6	7	4,96 6,13 7,28	18,9 23,1 27,1	1,95 1,94 1,93	1,69 1,74 1,78

Annex 3 (continued)

Equal leg angle steel rolled steel. Assortment. ГОСТ 8509-86 (continued)

1	2	3	4	5	6	7	8
7	70	4,5	8,0	6,20	29,0	2,16	1,88
		5		6,86	31,9	2,16	1,90
		6		8,15	37,6	2,15	1,94
		7		9,42	43,0	2,14	1,99
		8		10,7	48,2	2,13	2,02
7,5	75	5	9	7,39	39,5	2,31	2,02
		6		8,78	46,6	2,30	2,06
		7		10,1	53,3	2,29	2,10
		8		11,5	59,8	2,28	2,15
		9		12,8	66,1	2,27	2,18
8	80	5,5	9	8,63	52,7	2,47	2,17
		6		9,38	57,0	2,47	2,19
		7		10,8	65,3	2,45	2,23
		8		12,3	73,4	2,44	2,27
9	90	6	10	10,6	82,1	2,78	2,43
		7		12,3	94,3	2,77	2,47
		8		13,9	106	2,76	2,51
		9		15,6	118	2,75	2,55
10	100	6,5	12	12,8	122	3,09	2,68
		7		13,8	131	3,08	2,71
		8		15,6	147	3,07	2,75
		10		19,2	179	3,05	2,83
		12		22,8	209	3,03	2,91
		14		26,3	237	3,00	2,99
		16		29,7	264	2,98	3,06
11	110	7	12	15,2	176	3,40	2,96
		8		17,2	198	3,39	3,00
12,5	125	8	14	19,7	294	3,87	3,46
		9		22,0	327	3,86	3,40
		10		24,3	360	3,85	3,45
		12		28,9	422	3,82	3,53
		14		33,4	482	3,80	3,61
		16		37,8	539	3,78	3,68
14	140	9	14	24,7	466	4,34	3,78
		10		27,3	512	4,33	3,82
		12		32,5	602	4,31	3,90

Annex 4



Unequal leg angle rolled steel. Assortment. ГОСТ 8510-86

Shape numbers	Dimensios				Section area A, sm^2	Reference values for axes					
						$x-x$		$y-y$		x_1-x_1	y_1-y_1
										Distance from the center of gravity	
	B	b	d	R		J_x	i_x	J_y	i_y	y_0	x_0
	mm					sm^4	sm	sm^4	sm	sm	sm
1	2	3	4	5	6	7	8	9	10	11	12
2,5/1,6	25	16	3	3,5	1,16	0,70	0,78	0,22	0,44	0,86	0,42
3,2/2	32	20	3	3,5	1,49	1,52	1,01	0,46	0,55	1,08	0,49
			4		1,94	1,93	1,00	0,57	0,54	1,12	0,53
4/2,5	40	25	3	4,0	1,89	3,06	1,27	0,93	0,70	1,32	0,59
			4		2,47	3,93	1,26	1,18	0,69	1,37	0,63
4,5/2,8	45	28	3	5	2,14	4,41	1,43	1,32	0,79	1,47	0,64
			4		2,80	5,68	1,42	1,69	0,78	1,51	0,68
5/3,2	50	32	3	5,5	2,42	6,17	1,60	1,99	0,91	1,60	0,72
			4		3,17	7,98	1,59	2,56	0,90	1,65	0,76
5,6/3,6	56	36	3,5	6,0	3,16	10,1	1,79	3,30	1,02	1,80	0,82
			4		3,58	11,4	1,78	3,70	1,02	1,82	0,84
			5		4,41	13,8	1,77	4,48	1,01	1,86	0,88
6,3/4,0	63	40	4	7,0	4,04	16,3	2,01	5,16	1,13	2,03	0,91
			5		4,98	19,9	2,00	6,26	1,12	2,08	0,95
			6		5,90	23,3	1,99	7,28	1,11	2,12	0,99
			8		7,68	29,6	1,96	9,15	1,09	2,20	1,07

Annex 4 (continued)

Unequal leg angle rolled steel**.** Assortment. ГОСТ 8510-86 continued)

1	2	3	4	5	6	7	8	9	10	11	12
7/4,5	70	45{	4,5 5	7,5	5,07 5,59	25,3 27,8	2,23 2,23	8,25 9,05	1,28 1,27	2,25 2,28	1,03 1,05
7,5/5	75	50{	5 6 8	8	6,11 7,25 9,47	34,8 40,9 52,4	2,39 2,38 2,35	12,5 14,6 18,5	1,43 1,42 1,40	2,39 2,44 2,52	1,17 1,21 1,29
8/5	80	50{	5 6	8	6,36 7,55	41,6 49,0	2,56 2,55	12,7 14,8	1,41 1,40	2,6 2,65	1,13 1,17
9/5,6	90	56{	5,5 6 8	9	7,86 8,54 11,18	65,3 70,6 90,9	2,88 2,88 2,85	19,7 21,2 27,1	1,58 1,58 1,56	2,92 2,95 3,04	1,26 1,28 1,36
10/6,3	100	63{	6 7 8 10	10	9,59 11,1 12,6 15,5	98,3 113 127 154	3,2 3,19 3,18 3,15	30,6 35,0 39,2 47,1	1,79 1,78 1,77 1,75	3,23 3,28 3,32 3,40	1,42 1,46 1,50 1,58
11/7	110	70{	6,5 7 8	10	11,4 12,3 13,9	142 152 172	3,53 3,52 3,51	45,6 48,7 54,6	2 1,99 1,98	3,55 3,57 3,61	1,58 1,6 1,54
12,5/8	125	80{	7 8 10 12	11	14,1 16 19,7 23,4	227 256 312 356	4,01 4 3,98 3,95	73,7 83,0 100 117	2,29 2,28 2,26 2,24	4,01 4,05 4,14 4,22	1,8 1,84 1,92 2
14/9	140	90{	8 10	12	18 22,2	364 444	4,49 4,47	120 146	2,58 2,56	4,49 4,58	2,02 2,12

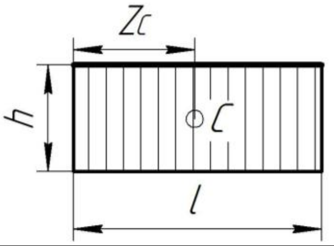
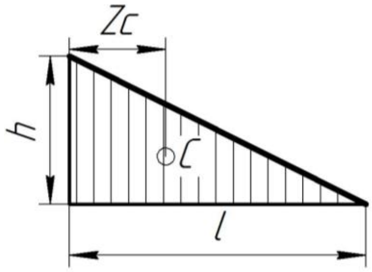
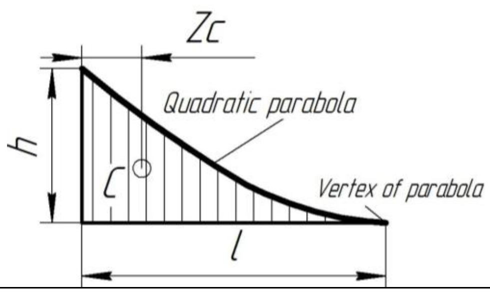
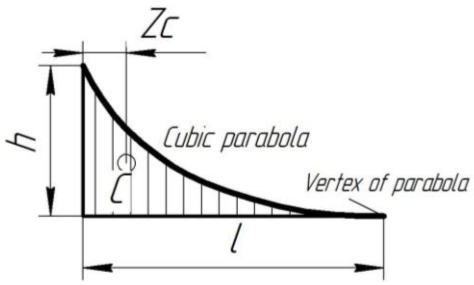
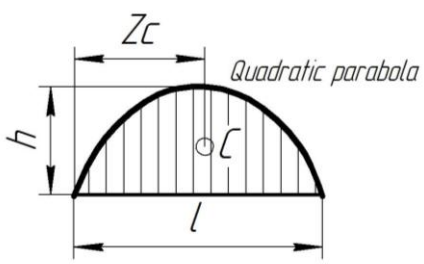
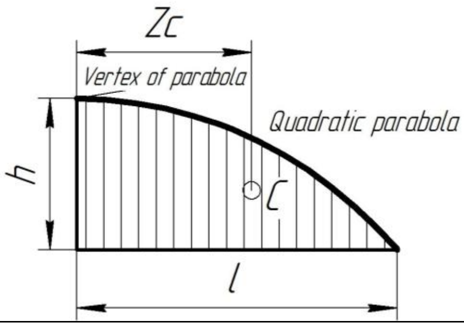
Annex 5

Coefficients φ of reduction of the main allowable stress

Flexibility of rods $\lambda = \frac{\mu \cdot l}{i}$	Steel types St.OC, St.2, St.3, St.4	Steel St.5	Steel	Cast iron	Wood
0	1,00	1,00	1,00	1,00	1,00
10	0,99	0,98	0,97	0,97	0,99
20	0,96	0,95	0,95	0,91	0,97
30	0,94	0,92	0,91	0,81	0,93
40	0,92	0,89	0,87	0,69	0,87
50	0,89	0,86	0,83	0,57	0,80
60	0,86	0,82	0,79	0,44	0,71
70	0,81	0,76	0,72	0,34	0,60
80	0,75	0,70	0,65	0,26	0,48
90	0,69	0,62	0,55	0,20	0,38
100	0,60	0,51	0,43	0,16	0,31
110	0,52	0,43	0,35	—	0,25
120	0,45	0,36	0,30	—	0,22
130	0,40	0,33	0,26	—	0,18
140	0,36	0,29	0,23	—	0,16
150	0,32	0,26	0,21	—	0,14
160	0,29	0,24	0,19	—	0,12
170	0,26	0,21	0,17	—	0,11
180	0,23	0,19	0,15	—	0,10
190	0,21	0,17	0,14	—	0,09
200	0,19	0,16	0,13	—	0,08

Annex 6

Areas ω and coordinates z_C of the gravity center of simple figures

Figure	ω	z_C
	$l \cdot h$	$\frac{1}{2}l$
	$\frac{1}{2}l \cdot h$	$\frac{1}{2}l$
	$\frac{1}{3}l \cdot h$	$\frac{1}{4}l$
	$\frac{1}{4}l \cdot h$	$\frac{1}{5}l$
	$\frac{2}{3}l \cdot h$	$\frac{1}{2}l$
	$\frac{2}{3}l \cdot h$	$\frac{3}{8}l$

MAIN DEFINITIONS OF STRENGTH OF MATERIALS

Strength of materials is the science of engineering methods for calculating the strength, rigidity and durability of machines and structures elements.

Strength is the ability of material or structure to withstand mechanical stress without fracture.

Rigidity is the ability of the structure and its elements to withstand elastic deformations, i. e. the ability to perceive external loading without changing the geometric dimensions and shape.

Durability is the ability of the structure or its elements to retain, under the action of given forces, the initial shape of the elastic equilibrium.

Rod (bar) is a body of prismatic shape where one size (length) is much bigger than the other two (transverse) dimensions.

Plate is the prismatic (cylindrical) body in which one size (thickness) is much smaller than two others.

Shell is a body restricted by two curvilinear surfaces, the distance between which (thickness) is small in comparison with other dimensions. This is a plate with curved middle surface. Examples: walls of thin-walled tanks, walls of boilers, domes of building structures, hulls of aircrafts, rockets, submarines.

Solid (massive body) is the body which dimensions are of the same order in all (three) directions. Examples: foundations of structures, retaining walls, foundations of powerful presses and machine tools.

Calculation scheme is the real object, free of insignificant features. More than one calculation scheme may be developed for the same object, depending on the load features and operating conditions.

Tensile-compressive is a type of deformation in which only ***longitudinal*** **(*axial*)** ***force*** N occurs in the cross sections of a straight bar.

Shear. Is the type of deformation, in which the cross-section of the rod (bar) only ***shear*** **(*cutting*)** ***force*** Q acts. The shear deformation results in material fracture. Rivets, bolts, keys, seams of welded joints undergo shear.

Torsion is type of deformation in which only ***torque moment*** M_{TR} , acts in the cross sections of the rod. The circular cross-section rod (bar) transmiting power during rotational motion is called the shaft. Torsion is often accompanied by bending or other deformation.

Direct lateral bending is type of deformation in which ***the bending moment*** M_{BN} and the shear (cutting) force Q occur at the cross sections of the beam. The bending rod (bar) is called the beam. This bending occurs in axes, bridge and floor beams, gear-wheel teeth, leaf springs.

Complex strength is the combination of two or more simple types of deformation, such as: ***bending + torsion; compression + bending***, etc.

Diagram is the graph showing the distribution of internal forces factors or displacements along the axis of the rod. **Diagrams are lined perpendiculary to the axis of the rod (bar).**

The tangential (*shear*) stress is the intensity of the tangent forces at the given point of section.

The normal stress is the intensity of normal forces at the given point of section.

Tension (compression) is the type of deformation (type of resistance) in which only **longitudinal (axial, normal) force** N or N_X directed along the axis of the rod (bar) and applied at the center of cross-section gravity occurs.

The limit of proportionality, in this section deformation is proportional to the load, the highest stress, at which Hooke law is correct.

The limit of elasticity, up to this stress the material retains its elastic properties (no residual deformations occur in the sample at load removal).

The yield strength is the stress at which the increase of plastic deformation of the sample at constant load occurs, this is the main mechanical characteristic for evaluation the durability of plastic materials (steels).

The tensile strength is the stress at which the fracture of the sample material occurs, that is, the conditional stress that corresponds to the highest load that the sample can withstand up to fracture.

The allowable stresses are those in which the safe work of the part is guaranteed.

The static moment of the plane figure area with respect to the axis lying in the same plane is the sum of the products of the areas of elementary planes at their distance from that axis.

The axial moment of inertia of a plane figure with respect to the axis lying in the same plane is the sum products over the whole area by the elementary areas squared by their distance from that axis.

Central axes are the axes that pass through the center of gravity of the plane figure.

The polar moment of inertia of the plane figure with respect to the pole lying in the same plane is the sum of the product of the areas of the elementary plane by the squares of their distances from the pole.

Main axes of inertia are axes in relation to which the axial moments of inertia of the section (plane figure) reach the maximum and minimum values.

The main moments of inertia of the section are the axial moments of inertia relatively to the principal axes.

The principal central axes are the main axes that pass through the center of gravity of the section (plane figure). If the figure has at least one axis of symmetry, then this axis will always be one of the main central axes.

The main central moments of inertia of the section (plane figure) are the moments of inertia with respect to the principal central axes.

Shear is a type of deformation in which at any cross-section of the bar only shear (cutting) force Q acts.

The shear deformation resulting in material fracture is **shear**.

Torsion is a type of deformation in which ***only torque moment*** M_{TR} occurs at any cross-section of the bar.

The circular cross-section bar, which operates for torsional deformation, is called the **shaft**.

The torque diagram is the graph showing the law of torque change along the bar length is called.

Complex stressed condition. The set of normal and tangential (shear) stresses occurring on planes crossing the given point characterize the stressed of the body at that point.

Bending is the bar resistance state in which bending or change of the curvature of its axis occurs. The bar that works in bending is called the ***beam***.

Flat, or **straight**, **bending** is the case of bending in which the beam axis is curved in the direction of external forces and loads, i.e. in the same plane with external forces.

Straight transverse bending is a type of deformation in which the **shear (cutting) force** Q and **bending moment** M_{BN} occur in the cross-sections of the beam. If the shear force does not occur, then it is the **pure bending**.

All forces, active and reactive are the beam loads.

Shear (**cutting**) **force** at any cross-section of the beam is equal to the algebraic sum of the projections of all external forces acting on the right or left of the intersection on the axis perpendicular to the axis of the beam.

Bending moment at any cross section of the beam is equal to the algebraic sum of the moments of all external forces acting to the right or left of the intersection relatively to the center of gravity of the section.

Linear displacement $y_A = y(x_A)$ of the gravity center of the section in the direction perpendicular to the undeformed axis of the beam, which is referred to as deflection.

Angular displacement $\Theta_A = \Theta(x_A)$ is a slope of the elastic curve around the neutral axis of the section relative to its initial position.

Redundant (auxiliary) beam is a given beam without external loads.

Statically indeterminate systems are systems in which the reactions of junctions and internal forces are impossible to determine by the equilibrium equations only.

The main system is statically determinate geometrically unchangeable system made of statically indetermitate one is defined.

Oblique bending is a complex type of deformation. It occurs when the plane of absolute bending moment action does not coincide with any of its main planes, i.e. planes drawn through the beam axis and the main axis of cross-cut inertia.

The neutral (zero) **section line is a geometric place of the points where normal stresses equal zero.** This line must run through the weight centre of the cross-cut.

Joint action of bending with torsion is a type of resistance to combined stress in which external forces acting on the beam cause the following internal force factors: ***torque, bending moments and shear*** **(*cutting*)** ***forces***.

The critical force is the largest value of the compressive force applied centrally, to which the rectilinear form of equilibrium of the rod is stable. The bend caused by the loss of stability of the rectilinear shaped rod is called ***the longitudinal bend***.

The smallest value of the compressive force at which the rod loses the ability to keep a rectilinear shape is called critical and is indicated F_{CR} .

Dynamic load ***is load which is partially or completely caused by the forces of inertia*** (at accelerated movement of parts, during their rotation and oscillation), as well as ***at instantaneous load and impact.***

Mechanical characteristic of material, which reflects its ability to resist impact loads, is called impact viscosity.

MAIN FORMULAS OF STRENGTH OF MATERIALS

Hooke law

$$E = \sigma / \varepsilon .$$

Tensile-compression strength condition

$$\sigma = \frac{N}{A} \le [\sigma].$$

Condition of shearing strength

$$\tau = \frac{Q}{A} \le [\tau]_{ss} .$$

Hooke's shear law

$$\tau = G \cdot \gamma .$$

Condition of tensile strength (torsion)

$$\tau_{\max} = \frac{M_{TR}}{W_P} \le [\tau].$$

Hooke's shear law (torsion)

$$\varphi = \frac{M_{TR} \cdot l}{G \cdot I_P} \; [\text{rad}]; \qquad \varphi = \frac{M_{TR} \cdot l}{G \cdot I_P} \cdot \frac{180^\circ}{\pi} \; [\text{degree}].$$

Condition of rigidity of the shaft at rotation (torsion)

$$\theta = \frac{M_{TR}}{G \cdot I_P} \cdot \frac{180^\circ}{\pi} \le [\theta].$$

Bending strength condition under normal stresses

$$\sigma_{\max} = \frac{M_{BN.\max}}{W_X} \le [\sigma] .$$

Tensile strength condition D.I. Zhuravsky formula (bending)

$$\tau = \frac{Q_y \cdot S_X(y)}{b(y) \cdot I_X}.$$

Strength conditions under bending according to shear stresses

$$\tau_{\max} = \frac{Q_{y\max} \cdot S_X(y)_{\max}}{b(y) \cdot I_X} \le [\tau] .$$

Approximate differential equation of the bent axis of the beam

$$E \cdot I_0 \cdot y''(x) = M_{BN}(x) .$$

Equation of the curved axis of the beam

$$y(x) = \frac{1}{E \cdot I} \int dx \int M(x) dx + C \cdot x + D .$$

Mohr integral which spans all the length of the beam

$$\Delta(y,\ \Theta) = \sum \int_l \frac{M(x) \cdot \overline{M}(x)}{E \cdot I} dx$$

General formula for determining displacements under bending

$$\Delta(y, \Theta) = \sum \frac{1}{E \cdot I} \int_l M(x) \cdot \overline{M}(x) dx = \sum_{i=1}^{n} \frac{\omega_i \cdot \overline{M}_{Ci}}{E \cdot I}$$

Writing down the equation of strain compatability (continuity of displacements) **for the equivalent system** as **the condition of zero equality of displacements by the direction of any removed junction**

$$\Delta_i = X_1 \cdot \delta_{i1} + X_2 \cdot \delta_{i2} + ... + X_{in} \cdot \delta_{in} + \Delta_i(F, q, M) = 0 .$$

System of canonic (the simplest) equations:

$$\cdot \begin{cases} X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} + \cdots + X_n \cdot \delta_{1n} + \Delta_1(F) = 0; \\ X_2 \cdot \delta_{21} + X_2 \cdot \delta_{22} + \cdots + X_n \cdot \delta_{2n} + \Delta_2(F) = 0; \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ X_1 \cdot \delta_{n1} + X_2 \cdot \delta_{n2} + \cdots + X_n \cdot \delta_{nn} + \Delta_n(F) = 0. \end{cases}$$

Normal stress at oblique bending at any cross-cut point, e.g. at point C with coordinates x_C and y_C, **is found as algebraic sum of normal stresses** from the components of the bending moment M_X and M_Y,

$$\sigma_{Z\ sum} = \sigma_Z(M_X) + \sigma_Z(M_Y) = -\left(\frac{M_X}{I_X}\cdot y_C + \frac{M_Y}{I_Y}\cdot x_C\right).$$

Equation of the neutral line at oblique bending

$$\frac{M_X}{I_X}\cdot y_0 + \frac{M_Y}{I_Y}\cdot x_0 = 0\ .$$

Stresses at oblique bending

$$\sigma_{\substack{\max \\ \min}} = \pm\left(\frac{M_X}{W_X} + \frac{M_Y}{W_Y}\right)\ .$$

Strength condition by normal stresses at oblique bending is

$$\sigma_{\max} = \frac{M_X}{W_X} + \frac{M_Y}{W_Y} \le [\sigma].$$

Shearing stresses at oblique bending are determined as a sum of shearing stresses τ_X, τ_Y obtained from the cross-cut forces Q_X, Q_Y

$$\tau = \sqrt{\tau_X^2 + \tau_Y^2}\ .$$

Absolute bending of the beam

$$f = \sqrt{f_X^2 + f_Y^2}\ .$$

Direction of absolute bending is determined by angle

$$\gamma = arctg\left(\frac{f_X}{f_Y}\right).$$

Condition of strength under the joint action of bending with torsion

$$\sigma_{eqv} = \frac{M_R}{W_0} \le [\sigma].$$

According to the third theory of strength (maximum tangential stresses)

$$M_R = \sqrt{M_{BN}^2 + M_{TR}^2}.$$

According to the third theory of strength (energetic)

$$M_R = \sqrt{M_{BN}^2 + 0,75\,M_{TR}^2}.$$

Euler's formula

$$\sigma_{CR} \le \sigma_{pr}, \qquad \sigma_{CR} = \frac{\pi^2 \cdot E \cdot I_{\min}}{(\mu \cdot l)^2 \cdot A} = \frac{\pi^2 \cdot E}{\lambda_{\max}^2} \le \sigma_{pr},$$

$$F_{CR} = \frac{\pi^2 E \cdot I_{\min}}{(\mu \cdot l)^2}.$$

Yasinsky's formula

$$\sigma_{CR} = a - b \cdot \lambda_{\max}.$$

Maximum flexibility of the rod

$$\lambda_{\max} = \frac{\mu \cdot l}{i_{\min}}.$$

Allowable stability stress

$$[\sigma_{ST}] = \sigma_{CR} / n_{cm}.$$

The condition of stability

$$\sigma = \frac{F_{\max}}{\varphi \cdot A} \le [\sigma], \qquad \sigma = \frac{F_{\max}}{A} \le [\sigma_{ST}].$$

PERSONALITIES

Robert Hooke

Robert Hooke (28 July [O.S. 18 July] 1635 – 3 March 1703) was an English scientist and architect, a polymath, recently called "England's Leonardo", who, using a microscope, was the first to visualize a microorganism. An impoverished scientific inquirer in young adulthood, he found wealth and esteem by performing over half of the architectural surveys after London's great fire of 1666. Hooke was also a member of the Royal Society, by now the world's oldest continuously operating scientific society, and since 1662 was its curator of experiments. Hook was also the Professor of Geometry at Gresham College.

As an assistant to physician Thomas Willis and to physical scientist Robert Boyle, Hooke built the vacuum pumps used in Boyle's experiments on gas law, and himself conducted experiments. In 1673, Hooke built the earliest Gregorian telescope, and then he observed the rotations of the planets Mars and Jupiter. Hooke's 1665 book Micrographia spurred microscopic investigations. Thus observing microscopic fossils, Hooke endorsed biological evolution. Investigating in optics, specifically light refraction, he inferred a wave theory of light. And his is the first recorded hypothesis of heat expanding matter, air's composition by small particles at larger distances, and heat as energy.

In physics, he approximated experimental confirmation that gravity heeds an inverse square law, and first hypothesised such a relation in planetary motion, too, a principle furthered and formalised by Isaac Newton in Newton's law of universal gravitation. Priority over this insight contributed to the rivalry between Hooke and Newton, who thus antagonized Hooke's legacy. In geology and paleontology, Hooke originated the theory of a terraqueous globe, disputed the literally Biblical view of the Earth's age, hypothesised the extinction of organism species, and argued that fossils atop hills and mountains had become elevated by geological processes. Hooke's pioneering work in land surveying and in mapmaking aided development of the first modern plan-form map, although his grid-system plan for London was rejected in favour of rebuilding along existing routes. Even so, Hook was key in devising for London a set of planning controls that remain influential.

Life and works

Much of what is known of Hooke's early life comes from an autobiography that he commenced in 1696 but never completed. Richard Waller mentions it in his introduction to The Posthumous Works of Robert Hooke, M.D. S.R.S., printed in 1705. In the chapter Of Dr. Dee's Book of Spirits, Hooke argues that John Dee made use of Trithemian steganography, to conceal his communication with Queen Elizabeth I. The work of Waller, along with John Ward's Lives of the Gresham Professors (with a list of his major works) and John Aubrey's Brief Lives, form the major near-contemporaneous biographical accounts of Hooke.

Early life

Robert Hooke was born in 1635 in Freshwater on the Isle of Wight to Cecily Gyles and John Hooke, a Church of England priest, the curate of Freshwater's Church of All Saints. Father John Hooke's two brothers, Robert's paternal uncles, were also ministers. A royalist, John Hooke likely was among a group that went to pay respects to Charles I as he escaped to the Isle of Wight. Expected to join the church, Robert, too, would become a staunch monarchist. Robert was the youngest, by seven years, of four siblings, two boys and two girls. Their father led a local school as well, yet at least partly homeschooled Robert, frail in health. The young Robert Hooke was fascinated by observation, mechanical works, and drawing. He dismantled a brass clock and built a wooden replica that reportedly worked "well enough". He made his own drawing materials from coal, chalk, and ruddle (iron ore).

On his father's death in 1648, Robert inherited 40 pounds. With it, he bought an apprenticeship. Although he went to London to begin apprenticeship, he studied briefly with Samuel Cowper and Peter Lely, and soon entered Westminster School, in London, under Dr. Richard Busby. Hooke quickly mastered Latin and Greek, studied Hebrew some, mastered Euclid's Elements, and began his lifelong study of mechanics.

Hooke may have been among a group of students that Busby taught in parallel to the school's main courses. Contemporary accounts call him "not much seen" in school, apparently true of others positioned similarly. Busby, an ardent and outspoken royalist (he had the school observe a fast-day on the anniversary of the King's beheading), was by all accounts trying to preserve the nascent spirit of scientific inquiry that had begun to flourish in Carolean England but which was at odds with the literal Biblical teachings of the Protectorate. To Busby and his select students the Anglican Church was a framework to support the spirit of inquiry into God's work, those who were able were destined by God to explore and study His creation, and the priesthood functioned as teachers to explain it to those who were less able. This was exemplified in the person of George Hooper, the Bishop of Bath and Wells, whom Busby described as "the best scholar, the finest gentleman and will make the completest bishop that ever was educated at Westminster School".

Science

Mechanics

In 1660, Hooke discovered the law of elasticity which bears his name and which describes the linear variation of tension with extension in an elastic spring. He first described this discovery in the anagram "ceiiinosssttuv", whose solution he published in 1678 as "Ut tensio, sic vis" meaning "As the extension, so the force." Hooke's work on elasticity culminated, for practical purposes, in his development of the balance spring or hairspring, which for the first time enabled a portable timepiece – a watch – to keep time with reasonable accuracy. A bitter dispute between Hooke and Christiaan Huygens on the priority of this invention was to continue for centuries after the death of both; but a note dated 23 June 1670 in the Hooke Folio (see External linksbelow), describing a demonstration of a balance-controlled watch before the Royal Society, has been held to favour Hooke's claim.

It is interesting[to whom?] from a twentieth-century vantage point that Hooke first announced his law of elasticity as an anagram. This was a method sometimes used by scientists, such as Hooke, Huygens, Galileo, and others, to establish priority for a discovery without revealing details.

Hooke became Curator of Experiments in 1662 to the newly founded Royal Society, and took responsibility for experiments performed at its weekly meetings. This was a position he held for over 40 years. While this position kept him in the thick of science in Britain and beyond, it also led to some heated arguments with other scientists, such as Huygens (see above) and particularly with Isaac Newton and the Royal Society's Henry Oldenburg. In 1664 Hooke also was appointed Professor of Geometry at Gresham College in London and Cutlerian Lecturer in Mechanics.

On 8 July 1680, Hooke observed the nodal patterns associated with the modes of vibration of glass plates. He ran a bow along the edge of a glass plate covered with flour, and saw the nodal patterns emerge. In acoustics, in 1681 he showed the Royal Society that musical tones could be generated from spinning brass cogs cut with teeth in particular proportions.

Blaise Pascal

Blaise Pascal (19 June 1623 – 19 August 1662) was a French mathematician, physicist, inventor, writer and Catholic theologian. He was a child prodigy who was educated by his father, a tax collector in Rouen. Pascal's earliest work was in the natural and applied sciences, where he made important contributions to the study of fluids, and clarified the concepts of pressure and vacuum by generalising the work of Evangelista Torricelli. Pascal also wrote in defence of the scientific method.

In 1642, while still a teenager, he started some pioneering work on calculating machines. After three years of effort and 50 prototypes, he built 20 finished machines (called Pascal's calculators and later Pascalines) over the following 10 years, establishing him as one of the first two inventors of the mechanical calculator.

Pascal was an important mathematician, helping create two major new areas of research: he wrote a significant treatise on the subject of projective geometry at the age of 16, and later corresponded with Pierre de Fermat on probability theory, strongly influencing the development of modern economics and social science. Following Galileo Galilei and Torricelli, in 1647, he rebutted Aristotle's followers who insisted that nature abhors a vacuum. Pascal's results caused many disputes before being accepted.

In 1646, he and his sister Jacqueline identified with the religious movement within Catholicism known by its detractors as Jansenism. Following a religious experience in late 1654, he began writing influential works on philosophy and theology. His two most famous works date from this period: the Lettres provinciales and the Pensées, the former set in the conflict between Jansenists and Jesuits. In that year, he also wrote an important treatise on the arithmetical triangle. Between 1658 and 1659, he wrote on the cycloid and its use in calculating the volume of solids.

Throughout his life, Pascal was in frail health, especially after the age of 18; he died just two months after his 39th birthday.

Early life and education

Pascal was born in Clermont-Ferrand, which is in France's Auvergne region. He lost his mother, Antoinette Begon, at the age of three. His father, Étienne Pascal (1588–1651), who also had an interest in science and mathematics, was a local judge and member of the "Noblesse de Robe". Pascal had two sisters, the younger Jacqueline and the elder Gilberte.

In 1631, five years after the death of his wife, Étienne Pascal moved with his children to Paris. The newly arrived family soon hired Louise Delfault, a maid who eventually became an instrumental member of the family. Étienne, who never remarried, decided that he alone would educate his children, for they all showed extraordinary intellectual ability, particularly his son Blaise. The young Pascal showed an amazing aptitude for mathematics and science.

Particularly of interest to Pascal was a work of Desargues on conic sections. Following Desargues' thinking, the 16-year-old Pascal produced, as a means of proof, a short treatise on what was called the "Mystic Hexagram", Essai pour les coniques ("Essay on Conics") and sent it – his first serious work of mathematics – to Père Mersenne in Paris; it is known still today as Pascal's theorem. It states that if a hexagon is inscribed in a circle (or conic) then the three intersection points of opposite sides lie on a line (called the Pascal line).

Pascal's work was so precocious that Descartes was convinced that Pascal's father had written it. When assured by Mersenne that it was, indeed, the product of the son and not the father, Descartes dismissed it with a sniff: "I do not find it strange that he has offered demonstrations about conics more appropriate than those of the ancients," adding, "but other matters related to this subject can be proposed that would scarcely occur to a 16-year-old child."

In France at that time offices and positions could be – and were – bought and sold. In 1631, Étienne sold his position as second president of the Cour des Aides for 65,665 livres. The money was invested in a government bond which provided, if not a lavish, then certainly a comfortable income which allowed the Pascal family to move to, and enjoy, Paris. But in 1638 Richelieu, desperate for money to carry on the Thirty Years' War, defaulted on the government's bonds. Suddenly Étienne Pascal's worth had dropped from nearly 66,000 livres to less than 7,300.

Like so many others, Étienne was eventually forced to flee Paris because of his opposition to the fiscal policies of Cardinal Richelieu, leaving his three children in the care of his neighbour Madame Sainctot, a great beauty with an infamous past who kept one of the most glittering and intellectual salons in all France. It was only when Jacqueline performed well in a children's play with Richelieu in attendance that Étienne was pardoned. In time, Étienne was back in good graces with the cardinal and in 1639 had been appointed the king's commissioner of taxes in the city of Rouen – a city whose tax records, thanks to uprisings, were in utter chaos.

In 1642, in an effort to ease his father's endless, exhausting calculations, and recalculations, of taxes owed and paid (into which work the young Pascal had been recruited), Pascal, not yet 19, constructed a mechanical calculator capable of addition and subtraction, called Pascal's calculator or the Pascaline. Of the eight Pascalines known to have survived, four are held by the Musée des Arts et Métiers in Paris and one more by the Zwinger museum in Dresden, Germany, exhibit two of his original mechanical calculators. Although these machines are pioneering forerunners to a further 400 years of development of mechanical methods of calculation, and in a sense to the later field of computer engineering, the calculator failed to be a great commercial success. Partly because it was still quite cumbersome to use in practice, but probably primarily because it was extraordinarily expensive, the Pascaline became little more than a toy, and a status symbol, for the very rich both in France and elsewhere in Europe. Pascal continued to make improvements to his design through the next decade, and he refers to some 50 machines that were built to his design.

Philosophy of mathematics

Pascal's major contribution to the philosophy of mathematics came with his De l'Esprit géométrique ("Of the Geometrical Spirit"), originally written as a preface to a geometry textbook for one of the famous "Petites-Ecoles de Port-Royal" ("Little Schools of Port-Royal"). The work was unpublished until over a century after his death. Here, Pascal looked into the issue of discovering truths, arguing that the ideal of such a method would be to found all propositions on already established truths. At the same time, however, he claimed this was impossible because such established truths would require other truths to back them up – first principles, therefore, cannot be reached. Based on this, Pascal argued that the procedure used in geometry was as perfect as possible, with certain principles assumed and other propositions developed from them. Nevertheless, there was no way to know the assumed principles to be true.

Pascal also used De l'Esprit géométrique to develop a theory of definition. He distinguished between definitions which are conventional labels defined by the writer and definitions which are within the language and understood by everyone because they naturally designate their referent. The second type would be characteristic of the philosophy of essentialism. Pascal claimed that only definitions of the first type were important to science and mathematics, arguing that those fields should adopt the philosophy of formalism as formulated by Descartes.

In De l'Art de persuader ("On the Art of Persuasion"), Pascal looked deeper into geometry's axiomatic method, specifically the question of how people come to be convinced of the axioms upon which later conclusions are based. Pascal agreed with Montaigne that achieving certainty in these axioms and conclusions through human methods is impossible. He asserted that these principles can be grasped only through intuition, and that this fact underscored the necessity for submission to God in searching out truths.

Contributions to the physical sciences

Pascal's work in the fields of the study of hydrodynamics and hydrostatics centered on the principles of hydraulic fluids. His inventions include the hydraulic press (using hydraulic pressure to multiply force) and the syringe. He proved that hydrostatic pressure depends not on the weight of the fluid but on the elevation difference. He demonstrated this principle by attaching a thin tube to a barrel full of water and filling the tube with water up to the level of the third floor of a building. This caused the barrel to leak, in what became known as Pascal's barrel experiment.

By 1647, Pascal had learned of Evangelista Torricelli's experimentation with barometers. Having replicated an experiment that involved placing a tube filled with mercury upside down in a bowl of mercury, Pascal questioned what force kept some mercury in the tube and what filled the space above the mercury in the tube. At the time, most scientists contended that, rather than a vacuum, some invisible matter was present. This was based on the Aristotelian notion that creation was a thing of substance, whether visible or invisible; and that this substance was forever in motion. Furthermore, "Everything that is in motion must be moved by something," Aristotle declared. Therefore, to the Aristotelian trained scientists of Pascal's time, a vacuum was an impossibility. How so? As proof it was pointed out:

Light passed through the so-called "vacuum" in the glass tube.

Aristotle wrote how everything moved, and must be moved by something.

Therefore, since there had to be an invisible "something" to move the light through the glass tube, there was no vacuum in the tube. Not in the glass tube or anywhere else. Vacuums – the absence of any and everything – were simply an impossibility.

Following more experimentation in this vein, in 1647 Pascal produced Experiences nouvelles touchant le vide ("New experiments with the vacuum"), which detailed basic rules describing to what degree various liquids could be supported by air pressure. It also provided reasons why it was indeed a vacuum above the column of liquid in a barometer tube. This work was followed by Récit de la grande expérience de l'équilibre des liqueurs ("Account of the great experiment on equilibrium in liquids") published in 1648.

The Torricellian vacuum found that air pressure is equal to the weight of 30 inches of mercury. If air has a finite weight, Earth's atmosphere must have a maximum height. Pascal reasoned that if true, air pressure on a high mountain must be less than at a lower altitude. He lived near the Puy de Dôme mountain, 4,790 feet (1,460 m) tall, but his health was poor so could not climb it.

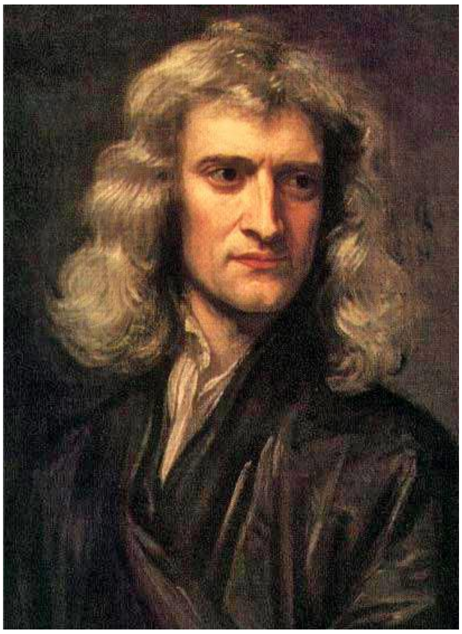
Legacy

In honour of his scientific contributions, the name Pascal has been given to the SI unit of pressure, to a programming language, and Pascal's law (an important principle of hydrostatics), and as mentioned above, Pascal's triangle and Pascal's wager still bear his name.

Pascal's development of probability theory was his most influential contribution to mathematics. Originally applied to gambling, today it is extremely important in economics, especially in actuarial science. John Ross writes, "Probability theory and the discoveries following it changed the way we regard uncertainty, risk, decision-making, and an individual's and society's ability to influence the course of future events." However, Pascal and Fermat, though doing important early work in probability theory, did not develop the field very far. Christiaan Huygens, learning of the subject from the correspondence of Pascal and Fermat, wrote the first book on the subject. Later figures who continued the development of the theory include Abraham de Moivre and Pierre-Simon Laplace.

In France, prestigious annual awards, Blaise Pascal Chairs are given to outstanding international scientists to conduct their research in the Ile de France region. One of the Universities of Clermont-Ferrand, France – Université Blaise Pascal – is named after him. The University of Waterloo, Ontario, Canada, holds an annual math contest named in his honour.

Sir Isaac Newton



Sir Isaac Newton (25 December 1642 – 20 March 1726/27[a]) was an English mathematician, physicist, astronomer, theologian, and author (described in his own day as a "natural philosopher") who is widely recognised as one of the most influential scientists of all time and as a key figure in the scientific revolution. His book Philosophiæ Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy), first published in 1687, laid the foundations of classical mechanics. Newton also made seminal contributions to optics, and shares credit with Gottfried Wilhelm Leibniz for developing the infinitesimal calculus.

In Principia, Newton formulated the laws of motion and universal gravitation that formed the dominant scientific viewpoint until it was superseded by the theory of relativity. Newton used his mathematical description of gravity to prove Kepler's laws of planetary motion, account for tides, the trajectories of comets, the precession of the equinoxes and other phenomena, eradicating doubt about the Solar System's heliocentricity. He demonstrated that the motion of objects on Earth and celestial bodies could be accounted for by the same principles. Newton's inference that the Earth is an oblate spheroid was later confirmed by the geodetic measurements of Maupertuis, La Condamine, and others, convincing most European scientists of the superiority of Newtonian mechanics over earlier systems.

Newton built the first practical reflecting telescope and developed a sophisticated theory of colour based on the observation that a prism separates white light into the colours of the visible spectrum. His work on light was collected in his highly influential book Opticks, published in 1704. He also formulated an empirical law of cooling, made the first theoretical calculation of the speed of sound, and introduced the notion of a Newtonian fluid. In addition to his work on calculus, as a mathematician Newton contributed to the study of power series, generalised the binomial theorem to non-integer exponents, developed a method for approximating the roots of a function, and classified most of the cubic plane curves.

Newton was a fellow of Trinity College and the second Lucasian Professor of Mathematics at the University of Cambridge. He was a devout but unorthodox Christian who privately rejected the doctrine of the Trinity. Unusually for a member of the Cambridge faculty of the day, he refused to take holy orders in the Church of England. Beyond his work on the mathematical sciences, Newton dedicated much of his time to the study of alchemy and biblical chronology, but most of his work in those areas remained unpublished until long after his death. Politically and personally tied to the Whig party, Newton served two brief terms as Member of Parliament for the University of Cambridge, in 1689–90 and 1701–02. He was knighted by Queen Anne in 1705 and spent the last three decades of his life in London, serving as Warden (1696–1700) and Master (1700–1727) of the Royal Mint, as well as president of the Royal Society (1703–1727).

Early life

Isaac Newton was born (according to the Julian calendar, in use in England at the time) on Christmas Day, 25 December 1642 (NS 4 January 1643[a]) "an hour or two after midnight", at Woolsthorpe Manor in Woolsthorpe-by-Colsterworth, a hamlet in the county of Lincolnshire. His father, also named Isaac Newton, had died three months before. Born prematurely, Newton was a small child; his mother Hannah Ayscough reportedly said that he could have fit inside a quart mug. When Newton was three, his mother remarried and went to live with her new husband, the Reverend Barnabas Smith, leaving her son in the care of his maternal grandmother, Margery Ayscough (née Blythe). Newton disliked his stepfather and maintained some enmity towards his mother for marrying him, as revealed by this entry in a list of sins committed up to the age of 19: "Threatening my father and mother Smith to burn them and the house over them." Newton's mother had three children (Mary, Benjamin and Hannah) from her second marriage.

From the age of about twelve until he was seventeen, Newton was educated at The King's School, Grantham, which taught Latin and Greek and probably imparted a significant foundation of mathematics. He was removed from school and returned to Woolsthorpe-by-Colsterworth by October 1659. His mother, widowed for the second time, attempted to make him a farmer, an occupation he hated. Henry Stokes, master at The King's School, persuaded his mother to send him back to school. Motivated partly by a desire for revenge against a schoolyard bully, he became the top-ranked student, distinguishing himself mainly by building sundials and models of windmills.

In June 1661, he was admitted to Trinity College, Cambridge, on the recommendation of his uncle Rev William Ayscough, who had studied there. He started as a subsizar – paying his way by performing valet's duties – until he was awarded a scholarship in 1664, guaranteeing him four more years until he could get his MA. At that time, the college's teachings were based on those of Aristotle, whom Newton supplemented with modern philosophers such as Descartes, and astronomers such as Galileo and Thomas Street, through whom he learned of Kepler's work. He set down in his notebook a series of "Quaestiones" about mechanical philosophy as he found it. In 1665, he discovered the generalised binomial theorem and began to develop a mathematical theory that later became calculus. Soon after Newton had obtained his BA degree in August 1665, the university temporarily closed as a precaution against the Great Plague. Although he had been undistinguished as a Cambridge student, Newton's private studies at his home in Woolsthorpe over the subsequent two years saw the development of his theories on calculus, optics, and the law of gravitation.

In April 1667, he returned to Cambridge and in October was elected as a fellow of Trinity. Fellows were required to become ordained priests, although this was not enforced in the restoration years and an assertion of conformity to the Church of England was sufficient. However, by 1675 the issue could not be avoided and by then his unconventional views stood in the way. Nevertheless, Newton managed to avoid it by means of special permission from Charles II.

His studies had impressed the Lucasian professor Isaac Barrow, who was more anxious to develop his own religious and administrative potential (he became master of Trinity two years later); in 1669 Newton succeeded him, only one year after receiving his MA. He was elected a Fellow of the Royal Society (FRS) in 1672.

Middle years. Mathematics

Newton's work has been said "to distinctly advance every branch of mathematics then studied." His work on the subject usually referred to as fluxions or calculus, seen in a manuscript of October 1666, is now published among Newton's mathematical papers. The author of the manuscript De analysi per aequationes numero terminorum infinitas, sent by Isaac Barrow to John Collins in June 1669, was identified by Barrow in a letter sent to Collins in August of that year as "[...] of an extraordinary genius and proficiency in these things."

Newton later became involved in a dispute with Leibniz over priority in the development of calculus (the Leibniz–Newton calculus controversy). Most modern historians believe that Newton and Leibniz developed calculus independently, although with very different mathematical notations. Occasionally it has been suggested that Newton published almost nothing about it until 1693, and did not give a full account until 1704, while Leibniz began publishing a full account of his methods in 1684. Leibniz's notation and "differential Method", nowadays recognised as much more convenient notations, were adopted by continental European mathematicians, and after 1820 or so, also by British mathematicians.

Such a suggestion fails to account for the calculus in Book 1 of Newton's Principia itself and in its forerunner manuscripts, such as De motu corporum in gyrum of 1684; this content has been pointed out by critics [Like whom?] of both Newton's time and modern times.

His work extensively uses calculus in geometric form based on limiting values of the ratios of vanishingly small quantities: in the Principia itself, Newton gave demonstration of this under the name of "the method of first and last ratios" and explained why he put his expositions in this form, remarking also that "hereby the same thing is performed as by the method of indivisibles."

Because of this, the Principia has been called "a book dense with the theory and application of the infinitesimal calculus" in modern times and in Newton's time "nearly all of it is of this calculus." His use of methods involving "one or more orders of the infinitesimally small" is present in his De motu corporum in gyrum of 1684 and in his papers on motion "during the two decades preceding 1684".

Newton had been reluctant to publish his calculus because he feared controversy and criticism. He was close to the Swiss mathematician Nicolas Fatio de Duillier. In 1691, Duillier started to write a new version of Newton's

Principia, and corresponded with Leibniz. In 1693, the relationship between Duillier and Newton deteriorated and the book was never completed.

Starting in 1699, other members [who?] of the Royal Society accused Leibniz of plagiarism. The dispute then broke out in full force in 1711 when the Royal Society proclaimed in a study that it was Newton who was the true discoverer and labelled Leibniz a fraud; it was later found that Newton wrote the study's concluding remarks on Leibniz. Thus began the bitter controversy which marred the lives of both Newton and Leibniz until the latter's death in 1716.

Newton is generally credited with the generalised binomial theorem, valid for any exponent. He discovered Newton's identities, Newton's method, classified cubic plane curves (polynomials of degree three in two variables), made substantial contributions to the theory of finite differences, and was the first to use fractional indices and to employ coordinate geometry to derive solutions to Diophantine equations. He approximated partial sums of the harmonic series by logarithms (a precursor to Euler's summation formula) and was the first to use power series with confidence and to revert power series. Newton's work on infinite series was inspired by Simon Stevin's decimals.

When Newton received his MA and became a Fellow of the "College of the Holy and Undivided Trinity" in 1667, he made the commitment that "I will either set Theology as the object of my studies and will take holy orders when the time prescribed by these statutes [7 years] arrives, or I will resign from the college." Up until this point he had not thought much about religion and had twice signed his agreement to the thirty-nine articles, the basis of Church of England doctrine.

He was appointed Lucasian Professor of Mathematics in 1669, on Barrow's recommendation. During that time, any Fellow of a college at Cambridge or Oxford was required to take holy orders and become an ordained Anglican priest. However, the terms of the Lucasian professorship required that the holder not be active in the church – presumably,[weasel words] so as to have more time for science. Newton argued that this should exempt him from the ordination requirement, and Charles II, whose permission was needed, accepted this argument. Thus a conflict between Newton's religious views and Anglican orthodoxy was averted.

Mechanics and gravitation

In 1679, Newton returned to his work on celestial mechanics by considering gravitation and its effect on the orbits of planets with reference to Kepler's laws of planetary motion. This followed stimulation by a brief exchange of letters in 1679–80 with Hooke, who had been appointed to manage the Royal Society's correspondence, and who opened a correspondence intended to elicit contributions from Newton to Royal Society transactions.

Newton's reawakening interest in astronomical matters received further stimulus by the appearance of a comet in the winter of 1680–1681, on which he corresponded with John Flamsteed. After the exchanges with Hooke, Newton worked out proof that the elliptical form of planetary orbits would result from a centripetal force inversely proportional to the square of the radius vector. Newton communicated his results to Edmond Halley and to the Royal Society in De motu corporum in gyrum, a tract written on about nine sheets which was copied into the Royal Society's Register Book in December 1684. This tract contained the nucleus that Newton developed and expanded to form the Principia.

The Principia was published on 5 July 1687 with encouragement and financial help from Edmond Halley. In this work, Newton stated the three universal laws of motion. Together, these laws describe the relationship between any object, the forces acting upon it and the resulting motion, laying the foundation for classical mechanics. They contributed to many advances during the Industrial Revolution which soon followed and were not improved upon for more than 200 years. Many of these advancements continue to be the underpinnings of non-relativistic technologies in the modern world. He used the Latin word gravitas (weight) for the effect that would become known as gravity, and defined the law of universal gravitation.

In the same work, Newton presented a calculus-like method of geometrical analysis using "first and last ratios", gave the first analytical determination (based on Boyle's law) of the speed of sound in air, inferred the oblateness of Earth's spheroidal figure, accounted for the precession of the equinoxes as a result of the Moon's gravitational attraction on the Earth's oblateness, initiated the gravitational study of the irregularities in the motion of the Moon, provided a theory for the determination of the orbits of comets, and much more.

Newton made clear his heliocentric view of the Solar System–developed in a somewhat modern way because already in the mid-1680s he recognised the "deviation of the Sun" from the centre of gravity of the Solar System. For Newton, it was not precisely the centre of the Sun or any other body that could be considered at rest, but rather "the common centre of gravity of the Earth, the Sun and all the Planets is to be esteem'd the Centre of the World", and this centre of gravity "either is at rest or moves uniformly forward in a right line" (Newton adopted the "at rest" alternative in view of common consent that the centre, wherever it was, was at rest).

Newton's postulate of an invisible force able to act over vast distances led to him being criticised for introducing "occult agencies" into science. Later, in the second edition of the Principia (1713), Newton firmly rejected such criticisms in a concluding General Scholium, writing that it was enough that the phenomena implied a gravitational attraction, as they did; but they did not so far indicate its cause, and it was both unnecessary and improper to frame hypotheses of things that were not implied by the phenomena. (Here Newton used what became his famous expression "hypotheses non-fingo"). With the Principia, Newton became internationally recognised. He acquired a circle of admirers, including the Swiss-born mathematician Nicolas Fatio de Duillier.

Later life

In the 1690s, Newton wrote a number of religious tracts dealing with the literal and symbolic interpretation of the Bible. A manuscript Newton sent to John Locke in which he disputed the fidelity of 1 John 5:7 the Johannine Comma and its fidelity to the original manuscripts of the New Testament, remained unpublished until 1785.

Newton was also a member of the Parliament of England for Cambridge University in 1689 and 1701, but according to some accounts his only comments were to complain about a cold draught in the chamber and request that the window be closed. He was, however, noted by Cambridge diarist Abraham de la Pryme to have rebuked students who were frightening locals by claiming that a house was haunted.

Newton moved to London to take up the post of warden of the Royal Mint in 1696, a position that he had obtained through the patronage of Charles Montagu, 1st Earl of Halifax, then Chancellor of the Exchequer. He took charge of England's great recoining, trod on the toes of Lord Lucas, Governor of the Tower, and secured the job of deputy comptroller of the temporary Chester branch for Edmond Halley. Newton became perhaps the best-known Master of the Mint upon the death of Thomas Neale in 1699, a position Newton held for the last 30 years of his life. These appointments were intended as sinecures, but Newton took them seriously. He retired from his Cambridge duties in 1701, and exercised his authority to reform the currency and punish clippers and counterfeiters.

As Warden, and afterwards as Master, of the Royal Mint, Newton estimated that 20 percent of the coins taken in during the Great Recoinage of 1696 were counterfeit. Counterfeiting was high treason, punishable by the felon being hanged, drawn and quartered. Despite this, convicting even the most flagrant criminals could be extremely difficult, however, Newton proved equal to the task.

Disguised as a habitué of bars and taverns, he gathered much of that evidence himself. For all the barriers placed to prosecution, and separating the branches of government, English law still had ancient and formidable customs of authority. Newton had himself made a justice of the peace in all the home counties. A draft letter regarding the matter is included in Newton's personal first edition of Philosophiæ Naturalis Principia Mathematica, which he must have been amending at the time. Then he conducted more than 100 cross-examinations of witnesses, informers, and suspects between June 1698 and Christmas 1699. Newton successfully prosecuted 28 coiners.

Newton was made President of the Royal Society in 1703 and an associate of the French Académie des Sciences. In his position at the Royal Society, Newton made an enemy of John Flamsteed, the Astronomer Royal, by prematurely publishing Flamsteed's Historia Coelestis Britannica, which Newton had used in his studies.

In April 1705, Queen Anne knighted Newton during a royal visit to Trinity College, Cambridge. The knighthood is likely to have been motivated by political considerations connected with the parliamentary election in May 1705, rather than any recognition of Newton's scientific work or services as Master of the Mint. Newton was the second scientist to be knighted, after Sir Francis Bacon.

As a result of a report written by Newton on 21 September 1717 to the Lords Commissioners of His Majesty's Treasury, the bimetallic relationship between gold coins and silver coins was changed by Royal proclamation on 22 December 1717, forbidding the exchange of gold guineas for more than 21 silver shillings. This inadvertently resulted in a silver shortage as silver coins were used to pay for imports, while exports were paid for in gold, effectively moving Britain from the silver standard to its first gold standard. It is a matter of debate as to whether he intended to do this or not. It has been argued that Newton conceived of his work at the Mint as a continuation of his alchemical work.

Newton was invested in the South Sea Company and lost some £20,000 (US$3 million in 2003) when it collapsed in around 1720. Toward the end of his life, Newton took up residence at Cranbury Park, near Winchester with his niece and her husband, until his death in 1727. His half-niece, Catherine Barton Conduitt, served as his hostess in social affairs at his house on Jermyn Street in London; he was her "very loving Uncle", according to his letter to her when she was recovering from smallpox.

Death

Newton died in his sleep in London on 20 March 1727 (OS 20 March 1726; NS 31 March 1727).[a] His body was buried in Westminster Abbey. Voltaire may have been present at his funeral. A bachelor, he had divested much of his estate to relatives during his last years, and died intestate. His papers went to John Conduitt and Catherine Barton. After his death, Newton's hair was examined and found to contain mercury, probably resulting from his alchemical pursuits. Mercury poisoning could explain Newton's eccentricity in late life.

Leonhard Euler

Leonhard Euler (15 April 1707 – 18 September 1783) was a Swiss mathematician, physicist, astronomer, geographer, logician and engineer who made important and influential discoveries in many branches of mathematics, such as infinitesimal calculus and graph theory, while also making pioneering contributions to several branches such as topology and analytic number theory. He also introduced much of the modern mathematical terminology and notation, particularly for mathema-tical analysis, such as the notion of a mathematical function. He is also known for his work in mechanics, fluid dynamics, optics, astronomy and music theory.

Euler was one of the most eminent mathematicians of the 18th century and is held to be one of the greatest in history. He is also widely considered to be the most prolific, as his collected works fill 92 volumes, more than anyone else in the field. He spent most of his adult life in Saint Petersburg, Russia, and in Berlin, then the capital of Prussia.

A statement attributed to Pierre-Simon Laplace expresses Euler's influence on mathematics: "Read Euler, read Euler, he is the master of us all."

Early life

Leonhard Euler was born on 15 April 1707, in Basel, Switzerland, to Paul III Euler, a pastor of the Reformed Church, and Marguerite née Brucker, another pastor's daughter. He had two younger sisters, Anna Maria and Maria Magdalena, and a younger brother, Johann Heinrich. Soon after the birth of Leonhard, the Eulers moved from Basel to the town of Riehen, Switzerland, where Leonhard spent most of his childhood. Paul was a friend of the Bernoulli family; Johann Bernoulli, then regarded as Europe's foremost mathematician, would eventually be the most important influence on young Leonhard.

Euler's formal education started in Basel, where he was sent to live with his maternal grandmother. In 1720, at age thirteen, he enrolled at the University of Basel. In 1723, he received a Master of Philosophy with a dissertation that compared the philosophies of Descartes and Newton. During that time, he was receiving Saturday afternoon lessons from Johann Bernoulli, who quickly discovered his new pupil's incredible talent for mathematics. At that time Euler's main studies included theology, Greek and Hebrew at his father's urging to become a pastor, but Bernoulli convinced his father that Leonhard was destined to become a great mathematician.

In 1726, Euler completed a dissertation on the propagation of sound with the title De Sono. At that time, he was unsuccessfully attempting to obtain a position at the University of Basel. In 1727, he first entered the Paris Academy Prize Problem competition; the problem that year was to find the best way to place the masts on a ship. Pierre Bouguer, who became known as "the father of naval architecture", won and Euler took second place. Euler later won this annual prize twelve times.

Career

Saint Petersburg

Around this time Johann Bernoulli's two sons, Daniel and Nicolaus, were working at the Imperial Russian Academy of Sciences in Saint Petersburg. On 31 July 1726, Nicolaus died of appendicitis after spending less than a year in Russia. When Daniel assumed his brother's position in the mathematics/physics division, he recommended that the post in physiology that he had vacated be filled by his friend Euler. In November 1726 Euler eagerly accepted the offer, but delayed making the trip to Saint Petersburg while he unsuccessfully applied for a physics professorship at the University of Basel.

Euler arrived in Saint Petersburg on 17 May 1727. He was promoted from his junior post in the medical department of the academy to a position in the mathematics department. He lodged with Daniel Bernoulli with whom he often worked in close collaboration. Euler mastered Russian and settled into life in Saint Petersburg. He also took on an additional job as a medic in the Russian Navy.

The Academy at Saint Petersburg, established by Peter the Great, was intended to improve education in Russia and to close the scientific gap with Western Europe. As a result, it was made especially attractive to foreign scholars like Euler. The academy possessed ample financial resources and a comprehensive library drawn from the private libraries of Peter himself and of the nobility. Very few students were enrolled in the academy to lessen the faculty's teaching burden. The academy emphasized research and offered to its faculty both the time and the freedom to pursue scientific questions.

The Academy's benefactress, Catherine I, who had continued the progressive policies of her late husband, died on the day of Euler's arrival. The Russian nobility then gained power upon the ascension of the twelve-year-old Peter II. The nobility, suspicious of the academy's foreign scientists, cut funding and caused other difficulties for Euler and his colleagues.

Conditions improved slightly after the death of Peter II, and Euler swiftly rose through the ranks in the academy and was made a professor of physics in 1731. Two years later, Daniel Bernoulli, who was fed up with the censorship and hostility he faced at Saint Petersburg, left for Basel. Euler succeeded him as the head of the mathematics department.

On 7 January 1734, he married Katharina Gsell (1707–1773), a daughter of Georg Gsell, a painter from the Academy Gymnasium. The young couple bought a house by the Neva River. Of their thirteen children, only five survived childhood.

Berlin

Concerned about the continuing turmoil in Russia, Euler left St. Petersburg on 19 June 1741 to take up a post at the Berlin Academy, which he had been offered by Frederick the Great of Prussia. He lived for 25 years in Berlin, where he wrote over 380 articles. In Berlin, he published the two works for which he would become most renowned: the Introductio in analysin infinitorum, a text on functions published in 1748, and the Institutiones calculi differentialis, published in 1755 on differential calculus. In 1755, he was elected a foreign member of the Royal Swedish Academy of Sciences.

In addition, Euler was asked to tutor Friederike Charlotte of Brandenburg-Schwedt, the Princess of Anhalt-Dessau and Frederick's niece. Euler wrote over 200 letters to her in the early 1760s, which were later compiled into a best-selling volume entitled Letters of Euler on different Subjects in Natural Philosophy Addressed to a German Princess. This work contained Euler's exposition on various subjects pertai-ning to physics and mathematics, as well as offering valuable insights into Euler's personality and religious beliefs. This book became more widely read than any of his mathematical works and was published across Europe and in the United States. The popularity of the "Letters" testifies to Euler's ability to communicate scientific matters effectively to a lay audience, a rare ability for a dedicated research scientist.

Despite Euler's immense contribution to the Academy's prestige, he eventually incurred the ire of Frederick and ended up having to leave Berlin. The Prussian king had a large circle of intellectuals in his court, and he found the mathematician unsophisticated and ill-informed on matters beyond numbers and figures. Euler was a simple, devoutly religious man who never questioned the existing social order or conventional beliefs, in many ways the polar opposite of Voltaire, who enjoyed a high place of prestige at Frederick's court. Euler was not a skilled debater and often made it a point to argue subjects that he knew little about, making him the frequent target of Voltaire's wit. Frederick also expressed disappointment with Euler's practical engineering abilities:

I wanted to have a water jet in my garden: Euler calculated the force of the wheels necessary to raise the water to a reservoir, from where it should fall back through channels, finally spurting out in Sanssouci. My mill was carried out geometrically and could not raise a mouthful of water closer than fifty paces to the reservoir. Vanity of vanities! Vanity of geometry!

Personal life

Eyesight deterioration

Euler's eyesight worsened throughout his mathematical career. In 1738, three years after nearly expiring from fever, he became almost blind in his right eye, but Euler rather blamed the painstaking work on cartography he performed for the St. Petersburg Academy for his condition. Euler's vision in that eye worsened throughout his stay in Germany, to the extent that Frederick referred to him as "Cyclops". Euler remarked on his loss of vision, "Now I will have fewer distractions." He later developed a cataract in his left eye, which was discovered in 1766. Just a few weeks after its discovery, a failed surgical restoration rendered him almost totally blind. He was 59 years old then. However, his condition appeared to have little effect on his productivity, as he compensated for it with his mental calculation skills and exceptional memory. For example, Euler could repeat the Aeneid of Virgil from beginning to end without hesitation, and for every page in the edition he could indicate which line was the first and which the last. With the aid of his scribes, Euler's productivity on many areas of study actually increased. He produced, on average, one mathematical paper every week in the year 1775. The Eulers bore a double name, Euler-Schölpi, the latter of which derives from schelb and schief, signifying squint-eyed, cross-eyed, or crooked. This suggests that the Eulers may have had a susceptibility to eye problems.

Return to Russia and death

In 1760, with the Seven Years' War raging, Euler's farm in Charlottenburg was ransacked by advancing Russian troops. Upon learning of this event, General Ivan Petrovich Saltykov paid compensation for the damage caused to Euler's estate, with Empress Elizabeth of Russia later adding a further payment of 4000 roubles – an exorbitant amount at the time. The political situation in Russia stabilized after Catherine the Great's accession to the throne, so in 1766 Euler accepted an invitation to return to the St. Petersburg Academy. His conditions were quite exorbitant – a 3000 ruble annual salary, a pension for his wife, and the promise of high-ranking appointments for his sons. All of these requests were granted. He spent the rest of his life in Russia. However, his second stay in the country was marred by tragedy. A fire in St. Petersburg in 1771 cost him his home, and almost his life. In 1773, he lost his wife Katharina after 40 years of marriage.

Three years after his wife's death, Euler married her half-sister, Salome Abigail Gsell (1723–1794). This marriage lasted until his death. In 1782 he was elected a Foreign Honorary Member of the American Academy of Arts and Sciences.

In St. Petersburg on 18 September 1783, after a lunch with his family, Euler was discussing the newly discovered planet Uranus and its orbit with a fellow academician Anders Johan Lexell, when he collapsed from a brain hemorrhage. He died a few hours later. Jacob von Staehlin-Storcksburg wrote a short obituary for the Russian Academy of Sciences and Russian mathematician Nicolas Fuss, one of Euler's disciples, wrote a more detailed eulogy, which he delivered at a memorial meeting. In his eulogy for the French Academy, French mathematician and philosopher Marquis de Condorcet, wrote:

il cessa de calculer et de vivre – ... he ceased to calculate and to live.

Euler was buried next to Katharina at the Smolensk Lutheran Cemetery on Goloday Island. In 1785, the Russian Academy of Sciences put a marble bust of Leonhard Euler on a pedestal next to the Director's seat and, in 1837, placed a headstone on Euler's grave. To commemorate the 250th anniversary of Euler's birth, the headstone was moved in 1956, together with his remains, to the 18th-century necropolis at the Alexander Nevsky Monastery.

Contributions to mathematics and physics

Euler worked in almost all areas of mathematics, such as geometry, infinitesimal calculus, trigonometry, algebra, and number theory, as well as continuum physics, lunar theory and other areas of physics. He is a seminal figure in the history of mathematics; if printed, his works, many of which are of fundamental interest, would occupy between 60 and 80 quarto volumes. Euler's name is associated with a large number of topics.

Euler is the only mathematician to have two numbers named after him: the important Euler's number in calculus, e, approximately equal to 2.71828, and the Euler–Mascheroni constant γ (gamma) sometimes referred to as just "Euler's constant", approximately equal to 0.57721. It is not known whether γ is rational or irrational.

Mathematical notation

Euler introduced and popularized several notational conventions through his numerous and widely circulated textbooks. Most notably, he introduced the concept of a function and was the first to write $f(x)$ to denote the function f applied to the argument x. He also introduced the modern notation for the trigonometric functions, the letter e for the base of the natural logarithm (now also known as Euler's number), the Greek letter Σ for summations and the letter i to denote the imaginary unit. The use of the Greek letter π to denote the ratio of a circle's circumference to its diameter was also popularized by Euler, although it originated with Welsh mathematician William Jones.

Jean-Baptiste le Rond d'Alembert

Jean-Baptiste le Rond d'Alembert (16 November 1717 – 29 October 1783) was a French mathematician, mechanician, physicist, philosopher, and music theorist. Until 1759 he was, together with Denis Diderot, a co-editor of the Encyclopédie. D'Alembert's formula for obtaining solutions to the wave equation is named after him. The wave equation is sometimes referred to as d'Alembert's equation, and the Fundamental theorem of algebra is named after d'Alembert in French.

Early years

Born in Paris, d'Alembert was the natural son of the writer Claudine Guérin de Tencin and the chevalier Louis-Camus Destouches, an artillery officer. Destouches was abroad at the time of d'Alembert's birth. Days after birth his mother left him on the steps of the Saint-Jean-le-Rond de Paris [fr] church. According to custom, he was named after the patron saint of the church. D'Alembert was placed in an orphanage for foundling children, but his father found him and placed him with the wife of a glazier, Madame Rousseau, with whom he lived for nearly 50 years. She gave him little encouragement. When he told her of some discovery he had made or something he had written she generally replied,

You will never be anything but a philosopher - and what is that but an ass who plagues himself all his life, that he may be talked about after he is dead.

Destouches secretly paid for the education of Jean le Rond, but did not want his paternity officially recognised.

Studies and adult life

D'Alembert first attended a private school. The chevalier Destouches left d'Alembert an annuity of 1200 livres on his death in 1726. Under the influence of the Destouches family, at the age of 12 d'Alembert entered the Jansenist Collège des Quatre-Nations (the institution was also known under the name "Collège Mazarin"). Here he studied philosophy, law, and the arts, graduating as baccalauréat en arts in 1735.

In his later life, d'Alembert scorned the Cartesian principles he had been taught by the Jansenists: "physical promotion, innate ideas and the vortices". The Jansenists steered d'Alembert toward an ecclesiastical career, attempting to deter him from pursuits such as poetry and mathematics. Theology was, however, "rather unsubstantial fodder" for d'Alembert. He entered law school for two years, and was nominated avocat in 1738.

He was also interested in medicine and mathematics. Jean was first registered under the name "Daremberg", but later changed it to "d'Alembert". The name "d'Alembert" was proposed by Frederick the Great of Prussia for a suspected (but non-existent) moon of Venus.

Career

In July 1739 he made his first contribution to the field of mathematics, pointing out the errors he had detected in Analyse démontrée (published 1708 by Charles-René Reynaud) in a communication addressed to the Académie des Sciences. At the time L'analyse démontrée was a standard work, which d'Alembert himself had used to study the foundations of mathematics. D'Alembert was also a Latin scholar of some note and worked in the latter part of his life on a superb translation of Tacitus, for which he received wide praise including that of Denis Diderot.

In 1740, he submitted his second scientific work from the field of fluid mechanics Mémoire sur la réfraction des corps solides, which was recognised by Clairaut. In this work d'Alembert theoretically explained refraction.

In 1741, after several failed attempts, d'Alembert was elected into the Académie des Sciences. He was later elected to the Berlin Academy in 1746 and a Fellow of the Royal Society in 1748.

In 1743, he published his most famous work, Traité de dynamique, in which he developed his own laws of motion.

When the Encyclopédie was organised in the late 1740s, d'Alembert was engaged as co-editor (for mathematics and science) with Diderot, and served until a series of crises temporarily interrupted the publication in 1757. He authored over a thousand articles for it, including the famous Preliminary Discourse. D'Alembert "abandoned the foundation of Materialism" when he "doubted whether there exists outside us anything corresponding to what we suppose we see." In this way, d'Alembert agreed with the Idealist Berkeley and anticipated the transcendental idealism of Kant.[citation needed]

In 1752, he wrote about what is now called D'Alembert's paradox: that the drag on a body immersed in an inviscid, incompressible fluid is zero.

In 1754, d'Alembert was elected a member of the Académie des sciences, of which he became Permanent Secretary on 9 April 1772.

In 1757, an article by d'Alembert in the seventh volume of the Encyclopedia suggested that the Geneva clergymen had moved from Calvinism to pure Socinianism, basing this on information provided by Voltaire. The Pastors of Geneva were indignant, and appointed a committee to answer these charges. Under pressure from Jacob Vernes, Jean-Jacques Rousseau and others, d'Alembert eventually made the excuse that he considered anyone who did not accept the Church of Rome to be a Socinianist, and that was all he meant, and he abstained from further work on the encyclopaedia following his response to the critique.

He was elected a Foreign Honorary Member of the American Academy of Arts and Sciences in 1781.

Legacy

In France, the fundamental theorem of algebra is known as the d'Alembert/Gauss theorem, as an error in d'Alembert's proof was caught by Gauss.

He also created his ratio test, a test to see if a series converges.

The D'Alembert operator, which first arose in D'Alembert's analysis of vibrating strings, plays an important role in modern theoretical physics.

While he made great strides in mathematics and physics, d'Alembert is also famously known for incorrectly arguing in Croix ou Pile that the probability of a coin landing heads increased for every time that it came up tails. In gambling, the strategy of decreasing one's bet the more one wins and increasing one's bet the more one loses is therefore called the D'Alembert system, a type of martingale.

Thomas Young

Thomas Young (13 June 1773 – 10 May 1829) was a British polymath who made notable contributions to the fields of vision, light, solid mechanics, energy, physiology, language, musical harmony, and Egyptology. He "made a number of original and insightful innovations" in the decipherment of Egyptian hieroglyphs (specifically the Rosetta Stone) before Jean-François Champollion eventually expanded on his work.

Young has been described as "The Last Man Who Knew Everything". His work informed that later done by William Herschel, Hermann von Helmholtz, James Clerk Maxwell, and Albert Einstein. Young is credited with establishing the wave theory of light, in contrast to the particle theory of Isaac Newton. Young's work was subsequently supported by the work of Augustin-Jean Fresnel.

Biography

Young belonged to a Quaker family of Milverton, Somerset, where he was born in 1773, the eldest of ten children. At the age of fourteen Young had learned Greek and Latin and was acquainted with French, Italian, Hebrew, German, Aramaic, Syriac, Samaritan, Arabic, Persian, Turkish and Amharic.

Young began to study medicine in London at St Bartholomew's Hospital in 1792, moved to the University of Edinburgh Medical School in 1794, and a year later went to Göttingen, Lower Saxony, Germany, where he obtained the degree of doctor of medicine in 1796 from the University of Göttingen. In 1797 he entered Emmanuel College, Cambridge. In the same year he inherited the estate of his grand-uncle, Richard Brocklesby, which made him financially independent, and in 1799 he established himself as a physician at 48 Welbeck Street, London (now recorded with a blue plaque). Young published many of his first academic articles anonymously to protect his reputation as a physician.

In 1801, Young was appointed professor of natural philosophy (mainly physics) at the Royal Institution. In two years, he delivered 91 lectures. In 1802, he was appointed foreign secretary of the Royal Society, of which he had been elected a fellow in 1794. He resigned his professorship in 1803, fearing that its duties would interfere with his medical practice. His lectures were published in 1807 in the Course of Lectures on Natural Philosophy and contain a number of anticipations of later theories.

In 1811, Young became physician to St George's Hospital, and in 1814 he served on a committee appointed to consider the dangers involved in the general introduction of gas for lighting into London. In 1816 he was secretary of a commission charged with ascertaining the precise length of the second's or seconds pendulum (the length of a pendulum whose period is exactly 2 seconds), and in 1818 he became secretary to the Board of Longitude and superintendent of the HM Nautical Almanac Office.

Young was elected a Foreign Honorary Member of the American Academy of Arts and Sciences in 1822. A few years before his death he became interested in life insurance, and in 1827 he was chosen one of the eight foreign associates of the French Academy of Sciences. In 1828, he was elected a foreign member of the Royal Swedish Academy of Sciences.

In 1804, Young married Eliza Maxwell. They had no children.

Research

Wave theory of light

In Young's own judgment, of his many achievements the most important was to establish the wave theory of light. To do so, he had to overcome the century-old view, expressed in the venerable Newton's Opticks, that light is a particle. Nevertheless, in the early 19th century Young put forth a number of theoretical reasons supporting the wave theory of light, and he developed two enduring demonstrations to support this viewpoint. With the ripple tank he demonstrated the idea of interference in the context of water waves. With

Young's interference experiment, or double-slit experiment, he demonstrated interference in the context of light as a wave.

Young, speaking on 24 November 1803, to the Royal Society of London, began his now-classic description of the historic experiment:

The experiments I am about to relate ... may be repeated with great ease, whenever the sun shines, and without any other apparatus than is at hand to every one.

In his subsequent paper, titled Experiments and Calculations Relative to Physical Optics (1804), Young describes an experiment in which he placed a card measuring approximately 0,85 millimetres (0,033 in) in a beam of light from a single opening in a window and observed the fringes of colour in the shadow and to the sides of the card. He observed that placing another card in front or behind the narrow strip so as to prevent the light beam from striking one of its edges caused the fringes to disappear. This supported the contention that light is composed of waves.

Young performed and analysed a number of experiments, including interference of light from reflection off nearby pairs of micrometre grooves, from reflection off thin films of soap and oil, and from Newton's rings. He also performed two important diffraction experiments using fibres and long narrow strips. In his Course of Lectures on Natural Philosophy and the Mechanical Arts (1807) he gives Grimaldi credit for first observing the fringes in the shadow of an object placed in a beam of light. Within ten years, much of Young's work was reproduced and then extended by Augustin-Jean Fresnel.

Young's modulus

Young described the characterization of elasticity that came to be known as Young's modulus, denoted as E, in 1807, and further described it in his Course of Lectures on Natural Philosophy and the Mechanical Arts. However, the first use of the concept of Young's modulus in experiments was by Giordano Riccati in 1782 – predating Young by 25 years. Furthermore, the idea can be traced to a paper by Leonhard Euler published in 1727, some 80 years before Thomas Young's 1807 paper.

The Young's modulus relates the stress (pressure) in a body to its associated strain (change in length as a ratio of the original length); that is, stress = E × strain, for a uniaxially loaded specimen. Young's modulus is independent of the component under investigation; that is, it is an inherent material property (the term modulus refers to an inherent material property). Young's Modulus allowed, for the first time, prediction of the strain in a component subject to a known stress (and vice versa). Prior to Young's contribution, engineers were required to apply Hooke's F=kx relationship to identify the deformation (x) of a body subject to a known load (F), where the constant (k) is a function of both the geometry and material under consideration. Finding k required physical testing for any new component, as the F=kx relationship is a function of both geometry and material. Young's Modulus depends only on the material, not its geometry, thus allowing a revolution in engineering strategies.

Young's problems in sometimes not expressing himself clearly were shown by his own definition of the modulus: "The modulus of the elasticity of any substance is a column of the same substance, capable of producing a pressure on its base which is to the weight causing a certain degree of compression as the length of the substance is to the diminution of its length." When this explanation was put to the Lords of the Admiralty, their clerk wrote to Young saying "Though science is much respected by their Lordships and your paper is much esteemed, it is too learned ... in short it is not understood."

Vision and colour theory

Young has also been called the founder of physiological optics. In 1793 he explained the mode in which the eye accommodates itself to vision at different distances as depending on change of the curvature of the crystalline lens; in 1801 he was the first to describe astigmatism; and in his lectures he presented the hypothesis, afterwards developed by Hermann von Helmholtz, (the Young-Helmholtz theory), that colour perception depends on the presence in the retina of three kinds of nerve fibres. This foreshadowed the modern understanding of colour vision, in particular the finding that the eye does indeed have three colour receptors which are sensitive to different wavelength ranges.

Young-Laplace equation

In 1804, Young developed the theory of capillary phenomena on the principle of surface tension. He also observed the constancy of the angle of contact of a liquid surface with a solid, and showed how from these two principles to deduce the phenomena of capillary action. In 1805, Pierre-Simon Laplace, the French philosopher, discovered the significance of meniscus radii with respect to capillary action.

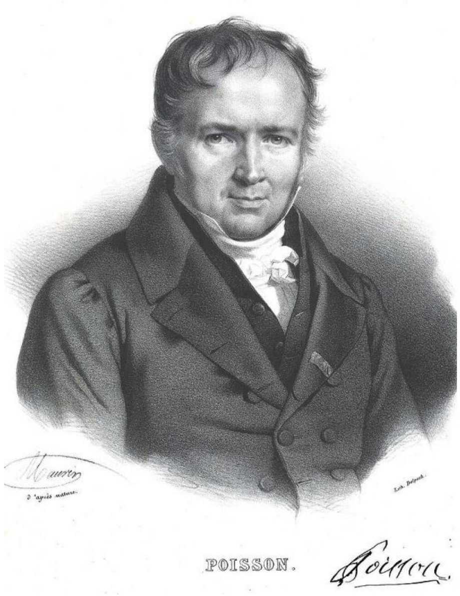
In 1830, Carl Friedrich Gauss, the German mathematician, unified the work of these two scientists to derive the Young-Laplace equation, the formula that describes the capillary pressure difference sustained across the interface between two static fluids.

Young was the first to define the term "energy" in the modern sense.

Young's equation and Young-Dupré equation

Young's equation describes the contact angle of a liquid drop on a plane solid surface as a function of the surface free energy, the interfacial free energy and the surface tension of the liquid. Young's equation was developed further some 60 years later by Dupré to account for thermodynamic effects, and this is known as the Young-Dupré equation.

Siméon Denis Poisson



Baron Siméon Denis Poisson (21 June 1781 – 25 April 1840) was a French mathematician, engineer, and physicist who made many scientific advances.

Biography

Poisson was born in Pithiviers, Loiret district in France, the son of Siméon Poisson, an officer in the French army.

In 1798, he entered the École Polytechnique in Paris as first in his year, and immediately began to attract the notice of the professors of the school, who left him free to make his own decisions as to what he would study. In 1800, less than two years after his entry, he published two memoirs, one on Étienne Bézout's method of elimination, the other on the number of integrals of a finite difference equation. The latter was examined by Sylvestre-François Lacroix and Adrien-Marie Legendre, who recommended that it should be published in the Recueil des savants étrangers, an unprecedented honor for a youth of eighteen. This success at once procured entry for Poisson into scientific circles. Joseph Louis Lagrange, whose lectures on the theory of functions he attended at the École Polytechnique, recognized his talent early on, and became his friend. Meanwhile, Pierre-Simon Laplace, in whose footsteps Poisson followed, regarded him almost as his son. The rest of his career, till his death in Sceaux near Paris, was nearly occupied by the composition and publication of his many works and in fulfilling the duties of the numerous educational positions to which he was successively appointed.

Immediately after finishing his studies at the École Polytechnique, he was appointed répétiteur (teaching assistant) there, a position which he had occupied as an amateur while still a pupil in the school; for his schoolmates had made a custom of visiting him in his room after an unusually difficult lecture to hear him repeat and explain it. He was made deputy professor (professeur suppléant) in 1802, and, in 1806 full professor succeeding Jean Baptiste Joseph Fourier, whom Napoleon had sent to Grenoble. In 1808 he became astronomer to the Bureau des Longitudes; and when the Faculté des sciences de Paris [fr] was instituted in 1809 he was appointed a professor of rational mechanics (professeur de mécanique rationelle). He went on to become a member of the Institute in 1812, examiner at the military school (École Militaire) at Saint-Cyr in 1815, graduation examiner at the École Polytechnique in 1816, councillor of the university in 1820, and geometer to the Bureau des Longitudes succeeding Pierre-Simon Laplace in 1827.

In 1817, he married Nancy de Bardi and with her, he had four children. His father, whose early experiences had led him to hate aristocrats, bred him in the stern creed of the First Republic. Throughout the Revolution, the Empire, and the following restoration, Poisson was not interested in politics, concentrating on mathematics. He was appointed to the dignity of baron in 1821; but he neither took out the diploma nor used the title. In March 1818, he was elected a Fellow of the Royal Society, in 1822 a Foreign Honorary Member of the American Academy of Arts and Sciences, and in 1823 a foreign member of the Royal Swedish Academy of Sciences. The revolution of July 1830 threatened him with the loss of all his honours; but this disgrace to the government of Louis-Philippe was adroitly averted by François Jean Dominique Arago, who, while his "revocation" was being plotted by the council of ministers, procured him an invitation to dine at the Palais-Royal, where he was openly and effusively received by the citizen king, who "remembered" him. After this, of course, his degradation was impossible, and seven years later he was made a peer of France, not for political reasons, but as a representative of French science.

As a teacher of mathematics Poisson is said to have been extraordinarily successful, as might have been expected from his early promise as a répétiteur at the École Polytechnique. As a scientific worker, his productivity has rarely if ever been equaled. Notwithstanding his many official duties, he found time to publish more than three hundred works, several of them extensive treatises, and many of them memoirs dealing with the most abstruse branches of pure mathematics, applied mathematics, mathematical physics, and rational mechanics. (Arago attributed to him the quote, "Life is good for only two things: doing mathematics and teaching it.")

A list of Poisson's works, drawn up by himself, is given at the end of Arago's biography. All that is possible is a brief mention of the more important ones. It was in the application of mathematics to physics that his greatest services to science were performed. Perhaps the most original, and certainly the most permanent in their influence, were his memoirs on the theory of electricity and magnetism, which virtually created a new branch of mathematical physics.

Next (or in the opinion of some, first) in importance stand the memoirs on celestial mechanics, in which he proved himself a worthy successor to Pierre-Simon Laplace. The most important of these are his memoirs Sur les inégalités séculaires des moyens mouvements des planètes, Sur la variation des constantes arbitraires dans les questions de mécanique, both published in the Journal of the École Polytechnique (1809); Sur la libration de la lune, in Connaissance des temps (1821), etc.; and Sur le mouvement de la terre autour de son centre de gravité, in Mémoires de l'Académie (1827), etc. In the first of these memoirs, Poisson discusses the famous question of the stability of the planetary orbits, which had already been settled by Lagrange to the first degree of approximation for the disturbing forces. Poisson showed that the result could be extended to a second approximation, and thus made an important advance in planetary theory. The memoir is remarkable inasmuch as it roused Lagrange, after an interval of inactivity, to compose in his old age one of the greatest of his memoirs, entitled Sur la théorie des variations des éléments des planètes, et en particulier des variations des grands axes de leurs orbites. So highly did he think of Poisson's memoir that he made a copy of it with his own hand, which was found among his papers after his death. Poisson made important contributions to the theory of attraction.

His is one of the 72 names inscribed on the Eiffel Tower.

Mathematics

In pure mathematics, his most important works were his series of memoirs on definite integrals and his discussion of Fourier series, the latter paving the way for the classic researches of Peter Gustav Lejeune Dirichlet and Bernhard Riemann on the same subject; these are to be found in the Journal of the École Polytechnique from 1813 to 1823, and in the Memoirs de l'Académie for 1823. He also studied Fourier integrals. We may also mention his essay on the calculus of variations (Mem. de l'acad., 1833), and his memoirs on the probability of the mean results of observations (Connaiss. d. temps, 1827, &c). The Poisson distribution in probability theory is named after him.

Mechanics

In his Traité de mécanique (2 vols. 8vo, 1811 and 1833), which was written in the style of Laplace and Lagrange and was long a standard work, he showed many novelties such as an explicit usage of momenta:

$$p_i = \frac{\partial T}{\partial \left(\frac{\partial q_i}{\partial t} \right)},$$

which influenced the work of Hamilton and Jacobi. A translation of Poisson's Treatise on Mechanics was published in London in 1842.

Other works

Besides his many memoirs, Poisson published a number of treatises, most of which were intended to form part of a great work on mathematical physics, which he did not live to complete. Among these may be mentioned:

Nouvelle théorie de l'action capillaire (4to, 1831);

Théorie mathématique de la chaleur (4to, 1835);

Supplement to the same (4to, 1837);

Recherches sur la probabilité des jugements en matières criminelles et matière civile (4to, 1837), all published at Paris.

In 1815 Poisson studied integrations along paths in the complex plane. In 1831 he derived the Navier–Stokes equations independently of Claude-Louis Navier.

Dmitrii Ivanovich Zhuravskii

Dmitrii Ivanovich Zhuravskii (Dec. 17 (29), 1821 – Nov. 18 (30), 1891) (1821–1891) was a Russian engineer who was one of the pioneers of bridge construction and structural mechanics in Russia.

Zhuravskii attended the Nezhin lycée and entered the St. Petersburg Institute of the Corps of Railroad Engineers where he was influenced by the academician Mikhail Ostrogradsky. He graduated from the institute as first in his class in 1842.

In the beginning of his career he took part in the surveying and planning of the Moscow – Saint Petersburg Railway. In 1857-58 he led the reconstruction of the Peter and Paul Cathedral in Saint Petersburg. In 1871–76 he took part in the reconstruction of the Mariinsky Canal System.

He was awarded the prestigious Demidov Prize in 1855 by the Russian Academy of Sciences.

The Zhuravskii Shear Stress formula is named after him (derived it in 1855):

$$\tau = \frac{V \cdot Q}{I \cdot t},$$

where

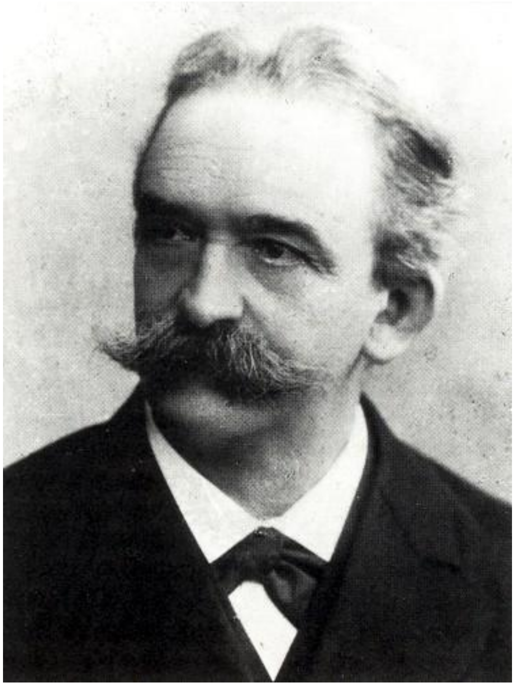
V – total shear force at the location in question;

Q – statical moment of area;

t – thickness in the material perpendicular to the shear;

I – moment of Inertia of the entire cross sectional area.

Christian Otto Mohr



Christian Otto Mohr (8 October 1835 – 2 October 1918) was a German civil engineer.

He was born on 8 October 1835 to a landowning family in Wesselburen in the Holstein region. At the age of 16 attended the Polytechnic School in Hannover.

Starting in 1855, his early working life was spent in railroad engineering for the Hanover and Oldenburg state railways, designing some famous bridges and making some of the earliest uses of steel trusses.

Even during his early railway years, Mohr had developed an interest in the theories of mechanics and the strength of materials. In 1867, he became professor of mechanics at Stuttgart Polytechnic, and in 1873 at Dresden Polytechnic. Mohr had a direct and unpretentious lecturing style that was popular with his students. In addition to a lone textbook, Mohr published many research papers on the theory of structures and strength of materials.

In 1874, Mohr formalised the idea of a statically indeterminate structure.

Mohr was an enthusiast for graphical tools and developed the method, for visually representing stress in three dimensions, previously proposed by Carl Culmann. In 1882, he famously developed the graphical method for analysing stress known as Mohr's circle and used it to propose an early theory of strength based on shear stress. He also developed the Williot-Mohr diagram for truss displacements and the Maxwell-Mohr method for analysing statically indeterminate structures, it can also be used to determine the displacement of truss nodes and forces acting on each member.

The Maxwell-Mohr method is also referred to as the virtual force method for redundant trusses.

He retired in 1900, yet continued his scientific work in Dresden until his death on 2 October 1918.

Stepan Prokofyevich Timoshenko

Stepan Prokofyevich Timoshenko (December 23, 1878 – May 29, 1972), was a Ukrainian, Russian and later, an American engineer and academician. He is considered to be the father of modern engineering mechanics. An inventor and one of the pioneering mechanical engineers at the St. Petersburg Polytechnic University. A founding member of the Ukrainian Academy of Sciences, Timoshenko wrote seminal works in the areas of engineering mechanics, elasticity and strength of materials, many of which are still widely used today. Having started his scientific career in the Russian Empire, Timoshenko emigrated to the Kingdom of Serbs, Croats and Slovenes during the Russian Civil War and then to the United States.

Biography

Timoshenko was born in the village of Shpotovka in the Chernigov Governorate which at that time was a territory of the Russian Empire (today in Konotop Raion, Ukraine). He studied at a Realschule in Romny, Poltava Governorate (now in Sumy Oblast) from 1889 to 1896. In Romny his schoolmate and friend was future famous semiconductor physicist Abram Ioffe. Timoshenko continued his education towards a university degree at the St Petersburg Institute of engineers Ways of Communication. After graduating in 1901, he stayed on teaching in this same institution from 1901 to 1903 and then worked at the Saint Petersburg Polytechnical Institute under Viktor Kirpichov 1903-1906. In 1905 he was sent for one year to the University of Göttingen where he worked under Ludwig Prandtl.

In the fall of 1906 he was appointed to the Chair of Strengths of Materials at the Kyiv Polytechnic Institute. The return to his native Ukraine turned out to be an important part of his career and also influenced his future personal life. From 1907 to 1911 as a professor at the Polytechnic Institute he did research in the earlier variant of the Finite Element Method of elastic calculations, the so-called Rayleigh method. During those years he also pioneered work on buckling, and published the first version of his famous Strength of Materials textbook. He was elected dean of the Division of Structural Engineering in 1909.

In 1911 he signed a protest against Minister for Education Kasso and was fired from the Kiev Polytechnic Institute. In 1911 he was awarded the D.I. Zhuravski prize of the St.Petersburg Ways of Communication Institute that helped him survive after losing his job. He went to St Petersburg where he worked as a lecturer and then a Professor in the Electrotechnical Institute and the St Petersburg Institute of the Railways (1911–1917). During that time he developed the theory of elasticity and the theory of beam deflection, and continued to study buckling. In 1918 he returned to Kiev and assisted Vladimir Vernadsky in establishing the Ukrainian Academy of Sciences – the oldest academy among the Soviet republics other than Russia. In 1918–1920 Timoshenko headed the newly established Institute of Mechanics of the Ukrainian Academy of Sciences, which today carries his name. Younger brother of Stephen, Serhiy Tymoshenko, was a Ukrainian Minister of Communication and participated in the Second Winter Campaign against the Soviet regime.

After the Armed Forces of South Russia of general Denikin had taken Kiev in 1919, Timoshenko moved from Kiev to Rostov-on-Don. After travel via Novorossiysk, Crimea and Constantinople to the Kingdom of Serbs, Croats and Slovenes, he arrived in Zagreb, where he got professorship at the Zagreb Polytechnic Institute. In 1920, during the brief liberation of Kiev from Bolsheviks, Timoshenko travelled to the city, reunited with his family and returned with his family to Zagreb.

He is remembered for delivering lectures in Russian while using as many words in Croatian as he could; the students were able to understand him well.

United States

In 1922 Timoshenko moved to the United States where he worked for the Westinghouse Electric Corporation from 1923 to 1927, after which he became a faculty professor in the University of Michigan where he created the first bachelor's and doctoral programs in engineering mechanics. His textbooks have been published in 36 languages. His first textbooks and papers were written in Russian; later in his life, he published mostly in English. In 1928 he was an Invited Speaker of the ICM in Bologna. From 1936 onward he was a professor at Stanford University.

In 1957 ASME established a medal named after Stephen Timoshenko; he became its first recipient. The Timoshenko Medal honors Stephen P. Timoshenko as the world-renowned authority in the field of mechanical engineering and it commemorates his contributions as author and teacher. The Timoshenko Medal is given annually for distinguished contributions in applied mechanics.

In addition to his textbooks, Timoshenko wrote Engineering Education in Russia and an autobiography, As I Remember, the latter first published in

Russian in 1963 with its English translation appearing in 1968.

In 1960 he moved to Wuppertal (Western Germany) to be with his daughter. He died in 1972 and his ashes are buried in Alta Mesa Memorial Park, Palo Alto, California. In 1963 Timoshenko wrote a book As I Remember in the Russian language. It was translated into English in 1968 by sponsorship of the Stanford University. Jacob Pieter den Hartog (1901-1989), who was Timoshenko's co-worker in early 1920s at Westinghouse, wrote a review in the magazine Science stating that "… Between 1922 and 1962 he [S.P. Timoshenko] wrote a dozen books on all aspects of engineering mechanics, which are in their third or fourth U.S. edition and which have been translated into half a dozen foreign languages each, so that his name as an author and scholar is known to nearly every mechanical and civil engineer in the entire world. Then, Den Hartog stressed: "There is no question that Timoshenko did much for America. It is an equally obvious truth that America did much for Timoshenko, as it did for millions of other immigrants for all over the world. However, our autobiographer has never admitted as much to his associates and pupils who, like myself often have been pained by his casual statements in conversation. That pain is not diminished by reading these statements on the printed page and one would have wished for a little less acid and a little more human kindness."

It should be emphasized that the celebrated theory that takes into account shear deformation and rotary inertia was developed by Timoshenko in collaboration with Paul Ehrenfest (1880-1933), famous Austrian-Dutch physicist, as the recent handbook by Elishakoff shows, and thus, should be referred to as Timoshenko-Ehrenfest beam theory. This fact was testified by Timoshenko. The interrelation between Timoshenko-Ehrenfest beam and Euler-Bernoulli beam theories was investigated in the book by Wang, Reddy and Lee.

Heorhij Stepanowytsch Pyssarenko

Heorhij Stepanowytsch Pyssarenko (* 30. Oktoberjul./ 12. November 1910greg. in Poltawa, Russisches Kaiserreich; 9. Januar 2001 in Kiew, Ukraine) war ein sowjetisch-ukrainischer Bauingenieur.

Biografie

Pyssarenko stammte aus einer Kosakenfamilie und studierte Schiffbau am Industrie-Institut in Gorki mit dem Abschluss 1936. Ab 1939 war er zu weiteren Studien am Polytechnikum in Kiew, an dem er 1948 promoviert wurde. Außerdem war er ab 1939 am Institut für Baustatik der Akademie der Wissenschaften der Ukrainischen SSR, das er 1966 bis 1988 leitete. 1952 bis 1984 leitete er die Abteilung Festigkeitslehre am Polytechnikum in Kiew.

Er gründete eine international bekannte Schule der Festigkeitslehre, insbesondere forschte er seit seiner Dissertation über Festigkeit unter extremen Bedingungen.

1957 wurde er korrespondierendes und 1964 volles Mitglied der Ukrainischen Akademie der Wissenschaften. 1962 bis 1966 war er deren Generalsekretär und 1970 bis 1978 deren Vizepräsident. Im wurde der Leninorden verliehen, 1969 und 1980 erhielt er den Staatspreis der Ukraine und 1982 den sowjetischen Staatspreis.

MAIN SYMBOLS OF STRENGTH OF MATERIALS

Terms	Symbols	Measurements
Сила Force longitudinal (axial, normal) force, shear (cutting) force, critical force	F , Q , N	N
Момент Moment bending moment, torque moment	M , T	Nm
Абсолютне видовження Total longitudinal elongation of the rod (absolute longitudinal deformation, linear elongation, linear deformation)	Δl	m
Відносна деформація Relative longitudinal deformation	ε	-
Модуль пружності I-го роду Modulus of elasticity (modulus of elasticity of the first kind, Young's modulus, normal elastic modulus, longitudinal elastic modulus)	E	MPa
Модуль пружності II-го роду The shear modulus or modulus of elasticity of the second type, characterizing the material rigidity	G	MPa
Коефіцієнт Пуассона Mechanical characteristic of the material (coefficient of transverse deformation or Poisson)	μ	-
Потужність Power	P	kW
Лінійне переміщення Linear displacement	y_A , f_A	m
Кутове переміщення Angular displacement	Θ_A	-

Terms		Symbols	Measurements
Напруження: Stresses:	нормальні the normal stresses	σ	MPa
	дотичні the tangential (shear) stresses	τ	MPa
	допустимі the allowable stresses	$[\sigma]$, $[\tau]$	MPa
Границі: Limits (strength):	міцності the tensile limit	σ_t	MPa
	пружності of elasticity limit	σ_{el}	MPa
	пропорційності of proportionality limit	σ_{pr}	MPa
	текучості the yield limit	σ_{ye}	MPa
Геометричні характеристики поперечних перетинів Geometric characteristics of transverse sections	площа the area	A	m^2
	статичний момент площі static moments of the section area	S_X, S_Y	m^3
	осьовий момент інерції the axial moment of inertia	I_O	m^4
	полярний момент інерції the polar moment of inertia	I_P	m^4
	осьовий момент опору the axial moment of resistance	W_O	m^3
	полярний момент опору the polar moment of resistance	W_P	m^3

UKRAINIAN-ENGLISH VOCABULARY OF BASIC TERMS

ОСНОВНІ ПОНЯТТЯ ОПОРУ МАТЕРІАЛІВ BASIC CONCEPTS OF STRENGTH OF MATERIALS		
опір матеріалів	–	strength of materials
теорія міцності	–	theory of strength
припущення (гіпотези)	–	assumption (hypotheses)
гіпотеза про суцільність матеріалу	–	hypothesis of the material continuity
гіпотеза про однорідність та ізотропність	–	hypothesis of homogeneity and isotropy
гіпотеза про ідеальну пружність та природну не напруженість матеріалу	–	hypothesis of the ideal elasticity and natural tension of the material
площа	–	area
міцність	–	strength
жорсткість, достатня жорсткість	–	rigidity, sufficient rigidity
стійкість	–	durability
стрижень (стержень)	–	rod
прямий стрижень	–	direct rod
стрижнева система	–	rod system
пластина	–	plate
оболонка	–	shell
масив (масивне тіло)	–	solid (massive body)
навантаження	–	load
зовнішнє навантаження	–	external load
статичне навантаження	–	static load
динамічне навантаження	–	dynamic load
рівномірно розподілене навантаження	–	evenly distributed load
навантаження розподілені на лінії	–	distributed on line load
інтенсивність розподіленого навантаження	–	intensity of the distributed load
силовий фактор	–	force factor
внутрішній силовий фактор	–	internal force factor
внутрішнє зусилля	–	internal force
зосереджена сила	–	concentrated force
рівнодійний	–	equivalent

критична сила	–	critical force
сила інерції	–	inertia force
сила тиску	–	pressure force
момент	–	moment
розрахункова схема	–	calculation scheme
деформація (переміщення)	–	deformation (displacement)
лінійна деформація (переміщення)	–	linear deformation (displacement)
відносна деформація	–	relative deformation
відносна зміна об’єму	–	relative change in volume
поздовжня деформація	–	longitudinal deformation
поперечна деформація	–	transverse deformation
плоска система сил	–	plane system of forces
система паралельних сил	–	system of parallel forces
правило знаків	–	sign rule
опуклість	–	convexity
зразок	–	speciment
площадка, площина	–	plane
запас міцності (коефіцієнт запасу міцності)	–	margin of safety
вибір, підбір	–	choice
прямокутна система координат	–	rectangular coordinate system
початок координат	–	coordinate origin
поточна координата	–	current coordinate
взаємно перпендикулярні площадки	–	mutually perpendicular planes
лінійна залежність	–	linear relationship
закон розподілу	–	law of distribution
прискорення	–	acceleration
ступінь	–	degree
абсолютно жорстке тіло	–	absolutely rigid body
пружна деформація	–	elastic deformation
пружна система	–	elastic system
конструкція	–	construction
рівняння статики	–	static equation
рівняння рівноваги	–	equilibrium equation
рівновага	–	equilibrium
значення, величина	–	value
форма (перетину)	–	shape
відома величина	–	known value
рівень	–	level
шар волокон	–	fiber layer
нейтральний шар	–	neutral layer

верхній шар	–	upper layer
нижній шар	–	lower layer
одиниця вимірювання, розмірність	–	measurement
перевага	–	advantage
недолік	–	drawback
особливість	–	peculiarity
рама	–	frame
сумісна дія	–	joint action
дія	–	action
розв’язування (розв’язок)	–	solution
двотавр	–	I-beam
швелер	–	U-beam
коротка балка	–	short beam
точний	–	exact
умова	–	condition
точка	–	point
вектор	–	vector
вузол	–	nod
нахил	–	slope
вгору	–	up
вниз	–	down (downwards)
кривизна	–	curvature
плоска крива	–	plane curve
руйнівний	–	destructive
метод перетинів	–	section method
плоский поперечний перетин	–	plane cross-section (section)
нормальний (поперечний) перетин	–	normal (shear) section
довільний (косий або похилий) перетин	–	random (oblique or inclined) cross-section
умовний (уявний) перетин	–	imaginary section
небезпечний перетин	–	dangerous section
діаграма, епюра	–	diagram
розмір	–	dimension
висота	–	height
ширина	–	width
довжина	–	length
діаметр	–	diameter
коло	–	circle
кільце	–	ring
круг	–	round
центр кола	–	circle center

прямокутник	–	rectangle
об’єм	–	volume
вага	–	weight
центр ваги	–	center of gravity (weight)
ЦЕНТРАЛЬНИЙ РОЗТЯГ-СТИСК ПРЯМИХ СТРИЖНІВ CENTRAL TENSION AND COMPRESSION OF DIRECT RODS (BARS)		
центральний розтяг-стиск	–	central tension and compression
поздовжня (нормальна осьова) сила	–	longitudinal (normal, axial) force
модуль пружності (модуль Юнга)	–	modulus of elasticity (Young’s modulus)
коефіцієнт поперечної деформації (коефіцієнт Пуассона)	–	coefficient of transverse deformation or Poisson
зсув (зріз)	–	shear
поперечна (перерізуюча) сила	–	cross-cut, shear (cutting) force
модуль зсуву	–	shear modulus
сколювання	–	chipping
пластичний матеріал	–	plastic material
крихкий матеріал	–	brittle material
сталь	–	steel
чавун	–	cast iron
дерево	–	wood
діаграма розтягу	–	stress-strain diagram
границя пропорційності	–	limit of proportionality
границя пружності	–	limit of elasticity
границя текучості	–	yield (strength) limit
границя міцності	–	tensile strength
потенціальна енергія деформації	–	potential deformation energy
розрахунок на міцність	–	strength calculation
напруження	–	stress
нормальне напруження	–	normal stress
дотичне напруження	–	tangential (shear) stress
головне напруження	–	main (principal) stress
робоче (фактичне) напруження	–	working (actual) stress
граничне напруження	–	boundary stress
допустиме напруження	–	allowable stress
коефіцієнт запасу міцності	–	strength factor
умова міцності	–	strength condition
проектний розрахунок	–	design calculation
перевірний розрахунок	–	validation calculation

наближений розрахунок	–	approximate calculation
ГЕОМЕТРИЧНІ ХАРАКТЕРИСТИКИ ПЛОСКИХ ПЕРЕТИНІВ GEOMETRIC CHARACTERISTICS OF PLANE SECTIONS		
геометрична характеристика плоских перетинів	–	geometric characteristics of plane sections
площа довільної форми	–	area of arbitrary shape
елементарна площа	–	elementary plane
статичний момент площі	–	static moment of the area
полярний момент інерції	–	polar moment of inertia
осьовий момент інерції	–	axial moment of inertia
центральна вісь	–	central axis
нейтральна вісь	–	neutral axis
центральний момент інерції	–	central moment of inertia
головна вісь інерції	–	main axis of inertia
головний момент інерції	–	main moment of inertia
головний центральний момент інерції	–	main central moment of inertia
головна центральна вісь	–	principal central axis
осьовий момент опору	–	axial moment of resistance
полярний момент опору	–	polar moment of intersection resistance
ЗСУВ. КРУЧЕННЯ SHEAR. TORSION		
кручення	–	torsion
крутний момент	–	torque moment
обертаючий момент	–	rotating moment
вал	–	shaft
шків (диск)	–	pulley
потужність	–	power
частота обертання вала	–	shaft rotation frequency
кут закручування	–	twist angle
відносний кут закручування	–	relative twist angle
ПРЯМИЙ ПОПЕРЕЧНИЙ ЗГИН STRAIGHT TRANSVERSE BENDING		
прямий поперечний згин	–	direct lateral bending
напруження при згині	–	bending stress
згинальний момент	–	bending moment
консольна балка	–	cantilever beam
балка на опорах	–	supported beam

лінійний закон	–	linear law
нижні волокна матеріалу	–	lower fibers of material
верхні волокна матеріалу	–	upper fibers of material
нейтральні волокна матеріалу	–	neutral fibers of material
реакція	–	reaction
опорна реакція	–	support reaction
відкинута в’язь	–	rejected link
активна сила	–	active force
реактивна сила	–	reactive force
вільний (незакріплений) кінець	–	free end
опора	–	support
шарнірно-рухома опора	–	hinged-movable support
шарнірно-нерухома опора	–	hinged-fixed support
жорстке закріплення затиснення (защемлення)	–	rigid fastening, rigidly fixed (clamping)
злом (епюри)	–	breaking
закріплення на опорах	–	fixation on supports
перевірка	–	validation, verification
правильність	–	correctness
межа, границя	–	boundary
функція	–	function
ліва сторона	–	left side
права сторона	–	right side
екстремальний момент	–	extreme moment
лінія нахилена до осі	–	line inclined to the axis
лінія паралельна осі	–	line parallel to the axis
стрибок	–	jump
квадратична парабола	–	quadratic parabola
зростати	–	increase
спадати	–	decrease
диференціальна залежність	–	differential dependency
похідна	–	derivative
недеформована вісь	–	undeformed axis
зігнута вісь	–	bent axis
вигнута вісь	–	curved axis
пружна лінія	–	elastic line
викривлення (спотворення)	–	distortion
прогин	–	deflection
кутове переміщення	–	angular displacement
кут повороту	–	slope of the elastic curve
косий згин	–	oblique bending
чистий косий згин	–	pure oblique bending

нейтральна лінія перетину	–	neutral (zero) crossing line
СКЛАДНИЙ НАПРУЖЕНИЙ СТАН COMPLEX STRESSED STATE		
складний опір	–	complex strength (resistance to combined stress)
складний напружений стан	–	complex stressed state
головний елемент	–	main element
головна площина (площадка)	–	main plane
головне напруження	–	main stress
більше з головних напружень	–	maximum main stress
менше з головних напружень	–	minimum main stress
розрахункове напруження	–	calculated stress
лінійний напружений стан	–	linear stress state
плоский напружений стан	–	plane stress state
СТАТИЧНО-НЕВИЗНАЧУВАНІ СИСТЕМИ STATICALLY INDETERMINATE SYSTEMS		
статично невизначувана система	–	statically indeterminate system
ступінь статичної невизначеності	–	degree of static indeterminance
зайвий (надлишковий) зв’язок	–	redundant (auxiliary) junction
відкинутий зв’язок	–	removed junction
незмінна система	–	unchangeable system
основна система	–	main system
еквівалентна система	–	equivalent system
умова нерозривності	–	condition of continuity
умова сумісності деформацій	–	condition of strain compatability
гіперстатична система	–	hyperstatic system
одиничне навантаження	–	singular load
переміщення від одиничного навантаження (сили або моменту)	–	singular displacement
одинична сила	–	singular force
одиничний момент	–	singular moment
канонічне рівняння	–	canonic equation
вільний член рівняння	–	absolute term of equation
невідома сила	–	unknown force
повне переміщення	–	complete displacement
диференціальне рівняння	–	differential equation
безпосереднє інтегрування	–	direct integration
наближене диференціальне рівняння	–	approximate differential equation
рівняння пружної лінії	–	equation of the elastic line
викривлення осі (зміна кривизни осі)	–	curvature of axis

допоміжна балка	–	redundant (auxiliary) beam
многочлен	–	polynomial
перемноження	–	multiplication
розшаровано (побудова епюр окремих фігур)	–	layered form
адитивність (безперервність) функції	–	additivity (continuity) of the function
СТІЙКІСТЬ ЦЕНТРАЛЬНО СТИСНЕНИХ СТРИЖНІВ STABILITY OF CENTRALLY-COMPRESSED RODS		
критичний стан	–	critical state
поздовжній згин	–	longitudinal bending
рівновага стійка	–	stable equilibrium
рівновага байдужа	–	indifferent equilibrium
рівновага нестійка	–	unstable equilibrium
коефіцієнт запасу стійкості	–	stability margin factor
гнучкість стрижня	–	flexibility of the rod
коефіцієнт зменшення основного допустимого напруження	–	coefficient of reduction of the main allowable strength stress
недонапруження	–	understressing
перенапруження	–	overstressing
дійсне напруження	–	actual stress
УДАРНІ НАВАНТАЖЕННЯ. ВИЗНАЧЕННЯ НАПРУЖЕНЬ І ПЕРЕМІЩЕНЬ ПРИ УДАРІ DYNAMIC LOADS. DETERMINING IMPACT STRESSES AND DISPLACEMENTS		
наближена теорія удару	–	approximated theory of impact
удар	–	impact
осьовий удар	–	axial impact
коливання	–	vibration (oscillation)
миттєве навантаження	–	instantaneous load
ударне навантаження	–	impact load
ударна в’язкість	–	impact viscosity
розсіювання енергії	–	energy dissipation
абсолютно тверде тіло	–	perfectly rigid body
вільно падаюче тіло	–	free-falling body (falling body)
рівняння балансу енергії	–	equation of energy balance
динамічне вкорочення (переміщення)	–	dynamic shortening
вкорочення (переміщення) колони (при ударі)	–	shortening of the column
коефіцієнт динамічності	–	coefficient of impact

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