SOME MODELS OF FINANCIAL DECISIONS

Danuta Seretna-Sałamaj[1], Agnieszka Szpara[2]
[1]University of Applied Sciences in Nysa, Nysa, Poland, danuta.seretna-salamaj@pwsz.nysa.pl
[2]University of Applied Sciences in Nysa, Nysa, Poland, agnieszka.szpara@pwsz.nysa.pl

Abstract
The article is devoted to risk modeling in prudent operators or investors, whose decisions are characterized by a trade-off between loss risk and reproduction function. Their attitude may be covered by the combined use of quantitative risk measures. Show the approach to risk modeling, which we will move to the traditional theory of maximizing the possibility of using service functions. Investors who engage their capital are always at risk because they make changes in the structure of their assets when investing. The risk of investing is identified with a possible threat or chance of achieving the expected benefits and is associated with the risk of an investment effect not being expected. This effect may be worse or better than previously assumed. The need to identify and verify the risk results from the possibility of achieving the expected benefits of the investor or avoiding losses. When making investment decisions, we can distinguish three types of investor behavior: Preference for risk and its effects (gambler) - the investor makes decisions even when the probability of loss exceeds the probability of profit. The investor is willing to incur higher expenses in order to make a decision about a higher risk. Risk neutrality - the investor does not make decisions when the probability of making a profit is too low. When making decisions, the investor does not pay attention to the amount of risk. Risk aversion - the investor expects the probability of profit to be greater than loss. An investor takes a risk when he expects to receive bonus compensation. Risk aversion also depends on the investor's resources. The richer the investor, the easier it will be for him to accept the loss. The models described in the article assume that investors act rationally and are characterized by risk aversion.

Keywords: utility function, risk, certainty equivalent, risk aversion.

INTRODUCTION

Nowadays, free market business is a natural space for entrepreneurs. The basic condition for the development of any enterprise in such an environment is the development of a proper investment strategy. It aims to bring improvement in business efficiency, strengthen the company's market position and improve its financial result [36]. Changes taking place in contemporary markets and the growing dynamics of their development do not make it easier for entrepreneurs to do the task. The final effect of the investment can be influenced not only by the internal conditions of the enterprise such as its structure, management staff, human capital, but also external factors. The most important of them are market globalization, information flows, very high competitiveness and finally the development of new technologies. Therefore, the process of identifying threats and effective attempts to reduce the adverse effects of decisions taken in an atmosphere of uncertainty are a prerequisite for the company's survival on the market. The profitability analysis of investment projects should therefore focus not only on micro- and macroeconomic factors, but also take into account global factors. Therefore, their identification is one of the basic tasks of the company. Making an investment decision is one of the most difficult tasks of the company. The investment implementation itself is the result of a long and arduous process of analyzing investment profitability. Guided by the subject of investment, the following groups can be distinguished: material investments, financial investments and investments in human capital. This first class is the enterprise's fixed assets and includes purchases of machinery, technical equipment, land, real estate, etc. The second group includes purchases of securities or opening of bank deposits. Traditional investment profitability testing methods are always based on the assumption of stable investment conditions, i.e. future cash flows are based on projections that may prove out of date in the future. This is obviously due to the uncertainty or unpredictability of the market and concerns material and financial investments. The classical method of updated current value assumes that the basic criterion for choosing an appropriate investment project is to maximize the expected value of future discounted cash inflows related to the project implementation. However, this method ignores changes in investment conditions that make some investments no longer profitable and others become. Therefore, the article deals with issues related to the process of investment profitability taking into account risk factors.

RESEARCH RESULTS AND DISCUSSION

1. Utility functions

In this section will be considered an entrepreneur having the opportunity to invest his capital, or broadly some good. Of course, these possibilities affect the state of ownership at the end of the investment period. The investor's goal is to choose the alternative or option that would bring the highest possible level of good. This good can mean money, or financial profit, but it can also mean intangible assets (e.g. acquiring new business partners, ease of cooperation, raising employees' qualifications). If the results of these investments are known, then it is easy to determine the ranking of alternatives. However, in the random case, i.e. when the level of good at the end of the period is not known and can be described by a random variable, determining the best alternative is not obvious. Therefore, a method is needed that would help construct a certain ranking in the set of random variables. Such a tool is utility function. Formally, the utility function *U* is defined on a set of real numbers. Then the ranking list is created according to the von Neuman - Morgenstern criterion, i.e. the criterion of maximizing the expected value [16; 46].

The *a* alternative is no worse than the *b* alternative, if $EU(X_a) \geq EU(X_b)$; where X_i is a random payout or random profit at the end of the investment period after selecting $i = a; b$. The following designation is used [7; 11; 25; 39]:

$$X_a \succcurlyeq X_b \Leftrightarrow EU(X_a) \geq EU(X_b):$$

Thus, this operation allows you to determine the order in a set of random variables. Utility functions used by the entrepreneur or decision-maker depend on his individual risk tolerance, his financial background, psychological conditions and material situation. The simplest utility function is the linear function $U(x) = x$: The investor using this utility function is called the investor it risk neutral, because this function only takes into account the expected value of future revenues. The only assumption that the function is a utility function is monotonicity (the function must be increasing) and continuity. Classical usability theory says that utility functions should be differentiable even twice [12]. These properties imply that the functions are convenient in calculations and the models based on them are used, e.g. in microeconomics, finance and analysis of consumer decisions [45]. Figure 1 presents the utility functions most commonly found in practice.

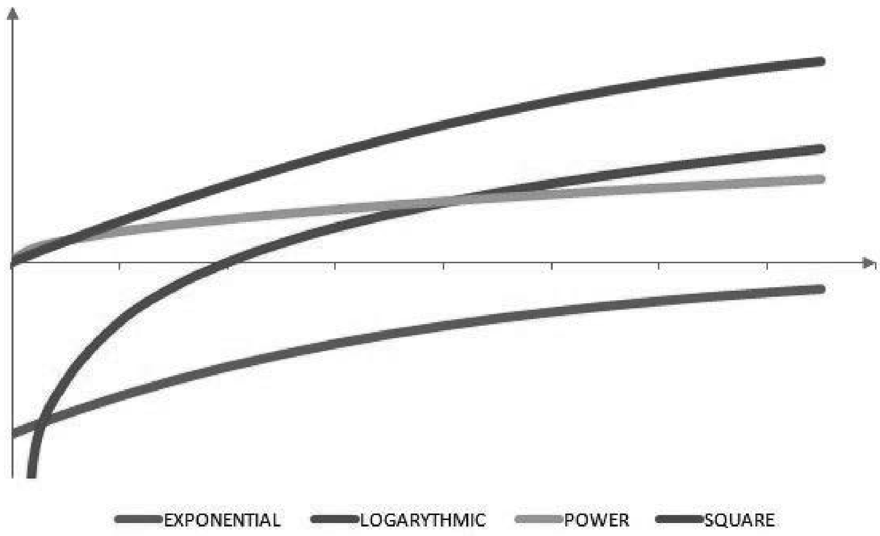

Figure 1. Examples of utility functions: exponential $\boldsymbol{U(x)} = -\boldsymbol{2e^{-0,5x}}$ *logarithmic* $\boldsymbol{U(x) = ln\,x}$*; power* $\boldsymbol{U(x)}$ *=0,5*$\boldsymbol{x^{0,5}}$*, square* $\boldsymbol{U(x) = x - 0,1x^2}$.

Source: own study

The following are four classes of utility functions found in the [31] literature. The domain of this function, i.e. the set in which it is specified, is denoted by DU.

[A] The exponential function $U(x) = -\frac{1}{\gamma}e^{-\gamma x}$; where $\gamma > 0$ is a certain parameter $DU = (-\infty,\infty)$:

[B] Logarithmic function $U(x) = ln\,x$, $DU = (0;+\infty)$:

It is easy to notice that although the function is specified for $x > 0$; in the event that the investor expects to be bankrupt with a positive probability, the use of such a utility function results in the expected usefulness of random withdrawal being $-\infty$.

[C] Power function $U(x) = \gamma x^{\gamma}$; where $\gamma < 1$ is a certain parameter. If $\gamma \epsilon (0;1)$; $DU = [0;+\infty)$: If $\gamma < 0$ then $DU = (0;+\infty)$:

[D] Quadratic function $U(x) = x - \gamma x^2$; where $\gamma > 0$ is a certain parameter, $DU = (-\infty; \frac{1}{2\gamma})$. This function is increasing for $x < \frac{1}{2\gamma}$.

It is worth emphasizing here that, although the utility function is a useful tool for creating a ranking of investments with random payments, its numerical value has no interpretation. Therefore, adding a constant to a utility function or multiplying it by a constant $k > 0$ does not change the ranking of alternatives. Therefore, the functions $U(x)$ and $\widehat{U}(x) = k_0 U(x) + k_1$ are considered equivalent because

$$X_a \succcurlyeq Xb \Leftrightarrow EU(X_a) \geq EU(X_b) \Leftrightarrow E\widehat{U}(X_a) \geq E\widehat{U}(X_b).$$

The rationale for using the criterion of maximizing the expected value from the utility function is the fact that this approach can be clearly described by the axioms [16]. It should also be mentioned that creating a ranking is only possible if the utility function is concave.

2. Risk aversion and utility function

The U function defined on the segment $[a;b]$ is concave [15, 29, 30], if for each $\alpha \epsilon\ [0;1]$ and $x;y \in [a;b]$ an inequality is satisfied

$$U(\alpha x + (1-\alpha)y) \geq \alpha U(x) + (1-\alpha)U(y).$$

This concave utility function U reflects the risk aversion of the decision maker. This property is illustrated in Figure 2.

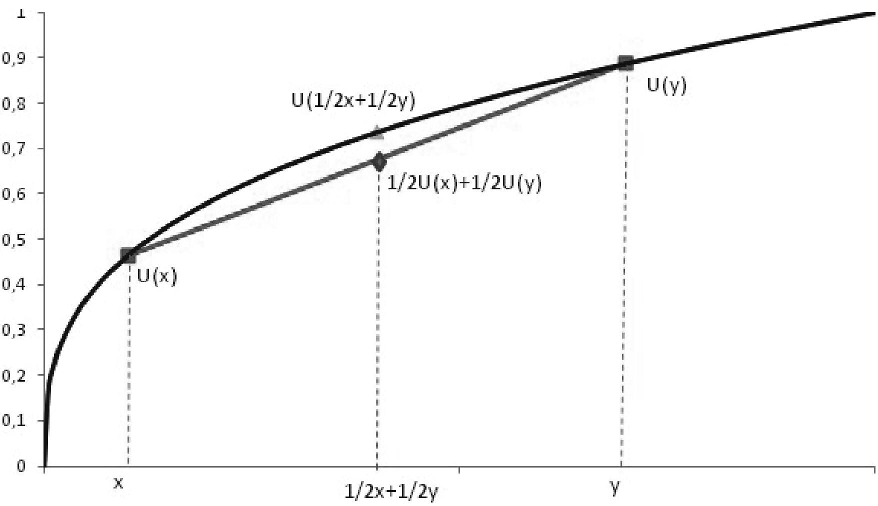

Figure 2 Concave utility function.

Source: own study

It was assumed that the decision maker has two alternatives to choose from:

(1) at the end of the period will receive x or y with probability $\frac{1}{2}$,

(2) receive $\frac{1}{2}x + \frac{1}{2}y$.

The expected utility of the first alternative is the weight of two utility function values $\frac{1}{2}U(x) + \frac{1}{2}U(y)$.The expected utility of the second option (not including risk) is equal to the value of the utility function in point $\frac{1}{2}x + \frac{1}{2}y$. This value is greater than the value of the first alternative when the utility function is concave. So the decision maker will choose the second alternative.

3. Risk aversion coefficient

The degree of risk aversion of the concave utility function is related to the fact how “strongly” such a function is concave. Formally, the degree of risk aversion is measured by a factor defined by Arrow and Pratt [2, 22, 31, 34]. To give the formula for the risk factor, it must be assumed that the utility function has a second derivative. If $U(x)$ is concave then $U''(x) < 0$ [15, 29, 30].

The Arrow-Pratt absolute risk aversion coefficient (Arrow-Pratt index)

$$\hat{\gamma}(x) = -\frac{U''(x)}{U'(x)}$$

Table 1 presents the most common utility functions and the corresponding Arrow-Pratt coefficient. Factor $U'(x)$ appearing in the denominator plays the role of a normalizing factor. Coefficient $\hat{\gamma}(x)$ illustrates the change in risk aversion along with the changing level of good. Most often, the risk ratio decreases as capital (assets) increases. This reflects the situation that an investor is able to take more risks if he feels more financially secured.

Table 1 The Arrow-Pratt coefficient for selected utility functions

Lp.	**Utility function** $U(x)$	**The Arrow-Pratt Coefficient**	**Coefficient properties**
[A]	$U(x) = -\frac{1}{\gamma}e^{-\gamma x}$	$\hat{\gamma}(x) = \gamma$	constant for each value of x
[B]	$U(x) = \ln x$	$\hat{\gamma}(x) = \frac{1}{x}$	decreases with increasing of x
[C]	$U(x) = \gamma x^{\gamma}$	$\hat{\gamma}(x) = \frac{\gamma - 1}{x}$	decreases with increasing of x
[D]	$U(x) = x - \gamma x^2$	$\hat{\gamma}(x) = \frac{2\gamma}{2\gamma x - 1}$	decreases with increasing of x

Source: own study

If the parameter γ tends to 0, in the case of utility [A] and [D] in Table 1, the decision-maker becomes increasingly risk-neutral [3, 4, 5, 44]. The same situation applies to the power utility [C] if γ is very close to 1.

4. Certainty equivalent

Although the expected value of the usefulness of a random good doesn't matter except comparing it to another alternative, you can define new concepts that have an intuitive meaning. This concept is the certainty equivalent [33], which for random profit is defined as the constant c such that

$$U(c) = E[U(X)].$$

In other words, it is a guaranteed value of a good, without any risk, for which the utility is the same as the expected value of the utility of the random good X [23]. If U is an increasing function, then there is an inverse function U^{-1} to the function U and you can write that

$$c = U^{-1}(E[U(X)]).$$

The certainty equivalent of a random variable for equivalent utility functions is the same and is measured in units of good value. Let $U(x)$ be a concave function of utility. The c constant is such a number that $U(c)$ equals the expected value of $U(X)$. In other words, it is such a value that the decision maker or company treats as a guaranteed withdrawal without investing in the X portfolio (which can be a loss or a profit). By definition of the equivalent of certainty and from Jensen's inequality [28], the inequality occurs

$$U(c) = E[U(X)] \leq U(EX).$$

Since U is an increasing function, $c \leq EX$. This fact is shown in Figure 3.

It has been assumed that the following investment is under consideration. The decision maker receives z and y payouts with probability $\frac{1}{2}$. Thus, the value of $E[U(X)]$ is halfway between the points $U(z)$ and $U(y)$, and the utility of the certainty equivalent is the intersection point of the function U and the horizontal straight line passing through the point $E[U(X)]$.

$$U(c) = E[U(X)] = \frac{1}{2}U(z) + \frac{1}{2}U(y).$$

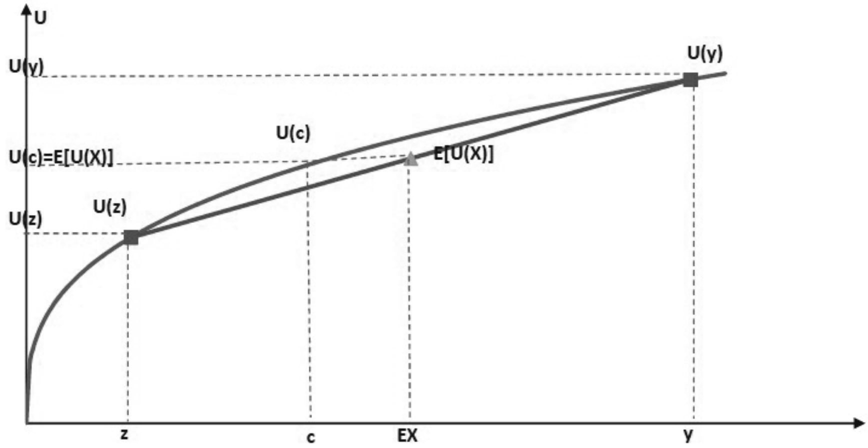

Figure 3 Certainty equivalent

Source: own study

Thus, it is easy to see that in the case of the concave utility function c is smaller than the expected value $E[X]$. It is clear that the stronger the function is concave (which corresponds to a more cautious investor), the number c lies further to the left of the value $E[U(X)]$ [17]. Mathematically, this is a consequence of Jensen's inequality. The value of $Premium = EX - c$ is called risk premium. In other words, the risk premium is an excess of return on investment over the risk-free amount [44]. In addition, [6] can be demonstrated that the risk premium is proportional to the random payout multiplied by a certain factor. More precisely, this coefficient is $\frac{1}{2}\hat{\gamma}(x)$.

The power utility function $U(x) = \gamma x^{\gamma}$ with parameter $\gamma \epsilon (0;1)$ was considered. Then $\gamma x^{\gamma} = \gamma E[X^{\gamma}]$; thus $c^{\gamma} = E[X^{\gamma}]$, or $c = (E[X^{\gamma}])^{\frac{1}{\gamma}}$. The certainty equivalent of this form is known in the literature as the Kreps-Porteus equivalent [27]. Example 1 shows the use of this equivalent for withdrawals with a uniform distribution.

5. Examples

EXAMPLE 1 It has been assumed that a random payment of X has a uniform distribution over the range of $[0;1]$. Then the utility function U

$$U(x) = x^{\gamma}\ ;\ \gamma \epsilon (0;1);$$

is specified for $x \in [0;1]$. The g parameter expresses the risk aversion of the decision maker. Since

$$E[X^\gamma] = \int_0^1 x^\gamma dx = \left[\frac{x^{1+\gamma}}{1+\gamma}\right]_0^1 = \frac{1}{1+\gamma}$$

from Table 1 it was obtained that

$$c = \left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}} = \frac{1}{(1+\gamma)^{\frac{1}{\gamma}}}.$$

It's easy to see that if γ is close to 1, then c is approaching value $\frac{1}{2}$ and $EX = \frac{1}{2}$. This borderline case means that risk aversion disappears. On the other hand, if $\gamma > 0$ and γ is close to 0, then $\theta = \frac{1}{\gamma}$ becomes any value.

Hence

$$c = \frac{1}{(1+\gamma)^{\frac{1}{\gamma}}} = \frac{1}{\left(1+\frac{1}{\theta}\right)^{\theta}} \to \frac{1}{e} \approx 0{,}37,$$

when θ tends to infinity.

This means that for getting rid of randomness, the decision-maker is willing to accept a smaller payout, and this payout decreases with g: In other words, the smaller the $\gamma$the lower the certainty equivalent and the greater the risk aversion. For example, for $\gamma = \frac{1}{2}$ received

$$c = \left(EX^{1/2}\right)^2 = \left(E\sqrt{X}\right)^2 = \left(\int_0^1 \sqrt{x}dx\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9} < EX = 0{,}5.$$

EXAMPLE 2 An investor was considered to decide on investment in a risky project. It has been estimated that this project will bring a profit of PLN 100,000 with a probability of about 5%, PLN 50,000 with a probability of 50%, will not bring a profit with a probability of 20% and with a probability of 25% will bring a loss of PLN 10,000. So let X be a random payout from this project.

$$X = \begin{cases} 10^5 \text{ with probability } 0{,}05 \\ 5 \cdot 10^4 \text{ with probability } 0{,}5, \\ 0 \text{ with probability } 0{,}2 \\ -10^4 \text{ with probability } 0{,}25. \end{cases}$$

The investor has a square utility function

$$U(x) = x - \frac{1}{2 \cdot 10^6} x^2, x < 10^6.$$

It was assumed that the decision maker faces two alternatives. In the first case, the probability $\frac{1}{2}$ receives a payout of 10, or a payout of 0 with the probability $\frac{1}{2}$ .In the second case, the player receives the amount A.

Then

$$E[U(X)] = \frac{1}{20}\left(10^5 - \frac{1}{2 \cdot 10^6} 10^{10}\right) + \frac{1}{2}\left(10^4 \cdot 5 - \frac{1}{2 \cdot 10^6} 25 \cdot 10^8\right) + \frac{1}{4}\left(-10^4 - \frac{1}{2 \cdot 10^6} 10^8\right).$$

$$E[U(X)] \approx 26612;5$$

Then c was calculated such that $c - \frac{1}{2 \cdot 10^6} c^2 = 26612{,}5$.

The quadratic equation was solved, obtaining $c = 26;976{:}36$.

So if a competitor compensates the investor with 26,976.36 PLN, the investor will be willing to surrender the project to a competitor.

Table 2 contains inverse functions and equivalence equivalents for selected utility functions.

Table 2. Inverse functions and certainty equivalents for selected utility functions

Utility function $U(x)$	**Parameter conditions and domain $U(x)$**	**Inverse function $U^{-1}(x)$**	**Certainty equivalent c**
$U(x) = lnx$	$x \in (0, \infty)$	$U^{-1}(x) = e^x$ $x \in (-\infty, \infty)$	$c = e^{E[lnX]}$
$U(x) = -\frac{1}{\gamma} e^{-\gamma x}$	$x \in (-\infty, \infty)$ $\gamma \epsilon (0, \infty)$	$U^{-1}(x) = -\frac{1}{\gamma} \ln(-\gamma x)$ $x \in (-\infty, 0)$	$c = -\frac{1}{\gamma} lnE[e^{-\gamma X}]$
$U(x) = \gamma x^\gamma$	$x \in [0, \infty)$ $\gamma \epsilon (0,1)$ lub $x \in (0, \infty)$ $\gamma < 0$	$U^{-1}(x) = \left(\frac{x}{\gamma}\right)^{\frac{1}{\gamma}}$ $x \in [0, \infty)$	$c = (E[X]^\gamma)^{\frac{1}{\gamma}}$
$U(x) = x - \gamma x^2$	$x \in \left(0, \frac{1}{2\gamma}\right)$ $\gamma \epsilon (0, \infty)$	$U^{-1}(x) = \frac{1 - \sqrt{1 - 4x\gamma}}{2\gamma}$ $x \in \left(-\infty, \frac{1}{4\gamma}\right)$	$c = \frac{1 - \sqrt{1 + 4\gamma EX - 4\gamma^2 EX}}{2b}$ EX is expected value of the investment

Source: own study based on [5; 31]

6. Methods for selecting utility functions

The choice of utility functions for the investor interested is a significant problem. One of the ways is to assign the investor the form of service functions and perform parameter estimation based on the conducted experiments among the examined group of people. The second use is to search for service functions. Since both characters and utility function parameters affect the value of the assessment, proper assessment is important in the [13] decision-making process. A set of standard procedures assigned to services functions for investors, decision makers or the entire company. Below are some ways to use it in practice.

1. Certainty Equivalent Method

One way to determine the utility of a decision maker is to assign a certainty equivalent of various risky alternatives. An elegant method is the organization of a lottery in which the decision maker knows the payday is A with a probability of p, or B with a probability of $1-p$. For different values of p the investor determines the price *c* (certainty equivalent) za for departing from the lottery. The expected value of such a lottery is $h = Ap + B(1-p)$. So if the decision maker is risk sensitive then the certainty equivalent *c* must be less than *h*.

2. Parameter selection method

Another method for determining the utility of a decision maker is to assign a given utility function from the appropriate class, followed by estimating a parameter. This method was proposed by Tversky and Kahneman [24]. It assumes that the utility function is exponential $U(x) = -\frac{1}{\gamma}e^{-\gamma x}$, because as research confirms, [8, 43] best characterizes the preferences of decision-makers. The $\gamma > 0$parameter can be set as a result of a simple lottery. The decision maker determines the equivalent of certainty c, which is the value he is able to accept for giving up participation in a certain lottery. The following lottery was proposed. The investor wins 2 with a probability of $\frac{2}{3}$ or loses 1 with a probability of $\frac{1}{3}$. If $c = 1{,}5$ for this decision maker then

$$-e^{-1{,}5\gamma} = -\frac{2}{3}e^{-2\gamma} - \frac{1}{3}e^{-\gamma}$$

The solution of the equation is $\gamma = 1{,}38629$.

3. Questionnaire

The basis of research on economic behavior is business psychology. It focuses on consumer behavior, studies financial behavior, deals with risk-taking and decision psychology [14, 40, 42]. Decision theory assumes individual decision-making preferences in relation to risk. However, there are many situations where it is desirable to determine the individual's risk / risk attitude. For example, banks would like to adjust the risk level of proposed investments by offering various investment instruments to the level of risk accepted by customers. Interesting to banks may also have an attitude to the risk of employees granting loans [41] (this is the so-called operational risk that may arise as a result of human error). The attitude to the entity's risk depends on the individual's perception of risk, his current financial position, future financial gains prospects, obligations and the person's age. One of the methods of estimating the appropriate risk factor and the entity's utility function is to conduct an appropriate survey. It gives a good qualitative assessment and the results can be used to determine the utility function. In the questionnaire, one question focuses on both the investor's financial position and the investor's approach to investing. The next questions characterize the market and relate to the value of the managed fund. This survey shows that risk tolerance is determined by the individual's perception of risk and by the investor's financial environment. The purpose of conducting such a survey is to determine a person's propensity to take investment risk. Such surveys are prepared in cooperation with psychologists. Investment firms use this type of questionnaire to research the client's investment profile because their propensity to risk affects which of the products offered to them is willing to accept. When a company knows the client's investment profile, it is able to offer products that best meet their needs.

CONCLUSION

The concept of expected utility enables formal analysis of economic behavior. A particular example of its application is the issue of choosing the optimal portfolio of shares. But since the theory of the value of expected utility has been formulated, there are discussions on its compliance with practice, with the observed behavior of individuals in a situation when a choice should be made. A number of experiments have been conducted which show that this approach is inefficient in many situations. Research by Kahneman and Tversky [24] showed that decision-makers evaluate the alternatives available to them on the basis of their own position, on their wealth, on their own experience. For positive forecasts, their utility function is concave, for negative forecasts convex (this is also confirmed by other researchers of human behavior [18, 21]). Very rare events are treated as impossible events, and the events with high probability of occurrence were treated as certain events. There are studies confirming that most people are risk averse when they focus on future profits and choose risk when they are facing losses. This phenomenon is known in the literature as theory of perspective [26, 35]. A person will choose a certain profit of 500 rather than a payout of 1000 with a probability of $\frac{1}{2}$. The same person will choose the risk of losing 1000 with a probability of $\frac{1}{2}$ than some loss of 500: It can be concluded that in the case of capital increase, the decision maker it is characterized by risk aversion, and in the case of capital decrease, in other words, losses are risk-sensitive [24].

REFERENCE

1. Allais M. (1953), Le comportement de l'homme rationnel devant le risque: critique des postulats et axioms de l'Ecole Americaine, Econometrica 21, 503-546.
2. Arrow K. J. (1965), Aspects of the Theory of Risk Bearing, The Theory of Risk Aversion, Helsinki: Yrjo Jahnssonin Saatio. Reprinted in: Essays in the Theory of Risk Bearing, Markham Publ. Co., Chicago, 1971, 90–109.
3. Bäuerle N., Ja´skiewicz A. (2015), Risk sensitive dividend problems, European Journal of Operational Research 242, 161-171.

4. Bäuerle N., Ja´skiewicz A. (2017), Dividend payout model with risk sensitive preferences, Insurance: Mathematics and Economics 73, 82-93.

5. Bäuerle N., Rieder U. (2011), Markov Decision Processes with Applications to Finance, Springer, Berlin.

6. Bäuerle N., Rieder U. (2017), Zero-sum risk-sensitive stochastic games, Stochastic Processes and their Applications 127, 622-642.

7. Begg D., Fisher S., Dornbusch R. (2003), Ekonomia. Mikroekonomia, Polskie Wydawnictwo Ekonomiczne, Warszawa.

8. Bell D.E. (1988), One-switch utility functions and a measure of risk, Management Science 34, 1416-1424.

9. Bernoulli D. (1954), Exposition of new theory on the measurement of risk, Econometrica 22, 23-36.

10. Camerer C. (1989), An experimental test of several generalized utility theories, Journal of Risk and Uncertainty 2, 61-104.

11. Czarny E. (2006), Mikroekonomia, Polskie Wydawnictwo Ekonomiczne, Warszawa.

12. Dror M., Hartman B. C. (1999), Stopping rules for utility functions and the St. Petersburg gamble, Applied Mathematics and Computation 98, 279-291.

13. Dudzińska-Baryła R. (2012), Wybrane metody szacowania funkcji użyteczności i funkcji wartości, W: Modelowanie preferencji a ryzyko '12, Zeszyty naukowe wydziałowe Uniwersytetu Ekonomicznego w Katowicach, Katowice.

14. Falkowski A., Tyszka T. (2006), Psychologia zachowań konsumenckich, Gdańskie Wydawnictwo Psychologiczne, Gdańsk.

15. Fichtenholz G. M. (2011), Rachunek ró˙zniczkowy i całkowy, Wydawnictwo Naukowe PWN, Warszawa.

16. Fishburn P. C. (1982), The Foundations of Expected Utility, Springer, Dordrecht.

17. Forlicz S., Jasi´nski M. (2000), Mikroekonomia, Wydawnictwo WSB, Pozna´n.

18. Frankenhuis,W. E., del Giudice, M. (2012), When do adaptive developmental mechanisms yield maladaptive outcomes?, Developmental Psychology 48, 628–642.

19. Gluzicka A. (2013), Problem naruszania zasad teorii oczekiwanej użyteczności na przykładzie paradoksu Allais, Studia ekonomiczne, Analiza i wspomaganie decyzji 135, 51-63.

20. Huck S., Muller W. (2007), Allais for all: revisiting the paradox, ELSE Working Papers 289.

21. Houston A.I, Fawcett T.W., Mallpress D.E., McNamara J.M. (2014), Clarifying relationship between prospect theory and risk-sensitive foraging theory, Evolution and Human Behavior 35, 502-506.

22. Jajuga K., Jajuga T. (1998), Inwestycje. Instrumenty finansowe, ryzyko finansowe, inżynieria finansowa, Wydawnictwo Naukowe PWN, Warszawa.

23. Jajuga K. (2004), O pewnych modelach decyzji finansowych, Decyzje 1, 37-54.

24. Kahneman D., Tversky, A. (1979), Prospect theory: an analysis of decision under risk, Econometrica 47, 263-291.

25. Klimczak B. (2006), Mikroekonomia, Wydawnictwo AE im. Oskara Langego we Wrocławiu, Wrocław.

26. Kopańska-Bródka D. (2012), Wzorzec modelu oczekiwanej użyteczności a rozwój teorii decyzji, W: Modelowanie preferencji a ryzyko '12, Zeszyty naukowe wydziałowe Uniwersytetu Ekonomicznego w Katowicach, Katowice.

27. Kreps D., Porteus E. (1978), Temporal resolution of uncertainty and dynamic choice theory, Econometrica 46, str. 185-200.

28. Krysicki W., Bartos J., Dyczka W., Królikowska K., Wasilewski M. (1999), Rachunek prawdopodobieństwa i statystyka matematyczna w zadaniach. Część I Rachunek prawdopodobieństwa, Wydawnictwo Naukowe PWN, Warszawa.

29. Krysicki W., Włodarski L., (2011), Analiza matematyczna w zadaniach cz 1,Wydawnictwo Naukowe PWN, Warszawa.

30. Krysicki W., Włodarski L., (2011), Analiza matematyczna w zadaniach cz 2,Wydawnictwo Naukowe PWN, Warszawa.

31. Luenberger D. (1998), Investment Science, Oxford University Press, New York Oxford, 229-238.

32. Machina M.J. (2004), Nonexpected Utility Theory,W: Encyclopedia Of Actuarial Science, Red. J.L. Teugels, B. Sundt, John Wiley and Sons, Chichester.

33. Merton R. C. (1990), Continuous-Time Finance, Blackwell Publishers, Cambridge.

34. Pratt, J. W. (1964), Risk aversion in the small and in the large, Econometrica 32, 122-136.

35. Rabin M. (2000), Risk aversion and expected utility theory: a calibration theorem, Econometrica 68, 1281-1292.

36. Rudkowski A. (2010), Zarządzanie finansami, Wydawnictwo Naukowe PWN, Warszawa.

37. Samuelson P. (1977), St. Petersburg paradoxes: defanged, dissected, and historically described, Journal of Economic Literature 15, 24-55.

38. Seidl C. (2013), The St. Petersburg Paradox at 300, Journal of Risk and Uncertainty 46, 247-264.

39. Tokarski T. (2011), Ekonomia matematyczna, Polskie Wydawnictwo Ekonomiczne, Warszawa.

40. Tyszka T. (1986), Analiza decyzyjna i psychologia decyzji, Wydawnictwo Naukowe PWN, Warszawa.

41. Tyszka T., Domurat A. (2004), Czy istnieje ogólna skłonność jednostki do ryzyka?, Decyzje 2, 85-103.

42. Tyszka T., Zaleśkiewicz T. (2001), Racjonalność decyzji, Polskie Wydawnictwo Ekonomiczne, Warszawa.

43. Weber M. (1990), Risikoenscheidungskalkule in der Finanzierungstheorie, C. E. Poeschel Verlag, Stuttgart.

44. Whittle P. (1990), Risk-Sensitive Optimal Control, Wiley, New York.

45. Varian H.R. (1992), Microeconomic Analysis, W.W. Norton, Baltimore.

46. von Neumann J., Morgenstern O. (1944), Theory of Games and Economic Behavior, Princeton University Press, Princeton, New Jersey.

47. Zhang Z.(2010), Certainty equivalent, risk premium and asset pricing, Higher Education Press and Springer - Verlag.