Вісник Тернопільського національного технічного університету
https://doi.org/10.33108/visnyk_tntu
Scientific Journal of the Ternopil National Technical University
2019, № 4 (96) https://doi.org/10.33108/visnyk_tntu2019.04
ISSN 2522-4433. Web: ***visnyk.tntu.edu.ua***

UDC 539.3

INTERPHASE CROSS-SECTIONS IN THE INFINITY PLATE WITH CURVILINEAR CONTOUR REINFORCED BY CLOSED RIB

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Summary. *Under the conditions of generalized plane stress state, caused by forces uniformly distributed at infinity, mixed contact problem for infinity isotropic plate with curvilinear hole which contour is reinforced by closed elastic rib is considered when there are two symmetric interphase cross-sections with zero width on the boundary of the plate and rib materials. The system of singular integral-differential equations for determining the contact forces between the plate and the rib and the internal forces and moments in the rib is constructed due to the reinforced rib modelling by closed curved rod of the constant rectangular cross-section, the middle surface of which does not coincide with the plate hole surface, and the combination of the plate and the rib by ideal mechanical contact. The problem boundary conditions are formulated in the form of conditions for plate and rib joint deformation. In order to calculate the initial parameters in the statically indeterminate reinforcing rib, the conditions for the displacement uniqueness of the points of its axis and the cross sections rotation angles are used. Using a special approach to the representation of components of the stress state in a conditionally cut rib, the structure of the searched functions at the ends of the connection area of the plate and the rib are established. The approximate problem solution is constructed by mechanical quadratures and collocation method, by which the influence of the rib physical-geometric parameters and the type of external load on the stresses distribution in the plate and the reinforcing rib are investigated. It is determined that the components of the stress-strain deformed state in the rib at the ends of the junction area have restricted values.*

Keywords: *interphase cross-sections, isotropic plate, elastic rib, transverse and longitudinal forces, bending moment, singular integral equations, contact efforts.*

https://doi.org/10.33108/visnyk_tntu2019.04.023 *Received 10.01.2020*

Introduction. Thin-walled plates with curvilinear holes, which contours are reinforced by closed elastic ribs of constant cross-section, are widely used in modern structures, machines and structures [1–3].

Currently, the most common reinforcing rib model is the closed curved rod [3–6], which is used in papers [3, 4, 6] for investigation of a number of problems concerning the contact interaction of isotropic and orthotropic plates with curved hole and closed elastic rib at their combination providing guaranteed tension or welding method.

During production and operation of plates with strengthened curvilinear hole at the interface of heterogeneous materials, defects in the form of interphase cuts resulting in high stresses concentration and subsequent interphase fracture under the force loading action occur.

If one symmetrical interphase section between the plate and the closed resilient rib of constant rectangular cross-section included, the calculation problems of the stress-deformed state of the plate structure under generalized flat stress state conditions, are investigated in papers [7, 8]. For piecewise-homogeneous structures with two interphase sections at the interface of the plate and closed elastic rib, such problems are not considered.

The numerical-analytic solution of the mixed contact problem of curvilinear hole contour strengthening in the infinite isotropic plate with closed elastic edge is proposed for the case if there are two symmetrical sections at the interface.

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Statement of the problem. Let us consider the infinite isotropic plate with thickness $2h$ weakened by symmetric curvilinear hole, which Γ contour is reinforced by closed elastic rib of the constant rectangular cross-section with width 2η and thickness $2h_0$, being under the generalized flat stress state conditions, created by forces p and q, acting on infinities in two mutually perpendicular directions. The common median plane of the plate and the ribs is related to Cartesian $(x,\ y)$ and polar $(r,\ \delta)$ coordinate systems with the pole in the hole center. The reference systems are chosen in such a way that axis Ox coincides with the polar axis and determines the direction of force p action (Fig. 1).

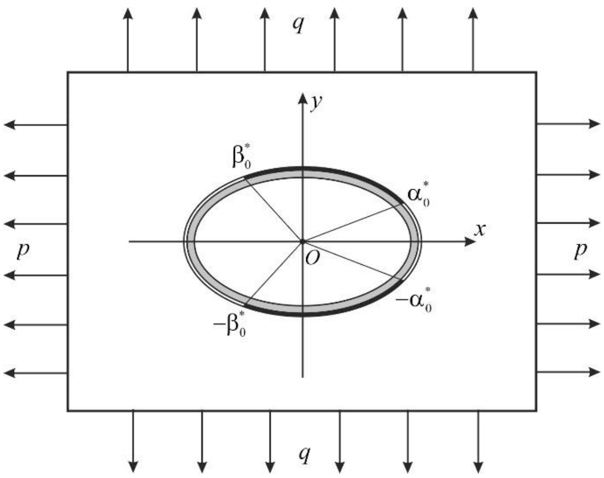

Figure 1. Calculation scheme of the plate with strengthened hole

Let us assume that in the course of structure manufacturing or its operation on the interconnection line of the plate and the ribs externally the sections $[-\beta_0^*,\ -\alpha_0^*]$, $[\alpha_0^*,\ \beta_0^*]$, (α_0^*, β_0^* – polar angles), there are two symmetrical relatively to axis Ox interphase sections, the edges of which do not contact during deformation.

The objective of the paper is to determine the stress state components on Γ contour in the plate, the reinforcing rib and to investigate the effect on these interphase sections values, the rib rigidity and the external load type.

Basic equations of the problem. Let us conditionally divide the considered structure into separate elements (infinite isotropic plate with curvilinear hole and reinforcing rib), replacing the action of one body with another by unknown contact forces.

The infinite isotropic plate is in equilibrium under the load action on infinites and normal T_ρ and tangent $S_{\rho\lambda}$ contact forces transmitted to Γ contour from the reinforcing rib.

Let the hole shape in the plate be described by function [7]

$$z = x + iy = \omega(\xi) = R_0\left(\xi + \frac{\varepsilon_1}{\xi} + \frac{\varepsilon_2}{\xi^2}\right), \tag{1}$$

which implements the conformal reflection of the form S^- of the single circle γ in plane $\xi = \tilde{\rho} e^{i\lambda}$ on the area occupied by the plate median plane.

Here $R_0 = 1$ is the characteristic hole size; $\varepsilon_1,\ \varepsilon_2$ are parameters characterizing the deviation of Γ contour shape from the circle; $i = \sqrt{-1}$; $(\tilde{\rho},\ \lambda)$ are polar coordinates of points in plane ξ.

The deformation of Γ contour in the plate at the given load is determined by formulas [7]

$$\varepsilon_\lambda = \frac{1}{2Eh(\alpha^2+\beta^2)}\left[(1-\nu)(\alpha^2+\beta^2)T_\rho(\lambda) - \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[\Phi_1(\lambda,\ t)T_\rho(t) - \Phi_2(\lambda,\ t)S_{\rho\lambda}(t)\right]dt + \right.$$

$$\left. + \alpha(\lambda)\tilde{\varepsilon}_\lambda^0 + \beta(\lambda)\tilde{V}^0\right]; \quad V = \frac{1}{2Eh(\alpha^2+\beta^2)}\left[(1-\nu)(\alpha^2+\beta^2)S_{\rho\lambda}(\lambda) - \right.$$

$$-\frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[\Phi_3(\lambda,t)S_{\rho\lambda}(t)+\Phi_4(\lambda,t)T_\rho(t)\right]dt+\alpha(\lambda)\tilde{V}^0-\beta(\lambda)\tilde{\varepsilon}_\lambda^0\Bigg],\ \ \lambda\in[0,\ \pi],\qquad(2)$$

where the following notation is introduced

$$\Phi_1(\lambda,\ t)=S_1(\lambda,t)+S_3(\lambda,t)+S_4(\lambda,t)\mathrm{ctg}\frac{\lambda+t}{2}-S_2(\lambda,t)\mathrm{ctg}\frac{\lambda-t}{2};$$
$$\Phi_2(\lambda,\ t)=S_2(\lambda,t)+S_4(\lambda,t)-S_3(\lambda,t)\mathrm{ctg}\frac{\lambda+t}{2}+S_1(\lambda,t)\mathrm{ctg}\frac{\lambda-t}{2};$$
$$\Phi_3(\lambda,\ t)=S_2(\lambda,t)-S_4(\lambda,t)+S_3(\lambda,t)\mathrm{ctg}\frac{\lambda+t}{2}+S_1(\lambda,t)\mathrm{ctg}\frac{\lambda-t}{2};$$
$$\Phi_4(\lambda,\ t)=S_1(\lambda,t)-S_3(\lambda,t)-S_4(\lambda,t)\mathrm{ctg}\frac{\lambda+t}{2}-S_2(\lambda,t)\mathrm{ctg}\frac{\lambda-t}{2};$$
$$S_1(\lambda,t)=\alpha(\lambda)\alpha(t)+\beta(\lambda)\beta(t);\ \ S_2(\lambda,t)=\alpha(\lambda)\beta(t)-\beta(\lambda)\alpha(t);$$
$$S_3(\lambda,t)=\alpha(\lambda)\alpha(t)-\beta(\lambda)\beta(t);\ \ S_4(\lambda,t)=\alpha(\lambda)\beta(t)+\beta(\lambda)\alpha(t);$$
$$\tilde{\varepsilon}_\lambda^0=p+q-2(p-q)\cos2\lambda+(p+q)(\varepsilon_1\cos2\lambda+2\varepsilon_2\cos3\lambda);$$
$$\tilde{V}^0=2(p-q)\sin2\lambda-(p+q)(\varepsilon_1\sin2\lambda+2\varepsilon_2\sin3\lambda);\qquad(3)$$

E, ν – are Young's modulus and Poisson's ratio of plate material Юнга; ε_λ, V are the relative elongation of Γ contour and the angle of the normal rotation to it; $[\alpha_0,\ \beta_0]$ is section $[\alpha_0^*,\ \beta_0^*]$ prototype under mapping (1); $\alpha+i\beta=\omega'(\sigma)$; $\sigma=e^{i\lambda}$.

The ring forces T_λ on Γ contour are determined from the relation [7]

$$T_\lambda=\nu T_\rho+2Eh\varepsilon_\lambda.\qquad(4)$$

The reinforcing rib is simulated by closed curvilinear rod of the constant rectangular cross-section $2h_0\times2\eta$, which is deformed by the contact forces transmitted to its outer lateral surface from the plate. The stress state calculation problem for such rod is statically indetermined.

Taking into account the problem symmetry relatively to axis Ox, we conditionally neglect the lower strengthening part, replacing it with the longitudinal forces N_0, N_1 and bending moments $L_b^{(0)}$, $L_b^{(1)}$ applied to the ends $\lambda=0$ and $\lambda=\pi$ (Fig. 2). As a result, we obtain the open statically determined rod, which is in equilibrium under the contact forces action at section $[\alpha_0^*,\ \beta_0^*]$ $([\theta_0;\theta_1])$ and loading at the ends.

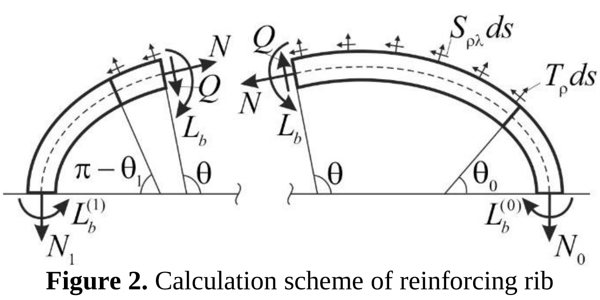

Figure 2. Calculation scheme of reinforcing rib

We describe the stress-strain state of the open rib is described by the equations of one-dimensional theory of curvilinear rods, taking into account the hypothesis of flat sections and transverse shear deformation [5, 9]:

- differential equations of rod element equilibrium are

$$T_\rho(\lambda)=\frac{N(\lambda)}{\rho}-\frac{dQ(\lambda)}{ds};\ S_{\rho\lambda}(\lambda)=-\frac{Q(\lambda)}{\rho}-\frac{dN(\lambda)}{ds};\quad\frac{dL_b(\lambda)}{ds}=\eta\frac{dN(\lambda)}{ds}+Q(\lambda);\qquad(5)$$

- physical dependencies for the outer longitudinal rod fiber, contacting with the plate are

$$\varepsilon_\lambda^{(\tilde{n})} = \frac{1}{E_0 F_0}\left[N(\lambda) + \frac{\eta + \eta_c}{\rho \eta_c} L_b(\lambda)\right]; \quad \frac{d\theta_b}{d\theta} = \frac{1}{E_0 F_0}\left[N(\lambda) + \frac{L_b(\lambda)}{\eta_c} - 2(1+\nu_0)\mu \frac{dQ(\lambda)}{d\theta}\right], \quad (6)$$

where $N(\lambda), Q(\lambda), L_b(\lambda)$ are the longitudinal and transverse forces and bending moment occurring in the rod cross-sections and related to its axis; $\varepsilon_\lambda^{(c)}$, θ_b is relative fiber elongation and the angle of the normal rotation to it; θ is the normal inclination angle at the point (x, y) of Γ contour to the axis Ox; E_0, ν_0 – are Young's modulus and Poisson's ratio of the rod material; $E_0 F_0$ is its tensile (compression) strength; μ is the constant (for rectangular section $\mu = 1.2$); η_c is the distance from the rod axis to the neutral one for the longitudinal fiber pure bending; $ds = \rho d\theta = |\omega'(\sigma)| d\lambda$; ρ is Γ contour curvature radius; $e^{i\theta} = e^{i\lambda}\omega'(\sigma)/|\omega'(\sigma)|$.

In addition to relations (5), (6), the conditions of uniqueness of the normal rotation angle and horizontal displacement must be met [7–9]

$$\int_0^\pi \left[N(\lambda) + \frac{L_b(\lambda)}{\eta_c}\right] d\theta = 0; \quad \int_0^\pi \left[N(\lambda)(x - \rho\cos\theta) + L_b(\lambda)\frac{x - (\eta + \eta_c)\cos\theta}{\eta_c}\right] d\theta = 0, \quad (7)$$

as well as the equilibrium conditions of the open rod shown in Fig. 2

$$\int_{\theta_0}^{\theta_1} \left[T_\rho(\lambda)(y\cos\theta - x\sin\theta) - S_{\rho\lambda}(\lambda)(y\sin\theta + x\cos\theta)\right] ds =$$

$$= L_b^{(0)} - L_b^{(1)} + (x(0) - \eta)N_0 + (x(\pi) + \eta)N_1; \quad \int_{\theta_0}^{\theta_1} \left(T_\rho(\lambda)\sin\theta + S_{\rho\lambda}(\lambda)\cos\theta\right) ds = N_0 + N_1. \quad (8)$$

Mathematical model of the problem. The boundary problem conditions are formulated in the form of conditions for the plate and ribs compatible deformation at their connection section

$$\varepsilon_\lambda(\lambda) = \varepsilon_\lambda^{(c)}(\lambda); \quad V(\lambda) = \theta_b(\lambda), \quad \lambda \in [\alpha_0; \beta_0]. \quad (9)$$

Substituting (2), (6) into condition (9), we obtain the system of singular integral equations with Hilbert kernels for functions $T_\rho(\lambda), S_{\rho\lambda}(\lambda)$, $N(\lambda)$, $Q(\lambda)$, $L_b(\lambda)$ and constants N_0, N_1, $L_b^{(0)}$, $L_b^{(1)}$ determination

$$(1-\nu)(\alpha^2 + \beta^2)T_\rho(\lambda) - \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[\Phi_1(\lambda, t)T_\rho(t) - \Phi_2(\lambda, t)S_{\rho\lambda}(t)\right]dt + \alpha\tilde{\varepsilon}_\lambda^0 + \beta\tilde{V}^0 =$$

$$= \frac{2Eh(\alpha^2 + \beta^2)}{E_0 F_0}\left[N(\lambda) + \frac{\eta + \eta_c}{\rho\eta_c}L_b(\lambda)\right];$$

$$(1-\nu)(\alpha^2 + \beta^2)S_{\rho\lambda}(\lambda) - \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[\Phi_3(\lambda, t)S_{\rho\lambda}(t) + \Phi_4(\lambda, t)T_\rho(t))\right]dt + \alpha\tilde{V}^0 - \beta\tilde{\varepsilon}_\lambda^0 =$$

$$= \frac{2Eh(\alpha^2 + \beta^2)}{E_0 F_0}\int_\pi^\theta\left[N(\lambda) + \frac{L_b(\lambda)}{\eta_c} - 2(1+\nu_0)\mu\frac{dQ(\lambda)}{d\theta}\right]d\theta, \quad \lambda \in [\alpha_0; \beta_0]. \quad (10)$$

Along with equations (5) and conditions (7), (8), it determines the mathematical model of the problem.

Approximate solution of the problem. The exact system solution (5), (7), (8), (10) cannot be found. In order to construct its approximate solution, it is necessary to establish the structure of the desired functions at the ends of the plate interface section and the reinforcing rib.

For this purpose let us consider the open rod cross-section, inclined to axis Ox at angle θ. It divides the rod into two parts that are in equilibrium under the action of forces N, Q, bending moment L_b, contact forces and end load (Fig. 2). From the equilibrium conditions each part we find expressions for internal force factors at characteristic rod sections

$$N(\theta)=N_0\cos\theta;\quad Q(\theta)=N_0\sin\theta;\quad L_b(\theta)=L^{*(0)}+\left[x(\theta_0)-x(\theta)+\eta(\cos\theta-\cos\theta_0)\right]N_0,$$
$$\theta\in[0;\theta_0];$$
$$N(\theta)=\tilde{N}(\theta)+\cos\theta\left(N_0\frac{\theta_1-\theta}{\theta_1-\theta_0}-N_1\frac{\theta-\theta_0}{\theta_1-\theta_0}\right);$$
$$Q(\theta)=\tilde{Q}(\theta)+\sin\theta\left(N_0\frac{\theta_1-\theta}{\theta_1-\theta_0}-N_1\frac{\theta-\theta_0}{\theta_1-\theta_0}\right);$$
$$L_b(\theta)=\tilde{L}_b(\theta)+L^{*(0)}\frac{\theta_1-\theta}{\theta_1-\theta_0}+L^{*(1)}\frac{\theta-\theta_0}{\theta_1-\theta_0},\ \theta\in[\theta_0;\theta_1];\qquad (11)$$
$$N(\theta)=-N_1\cos\theta;\quad Q(\theta)=-N_1\sin\theta;\quad L_b(\theta)=L^{*(1)}+\left[x(\theta)-x(\theta_1)-\eta(\cos\theta-\cos\theta_1)\right]N_1,$$
$$\theta\in[\theta_1;\pi].$$

Here $L^{*(0)}=L_b^{(0)}+\left[x(0)-x(\theta_0)+\eta(\cos\theta_0-1)\right]N_0$; $L^{*(1)}=L_b^{(1)}+\left[x(\theta_1)-x(\pi)-\eta(\cos\theta_0+1)\right]N_1$ are bending moments occurring in cross-sections $\theta=\theta_0$ and $\theta=\theta_1$ relatively; $\tilde{N}(\theta)$, $\tilde{Q}(\theta)$, $\tilde{L}_b(\theta)$ are limited and continuous at $[\theta_0;\theta_1]$ of the function for which conditions are fulfilled $\tilde{N}(\alpha_0)=\tilde{N}(\beta_0)=\tilde{Q}(\alpha_0)=\tilde{Q}(\beta_0)=\tilde{L}_b(\alpha_0)=\tilde{L}_b(\beta_0)=0$.

At the end section points $[\alpha_0,\beta_0]$, the contact forces have the root peculiarity, resulting from the boundary conditions character change at these points.

Substituting (11) into (5), (7), (8), (10), we obtain the equivalent equations system relatively to functions T_ρ, $S_{\rho\lambda}$, $\tilde{N}$, $\tilde{Q}$, $\tilde{L}_b$ and constants N_0, N_1, $L^{*(0)}$, $L^{*(1)}$ having the same structure as in the case of a single interphase section [7, 8]. This means that the method of mechanical quadratures and collocation of its approximate solution, proposed in [8], is transferred without unchanged.

If the solution of such system becomes known, the ring forces T_λ at Γ can be determined by the formula (4), and the internal force factors in the reinforcement can be determined from the relations (11). The normal stresses in the outer and inner longitudinal rib fibers and the greatest tangential stresses in the axial fiber are determined by the formulas [4]

$$\sigma^{(1)}=\frac{1}{F_0}\left[N+\frac{\eta+\eta_c}{\eta_c}\cdot\frac{L_b}{\rho}\right];\quad \sigma^{(2)}=\frac{1}{F_0}\left[N+\frac{\eta_c-\eta}{\eta_c}\cdot\frac{L_b}{\rho-2\eta}\right];\quad \tau_{\max}=\frac{3Q}{2F_0}.\qquad (12)$$

Analysis of numerical results. For infinite plate with egg-shaped hole ($\varepsilon_1=0.1$; $\varepsilon_2=-0.1$) and reinforcing rib with parameters

$$h_0/h=4/3;\qquad \eta/R_0=0.1;\qquad \alpha_0=30^\circ;\qquad \beta_0=150^\circ$$

the influence on the plate and rib stress state relatively to the reinforcement rigidity is investigated.

The results of numerical calculation of the values T_ρ, $S_{\rho\lambda}$, T_λ on Γ contour in the plate and $F_0\sigma^{(1)}$, $F_0\sigma^{(2)}$, $F_0\tau_{\max}$ – in the rib are shown in Fig. 3–5, ($p=1$, $q=0$) and Fig. 6–8 ($p=0$, $q=1$). Dashed lines are constructed for the case $E_0/E=1$; solid for $E_0/E=5$; dash-and-dot lines – for $E_0/E=10$.

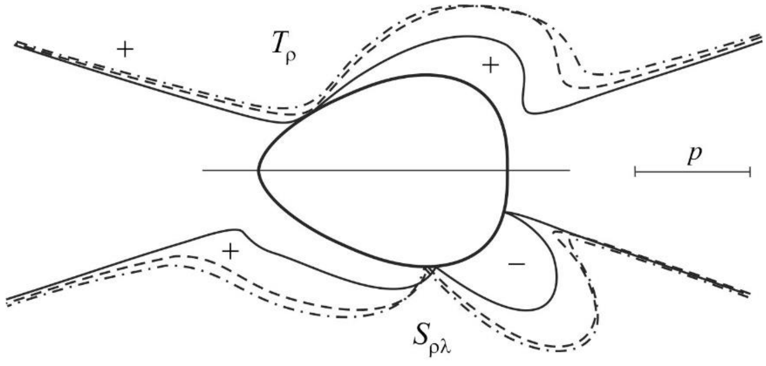

Figure 3. Diagrams of contact forces distribution on Γ contour in the plate

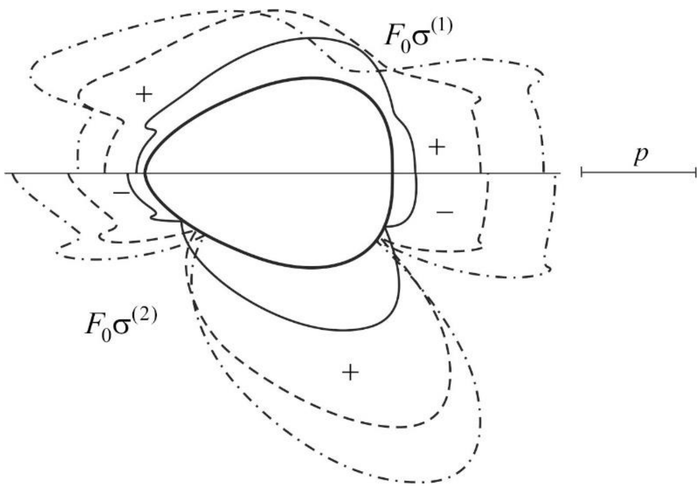

Figure 4. Diagrams of normal stresses distribution in the extreme fibers of the reinforcing rib

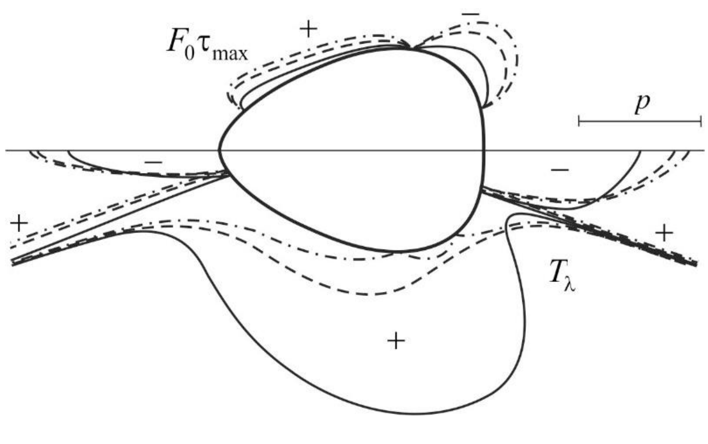

Figure 5. Diagrams of the maximum tangential stresses distribution f in the axial fiber of the reinforcing rib and ring forces on Γ contour in the plate

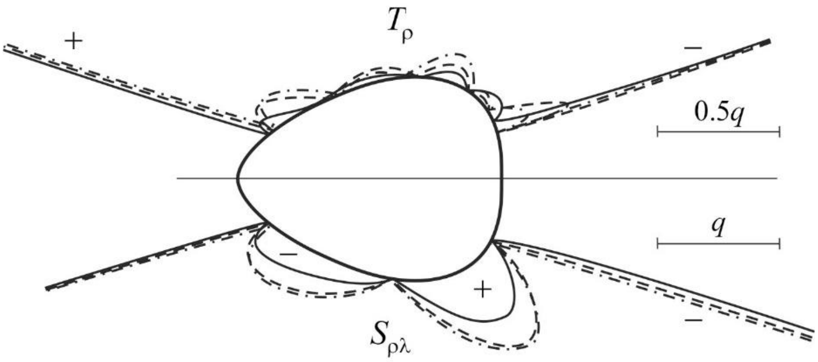

Figure 6. Diagrams of contact forces distribution on Γ contour in the plate

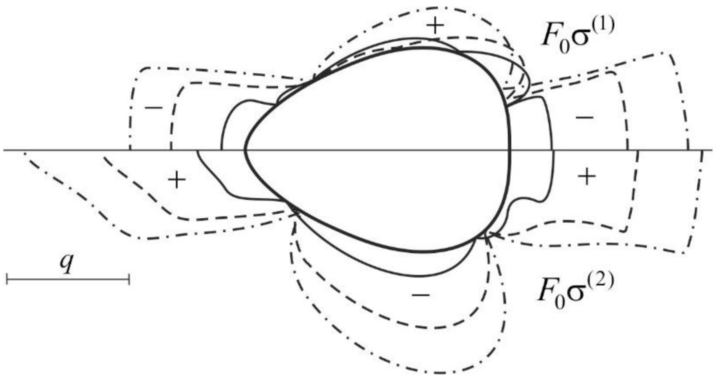

Figure 7. Diagrams of normal stresses distribution in the extreme fibers of the reinforcing rib

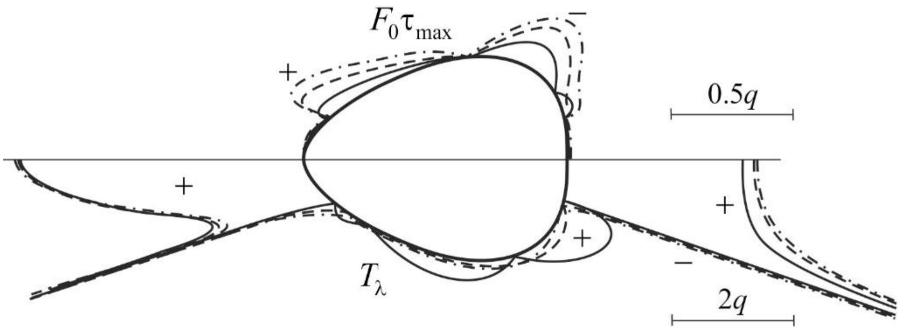

Figure 1. Diagram of the maximum tangential stresses distribution in the axial fiber of the reinforcing rib and ring forces on Γ contour in the plate

Analyzing the obtained results, we come to the following conclusions:

- the influence of transverse forces and shear deformations in the reinforcing rib on its stress-strain state is negligible and can be neglected in engineering calculations;
- the presence of unlimited stresses at the reinforcement section ends in the plate causes the occurrence of local plastic zones in the circle of these points, in which the problem solution loses physical meaning;
- the increase in the relative rigidity of the reinforcing rib results in the increase of contact efforts in the plate and reduction of ring efforts;

- at a distance from the area of reinforcing, the ring efforts is practically independent on rib rigidity;
- if the external load acts along the axis of symmetry of the hole, then the maximum normal stresses in the rib occur at the location of combination, and if perpendicular to the axis – in the location of cut.

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УДК 539.3

МІЖФАЗНІ РОЗРІЗИ В НЕСКІНЧЕННІЙ ПЛАСТИНЦІ З КРИВОЛІНІЙНИМ КОНТУРОМ, ПІДСИЛЕНИМ ЗАМКНЕНИМ РЕБРОМ

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Анотація. *В умовах узагальненого плоского напруженого стану, створеного рівномірно розподіленими на нескінченності зусиллями, розглянуто мішану контактну задачу для нескінченної ізотропної пластинки з криволінійним отвором, контур якого підсилений замкненим пружним ребром, за наявності на межі поділу матеріалів пластинки і ребра двох симетричних міжфазних розрізів нульової ширини. Моделюючи підсилювальне ребро замкненим криволінійним стрижнем сталого прямокутного поперечного перерізу, серединна поверхня якого не співпадає з поверхнею отвору пластинки, а сполучення пластинки і ребра – ідеальним механічним контактом, побудовано систему сингулярних інтегрально-диференціальних рівнянь для визначення контактних зусиль між пластинкою і ребром та внутрішніх сил і моментів у ребрі. Для розрахунку початкових параметрів у статично невизначеному підсилювальному ребрі використано умови однозначності зміщень точок його осі і кутів повороту поперечних перерізів. Використовуючи спеціальний підхід до подання компонент напруженого стану в умовно розрізаному ребрі, встановлено структуру шуканих функцій на кінцях ділянки сполучення пластинки і ребра. Наближений розв'язок задачі побудовано методом механічних квадратур і колокації, яким досліджено вплив фізико-геометричних параметрів ребра і виду зовнішнього навантаження на розподіл напружень в пластинці та підсилювальному ребрі. З'ясовано, що компоненти напружено деформованого стану в ребрі на кінцях ділянки сполучення приймають обмежені значення.*

Ключові слова: *міжфазні розрізи, ізотропна пластинка, пружне ребро, поперечна і поздовжня сили, згинальний момент, сингулярні інтегральні рівняння, контактні зусилля.*

https://doi.org/10.33108/visnyk_tntu2019.04.023 *Отримано 10. 01.2020*