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ELASTIC-PLASTIC DEFORMATION OF A HALF-LAYER WITH A NOTCH AT RIGID LOADING

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***Summary**. The stress-strain state of an ideally elastic-plastic half-band of finite width with a central section-crack was analyzed. The state of anti-plane deformation is caused by tangential displacements of the strip faces. The elastic-plastic problem was solved and a continual zone of plastic deformations was found. The problem of the development of plastic deformations along the incision in its plane was solved. It is shown that at low loads, the continual plastic zone is shaped like a circle centered on the extension of the section, distant at a distance equal to the radius of the circle from the top of the section. The shape of the plastic zone and the length of the plastic strip are determined on the basis of a linear model of the plastic zone, according to which its characteristics are definite by the stress intensity factor. Load limits for which the linear model of the plastic zone provides sufficient investigation accuracy are established.*

***Key words:** anti-plane deformation, section-crack, elastic-plastic problem, plastic zone, plastic band.*

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Problem statement. Investigation of strength and conditions of structural materials destruction requires analysis of development of plastic deformation zones. In the case of sufficiently brittle materials of the fracture process, the relatively small plastic deformation regions are preceding. Therefore, their approximate analysis is possible on the basis of elastic solution by stress intensity factor (SIF). The location of stress concentrators (proximity to each other or proximity to the boundary of the body) can significantly affect the stressed state (SDS) of the body as a whole, change the pattern of plastic deformations in the vicinity of stress concentrators, and affect the deformation characteristics and strength of the body. In this regard, it is relevant to study the limits of applicability of theories of strength and destruction conditions, based on SIF and studies of SDS of bodies under high loads, at which linear theories are not applicable [1].

Analysis of known research results. To date, the SIF calculation methodology is quite well developed for different methods of loads, different shapes of bodies and stress concentrators [2–4]. At the initial stage [5, 6] plastic zones in the vicinity of crack tops increase almost homothetically: their shape is not affected by body shape or presence of other stress concentrators. The shape of the plastic zone and its characteristics (length of the plastic strip at the continuation of the crack, opening at the top of the crack) at the initial stage of development can be determined on the basis of the corresponding elastic solution using a linear model of the plastic zone (LMPZ) [7].

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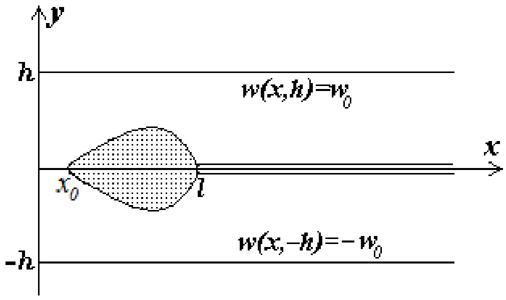

Figure 1. Half-layer with an incision. Doted region denotes plastic zone

Goal of the work. At the current state of development of theories of strength and destruction conditions based on SIF, the question arises about the limits of applicability of LMPZ: determination of load levels or dimensions of plastic zones, for which it is possible to predict the development of zones only on the basis of SIF. This applies to highly interacting stress concentrators and, in particular, close placed concentrators or placed near the body boundary. Then even a small load can significantly change the shape of plastic deformation zones and its characteristics. A significant effect on the shape and development of plastic zones can also be hoped when the distance of the crack top from the body boundary is much smaller against its own length.

Problem definition. Below we examine the development of plastic deformations in the half-layer $x \geq 0$, $-h \leq y \leq h$, $-\infty < z < \infty$, with incision $x \geq l$, $y = 0$, $-\infty < z < \infty$, (l – the distance of the incision apex from the end of the half-layer, $2h$ – the width of the half-layer) under the influence of constant and equal $\pm w_0$ shears along the axis Oz of its horizontal faces (Figure 1). Zone of plastic deformations is determined by classical solution of elastic-plastic problem and on the basis of assumption of localization of plastic deformations in crack plane. The body medium is considered resiliently perfectly plastic with the shear yield limit of the body k.

Formalization of a task. Under these conditions, an anti-plane of SDS will occur in the half-layer, which is described by two components of the stress tensor τ_{xz} and τ_{yz}, which in the elastic part of the body are described by an analytical function $\tau(\zeta) = \tau_{xz}(x, y) + i\tau_{yz}(x, y)$ $(\zeta = x + iy)$.

We will set the boundary problem in stresses in the area of elasticity of the half-layer. Let us also denote the Q and $-Q$ components of the main vector of forces acting along the lines $0 \leq x < +\infty, y = \pm h$.

From known [8] ratios $\tau_{xz} = \mu \partial w / \partial x$, $\tau_{yz} = \mu \partial w / \partial y$ ($w(x, y)$ – displacement along the axis Oz) we obtain:

$$\tau_{xz}(x, \pm h) = 0 \quad (0 \leq x < \infty). \tag{3}$$

Displacement along horizontal faces remains constant so

$$\tau_{xz}(0, y) = 0 \quad (-h \leq y \leq h). \tag{4}$$

Force Q and displacements w_0 are determined through stress tensor components by formulas

$$Q = \int_0^\infty \tau_{yz}(x, h)dx, \; w_0 = \frac{1}{\mu}\int_0^h \tau_{yz}(0, y)dy. \tag{5}$$

Taking the condition of stress absence on the borders of incision, we obtain:

$$\tau_{yz}(x,\pm 0)=0 \quad (x>l)\,. \tag{6}$$

The Genki ratio shall be performed at the boundary L of the continual plastic zone [9]:

$$(x-l)\tau_{xz}+y\tau_{yz}=0 \quad ((x,y)\in L). \tag{7}$$

In addition, in the plastic zone and on its boundary L, the condition of plasticity must be met

$$\tau_{xz}^2+\tau_{yz}^2=k^2\,. \tag{8}$$

The equals (3)–(8) express the setting of the problem in stresses for the case of a continual plastic zone. If the condition (7) is disengaged, and (8) take place only in the incision plane $l-d_1\le x\le l$, $y=0$, we get a boundary problem for the case when plastic deformations are concentrated in the cut plane (d_1 – length of plastic layer).

Owing to symmetry the boundary problem is enough to be considered only in the top half $0\le x<\infty$, $0\le y\le h$ a semi-layer having put

$$\tau_{xz}(x,0)=0 \quad (0\le x\le x_0)\,, \tag{9}$$

where x_0 is the plastic zone end coordinate on the abscissa axis.

For a continual plastic zone in the area D (a part of the top half of a semi-layer out of a plasticity zone) we receive such a boundary problem:

$$\operatorname{Im}\tau(\zeta)=0 \quad (\zeta=x+ih,\,0\le x<\infty\cup\zeta=iy,\,0\le y\le h\cup\zeta=x,\,0\le x\le x_0);$$

$$\operatorname{Im}(\zeta-l)\tau(\zeta)=0 \quad (\zeta\in L);\ |\tau(\zeta)|=k\ \ (\zeta\in L). \tag{10}$$

$$\frac{1}{\mu}\operatorname{Im}\int_0^{ih}\tau(\zeta)d\zeta=w_0 \ \text{ or }\ \operatorname{Re}\int_{ih}^{+\infty+ih}\tau(\zeta)d\zeta=Q\,. \tag{11}$$

If plastic deformations localized only in the cut plane we get a similar edge task, which differs from (10), (11) only by the condition of plasticity. In this case condition (7) should be disengaged, and (8) should be required only on line $\zeta=x,\ x_0\le x\le l$ ($x_0=l-d_1$).

Research of zones of plasticity. As in operation [10], the solution of problem (10), (11) will be searched in form ($\tau=\tau_1(t),\zeta=\zeta(t)$ ($\zeta\in H,\ H=\{\operatorname{Im}t>0\}$). Enter a new unknown function

$$\lambda(t)=\tau_1(t)(\zeta(t)-l) \tag{12}$$

and define an appropriate boundary problem.

Due to conditions (10), the function $\tau(\zeta)$ conformal mapping the plane D area ζ by a

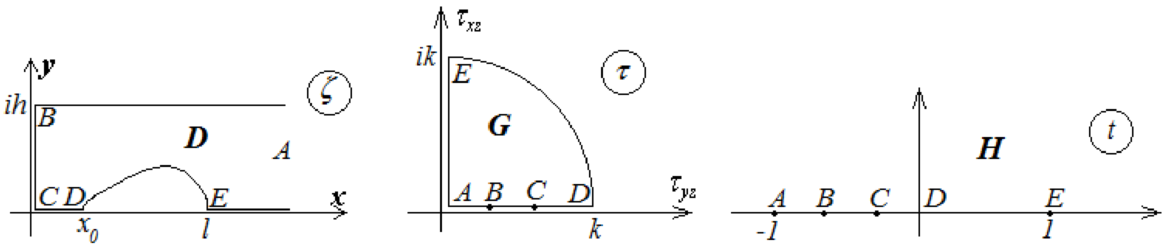

Figure 2. Conformal mapping

quarter of a circle $|\tau| < k, 0 < \arg\tau < \pi/2$ (area *G*) of the plane τ. The stress at the infinitely distant point of the half-layer, and at the points $(x_0, 0)$ and $(l, 0)$ are known: $\tau(\infty) = 0$, $\tau(x_0) = k$, $\tau(l) = ik$. By matching the points $0, k, ik$ of the boundary of the point $t = -1$, $t = 0$ i $t = 1$ area *G*, and in the plane *t* and drawing the corresponding conformal mapping (Figure 2), we get

$$\tau_1(t) = k\frac{\sqrt{1-t} - \sqrt{-2t}}{\sqrt{t+1}}. \qquad (13)$$

We will mark through t_B and t_C the points of the plane *t* at which the values acquire τ_B and τ_C: $\tau_1(t_B) = \tau_B$ i $\tau_1(t_C) = \tau_C$.

Thus we obtain the following boundary problem for a function $\lambda(t)$:

$$\operatorname{Re}\lambda(t) = 0 \ (t \in (-\infty, -1) \cup (1, +\infty)), \ \operatorname{Im}\lambda(t) = h\tau_1(t) \quad (t \in [-1, t_B]),$$

$$\operatorname{Re}\lambda(t) = -l\tau_1(t) \quad (t \in (t_B, t_C)), \ \operatorname{Im}\lambda(t) = 0 \quad (t \in [t_C, 1]). \qquad (14)$$

At points of change of type of boundary conditions (points $t_B, t_C, 1$ of the real axis), the function $\lambda(t)$ is limited due to the function $\tau_1(t)$ limitation and $\zeta(t)$. At point $t = -1$ $\tau_1(t) = k\sqrt{t+1}/2\sqrt{2} + o(\sqrt{t+1})$, and $\zeta(t)$ – logarithmically unlimited, so $\lambda(t)$ is also limited in the vicinity of this point. A limited solution to the Keldysh-Sedov problem (14) exists as provided below

$$h\int_{-1}^{t_B} F(\eta)d\eta = l\int_{t_B}^{t_C} F(\eta)d\eta, \qquad (15)$$

where $F(\eta) = \tau_1(\eta)/\sqrt{|P(\eta)|}$, $P(\eta) = (\eta+1)(\eta - t_B)(\eta - t_C)(\eta - 1)$ is expressed by the formula [11]

$$\lambda(t) = \frac{1}{\pi}\sqrt{P(t)}S(t). \qquad (16)$$

Here $S(t)=\left(h\int_{-1}^{t_B}F(\eta)\frac{d\eta}{\eta-t}-l\int_{t_B}^{t_C}F(\eta)\frac{d\eta}{\eta-t}\right)$, $\sqrt{P(t)}$ – analytical in H function, which at $t\to\infty$ is equal to $t^2+o(t^2)$.

Having provided condition (11), meaning by

$$\lambda(t_B)-\int_{-1}^{t_B}\frac{\lambda(t)\tau_1'(t)dt}{\tau_1(t)}=Q \text{ or } \frac{1}{\mu}\left|\lambda(t_C)-\lambda(t_B)-\int_{t_B}^{t_C}\frac{\lambda(t)\tau_1'(t)dt}{\tau_1(t)}\right|=w_0$$

and ratio (15), find values of both parameters t_B, t_C.

The plastic zone boundary is the image of a plane segment (0,1) when it is displayed by function $\zeta(t)$ so the plastic deformation zone boundary is described by the following equations

$$y(t)=\sqrt{2(t-t_B)(t-t_C)t(1-t)}S(t)/\pi,\ x(t)=l-\sqrt{(t-t_B)(t-t_C)}(1-t)S(t)/\pi,\ (t\in(0,1)). \quad (17)$$

We are now investigating the development of the plastic strip in the plane of the incision. In this case, the analytic function $\tau(\zeta)$ region boundary D_1 does not contain unknown areas: $D_1=\{0\le x<\infty,\ 0\le y\le h\}$. In this D_1 way the region in the plane τ is the region G, as for the continual zone. The function $\tau(\zeta)$ can be obtained by directly conformal mapping

$$\tau(\zeta)=k\frac{\sqrt{ch(\pi l/h)-ch(\pi\zeta/h)}-\sqrt{ch(\pi(l-d_1)/h)-ch(\pi\zeta/h)}}{\sqrt{ch(\pi l/h)-ch(\pi(l-d_1)/h)}}. \quad (18)$$

We will now associate the length of the plastic strip with the displacement of Q of horizontal faces or the value of the force Q acting on them.

From formulas (11), (18) we get

$$w_0=\frac{k}{\mu\sqrt{ch(\pi l/h)-ch(\pi(l-d_1)/h)}}\int_0^h\left(\sqrt{ch(\pi l/h)-\cos(\pi y/h)}-\sqrt{ch(\pi(l-d_1)/h)-\cos(\pi y/h)}\right)dy,$$

$$Q=\frac{k}{\mu\sqrt{ch(\pi l/h)-ch(\pi(l-d_1)/h)}}\left(\int_0^l\sqrt{ch(\pi l/h)-ch(\pi x/h)}dx-\int_0^{l-d_1}\sqrt{ch(\pi(l-d_1)/h)-ch(\pi x/h)}dx\right) \quad (19)$$

Opening of the top of the crack gives by the formula

$$g_1=\frac{2k}{\mu\sqrt{ch(\pi l/h)-ch(\pi(l-d_1)/h)}}\int_{l-d_1}^{l}\sqrt{ch(\pi x/h)-ch(\pi(l-d_1)/h)}dx.$$

For different relations between the width of the half-layer and the distance of the incision top from the end face of the half-layer, the dependences between the length of the

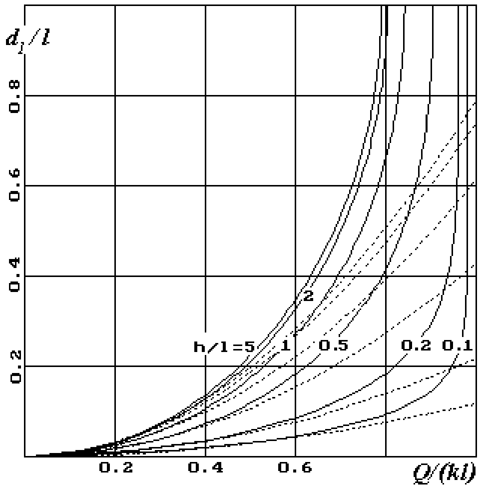

Figure 3. Plastic band length determined from LMPZ (dot line) and from model of localized plastic zone (firm lint)

plastic strip and the forces Q calculated according to the formula (19) are given in Figure 3 (solid curves). If the width of the half-layer does not exceed l, its horizontal faces, are significantly affecting on the growth of the plastic strip. For half-layer widths from l to $2l$ this influence becomes less significant, and for $h > 2l$ it is almost invisible. That is, with taking into account the given process of developing a plastic band the half-layer of width $h \geq 2l$ can be considered infinitely wide. For this particular case, the formulae (18), (19) are considerably simplified and become

$$\tau(\zeta) = k\frac{\sqrt{l^2-\zeta^2} - \sqrt{(l-d_1)^2-\zeta^2}}{\sqrt{2ld_1 - d_1^2}}, \qquad (20)$$

$$Q = (\pi k/4)\sqrt{2ld_1 - d_1^2}, \; d_1 = l - \sqrt{l^2 - 16Q^2/(\pi^2 k^2)}. \qquad (21)$$

The formulae (20), (21) also describe the development of plastic strips for two half-infinite cracks of shear lying in the same plane and whose vertices are spaced situated at distance $2l$ by an action of concentrated force of $2Q$. Plastic bands will couple if $Q = \pi kl/4$. The moment of coupling of continuous plastic zones depends only on the distance between cracks and occurs when Q reach a critical value $Q^{cr} = kl$. The value of Q^{cr} does not depend on width of semi-layer. At low loads, i.e. for small Q versus kl, length the plastic strip is defined by the formula

$$d_1 = 8Q^2/(\pi^2 k^2 l). \qquad (22)$$

We are now investigating the possibilities of analyzing the development of the plastic deformation zone by LMPZ. The stress function $\tau^{(e)}(\zeta)$ for this case can be obtained by solution an elastic-plastic problem for an environment with an infinitely large yield limit and the infinitely small in size plastic deformation zone. Thus, from formulae (18) and (19), we obtain

$$\tau^{(e)}(\zeta) = \frac{A}{\sqrt{ch(\pi l/h) - ch(\pi\zeta/h)}}, \; A = Q\Big/\int_0^l \left(ch(\pi l/h) - ch(\pi x/h)\right)^{-1/2} dx.$$

Since in the vicinity of the top of the shear crack $\tau^{(e)}(\zeta) = K_{III}\big/\sqrt{2\pi(\zeta - l)} + o\left(1\big/\sqrt{\zeta - l}\right)$, then

$$K_{III} = Q\sqrt{2h}\left(sh(\pi l/h)\right)^{-1/2}\left(\int_0^l \left(ch(\pi l/h) - ch(\pi x/h)\right)^{-1/2} dx\right)^{-1/2}. \qquad (23)$$

For infinitely large h/l formula (22) gives It can be shown that K_{III} increases with increases h while $K_{III}\sqrt{l}/Q$ asymptotically approaching to the value $2/\sqrt{\pi} \approx 1{,}128$. If $h \to 0$, then K_{III} also tends to zero. When varying h from 0 to ∞, the value $K_{III}\sqrt{l}/Q$ increases monotonically from 0 to $2/\sqrt{\pi}$. When h reaches level $2l$ value $K_{III}\sqrt{l}/Q$ reaches 96% of its maximum value, that is, as in the elastic-plastic state, the width of the half-layer $h \geq 2l$ can be considered almost unlimited.

Conclusions. Within the LMPZ, the plastic zone has the shape of a circle whose diameter is defined by formulae (1) and (23). The length of the plastic strip at this conditions is determined by formulae (2) and (23). In particular, for large h plastic strip length is given by formula (2) in which $K_{III} = 2Q/\sqrt{\pi l}$. And therefore, we get $d_1 = 8Q^2/(\pi^2 k^2 l)$ that coincides with the length of the plastic strip (22) directly determined by the solution of the problem of strip development. A comparison of the development of the plastic strip on the continuation of the crack determined by formulas (19) and by LMPZ (formulas (2), (23), dashed lines) is shown in Figure 3. The basic condition for the possibility of determining the length of the plastic strip by the elastic solution is the small length of the strip relative to the characteristic linear parameter of the problem. The low load, on the contrary, is not a necessary condition for the applicability of LMPZ to determination the length of the strip. Even for sufficiently large loads, LMPZ can give high accuracy.

We are now exploring the possibility of analyzing continual plastic zones within the framework of LMPZ. Figure 4 shows the continual plastic zones determined on the basis of the classical isolation of the elastic-plastic problem (formula (17)). The larger the width of the half-layer, the narrower continual plastic zone becomes. The extent of the zone is reduced, also as is the length of the strip at assuming the localization of plastic deformations in the crack plane. For half-layer width $h = 2l$, the exact and approximate boundaries of the plastic deformation zone are almost the same if $Q < 0{,}25kl$. As the load increases, the shape of the zone more and more differ from circular and the image accuracy with LMPZ decreases.

Within the framework of the performed studies on the basis of comparisons of the exact and approximate boundary of the continual plastic zone (Figure 4), as well as accurate and approximate dependencies for the length of the plastic strip on the load, it can be concluded that the accuracy of determining the length of the continual plastic zone by the LMPZ is not less than the accuracy of determining the length of the plastic strip based on the LMPZ.

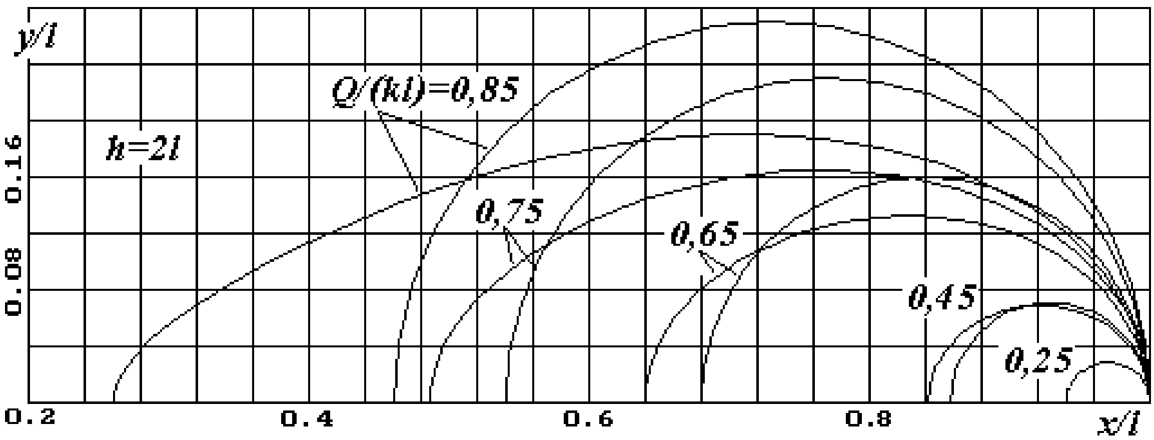

Figure 4. Continual plastic zones determined from solutions of elastoplastic problem and their comparison with zones determined from LMPZ

Since the study of the development of the plastic band is much simpler than the study of the continual zone, the result obtained can be the basis for the criterion of the applicability of LMPZ for the analysis of the continual plastic zone.

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ПРУЖНО-ПЛАСТИЧНЕ ДЕФОРМУВАННЯ ПІВШАРУ З РОЗРІЗОМ ПІД ЖОРСТКИМ НАВАНТАЖЕННЯМ

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Резюме. *Дослідження міцності й умов руйнування конструкційних матеріалів потребує аналізу розвитку зон пластичних деформацій. У випадку досить крихких матеріалів процесу руйнування передують відносно невеликі області пластичних деформацій. Тому їх наближений аналіз можливий на основі пружного розв'язку за коефіцієнтом інтенсивності напружень. Розташування концентраторів напружень (близькість між собою або близькість до межі тіла) може суттєво впливати на напружено-деформівний стан тіла в цілому, змінювати картину пластичних деформацій в околі концентраторів напружень, що визначально впливає на деформаційні характеристики й міцність тіла. У цьому зв’язку актуальним є дослідження меж застосовності теорій міцності й умов руйнування, що опираються на коефіцієнт інтенсивності напружень та дослідження напружено-деформівного стану тіл під вищими навантаженнями, за яких лінійні теорії незастосовні. Взаємодія концентраторів напружень між собою та їх взаємодія з межею тіла здатне спричинити значне збурення напружено-деформованого стану й вплинути на форму й розвиток пластичних зон. Аналогічно проявляється малість відстані вершини тріщини від межі тіла проти власної довжини тріщини. Проведено аналіз напружено-деформівного стану ідеально пружно-пластичного півшару скінченної ширини з центральним розрізом-тріщиною. Стан антиплоскої деформації спричинений тангенціальними зсувами граней півшару. Розв’язано пружно-пластичну задачу й знайдено континуальну зону пластичних деформацій. Розв’язано задачу про розвиток пластичних деформацій на продовженні розрізу в його площині. Показано, що за малих навантажень континуальна пластична зона має форму круга з центром на продовженні розрізу, віддаленого на відстань, що дорівнює радіусу круга від вершини розрізу. Встановлено форму пластичної зони й довжину пластичної смуги на основі лінійної моделі пластичної зони, за якою характеристики зони визначаються коефіцієнтом інтенсивності напружень. Встановлено межі навантажень, для яких лінійна модель пластичної зони забезпечує достатню точність дослідження.*

Ключові слова: *антиплоска деформація, розріз-тріщина, пружно-пластична задача, пластична зона, пластична смуга.*

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