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## OPTIMIZATION OF THE GEOMETRICAL PARAMETERS OF COUPLING IN THE CONNECTIVE UNIT. MODEL AND CALCULATIONS

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**Summary.** Within the plastic non-isothermal yielding theory the elastic-plastic thermal stresses has been investigated in a connective unit formed by means of thermal coupling shrinkage on tubes to be connected. The mathematical model for description of the thermomechanical processes has been proposed. The applicable mechanics problem has been formulated. The approximate approach for its solving based on the finite element method has been realized. The optimization of the coupling profile with variable thickness in axial direction has been fulfilled. A search of the optimal coupling variant has been executed for profiles restricted by piece-wise linear surfaces. The optimization criterion by minimization of inequality of normal contact pressure  $p_c$  distribution has been proposed.

**Keywords:** coupling, plastic non-isothermal yielding theory, finite element method, stresses, optimization.

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**Problem statement.** In the manufacture of engineering structures, the problem of pipe connection often arises. There are various technological methods for solving this problem, namely, connecting the pipes by welding [1, 2], thread cutting [3] or imposition the coupling and fitting it by preheating the connecting coupling and then thermally shrinkage it during cooling of the connective unit, etc. The latter method of connection is technologically simple, easily realizable, but requires high accuracy of connective unit manufacturing and preliminary strength calculations. Connecting couplings are often used when other methods are difficult to use, particularly in out-of-the-way places. Connections by couplings shall meet the conditions of tightness, reliability and stresses in the connecting area shall not cause damage.

The technical implementation of pipe connection by means of preheated couplings and their subsequent cooling is as follows. At ambient temperature  $T_A$ , the inner radius of the cylindrical sleeve is less than the outer radius of the pipes to be connected by a magnitude  $\Delta$  called tension. When the coupling is heated to a certain temperature  $T_C$ , the hole in the coupling increases and coupling can be freely imposed on the pipes to be connected. Subsequent cooling of connective unit to ambient temperature  $T_A$  is accompanied by thermal shrinkage of preheated coupling and setting of pipes. Tight contact between pipes and coupling is provided.

Such a connecting method is technologically not complex. However, the reliability of such a connection depends on the heating temperature  $T_C$  of the coupling, the geometric parameters of the connective unit, the cooling conditions, the thermomechanical properties of the materials from which the pipes and coupling are made. During thermal shrinkage of the coupling, stresses may occur that are dangerous to strength and reliability, including under the following operating conditions. At a sufficiently high stress level, plastic deformation of materials can occur. Therefore, predicting the thermomechanical behavior of such a coupling connection is an important theoretical and practical task focused on designing reliable in-service technical units. Searching for optimal coupling geometry is subordinated to this task.

**Analysis of known research results.** There are a number of theoretical [4 – 9] and experimental [10] works concerning estimation the resulting stress state in coupling connections under simulated operating conditions. Significantly less works related to optimization of coupling shapes in such connections [11] (optimization of chain couplings). At that stress state in connections is analyzed by means of couplings of constant thickness [2, 5].

New in this work are the follows: a) simulations within the thermoplasticity theory behavior of pipe connection by means of a preheated coupling bounded by a piece-linear boundary; b) optimization of coupling shape by varying parameters of piecewise-linear boundary; c) use of one of many versions of plasticity theories, which takes into account a number of important physically observed phenomena, it is to the tasks of stress and strain state (SSS) of coupling joints and optimization of their forms; d) theoretical and calculated approach to simulation of mechanical states of connection taking into account thermal sensitivity and nonlinear hardening of material. Taken together, aspects a), b), c), d), as well as aspects a), b), c) each particular are not considered in available literary sources.

The calculation of thermomechanical fields by analytical approaches is accompanied by significant mathematical difficulties or practically impossible due to the complexity of thermomechanical effects, the use of various materials in the joint, the possibility of their plastic deformation and other factors. The finite element method (FEM) is a powerful computational tool for solving a wide class of mechanics and mathematical physics problems. The application of this method to the evaluation of the resulting stress state in coupling joints enables to obtain solutions of specific problems without restrictions on the geometric configuration of the investigated domain, boundary conditions, nature of non-homogeneity of materials, for elastically and plastically deformable elements of the connective unit. There are no analytical methods of calculation based on mechanics tasks for coupling connections among a significant amount of processed literature. In the above-mentioned theoretical works [1 – 6, 11], FEM is a method of examining coupling connections implemented by various technological ways.

This work proposes the criterion of optimization of the coupling shape, which in the preheated state is superimposed on the connected pipes. Optimal version of coupling bounded by axisymmetric piece-linear surfaces at certain restrictions on their geometric parameters is established.

**Work purpose.** The most commonly used are couplings of the simplest shape, that is, constant thickness throughout the length. However, with axially variable-thickness couplings, there is a different stress state from that of the simplest couplings.

The aim of this work is to develop a theoretical approach to the evaluation of mechanical states in coupling joints under conditions of stationary thermal processes taking into account the possibility of plastic strains and piece non-homogeneity of the connective unit. Based on this approach, propose an optimal coupling shape in which the radial stresses drop along the contacting surfaces, i.e., the contact pressure drop, is minimal. Optimization shall be carried out under certain technologically expedient restrictions on the geometric parameters of the coupling.

**Formulation of mathematical problem about stress and strain state.** Solving of optimization problem on design of rational shape of coupling is based on previous theoretical estimates of arising stressed state in coupling joints of different geometrical configuration. Within the bounds of the proposed approach to stress and strain state (SSS) prediction, the problem of non-isothermal elastic-plastic yielding theory with isotropic-kinematic hardening [12, 13] is formulated and the method of its solution based on FEM calculation schemes is used.

It is taken into account that there are the such technological conditions under which it can be assumed that the temperature of the tubes is constant and equal to the ambient temperature  $T_A$  during cooling the coupling, and that the cooling of the coupling is close to uniform temperature change throughout the domain occupied by the coupling (heat exchange

between the tubes and the coupling is neglected). This assumption is based on the practical use of sufficiently thin elements in the connective unit. It is supposed also; that there is ideal mechanical contact on the contacting surfaces of the pipes and sleeve, and the resulting mechanical interaction can cause a plastic flow of materials. Rationally designed, the unit shall provide as close as possible to the uniform distribution of the contact pressure  $p_c$  at the given restrictions on the geometric parameters of the coupling.

The processes under consideration assume quasi-static and geometrically linear. Thermomechanical states are studied in the initial domain  $\Omega_0 = \Omega_{01} \cup \Omega_{02}$  occupied by the coupling and the pipes to be connected in the initially unstressed and strainless state. The domain  $\Omega_0$  is related to a Cartesian coordinate system and is bounded by a surface  $\Gamma_0$ . Here  $\Omega_{01}$  is the domain occupied by the coupling in the heated to temperature  $T_C$  state;  $\Omega_{02}$  is the domain occupied by the pipes at the beginning of the coupling superimposition. An ideal thermal contact between the sleeve and the pipes to be connected is assumed. During deformation, severing between connected pipes and coupling and their slip is excluded.

The task of elastic-plastic SSS of the considered unit fixed on the part  $\Gamma_{0u} \subset \Gamma_0$  and free from external mechanical effects is formulated. This task includes equation of equilibrium [14], geometric linear relation [14] and state equation of applied version of plastic yielding theory [12]

$$\begin{aligned} \{d\sigma\} = & \left( [D]^{(k)t+dt} - \frac{9}{4(\bar{\sigma}_i^t)^2} \cdot \frac{[D]^{(k)t+dt} \{\bar{s}\}^t \{\bar{s}\}^t [D]^{(k)t+dt}}{H^{(k)t} + 3G^{(k)t+dt}} \right) (\{d\varepsilon\} - \{d\varepsilon^T\}) + \\ & + \left( [dD]^{(k)} - \frac{9}{4(\bar{\sigma}_i^t)^2} \cdot \frac{[D]^{(k)t+dt} \{\bar{s}\}^t \{\bar{s}\}^t [dD]^{(k)}}{H^{(k)t} + 3G^{(k)t+dt}} \right) (\{\varepsilon\}^t - \{\varepsilon^P\}^t - \{\varepsilon^T\}^t) + \\ & + \frac{3}{2\bar{\sigma}_i^t} \cdot \frac{[D]^{(k)t+dt} \{\bar{s}\}^t \frac{\partial \bar{\sigma}_i^t}{\partial T}}{H^{(k)t} + 3G^{(k)t+dt}} dT \end{aligned} \quad (1)$$

with corresponding boundary conditions on part  $\Gamma_{0u} \subset \Gamma_0$  and on part  $\Gamma_{0\sigma} \subset \Gamma_0$  ( $\Gamma_{0\sigma} \cup \Gamma_{0u} = \Gamma_0$ ,  $\Gamma_{0\sigma} \cap \Gamma_{0u} = \emptyset$ ).

State equation (1) describes the behavior of plastically deformable heat-sensitive materials hardened during deforming. In relation (1) stress increment  $\{d\sigma\}$  functionally depends on complete  $\{\varepsilon\}$ , temperature  $\{\varepsilon^T\}$ , plastic strains  $\{\varepsilon^P\}$  respectively; from increments of strains  $\{d\varepsilon\}$ ,  $\{d\varepsilon^T\}$ ; increment of temperature  $dT$ ; matrix  $[D]$  of elastic constant material and matrix  $[dD]$  of elastic constant increments due to temperature change; from shear modulus of elasticity  $G$ ; current value  $H$  of the tangent of slope angle of the curve «stress intensity  $\sigma_i$  – strain intensity  $\varepsilon_i$ » material deforming; from the stress intensity  $\bar{\sigma}_i$  related to the center of the yield surface; from deviatoric stresses  $\{\bar{s}\}$  related to the center of the yield surface; from intensity  $\bar{\sigma}_i$  of Cauchy stresses [12]. In the vector representations of the respective tensorial values for strains, the components of the tensor are placed similarly to the vector

$\{\varepsilon\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{12}, 2\varepsilon_{13}, 2\varepsilon_{23}\}'$ . In vector representations for stresses components of a tensor are placed similar to placement in a vector  $\{\sigma\} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}\}'$ , where the «'» symbol means transposing operation. Upper indices  $t$  and  $t + dt$  refer to values at moments of deformation  $t$  and  $t + dt$  respectively. The upper indexes  $(k)$  indicate the step number when tracking the deformation process in the FEM calculating schemes.

Formulated task that is based on the state equation (1) is a task concerning unknown displacements  $\{u\}$ , strains  $\{\varepsilon\}$  and stresses  $\{\sigma\}$ . Solution of this plasticity problem in contrast to those of elasticity theory inaccurately describes the behavior of plastically deformable solids, because the equation of the state of this theory and other known plasticity theories are not accurate with respect to the plastic deformation criterion. This means that the stress intensity  $\sigma_i$  and strain intensity  $\varepsilon_i$  obtained by solving the formulated problem do not correspond to the experimental material deforming curve « $\sigma_i - \varepsilon_i$ » functionally described by the plastic deformation criterion. Therefore, the solution of this problem should be corrected with the condition of plasticity.

In this work the criterion of plastic deformation is condition [15]

$$\sqrt{\frac{3}{2} \{\bar{s}\}' \{\bar{s}\}^t} = \sigma_Y + \beta^* c (\varepsilon_i^{pt})^m \quad (0 \leq \beta^* \leq 1)$$

$$(\{\bar{s}\}^t = \{\bar{\sigma}\}^t - \{1, 1, 1, 0, 0, 0\}' \bar{\sigma}_0^t \quad \bar{\sigma}_0^t = \frac{1}{3} \{1, 1, 1, 0, 0, 0\} \{\bar{\sigma}\}^t),$$

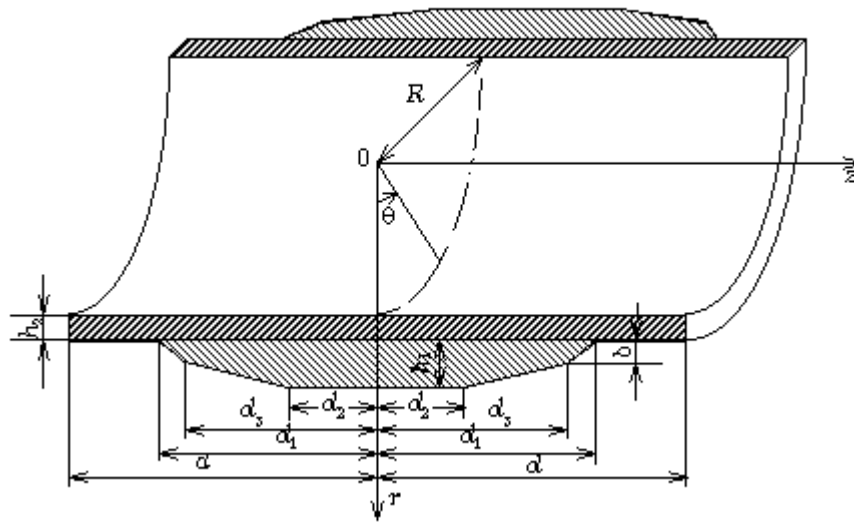
which recorded in a single expression based on the modified Mises criterion proposed by [12]. Here  $\sigma_Y$  is yield limit of material;  $\beta^*$ ,  $c$ ,  $m$  are parameters of material hardening.

**Technique of the solution of tasks.** The solution of the formulated problem about SSS is based on the use of FEM. Appropriate software has been developed for solving a wide class of two-dimensional problems of elasticity and elastoplasticity for piecewise homogeneous heat-sensitive solids [16]. The solids in general case of non-canonical form are hardenable during deformation. Such computational aspects as step-by-step approximation of the problem, linearization of the equation of state (1) by the method of variable parameters of elasticity or by the method of additional loads, organization of linearized iterative process, formation and solution of constitutive FEM equations, etc., are also in the work [16]. Mathematical formalism of designing a linearizing iterative process for a physically nonlinear problem, based on the particular case of theory [12, 13] and respectively on the particular case of state equation (1) presented by the author in [17].

The following is the search for the optimal coupling shape according to the mentioned below criterion with the specified permissible changes in geometric parameters of coupling. The SSS is examined for a certain specific domain that includes calculated domains for the pipes to be connected and for the coupling of a certain configuration specified by geometric parameters. Developed software [16] oriented on determination of mechanical state parameters in integration points of finite elements. Therefore, SSS, including radial stresses, are analyzed near the contacting surface. The following SSS calculations are performed for other geometric coupling shapes, which vary in the domain  $\Omega_0^*$  of specific permissible changes. At the same time approximate determination of contact pressure  $p_c$  consists in finding of radial component  $\sigma_r$  of stress vector near boundary of contact of pipes and coupling preheated to temperature  $T_C$ . Based on the comparison of the calculated results, the optimal coupling shape is determined

according to the optimization criterion mentioned below. For optimal shape of coupling, minimum difference in axial direction of radial stresses obtained near separating boundary of outer surface of pipes and inner surface of coupling is realized.

Coupling connection: geometric configuration of connection, specification of mathematical problem about SSS. Fig. 1 shows the connective unit to be designed. Before beginning of coupling cooling, each pipe has inner radius  $R$  and wall thickness  $h_2$ . Heated coupling has length  $2d_1$  and inner radius  $R_0$ . When calculating the stress state, it is also assumed that the length  $2d_1$  of the coupling is much smaller than the length  $2d$  of the pipes to be connected. For the connector shape in question determination of stress state and contact



**Figure 1.** Schematic representation of the axial section of the connective unit

pressure  $p_c$  was carried out by solving the axisymmetric problem of non-isothermal elastoplasticity in a cylindrical coordinate system  $(r, \theta, z)$ . The origin of the coordinate system  $(r, \theta, z)$  is at the center of the axial section of the connector, that is, at the pipe docking plane. In axial symmetry with respect to  $0z$ , the distribution of temperature, stresses and strains is independent of angle  $\theta$  and symmetrical with respect to  $z = 0$ . Therefore, a two-dimensional domain occupied by a fourth part of the axial section of the connector unit was investigated (Fig. 2).

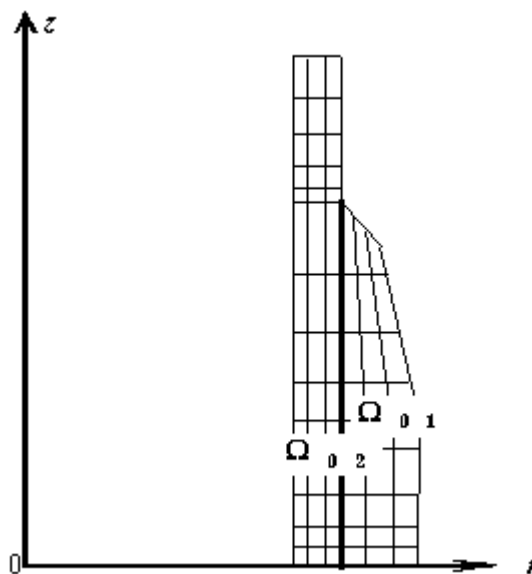
Stress and strain state analysis in the domain  $\Omega_0$  is performed at temperature load with initial condition

$$T(r, \theta, z, t) \Big|_{t=0} = \begin{cases} T_C, & (r, \theta, z) \in \Omega_{01} \\ T_A, & (r, \theta, z) \in \Omega_{02} \end{cases}, \quad (2)$$

where  $\Omega_{01}$ ,  $\Omega_{02}$  are the domains occupied by the fourth part of the axial sections of the coupling and the pipes to be connected, respectively. For the above assumptions about uniform change of coupling temperature and constant temperature of connected pipes, the temperature field at each moment of cooling time at  $t > 0$  is characterized by temperature distribution

$$T(r, \theta, z, t) = \begin{cases} T_A \leq T^* = \text{const} < T_C, & (r, \theta, z) \in \Omega_{01} \\ T_A, & (r, \theta, z) \in \Omega_{02} \end{cases}. \quad (3)$$

For simplicity, it is assumed that in the initial undeformed state of the coupling unit, the outer radius  $R + h_2$  of the pipes to be connected and the inner radius  $R_0$  of the coupling



**Figure 2.** Schematic representation of the calculated two-dimensional domain  $\Omega_0$  and its finite element discretization

are the same as:  $R_0 = R + h_2$ . If the inner radius of the preheated sleeve is greater than the outer radius of the pipes to be connected, then when the sleeve is cooled, there will be no mechanical pressure on the pipes to be connected until the sleeve is without thermally shrinkage, at which these radii will coincide. Further solving of the problem will not differ from solving of the problem with coincident radii of the composed system, which is characterized by known temperature of the coupling and temperature of the medium for pipes. Under this assumption of radii coincidence we have (Fig. 1, 2):

$$\begin{aligned} \Omega_0 &= \Omega_{01} \cup \Omega_{02}, \\ \Omega_{02} &= \{(r, \theta, z): R \leq r \leq R + h_2, \theta = \text{const}, 0 \leq z \leq d\}, \\ \Omega_{01} &= \left\{ (r, \theta, z): R + h_2 \leq r \leq R + h_2 + h_1, \theta = \text{const}, z \leq \frac{d_2 - d_3}{h_1 - b} [r - (R + h_2 + b)] + d_3 \cup \right. \\ &\quad \left. \cup z \leq \frac{d_3 - d_1}{b} [r - (R + h_2)] + d_1 \cup 0 \leq z \leq d_2, 0 \leq d_2 \leq d_3 \leq d_1 \ll d \right\}. \end{aligned} \quad (4)$$

8-node isoparametric elements of serendipal family are used to sample the domain  $\Omega_0$  [18].

Temperature stresses are investigated and on this basis contact pressure  $p_c$  is determined in connecting unit, free from initial stresses and strains, from external mechanical loads, with initial temperature distribution (2) and temperature distribution (3) at  $t > 0$ .

We specify the boundary conditions relative to the domain  $\Omega_0$  under consideration. On the line

$$\Gamma_{0u} = \{(r, \theta, z): R \leq r \leq R + h_2 + h_1, \theta = \text{const}, z = 0\},$$

which is allocated in the pipe docking plane, the kinematic boundary condition

$$u_z|_{\Gamma_{0u}} = 0 \quad (5)$$

is given. This condition means that the central cross section does not shift in the direction of the axis  $Oz$  during deformation and also set for symmetry reasons. On the rest of the bounding lines  $\Gamma_{0\sigma}$  boundary condition [14]

$$[n]' \{\sigma\}|_{\Gamma_{0\sigma}} = 0 \quad (6)$$

means that there are no force loads. In formula (6), the matrix  $[n]$  is the matrix of guide cosines of the outer normal  $\{n\}$  to the surface  $\Gamma_0$ . It is assumed that in the contact zone on the line  $S_{02}$  of the outer surface of the pipe and on the line  $S_{01}$  of the inner surface of the coupling, the components of the displacement vector in radial and axial directions should be equal to each other:

$$u_r^1|_{S_{01}} = u_r^2|_{S_{02}}, \quad u_z^1|_{S_{01}} = u_z^2|_{S_{02}}, \quad (7)$$

where

$$S_{01} = \{(r, \theta, z): r = R_0, \theta = \text{const}, 0 \leq z \leq d_1\} \subset \Omega_{01},$$

$$S_{02} = \{(r, \theta, z): r = R + h_2, \theta = \text{const}, 0 \leq z \leq d_1\} \subset \Omega_{02}.$$

Index «1» in formula (7) refers to displacements in coupling, and index «2» refers to displacements in pipes.

Formulated based on the equation of state (1), the mathematical problem with edge and contact conditions (5) – (7) is the thermoelastoplasticity problem for the system of solids occupying the domain  $\Omega_0$  (Fig. 2) given by expressions (4) under the temperature load (3) known at each cooling time.

Criterion for choice the optimal coupling shape. Formalizing the search for the optimal coupling profile will be carried out by choice the functional

$$I(r) = \left| \begin{array}{c} \max_{\substack{r=R^* \\ \theta=\text{const} \\ 0 \leq z \leq d}} \sigma_r(r, \theta, z) - \min_{\substack{r=R^* \\ \theta=\text{const} \\ 0 \leq z \leq d}} \sigma_r(r, \theta, z) \end{array} \right|, \quad (8)$$

which at  $r = R^* = R + h_2$  determines contact pressure  $p_c$  difference. Optimality criterion

$$I^*(r) = \min_{(r, \theta, z) \in \Omega_0^*} I(r) \quad (9)$$

means minimizing the stress drop  $\sigma_r$  (8) provided that the geometric parameters of the coupling profile are selected from a given class of domains  $\Omega_0^*$ . The task of finding conditions of equal strength, that is, conditions of providing as uniform distribution of stresses as possible, is one of the problems of resistance of materials. Minimum difference of contact stresses is technologically expedient for more efficient operation of structures [19]. The ways of achieving as uniform distribution of contact stresses as possible and obtaining of uniform -strength coupling joints are described in the works [20, 21].

**Results of a research.** Numerical calculations are made for pipes with inner radius  $R = 17$  mm and wall thickness  $h_2 = 3$  mm at length  $d = 35$  mm of considered pipe segment. When searching for optimal shape of coupling profile according to specified criterion of optimality (9) such dimensions as thickness of coupling  $h_1 = 5$  mm at  $z = 0$  and maximum

size of coupling in direction of axis  $Oz$   $d_1 = 25$  mm remain fixed. The search for the optimal coupling profile was obtained by varying the dimensions  $d_2, d_3, b$  ( $0 \leq d_2 \leq d_3 \leq d_1$ ,  $0 \leq b \leq 2h_1$ ), which together with the constant dimensions  $h_1$  and  $d_1$  define the domain  $\Omega_{01}^*$  occupied by the coupling and bounded by the piece-linear boundary.

The SSS was calculated in coupling connections for different versions of the geometric coupling profile with small changes of one of the dimensions  $d_2, d_3, b$  with a fixed value of the other two dimensions, which in the calculation of the following variants were also subject to change. As a result, about 60 tasks were solved to calculate SSS, which were direct in the process of solving the optimization task, and the task of finding a variant with the maximum value of functional (8). Then, radial stress  $\sigma_r$  differences in the connected pipes near the contacting surface were analyzed. On the basis of this, the optimal version of the coupling and the coupling with the maximum difference of approximate contact pressure  $p_c$  was established.

The restrictions  $\Omega_0^*$  under which the functional (8) was minimized were therefore as follows:

$$\Omega_0^* = \Omega_{01}^* \cup \Omega_{02}, \quad (10)$$

where

$$\Omega_{01}^* = \{(r, \theta, z) \in \Omega_{01} : h_1 = \text{const}, d_1 = \text{const}, 0 \leq d_2 \leq d_3 \leq d_1, 0 \leq b \leq 2h_1\}. \quad (11)$$

As an example, steel pipe connection by brass coupling is considered. Thermomechanical characteristics of the materials used in the connection: for steel Young's module  $E = 196$  GPa, Poisson's ratio  $\nu = 0,28$ , yield limit  $\sigma_Y = 422$  MPa, linear coefficient of thermal expansion  $\alpha_T = 11 \cdot 10^{-6}$  K<sup>-1</sup>; for brass  $E = 98$  GPa, Poisson's ratio  $\nu = 0,25$ , yield limit  $\sigma_Y = 255$  MPa, linear thermal expansion coefficient  $\alpha_T = 16 \cdot 10^{-6}$  K<sup>-1</sup>. Due to the lack of available reference data and the occurrence of plastic strains in the small, as the following calculations show, temperature range, hardening of the material is not essential. Therefore, the hardening effects are ignored, that is, steel and brass are assumed to deform as ideal elastic-plastic materials.

Initial coupling temperature  $T_C = 220^\circ\text{C}$ , ambient temperature  $T_A = 20^\circ\text{C}$ . In the present task, the temperature load, starting from the given value  $T_C = 220^\circ\text{C}$  for the coupling, changes with the corresponding increments during each deformation step, so that the final value  $T_A = 20^\circ\text{C}$  is taken at the end of the cooling process. Step-by-step temperature change when cooling is chosen as following:  $220^\circ\text{C} \rightarrow 170^\circ\text{C} \rightarrow 120^\circ\text{C} \rightarrow 70^\circ\text{C} \rightarrow 20^\circ\text{C}$ . With the temperature load step selected, the deformation history can be traced accurately enough.

A schematic representation of the sampled domain is shown in Fig. 2. The concrete variant of finite element mesh, the number of finite elements and nodes depended on the concrete configuration of the coupling profile.

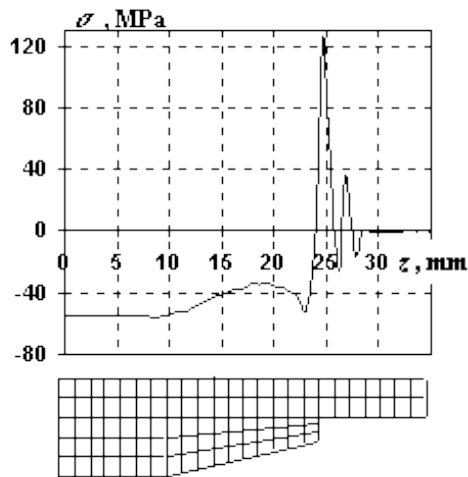
On the basis of computational experiments, the expediency of using a specific geometric grid and steps to trace the deformation process is established while maintaining the accuracy of calculations. That is, with practical coincidence of results on smaller and larger geometric meshes, on small and large steps of temperature load there is no need to count on smaller grids and smaller steps. For some variants, the geometric mesh and temperature steps could be larger. But since the calculated variants were many (about 60), the geometric mesh and temperature step, which provide accuracy of calculations, were chosen as priorities, although there are burdening for computations, i.e. smaller for some geometric profiles of the coupling.



Since the stress state is determined in the integration points of the finite elements, the value of the functional (8) was analyzed along the line

$$\Gamma_0^* = \{(r, \theta, z): r = R + h_2 - \delta, \theta = \text{const}, 0 \leq z \leq d\}, \quad (12)$$

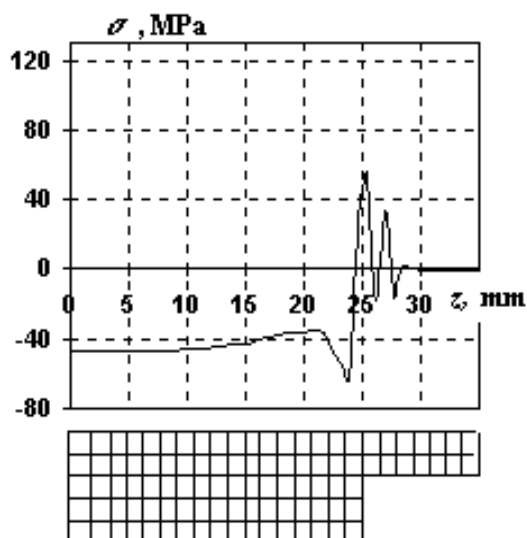
which passes in the domain  $\Omega_{02}$  of pipes to be connected at a distance  $\delta$  from their external surface through points of integration of finite elements ( $\delta = 0,284$  mm). At that, value of normal stresses  $\sigma_r$  along line (12) is sufficiently good approximation for stresses  $\sigma_r$  along butt line of connected pipes and coupling ( $r = R + h_2$ ), i.e. for contact pressure  $p_c$ .



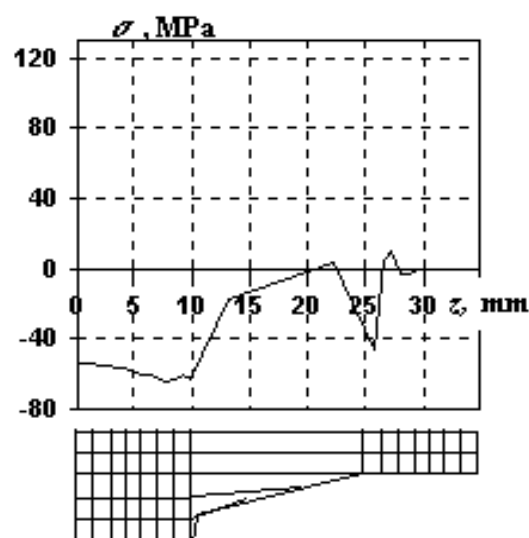
**Figure 3.** Stress  $\sigma_r$  distribution along the line  $\Gamma_0^*$  for the variant of the coupling with maximal functional value (8)  $I_1 = 181,19$  MPa and representation of the corresponded calculated discretized domain

On the basis of calculations it is obtained a coupling profile for which the value of functional (8) at  $r = R + h_2 - \delta$  and restrictions (10), (11) is maximum ( $d_2 = 10$  mm,  $d_3 = 22$  mm,  $b = 1,6$  mm) (Fig. 3):  $I_1 = 181,19$  MPa. Also distribution of stresses for the simplest in form coupling, i.e. for a hollow cylinder of finite length and constant thickness ( $d_2 = 10$  mm,  $d_3 = 25$  mm,  $b = 5$  mm) is calculated (Fig. 4). In this case  $I_2 = 117,33$  MPa. By solving of an optimizing task with an optimality criterion (9) at restrictions (10), (11) it is proposed the best in form coupling ( $d_2 = 10$  mm,  $d_3 = 10,9$  mm,  $b = 2,5$  mm)

for which difference of stresses  $\sigma_r$  in concordance with functional (8) along the line  $\Gamma_0^*$  set by expression (12) is minimum with value  $I_3 = 74,26$  MPa. In Fig. 3 – 5 it is given distributions of stresses  $\sigma_r$  along the line  $\Gamma_0^*$  and the profiles of connections corresponding to them with the images of meshes for a coupling configuration with maximum value of functional (8), for simplest and optimal versions of couplings. For the three cases we have considered  $I_1 : I_2 : I_3 = 2,44 : 1,58 : 1$ , where  $I_1, I_2, I_3$  are the values of the functional (8) on the line  $\Gamma_0^*$  at constraints (10), (11) for the coupling connections illustrated in Fig. 3, 4, 5 respectively. The domains in Fig. 3, 4 are divided into 99 elements using 356 nodes. The domain in Fig. 5 is sampled by 54 elements with 196 nodes. Plastic flow zones are observed for the coupling connections versions shown in Fig. 3 and 5. For the profile of the coupling of constant thickness (Fig. 4) deformation takes place according to the elastic law.



**Figure 4.** Stress  $\sigma_r$  distribution along the line  $\Gamma_0^*$  for the simplest variant of the coupling with functional value (8)  $I_2 = 117,33$  MPa and representation of the corresponded calculated discretized domain



**Figure 5.** Stress  $\sigma_r$  distribution along the line  $\Gamma_0^*$  for the optimal variant of the coupling with functional value (8)  $I_3 = 74,26$  MPa and representation of the corresponded calculated discretized domain

In addition, based on the analysis of the calculated results presented in Fig. 3 – 5, the following observed features in the distribution of contact stresses have been established. The most uniform contact pressure is the pressure in the vicinity of the central cross-section  $z = 0$  and is not significantly different at  $z = 0$  for the presented joint variants. The contact pressure drop  $p_c$  is significant at the contact point of the end of the coupling and the pipes to be connected, i.e. at  $z = d_1$ . The maximum stresses  $\sigma_r$  in optimal variant are compressive (Fig. 5). The significant reduction in the axial direction of contact pressure  $p_c$  and the reduction of stress  $\sigma_r$  level attained as a result of the optimization indicate that it is practical expediency to solve optimization problems for coupling connections.

**Conclusions.** Proposed is approach to prediction of mechanical processes caused by thermal shrinkage of couplings for connection of pipes, and criterion of optimization of geometric parameters of coupling. This approach for quantitative assessment of mechanical states is based on the theory of plastic non-isothermal yielding with isotropic-kinematic hardening [12, 13] and calculation schemes of FEM. When optimizing the coupling shape according to the above criterion (9) with restrictions (10), (11), for the geometric configuration of the connection minimization of the undesirable contact pressure  $p_c$  difference due to technological considerations is realized. That is, the difference in axial direction of stresses  $\sigma_r$  normal to the contacting surface is minimized. At the same time more uniform distribution of contact pressure compared to other forms of coupling is achieved. There is a significant predicted effect of designing an optimal coupling compared to a coupling variant in which the contact pressure drop is maximum and compared to a constant thickness coupling variant. In optimum, both compressive and tensile stresses  $\sigma_r$  are reduced (Fig. 5). In the optimal embodiment (Fig. 5), the tensile stresses  $\sigma_r$  are minor and substantially lower than the stresses  $\sigma_r$  for connection configuration with maximum difference of radial stresses (Fig. 3) and for the constant thickness coupling (Fig. 4).

A technique has been developed to solve mechanical problems for coupling connections and corresponding optimization problems can be used for a wider class of tasks in this direction. It is promising to minimize the difference in contact pressure  $p_c$  when varying the preheating temperature  $T_c$  of the couplings, when selecting couplings restricted by non-linear axisymmetric surfaces, made of other materials, couplings of different length and thickness. Similar optimization problems can be considered for welded coupling connections with acquired weld residual stresses. At the same time, taking into account the possibility of plastic deformation is essential.

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## ОПТИМІЗАЦІЯ ГЕОМЕТРИЧНИХ ПАРАМЕТРІВ МУФТИ У З'ЄДНУВАЛЬНОМУ ВУЗЛІ. МОДЕЛЬ ТА РОЗРАХУНКИ

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**Резюме.** В рамках теорії пластичного неізотермічного течіння досліджено пружно-пластичні температурні напруження в з'єднувальному вузлі, утвореному за рахунок термічної усадки муфти на з'єднуванні труби. Запропоновано математичну модель для прогнозування механічних процесів, спричинених тиском остигаючої муфти. З'єднувальний вузол, елементами якого є дві труби кругового перерізу та муфта, може бути утворений з використанням різних матеріалів. Сформульовано відповідну задачу механіки з врахуванням можливості виникнення пластичного деформування. Конкретизовано граничні умови для розгляданого з'єднувального вузла. Реалізовано наближений підхід до розв'язування задачі, який базується на розрахункових схемах методу скінченних елементів. Для сформульованої фізично нелінійної задачі еволюція механічних станів складеної кусково-однорідної системи прогнозується покроково відповідно до покрокової зміни температури. Розглядаються муфти змінної в осьовому напрямку товщини, які обмежені кусково-лінійними осесиметричними поверхнями. Запропоновано критерій оптимізації геометричних параметрів муфти із заданими обмеженнями стосовно форми та розмірів профілю муфти. Для шуканого оптимального варіанту мінімізується нерівномірність розподілу в осьовому напрямку контактного тиску  $p_c$  між трубами та муфтою. Проаналізовано ряд розрахункових результатів по визначенню напружено-деформованого стану для різних геометричних конфігурацій муфт змінної в осьовому напрямку товщини. Отримано варіант із найбільш нерівномірним розподілом тиску  $p_c$  та оптимальний варіант. Зроблено механічні висновки стосовно характеру поведінки контактного тиску. Здійснено кількісну оцінку ефекту від проектування муфт змінної товщини шляхом порівняння контактний тиску  $p_c$  для оптимального варіанту, для найпростішої муфти – постійної товщини та для профілю муфти з максимальним перепадом контактний тиску.

**Ключові слова:** муфта, теорія пластичного неізотермічного течіння, метод скінченних елементів, напруження, оптимізація.

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