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LOAD TRANSMISSION FROM THE ENDLESS STRINGER TO ONE AND TWO PRE-STRESSED STRIPES

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Summary*. The article is devoted to the research of problems of contact interaction of infinite elastic stringer with one and two identical clamped along one edge of pre-stressed strips. In general, the research was carried out for the theory of great initial and different variants of the theory of small initial deformations within the framework of linearized theory of elasticity with the elastic potential having arbitrary structure. The integral integer-differential equations are obtained using the integral Fourier transform. Their solution is represented in the form of quasiregular infinite systems of algebraic equations. In the article alsaw was investigated the influence of the initial (residual) stresses in strips on the law of distribution of contact stresses along the line of contact with an infinite stringer.*

Key words*: linearized elasticity theory, initial (residual) stresses, contact problem, integral Fourier transform, stringer.*

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Statement of the problem. Investigation of the problems of contact interaction of thin-walled elements in the form of overlays (stringers) and coatings of various geometric forms with massive deformed bodies is very important problem both in the theoretical and in the applied aspect. When creating structures and machines mechanisms for improving the strength characteristics and properties of parts, as well as the possibility of their use under high temperatures conditions, or in aggressive media presence, various coatings and reinforcements are widely used. Since such parts are often the structures responsible elements, which fracture can result in catastrophic effects, then their regular diagnosis is required. In theory, this problem can be reduced to the consideration of contact problems concerning the interaction of overlays and inclusions with elastic bodies of various forms. One of the important factors that significantly affect the reliability and durability of engineering structures and machine parts is the presence of their initial (residual) stresses.

Analysis of the available investigation results. Despite the fact that investigations concerning the impact of initial stresses have been actively carried out in our country and abroad only at the end of the XX century, it is possible to list many names, researches and publications related to this problem [1, 2]. In strict formulation of contact problems for elastic bodies with initial stresses [1, 2], it is necessary to involve the apparatus of nonlinear elasticity theory, which considerably complicates the analytic solutions construction of. But in case of large (finite) stresses (deformations) we can restrict ourselves to the consideration of linearized elasticity theory [1]. Historically, the investigation of contact problems in the framework of the linearized elasticity theory consisted of two directions. The first one is connected with the investigation of contact interaction of bodies with definite form of elastic potential [3]. In the second one, the problem is set up in general form for compressible (incompressible) bodies with the arbitrary structure potential based on the linearized elasticity theory [1, 2, 4−13].

The solutions of the contact problems concerning contact interaction of infinite stringer with one and two pre-stressed stripes using the linearized elasticity theory relations [1, 2] are presented in this paper. The investigation is carried out in general form for compressible and incompressible bodies for the theory of large (finite) initial deformations and two variants of the theory of small initial deformations with arbitrary elastic potential structure.

The objective of the paper is to investigate within the framework of the linearized elasticity theory, two flat contact problems concerning the load transmission from the infinite stringer to one and two identical stripes with initial (residual) stresses without taking into account frictional forces; to present the problem solutions in general form for the theory of large (finite) initial deformations and two versions of the theory of small initial deformations, for arbitrary elastic potential structure and to identify the effect of initial (residual) stresses in the strips on the law of contact stresses distribution along the contact line with the infinite stringer.

Statement of the problem. Let us keeping [2, 4, 14] carry out all investigations in the coordinates of the initial deformed state y_i, which are related to the Lagrangean coordinates x_i by relations $y_i = \lambda_i x_i \ (i = 1{,}2)$, where λ_i is the elongation coefficients determining the initial state displacement in the coordinate axes directions.

We regard that further four conditions which are fundamental in the theory of contact interaction of bodies with initial stress are always met and, therefore, determine the field of its application.

Condition 1. Contact interaction of elastic finite (infinite) overlays and elastic stripe with initial (residual) stresses occurs after occurrence of initial stressed state in the latter one.

Condition 2. The external load acting on the elastic thin overlay causes stresses with much lower values than the corresponding stressed state ones in elastic stripe with initial stresses.

Condition 3. The initial stressed state of one of the bodies being in contact interaction has the structure that, in the area of their contact, can be (approximately, with sufficient accuracy degree) considered as homogeneous initial stressed state.

Condition 4. The solution of linearized problems of the elasticity theory on the contact interaction of bodies with initial stresses is the only one.

While fulfilling conditions 1−4 in the contact area $L_k\{a_k, b_k\}$ for elastic overlays and elastic stripe with initial (residual) stresses, conditions are present at $y_2 = 0$

$$u(y_1) = u_1(y_1); \quad v(y_1) = u_2(y_1); \quad (y_2 = 0, \quad -\infty < y_1 < \infty). \tag{1}$$

$$\frac{du}{dy_1} = \frac{du_1}{dy_1}, \quad \frac{dv}{dy_1} = \frac{du_2}{dy_1}, \quad (y_2 = 0, \quad -\infty < y_1 < \infty). \tag{2}$$

The boundary conditions (1)−(2) along with conditions (1−4) and equilibrium conditions

$$p = \int_{a_k}^{y_1} \tau(t)\,dt \tag{3}$$

complete the linearized tasks concerning the contact interaction of elastic overlays (finite, infinite $(a_k = -\infty;\ b_k = +\infty)$) strengthening the elastic stripe.

Problem 1. Contact interaction of the infinite stringer with pre-stressed stripe. Let the elastic infinite stripe with initial stresses which has thickness t and pinched by one edge under plane deformation conditions, is reinforced by free end with thickness h due to the infinitely long stringer (Fig. 1).

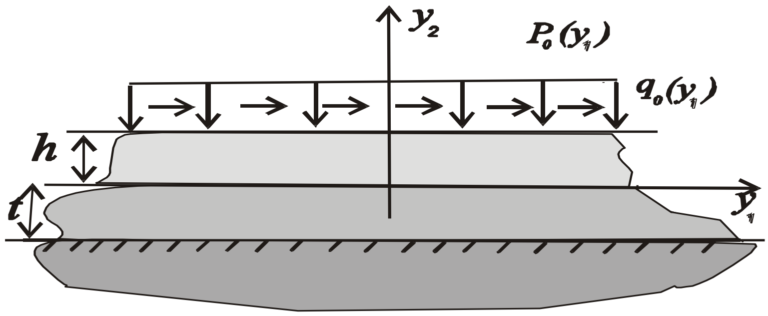

Figure 1. Action of forces on the reinforced strip

Let us assume that under the action of vertical and horizontal intensity forces, the stringer in the vertical direction bends as the ordinary beam, and in the horizontal stretches (compressed) as the one load stressed rod. Then you can write:

$$\frac{du^{(1)}(y_1)}{dy_1} = \frac{1}{E_1 h}\int_{-\infty}^{y_1}[q(t) - q_0(t)]dt \quad (-\infty < y_1 < \infty) \tag{4}$$

$$D\frac{d^4 v^{(1)}(y_1)}{dy_1^4} = p(y_1) - p_0(y_1) \quad (-\infty < y_1 < \infty) \tag{5}$$

where D – stringer stiffness coefficient, E_1 – Young's module, $p_0(y_1), p(y_1), q_0(y_1), q(y_1)$ – the intensity of the vertical and horizontal forces.

It should be noted in case of full contact, that the following conditions are to be met along the contact line:

$$\frac{\partial v^{(1)}(y_1)}{\partial y_1} = \frac{\partial u_2^{(2)}(y_1)}{\partial y_1}\ ,\ \frac{\partial u^{(1)}(y_1)}{\partial y_1} = \frac{\partial u_1^{(2)}(y_1)}{\partial y_1},\ (-\infty < y_1 > \infty) \tag{6}$$

where $u^{(1)}(y_1), v^{(1)}(y_1)$ – are the components of displacement vector in elastic stringer, $u_1^{(2)}(y_1), u_2^{(2)}(y_1)$ – are the components of displacement vector in elastic stripe with initial stresses.

Taking into account the contact conditions (6) together (5), as well as the expressions for vertical and horizontal displacements of the boundary points free of pinching, equation (4) for compressible and incompressible bodies, and keeping [4], will be as follows:

$$\begin{aligned} u_1(y_1) &= \int_{-\infty}^{\infty} h_{11}(|y_1 - t|)p(t)dt + \int_{-\infty}^{\infty} h_{12}(y_1 - t)q(t)dt. \\ u_2(y_1) &= \int_{-\infty}^{\infty} h_{21}(y_1 - t)\,p(t)dt + \int_{-\infty}^{\infty} h_{22}(|y_1 - t|)q(t)dt. \end{aligned} \tag{7}$$

where $h_{ij}(i, j = 1,2)$ are the influence functions for the elastic stripe with initial (residual) stresses, the expressions of which are given [4].

Taking into account (4)−(7), we get the following system of integral − differential equations:

$$D\frac{d^4}{dy_1^4}\left[\int_{-\infty}^{\infty} h_{11}(|y_1-\tau|)p(\tau)d\tau + \int_{-\infty}^{\infty} h_{12}(y_1-\tau)q(\tau)d\tau\right] = p(\tau) - p_0(\tau).$$
$$E_1h\frac{d}{dy_1}\left[\int_{-\infty}^{\infty} h_{21}(y_1-\tau)p(\tau)d\tau + \int_{-\infty}^{\infty} h_{22}(|y_1-\tau|)q(\tau)d\tau\right] = \int_{-\infty}^{\infty}[q(\tau) - q_0(\tau)]d\tau. \quad (8)$$

Assuming that only the vertical forces $p_0(y_1)$, a $q_0(y_1) = 0$ act on the overlay, then system (8) is reduced to one integral-differential equation:

$$D\frac{d^4}{dy_1^4}\left[\int_{-\infty}^{\infty} h_{11}(|y_1-\tau|)p(\tau)d\tau\right] = p(y_1) - p_0(y_1). \quad (9)$$

Equation (9) describes elastic overlay bending on the elastic stripe with initial (residual) stresses. In case when under the action of the horizontal forces $q_0(y_1)$ $(p_0(y_1) = 0)$ the elastic overlay only stretches, we get the following equation:

$$E_1h\frac{d}{dy_1}\left[\int_{-\infty}^{\infty} h_{22}(|y_1-\tau|)q(\tau)d\tau\right] = \int_{-\infty}^{y_1}[q(\tau) - q_0(\tau)]d\tau. \quad (10)$$

To solve the system of integral − differential equations (8), we use the Fourier integral transformations by variable y_1, and as the result we derive expressions for the contact stresses $p(y_1)$ and $q(y_1)$ finding:

$$p(y_1) = \frac{\mu}{\pi}\left[Q\int_0^{\infty} H_{21}^*(\alpha)\cdot H^{-1}(\alpha)\cdot\alpha^2 \sin\alpha y_1 d\alpha - P\int_0^{\infty} H_{22}^* H^{-1}(\alpha)\cos\alpha y_1 d\alpha\right].$$
$$q(y_1) = \frac{\mu}{\pi}\left[Q\int_0^{\infty} H_{11}^*(\alpha)\cdot H^{-1}(\alpha)\cos\alpha y_1 d\alpha - P\int_0^{\infty} H_{12}^* H^{-1}(\alpha)\sin\alpha y_1 d\alpha\right] \quad (11)$$

Here the values *P*, *Q* – vertical and horizontal external forces with which the elastic pad is loaded, H_{ij}^* $(i, j = 1,2)$ are expressed through known functions H_{ij} $(i, j = 1,2)$, which are defined for equal and unequal roots of the defining equation [2] in accordance with [4, 14]. We will perform similar actions with the integral − differential equations (9) and (10), will get the following contact stresses $p(y_1)$ and $q(y_1)$:

$$p(y_1) = \frac{\mu}{\pi}\int_{-\infty}^{\infty}\frac{H_{11}(\alpha)}{H(\alpha)}p_0(\alpha)e^{-i\alpha y_1}d\alpha;\quad q(y_1) = \frac{\mu}{\pi}\int_{-\infty}^{\infty}\frac{H_{11}(\alpha)}{H(\alpha)}q_0(\alpha)e^{-i\alpha y_1}d\alpha; \quad (12)$$

Problem 2. Contact interaction of the infinite stringer with two pre-stressed strips. Let the endless elastic strips be made of the same compressible or incompressible materials with the arbitrary structure potential. The same initial (residual) stresses act in the given stripes, moreover the stripes thickness is t. On the edges at $y = \pm t$, the stripes are pinched, and are under plane deformation conditions.

Let us assume that the infinite elastic strips are interconnected by the infinite elastic stringer with the modulus of material elasticity E_1 and Poisson factor ν_1. Also let the pre-stressed stripes be loaded with horizontal force $Q_0\delta(y_1)$ acting on the middle point of the stringer. We will use notation $\delta(y_1)$ − the known single Dirac delta function.

The investigation of this problem will be carried out in the coordinates of the initial (residual) deformed state Oy_1y_2 (Fig. 2)

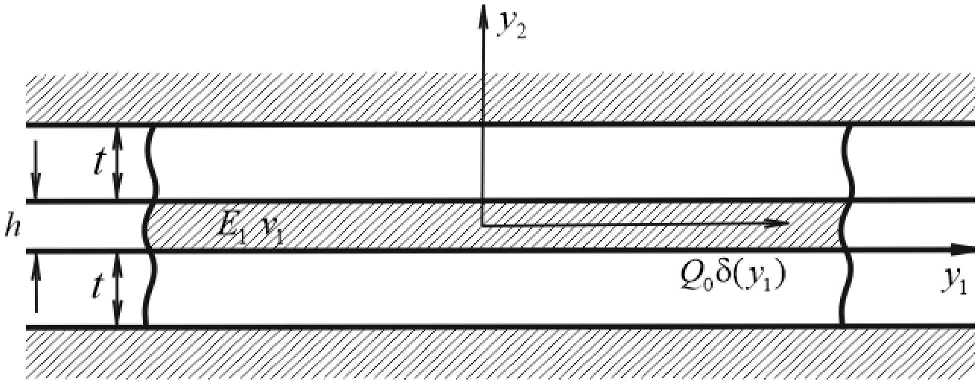

Figure 2. Force action on the strips

Let us determine the law of normal and tangential contact stress distribution along the stringer connection line with pre-stressed strips. Considering this problem, we assume that the interaction occurs when four well-known conditions are fulfilled, which are fundamental in the theory of contact interaction of bodies with initial stresses [2]. We denote the intensities of normal and tangential contact stresses through $p(y_1)$ and $q(y_1)$, and the vertical and horizontal stringer motions, respectively, $v^{(1)}(y_1)$ and $u^{(1)}(y_1)$. So, let's put down:

$$\frac{du^{(1)}(y_1)}{dy_1} = \frac{1}{E_1 h}\int_{-\infty}^{y_1}\left[2q(t) - Q_0\delta(t)\right]dt\ , \quad (-\infty < y_1 < \infty) \tag{13}$$

$$\frac{d\,\nu^{(1)}(y_1)}{dy_1} = 0\,, \qquad (-\infty < y_1 < \infty) \tag{14}$$

It should be noted in case of complete contact that the following conditions are to be met along the contact line:

$$\frac{d\nu^{(1)}(y_1)}{d\,y_1} = \frac{du_2^{(2)}(y_1)}{d\,y_1}, \qquad \frac{du^{(1)}(y_1)}{d\,y_1} = \frac{du_1^{(2)}(y_1)}{d\,y_1}\,, \ (-\infty < y_1 < \infty) \tag{15}$$

where $u^{(1)}(y_1), v^{(1)}(y_1) -$ are the components of vector displacement in elastic stringer, $u_1^{(2)}(y_1), u_2^{(2)}(y_1) -$ are the components of vector displacement in elastic stripes with initial stresses.

Taking into account the contact conditions (15) together with (13) and (14), as well as expressions for vertical and horizontal displacements of the boundary points free of pinching, which were obtained on the basis of the superposition principle in the case of equal and unequal roots of the determining equation [1, 2] for compressible and incompressible bodies, and taking into account [14], will be as follows:

$$\begin{aligned} u_1(y_1) &= \int_{-\infty}^{\infty} h_{11}(|y_1 - t|)p(t)dt + \int_{-\infty}^{\infty} h_{12}(y_1 - t)q(t)dt. \\ u_2(y_1) &= \int_{-\infty}^{\infty} h_{21}(y_1 - t)p(t)dt + \int_{-\infty}^{\infty} h_{22}(|y_1 - t|)q(t)dt. \end{aligned} \tag{16}$$

From (13)–(16) relatively to unknown contact stresses, we get the following system of integral − differential equations:

$$\frac{d}{dy_1}\left[\int_{-\infty}^{\infty} h_{11}(|y_1 - t|)p(t)dt + \int_{-\infty}^{\infty} h_{12}(y_1 - t)q(t)dt\right] = 0$$

$$(-\infty < y_1 < \infty) \quad (17)$$

$$\frac{d}{dy_1}\left[\int_{-\infty}^{\infty} h_{21}(y_1 - t)p(t)dt + \int_{-\infty}^{\infty} h_{22}(|y_1 - t|)q(t)dt\right] = \int_{-\infty}^{y_1}[2q(t) - Q_0\delta(t)]dt.$$

where h_{ij} $(i, j = 1, 2)$ are the influence functions for elastic stripe with initial (residual) stress [14]. Applying Cramer formula and inverse Fourier transform, we get the solution of the integral − differential equations system (17). This solution gives expressions for the desired contact stresses as

$$q(y_1) = \frac{Q_0}{\pi}\mu\int_0^{\infty}\frac{H_{11}^*(\alpha)}{H^*(\alpha)}\cos\alpha y_1 d\alpha\,,\ p(y_1) = \frac{Q_0}{\pi}\mu\int_0^{\infty}\frac{H_{11}^*(\alpha)}{H^*(\alpha)}\sin\alpha y_1 d\alpha\,,\ (-\infty < y_1 < \infty) \tag{18}$$

Examining the convergence of non-proper integrals included in (18), taking into account the values $H^*_{ij}(\alpha)$ and values $H_{ij}(\alpha)$ [14], as well as asymptotic formulas for $H_{ij}(\alpha)$, neglecting the lengthy elementary transformations for contact tangential stresses (18) from the action of horizontal external force $Q_0\delta(y_1)$, we obtain at $(-\infty < y_1 < \infty)$ the following:

$$q(y_1) = -\frac{Q_0}{2\pi}\cdot\left[c_2(\cos c_2 y_1 ci(c_2 y_1) + \sin c_2 y_1 si(c_2 y_1)) - \int_0^{\infty}\frac{2\mu(c_2+\alpha)H_{11}^*(\alpha) - c_2 H^*(\alpha)}{(c_2+\alpha)H^*(\alpha)}\cos\alpha y_1 d\alpha\right] \tag{19}$$

where $si(c_2 y_1) = -\int_{c_2 y_1}^{\infty}\frac{\sin\alpha}{\alpha}d\alpha;\quad ci(c_2 y_1) = -\int_{c_2 y_1}^{\infty}\frac{\cos\alpha}{\alpha}d\alpha$ – is, respectively, integral sinus and cosine.

Let us assume that elastic thin overlay is loaded with vertical external forces $p_0(y_1) = P\cdot\delta(y_1)$ and horizontal forces $q_0(y_1) = Q\delta(y_1)$. We denote $p_0(\alpha) = P;\ \ q_0(\alpha) = Q$

from expressions (12) we obtain expressions for determination of normal contact stresses $p(y_1)$ and tangential stresses $q(y_1)$

$$q(y_1)=\frac{\mu}{\pi}\left[Q\int_0^{\infty}H_{11}^{*}(\alpha)\cdot H^{-1}(\alpha)\cos\alpha y_1 d\alpha-P\int_0^{\infty}H_{12}^{*}H^{-1}(\alpha)\sin\alpha y_1 d\alpha\right]. \tag{20}$$

$$p(y_1)=\frac{\mu}{\pi}\left[Q\int_0^{\infty}H_{21}^{*}(\alpha)\cdot H^{-1}(\alpha)\cdot\alpha^2\sin\alpha y_1 d\alpha-P\int_0^{\infty}H_{22}^{*}H^{-1}(\alpha)\cos\alpha y_1 d\alpha\right]. \tag{21}$$

Analysis of numerical results and investigation results. On the basis of the formula (20)–(21) the numerical analysis [8] was carried out, its results are shown in the graphs (Fig. 3, 4). All results are obtained for the case of equal (harmonic potential, the potential of Bartenev-Khazanovich) and unequal (Treloar potential) roots of the defining equation [2].

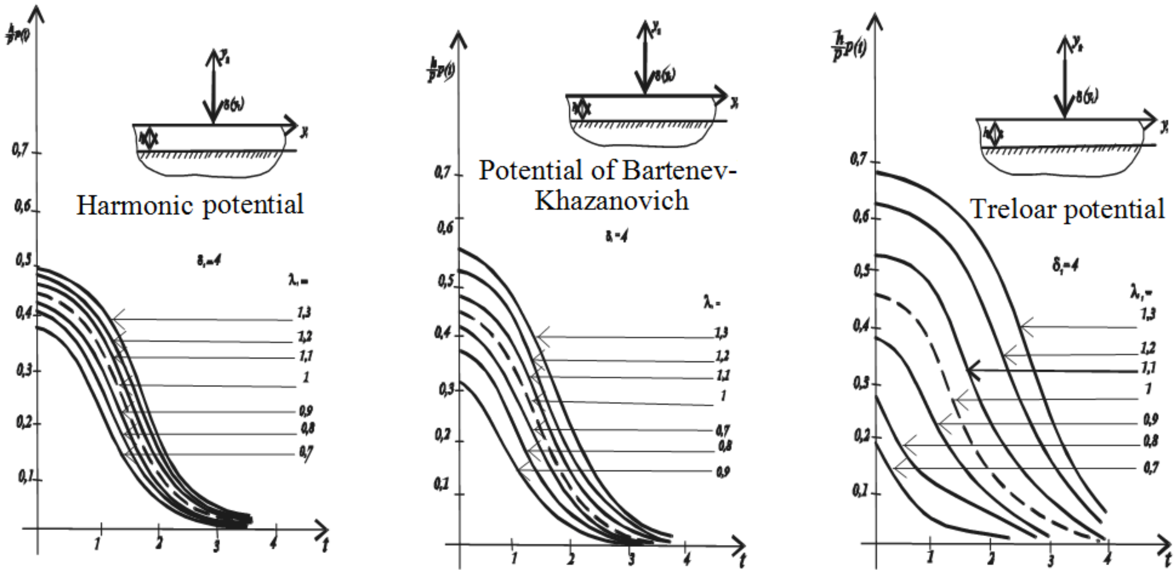

Figure 3. Intensity of normal contact stresses under the stringer

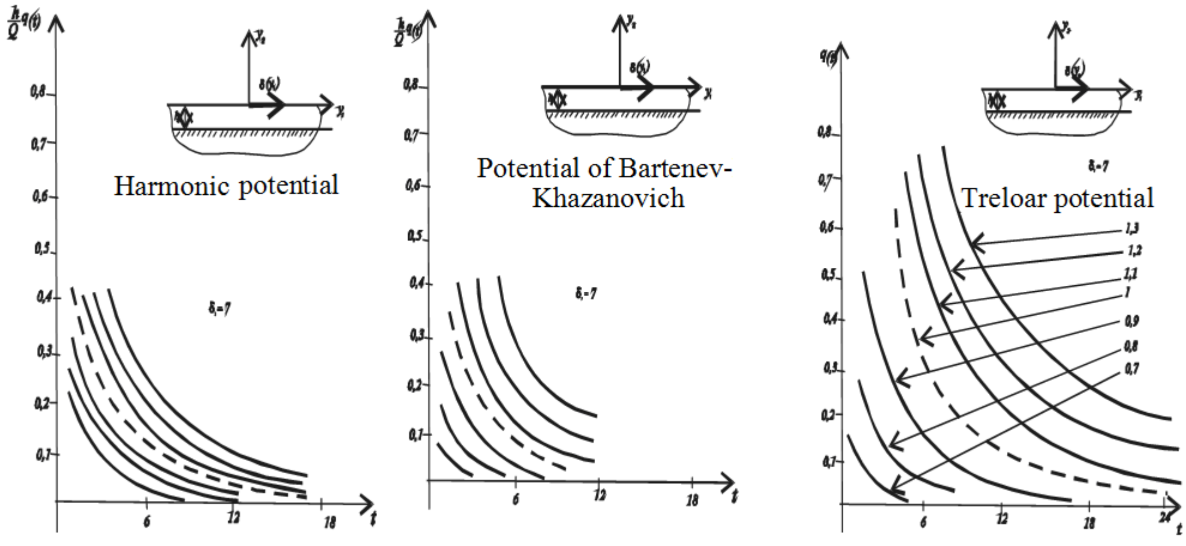

Figure 4. Intensity of tangential contact stresses under the stringer

The numerical calculations will be carried out on PC by licensed software Maple-8 [15], according to dimensionless quantities $hP^{-1}p(t)$ − normal contact stresses; $hQ^{-1}q(t)$ − tangential contact stresses.

The effect of initial stresses in elastic stripes on the law of contact stresses distribution under the stringer for dimensionless values $hQ^{-1}q(t)$ and $hP^{-1}p(t)$, where $hQ^{-1}q(t)$, $hP^{-1}p(t)$ is dimensionless contact tangential and normal stresses, respectively is shown in Fig. 3, 4. The value $\lambda_1 = 1$ (dotted line in Fig. 3.4) corresponds to the classical theory of elasticity and coincides with the results of paper [9]; $\lambda_1 = 0{,}7;0{,}8;0{,}9$ − correspond to the initial stresses of compression, and $\lambda_1 = 1{,}1;1{,}2;1{,}3$ – tensile stress; t − dimensionless coordinate of the initial stressed state in elastic stripes.

Conclusions. The obtained results of the carried out investigation are valuable for the calculation of structures and parts of machines with initial (residual) stresses, which are in contact interaction on strength, reliability and durability. This is due to the fact that they allow to predict more accurately the mechanical behavior of structures using reinforcement elements. The obtained results can be used to evaluate the application limits of the linearized elasticity theory. Within the framework of the linearized elasticity theory, the formulation is given and solutions of the problems of contact interaction of one and two identical elastic stripes with initial (residual) stresses reinforced by elastic infinite stringer are obtained in the general form for the theory of large (finite) and two versions of small initial deformations in the case of elastic potential arbitrary structure. New method of solving this type of contact problems for spripes with initial (residual) stresses reinforced by infinite elastic overlay using integral Fourier transforms is proposed. The main singular integral-differential equations for the class of problems under consideration are obtained. The solution of the obtained equations is presented as quasiregular infinite systems of algebraic equations.

The mechanical effect similar to the previous carried out investigations [2, 5−12], concerning the case when the elongation coefficient approaches the values of the material surface instability, there are resonant phenomena both in stripes and in stringer. This phenomenon occurs due to the fact that the stresses and displacements in the interacting bodies sharply change their values.

The analysis of numerical results shows that in the compression case $(\lambda_1 < 1)$, the presence of initial stresses in the elastic stripe results in significant reduction of contact stresses, in the case of tension $(\lambda_1 > 1)$ − in their increase. And it follows from the sown graphs (Figures 3, 4) that more significant impact of initial stresses is observed in highly elastic materials.

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ПЕРЕДАЧА НАВАНТАЖЕННЯ ВІД НЕСКІНЧЕННОГО СТРИНГЕРА ДО ОДНІЄЇ І ДО ДВОХ ПОПЕРЕДНЬО НАПРУЖЕНИХ СМУГ

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Резюме. *У рамках лінеаризованої теорії пружності розглянуто плоскі контактні задачі, що стосуються передавання навантаження від нескінченного стрингера до однієї і до двох однакових смуг із початковими (залишковими) напруженнями без урахування сил тертя. Дослідження проведено для теорії великих (скінченних) початкових деформацій та двох варіантів теорії малих початкових деформацій, для довільної структури пружного потенціалу. Зроблено припущення, що: 1) нескінченні пружні смуги виготовлені з однакових стисливих або нестисливих матеріалів з потенціалом довільної структури; 2) у смугах діють однакові початкові (залишкові) напруження; 3) під дією вертикальних і горизонтальних сил інтенсивності стрингер у вертикальному напрямку згинається як звичайна балка, а в горизонтальному − розтягується (стискається) як одновісно напружений стрижень. Дослідження даної задачі виконано в координатах початкового (залишкового) деформованого стану. За допомогою інтегрального перетворення Фур'є отримано основні інтегро − диференційні рівняння, розв'язок яких представлено у вигляді квазірегулярних нескінченних систем алгебраїчних рівнянь. Визначено закон розподілу нормальних і тангенціальних контактних напружень уздовж лінії з'єднання стрингера з попередньо напруженими смугами. Досліджено вплив наявності початкових (залишкових) напружень у смугах (смузі) на закон розподілу контактних напружень по лінії контакту з нескінченним стрингером. Проілюстровано вплив початкових напружень у пружних смугах на закон розподілу контактних напружень під стрингером від дії тангенціальної сили. Запропоновано новий спосіб розв'язування даного типу контактних задач для смуг із початковими (залишковими) напруженнями, які підкріплені нескінченою пружною накладкою (стрингером) із використанням інтегральних перетворень Фур'є.*

Ключові слова: *лінеаризована теорія пружності, початкові (залишкові) напруження, контактна задача, інтегральне перетворення Фур'є, стрингер.*

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