ICCPT 2019: Current Problems of Transport: Proceedings of the 1st International Scientific Conference, May 28-29, 2019, Ternopil, Ukraine

Optimization of machinery operation modes from the point of view of their dynamics

Evgeniy Kalinin *, **Mykhailo Shuliak**, **Ivan Koliesnik**

Kharkiv Petro Vasylenko National Technical University of Agriculture, Kharkiv, Alchevskyh str. 44, Kharkiv, 61002, Ukraine

* Corresponding author: kalininhntusg@gmail.com

Abstract: The analysis and synthesis of optimization methods of machinery dynamic modes are carried out in this work. Theoretical studies have shown that in order to find optimum, one must define a differential equation describing the motion of the system, which realization would ensure the most advantageous dynamic regime determined by the stationary value of the corresponding functionals. Thus, the problem of optimal dynamic modes lies in the fact that it is necessary to define such a differential equation, which realization would ensure the most favorable dynamic regime, determined by stationary value of functionals. The differential equation corresponding to the optimal mode must be defined in the process of machinery design, because its physical parameters and layout form the basis of these differential equations. The definition of this equation must be carried out while machinery construction is taking place, as its physical parameters and layout form the basis of these differential equations. Such approach requires the introduction of certain principles significantly affecting the development of optimization methods justified in this work. To solve the problem of optimal machinery modes, separation of complex motion by its dynamic properties is more suitable. Suppose that the complex motion can be devided into the motion of the machinery unit as a whole, to the static displacements of its elements as solid bodies, to the increasing and damping components of motion and to the vibrational component. Thus, the solution of the problem of optimal modes in the machinery dynamics consists in the following. The most advantageous machinery dynamic mode is determined by the conditions of the technological process, which would ensure its highest productivity, the lowest energy consumption and other optimal technical and economic indicators. This regime corresponds to the motion of the unit as a whole, that is, to the variation in the quasi-cyclic coordinates. The vector of external forces applied to the machinery is reduced to the initial conditions of its motion; homogeneous differential equations are considered further. The fundamental system of their solutions depends on the initial conditions of motion generated by external systems.

Keywords: machinery dynamics, operation system damping, technological process.

1. Introduction

The analysis techniques methods to study the motion of machines are critical in machine design process as such analyses should be performed on design concepts to optimize the motion of a machine arrangement. A focus is placed on the application of kinematic theories to real-world machinery. The main task is bridge the gap between a theoretical study of kinematics and the application to practical mechanisms [1-5]. Science and technology problems of machinery dynamics have been becoming increasingly important every year. Especially a lot of them arise while creating and operating heavy machinery that have significant linear dimensions, masses and moments of inertia of movable links that are under the influence of transient loading. Dynamic process simulation differs from purely steady-state simulation in that the former requires the mechanical construction of process items be taken into account; the amount of mechanical detail being dependent upon the particular application. The reason for this is that dynamic mass, energy and momentum balances have to be continuously updated [6-8]. In such conditions, even small accelerations of the movable links lead to the appearance of considerable inertia forces causing large dynamic loads on the elements of machinery and designs. The study of dynamic processes in machinery and the creation of methods for calculating machinery taking into account current dynamic loads and links elasticity acquire special importance with the increasing speed of modern machinery, which ensures their high productivity [9-11].

Machinery dynamics includes the complex tasks of modern machine building and, despite the rather wide coverage in the specialized literature [12-16], requires further comprehensive study both for explaining dynamic processes taking place in machinery, establishing their regularities, and for developing reliable calculation methods. The wide development of computer technology makes it much easier to solve these problems and makes many of them accessible to engineering practice. At the same time, much attention should be paid not only to the design of machinery, but also to their dynamic adaptation to the operating conditions by optimizing their operating modes according to dynamic criteria.

2. Materials and Methods

Suppose that machinery constructive elements be formalized by square matrices: inertial $K = \left\| k_{ij} \right\|_1^m$, stiffness of elastic elements $C = \left\| c_{ij} \right\|_1^m$ and attenuation coefficients $B = \left\| b_{ij} \right\|_1^m$. If the state of the machinery at a point of time t is determined by the column vector of the generalized coordinates q the column vector of the generalized velocities $\dot{q}$, then its energy properties will be expressed by quadratic forms of the form:

$$E = \frac{1}{2}\dot{q}^T K \dot{q}, \quad \Pi = \frac{1}{2} q^T C q, \quad \Phi = \frac{1}{2}\dot{q}^T B \dot{q}, \tag{1}$$

here T - a sign of transpose.

Suppose that Lagrangian corresponding to the forms (1) has the form:

$$L = L(q_1, \ldots, q_m; \dot{q}_1, \ldots, \dot{q}_m), \tag{2}$$

and let the determinant

$$\left| \frac{\partial^2 L}{\partial \dot{q}_s \partial q_i} \right| \neq 0, \quad (s = 1, \ldots, m; i = 1, \ldots, m). \tag{3}$$

Then the differential equations of machinery motion can be written in the form:

$$\ddot{q}_s = \varphi_s(t, q_i, \dot{q}_i), \quad (s = 1, \ldots, m; i = 1, \ldots, m). \tag{4}$$

If $q_1 = x_1, \dot{q}_1 = x_2, \ldots, q_m = x_{2m-1}, \dot{q}_m = x_{2m}$, then the system (4) will have a normal form of the form:

$$\dot{x}_i = f_i(x_1, \ldots x_n; t), \quad (n = 2m; i = 1, \ldots, n). \tag{5}$$

As coefficients, the system of equations (5) includes combinations of matrix elements K, C, B through forms (1), Lagrangian L and system (4). We represent these combinations in the form of a column vector $p \in P$, where P – parameter space with dimension k, bounded by a certain region. Let's give equations (5) a vector form:

$$\dot{x} = f(x, t, p), \tag{6}$$

here t belongs to some open interval $a < t < b$, which ends are real numbers; f – vector function completely defined in the region of $D(n+1)$ - dimensional space.

It has been supposed that requirements of the motion quality are formalized in the form of k functionals, depending on the machinery power mode and its structural elements in the form:

$$I_i = \int_{t_0}^{t_1} U_i[f(x,t,p)]dt\,, \quad (i=1,\dots,k; k<n)\,. \tag{7}$$

Functionals (7) can express in the mathematical form the conditions for the highest productivity of machinery, the smallest modules of the elastic forces of its links, the decay of transient processes in the shortest time, and many other important technological and dynamic conditions. So, if a machinery operating cycle is presented in the form of a cyclogram with a cycle time T, then $I=T$, and its maximum performance is achieved at $T \to \min$. If the functional (7) is written with respect to the largest maximum modulus of elastic forces developed during the transient process

$$I = \max_{1\le i\le n} \max_{0\le t\le T} |x_i|\,, \tag{8}$$

then, its smallest value

$$\min I = \min_{c} \max_{1\le i\le n} \max_{0\le t\le T} |x_i| \tag{9}$$

also corresponds to the optimal dynamic mode.

Usually a machinery operating process is determined by the decay time of the transitient component of elastic oscillations t_{tr}. Then $I=t_{tr}$, the achievement of $\min t_{tr}$ optimizes the operating process by the decay time. Functionals (7) can also be written with respect to the consumption of fuel or energy. In this case, energy modes of the machinery units are optimized.

Thus, the problem of optimal dynamic modes lies in the fact that it is necessary to define such a differential equation (6), which realization would ensure the most favorable dynamic regime, determined by stationary value of functionals (7).

The differential equation (6) corresponding to the optimal mode must be defined in the process of machinery design, because its physical parameters and layout form the basis of these differential equations.

3. Results

The mathematical and practical complexity of problem solving requires the introduction of some new principles that significantly affect the development of methods for optimizing processes.

3.1. The principle of generalized input

Let's define the following Euclidean norms for the equations of motion (5):

$$\|x\| = \sum_{i=1}^{n} |x_i|\,; \tag{10}$$

$$\|f(x)\| = \sum_{i=1}^{n} |f_i(x_1,\dots,x_n)|\,. \tag{11}$$

It has been supposed that on $f_i(x_1,\dots,x_n)$, $(i=1,\dots,n)$ are imposed conditions under which

$$\frac{\|f(x)\|}{\|x\|} \to 0 \quad \text{at} \quad \|x\| \to 0\,. \tag{12}$$

Then, on a certain interval of argument variation belonging to the entire numerical axis, we can pass to a linear differential equation of the form:

$$\dot{x} = Ax + F(t)\,. \tag{13}$$

In this equation $A = \|a_{ij}\|_1^n$ – a constant matrix of its coefficients, $F(t)$ – is a vector function of the external forces applied to machinery links.

The matrix A with its elements usually characterizes machinery, including the operation system. It is natural that different elements of the matrix correspond to a different quality of motion and that external forces influence this quality. In order to improve the machinery motion in a certain sense, the matrix A can be changed, or the vector function $F(t)$, or both.

It has been supposed that the design parameters of the machinery are changed so that they can be analytically represented in the form of the matrix $B_k = \left\| b_{ij} \right\|_1^n$. Let us write the equation of the unit motion in the form:

$$\dot{x} = B_k x . \tag{14}$$

We subordinate equations (13) и (14) to the same initial conditions, given in the form of a column x_0. We require that the solutions of these equations coincide everywhere on the interval $0 < t < \infty$. On this basis, we assume that the left-hand sides of equations (13) and (14) are equal to each other. Then:

$$(B_k - A)x = F(t) . \tag{15}$$

Let us show that when the equation (15) is satisfied, the solutions of equations (13) and (14) will be identical. Let us write the solution of the equation (13) in the form:

$$x = e^{At} x_0 + \int_0^t e^{A(t-\tau)} F(\tau) d\tau . \tag{16}$$

Substituting the value x from the equation (16), we reduce the equation (15) to the integral:

$$F(t) = (B_k - A)e^{At} x_0 + (B_k - A)\int_0^t e^{A(t-\tau)} F(\tau) d\tau . \tag{17}$$

As the kernel of the equation (17) is a function of the form

$$K(t,\tau) = (B_k - A)e^{A(t-\tau)} , \tag{18}$$

then its resolvent is written as

$$R(t,\tau) = \sum_{m=1}^{\infty} (B_k - A)\frac{(t-\tau)^{m-1}}{(m-1)!} = (B_k - A)e^{B_k(t-\tau)} . \tag{19}$$

If the matrices A and B_k commute, then the solution of the equation (16) will have the form:

$$F(t) = (B_k - A)e^{At} x_0 + \int_0^t (B_k - A)^2 e^{B_k(t-\tau)} e^{A\tau} x_0 d\tau . \tag{20}$$

After completing the quadrature, we get:

$$F(t) = (B_k - A)e^{B_k t} x_0 . \tag{21}$$

Comparing the results of (21) with the equality (15), we note that the initial coordinate x must simultaneously be the solution of equation (14), that is, equal to $e^{B_k t} x_0$.

Thus, the parameter variation of a machinery unit in a dynamic sense is equivalent to the variation of external forces acting on it. As the column vector $F(t)$ in the equation (13) is the system input, then the matrix equation (15), stating that the input is dynamically equivalent to its parameters variation, expresses the principle of generalized input. This principle, first of all, shows that the optimization of the power mode can be achieved through the rational choice of machinery design parameters.

3.2. The principle of motion separation by their dynamic properties

In classical mechanics, complex motion is devided into simple ones, as a rule, according to their geometric (kinematic) properties. To solve the problem of optimal machinery modes, separation of complex motion by its dynamic properties is more suitable. Suppose that the complex motion can be devided into the motion of the machinery unit as a whole, to the static displacements of its elements as solid bodies, to the increasing and damping components of motion and to the vibrational component.

The displacement of the object as a whole relatively to its center of inertia can be calculated with the help of equations (4) if they contain quasi-cyclic coordinates, understood as A.I. Lurie [17]. Let the generalized forces for quasi-cyclic coordinates have the form:

$$Q_{r+s} = Q_{r+s}(q_1, \ldots, q_r), \quad s = (1, \ldots, m-r), \tag{22}$$

but a quasi-cyclic pulse

$$p_{r+s} = \frac{\partial T}{\partial \dot{q}_{r+s}}, \quad s = (1, \ldots, m-r). \tag{23}$$

Then

$$\dot{P}_{r+s} = Q_{r+s}(q_1, \ldots, q_m), \quad s = (1, \ldots, m-r). \tag{24}$$

This system is solvable with respect to quasi-cyclic generalized velocities and its integration will determine the motion of the object as a whole.

Let the vector-column of external forces in equation (13) $F(t) \in W^{(r)}H^{(\alpha)}(M;a,b)$, that is, let them belong to some class of functions having $[a,b]$ derivatives of r, satisfying inequality

$$\left|F^{(r)}(t) - F^{(r)}(t')\right| \le M\left|(t-t')\right|^{\alpha}, \quad (t,t') \subset [a,b], \tag{25}$$

here $0 < \alpha \le 1$. In other words, $F(t)$ can be a polynomial, for example, of degree r:

$$F(t) = \sum_{i=0}^{r} F^{(i)}(0)\frac{t^i}{i!}. \tag{26}$$

Differentiating equation (26) $r+1=n$ times and setting $z = x^{(n)}$, we obtain:

$$\dot{z} = Az. \tag{27}$$

Defining the initial conditions $x(0) = x_0$ and $z(0) = z_0$, we have:

$$z_0 = x_0^{(n)}. \tag{28}$$

The solution of the equation (27) with the initial conditions (28) can be written in the form:

$$z = e^{At}z_0, \tag{29}$$

or

$$x^{(n)} = e^{At}x_0^{(n)}. \tag{30}$$

Integrating the equation (30) n times, we find:

$$x(t) = e^{At}\left[x_0 + \sum_{i=0}^{n-1} A^{-i-1}F^{(i)}(0)\right] - \sum_{i=0}^{n-1}\sum_{j=i}^{n-1} A^{-i-1-j}F^{(j)}(0)\frac{t^i}{i!}. \tag{31}$$

The first item on the right-hand side of this equation is the solution of the homogeneous equation (27) with the initial conditions (28):

$$z_0 = x_0 + \sum_{i=0}^{n-1} A^{-i-1}F^{(i)}(0). \tag{32}$$

The second item is a particular solution of the equation (13) if its right-hand member is the function $F(t)$. In fact, the general solution of the equation (13) has the form:

$$x = e^{At}x_0 + \int_0^t e^{A(t-\tau)}F(\tau)d\tau \,. \tag{33}$$

If $F^{(n)}(t) = 0$, then, integrating by parts, we get:

$$\int_0^t e^{A(t-\tau)}F(\tau)d\tau = e^{At}\sum_{i=0}^{n-1}F^{(i)}(0) - \sum_{i=0}^{n-1}A^{-i-1}F^{(i)}(t)\,. \tag{34}$$

Taking (26) into account, we have:

$$\sum_{i=0}^{n-1}A^{-i-1}F^{(i)}(t) = \sum_{i=0}^{n-1}\sum_{j=i}^{n-1}A^{-i-1-j}F^{(j)}(0)\frac{t^i}{i!}\,. \tag{35}$$

Putting in the equation (13) $\dot{x} = 0$, we define the so-called "quiescent" state, that is, the change in the coordinates of the system under the influence of the static action of the forces:

$$x = -A^{-1}F(t)\,. \tag{36}$$

If the law of external forces variation is given in the form

$$F(t) = const\,, \tag{37}$$

then (36) is a pure solution of the equation (13). Such a solution determines system deformability, understood in the most general sense, for example, as the elastic displacements of its elements. If external influences are variable in time, then the deformability of the object occurs at a certain rate.

Differentiating the equation (13) with respect to t, supposing that $x = 0$ and taking (35) into account, we obtain:

$$x = -A^{-1}F(t) - A^{-2}\dot{F}(t)\,. \tag{38}$$

Continuing deformability definition in the same order, i.e, equating to the zero all dominant derivatives, we find that the static displacements of the system are expressed by a particular solution (36).

Thus, if the external forces of the machinery unit belong to some fairly wide class of time functions, then the inhomogeneous differential equation (13) can be regarded as homogeneous with the initial conditions (32). By this there are distinguished the static displacements (35) and the dynamic component of the motion, determined by the solution (30) in the form:

$$x(t) = e^{At}\left[x_0 + \sum_{i=0}^{n-1}A^{-i-1}F^{(i)}(0)\right]. \tag{39}$$

The further separation of complex motion and its optimization, depending on the design parameters of the machinery unit, require the creation of a new form of a fundamental system for solving differential equations.

3.3. Fundamental system for solving differential equations in parameter space

As the external force vector, written in the differential equation (13), can be brought dynamically to the initial conditions (32) and to the static component of the motion (26), then we will consider the homogeneous equation (13). Suppose that there is a matrix of the form:

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -p_n & -p_{n-1} & -p_{n-2} & \cdots & -p_1 \end{pmatrix}. \tag{40}$$

Then the homogeneous differential equation (13) can be written as follows:

$$x^{(n)} + p_1 x^{(n-1)} + p_2 x^{(n-2)} + \ldots + p_n x = 0 . \tag{41}$$

If we replace the variable by setting

$$x = y e^{-\frac{p_1}{n} t} , \tag{42}$$

then the equation (41) takes the form:

$$y^{(n)} + b_2 y^{(n-2)} + b_3 y^{(n-3)} + \ldots + b_n y = 0 . \tag{43}$$

Then replacing t with the value $\tau = t\sqrt{b_2}$ we will have:

$$y^{(n)}(\tau) + y^{(n-2)}(\tau) + g_1 y^{(n-3)}(\tau) + \ldots + g_{n-2} y(\tau) = 0 , \tag{44}$$

here

$$g_j = \frac{b_{j+2}}{b_2\left(\sqrt{b_2}\right)^j} , \quad (j = 1, \ldots, n-2) . \tag{45}$$

We put in the equation (44)

$$y^{(n)}(\tau) = U(\tau) . \tag{46}$$

We reduce it to an integral equation of the form:

$$U(\tau) + \int_0^\tau K(\tau, y) U(\eta) d\eta = y^{(n-1)}(0) K(\tau) - y^{(n-2)}(0) K'(\tau) - \ldots - y^{(n-k)}(0) K^{(k-1)}(\tau) - \ldots - y(0) K^{(n-1)}(\tau) , \tag{47}$$

which core is

$$K(\tau, \eta) = \sum_{n=0}^{n-1} \frac{(\tau - \eta)^n}{n!} . \tag{48}$$

In the monograph [18] it was proved that the resolvent of the equation (47) has the form:

$$R(\tau,\eta) = \sum_{m=1}^{\infty} (-1)^m \sum_{i=0}^{m} \sum_{k=0}^{m-1} \ldots \sum_{p=0}^{m-i-\ldots-l} \binom{m}{i} \binom{m-i}{k} \ldots \binom{m-i-\ldots-l}{p} g_1^p g_2^l \ldots g_{n-2}^i \frac{\tau - \eta^{2m+p+2l+\ldots+(n-2)i-1}}{[2m+p+2l+\ldots+(n-2)i-1]!} . \tag{49}$$

As in the resolvent (49) there are coefficients g_j $(j = 1, \ldots, n-2)$, related to the parameters p_i $(i = 1, \ldots, n)$, then the solution of the equation (46) and, hence, of the equation (36) is written in terms of object parameters - this solution forms a fundamental system in the parameter space, which form is presented in the monograph [19].

3.4. Separation of motion in parameter space

In the work [18] it is proved that the core (48) and the resolvent (49) are divided into two items:

$$K(\tau, \eta) = K^*(\tau, \eta) + L(\tau, \eta) , \tag{50}$$

$$R(\tau, \eta) = R^*(\tau, \eta) + Q(\tau, \eta) , \tag{51}$$

here

$$K^*(\tau,\eta)=\frac{(\tau-\eta)}{1}+g_2\frac{(\tau-\eta)^3}{3!}+\ldots+g_{n-2}\frac{(\tau-\eta)^{n-1}}{(n-1)!},$$

$$R^*(\tau,\eta)=\sum_{m=1}^{\infty}(-1)^m\sum_{i=0}^{m}\sum_{k=0}^{m-1}\ldots\sum_{l=0}^{m-i-\ldots-p}\binom{m}{i}\binom{m-i}{k}\ldots\binom{m-i-\ldots-p}{l}\times$$

$$\times g_2^l g_4^p\ldots g_{n-2}^i\frac{(\tau-\eta)^{2m+2l+4p+\ldots+(n-2)i-1}}{[2m+2l+4p+\ldots+(n-2)i-1]!}. \quad (52)$$

If we substitute core and resolvent values into the integral equation (47), then its solution with respect to $y(\tau)$ will also consist of two parts:

$$y(\tau)=y_1(\tau)+y_2(\tau), \quad (53)$$

and the function $y_1(\tau)$ is formed by solving a differential equation of the form:

$$y_1^{(n)}+y_1^{(n-2)}+g_2y_1^{(n-4)}+\ldots+g_{n-2}y_1=0. \quad (54)$$

With a proper choice of the coefficients $g_2,g_4,\ldots,g_{n-2}$ the solution of the equation (54) will be an undamped, bounded by module almost periodic function of time. However, the original equation (43) has a time-increasing solution, as can be seen from its Hurwitz matrix

$$\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ g_1 & 1 & 0 & \cdots & 0 \\ g_3 & g_2 & g_1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & g_{n-2} \end{pmatrix}. \quad (55)$$

As the minors of this matrix are $\Delta_1=0$, $\Delta_2=-g_1<0$, $\Delta_3=-g_1^2<0$,… at positive g_j $(j=1,\ldots,n-2)$, then the solution of the equation (43) increases with time. But the solution of the equation (43) is the sum of two functions (53), in which $y_1(\tau)$ at certain conditions it does not increase by the module. Consequently, the function increasing in time is $y_2(\tau)$. Passing to the argument t and taking into account the substitution (42), we write:

$$x=[y_1(t)+y_2(t)]e^{-\frac{p_1}{n}i}. \quad (56)$$

Thus, the complex motion of machinery unit elements is divided according to the formula (56), into an increasing component of $y_2(t)$ and on a purely oscillatory component – $y_1(t)$.

3.5. Process, optimal by attenuation

If we take as the beginning of the transient process $t=0$, then by its duration we mean the time from the moment when the oscillations start to the moment of equilibrium onset. The duration of the damped oscillatory process depends essentially on the design parameters of the object. Consider a three-mass system with attenuation k_{12} and k_{23}, proportional to the speed of oscillations. The differential equation of the oscillatory process has the form:

$$x^{(4)}+p_1\dddot{x}+p_2\ddot{x}+p_3\dot{x}+p_4x=0, \quad (57)$$

here

$$p_1=k_{12}\frac{m_1+m_2}{m_1m_2}+k_{23}\frac{m_2+m_3}{m_2m_3}, \quad (58)$$

$$p_2=c_{12}\frac{m_1+m_2}{m_1m_2}+c_{23}\frac{m_2+m_3}{m_2m_3}+k_{12}k_{23}\frac{m_1+m_2+m_3}{m_1m_2m_3}, \quad (59)$$

$$p_3 = (k_{12}c_{23} + k_{23}c_{12})\frac{m_1 + m_2 + m_3}{m_1 m_2 m_3}, \tag{60}$$

$$p_4 = c_{12}c_{23}\frac{m_1 + m_2 + m_3}{m_1 m_2 m_3}. \tag{61}$$

In this equation discrete masses are denoted by m_i $(i = \overline{1,3})$ and the stiffnesses of the elastic links by c_{12} and c_{23}.

The attenuation of the process is determined by the coefficients p_1 and p_3. These coefficients, and hence the duration of the process, depend not only on k_{12} and k_{23}, but also on all parameters of the system, that is, on the machinery design. Therefore, even with sufficiently large attenuation coefficients, not optimal selection of machinery design parameters reduces their efficiency, and vice versa - with small attenuation coefficients, but with a suitable ratio of discrete masses and rigidities, it is possible to realize rapidly damped in time process. If we pass to the phase space, then the optimal process by attenuation is determined as follows.

The system makes free attenuating oscillations; its initial state is given by the vector $x(0)$, determining the position of the point in the $2n$-dimensional phase space. It is necessary to use the system parameters so that the transition of the phase point to the origin of coordinates proceeds in the shortest time interval.

To solve the problem, we turn to the equation (56). First of all, the function increasing in time should be suppressed $y_2(t)$. Such an operation is considered in detail in the monograph [20]. However, it is often possible to solve the problem correctly in a purely intuitive way. The condition $y_2(t) = 0$ takes place if the equation (44) becomes the equation (62). To do this, all odd coefficients g_i $(i = 1,3,5,\ldots)$ must be turned into zero. These coefficients are associated with system parameters by conditions (42) and (45). Then the solution of the differential equation of the transient, written in the form (56), will be the following:

$$x = y_1(t)e^{-\frac{p_1}{n}t}. \tag{62}$$

The second step in the process optimization is the choice

$$\max_{p_1 \in P} p_1. \tag{63}$$

Then it is necessary that the function $y_1(t)$ is bounded by module. In the general case the function $y_1(t)$ is the solution of the differential equation of small oscillations of conservative systems.

3.6. Optimization of the vibrational item

Let us consider a more general form of the equation (54). Having substituted the variable y with the variable x, we get:

$$x^{(2n)} + a_0 x^{(2n-2)} + a_1 x^{(2n-4)} + \ldots + a_{n-1}x = 0. \tag{64}$$

A similar equation is used in the machinery dynamics with elastic links without taking energy dissipation into account. Substituting the argument

$$\tau = \frac{t}{\sqrt{a_0}}, \tag{65}$$

the differential equation (64) is reduced to the form

$$x^{2n} + x^{(2n-2)} + c_1 x^{(2n-4)} + \ldots + c_{n-2}x = 0, \tag{66}$$

here

$$c_i = \frac{a_i}{\left(\sqrt{a_0}\right)^i}, \quad (i = 1, \ldots, n-2). \tag{67}$$

As the coefficients c_i $(i = 1, \ldots, n-2)$ are connected by means of the formula (67) with the system parameters, then, defining them in the form of inequalities

$$\begin{aligned} & c_1 < \frac{n-1}{2!n}, \\ & c_2 < \frac{(n-1)(n-2)}{3!n^2}, \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ & c_k < \frac{(n-1)(n-2)\ldots(n-k)}{(k+1)!n^k}, \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ & c_{n-1} < \frac{1}{n^n} \end{aligned} \tag{68}$$

here $2n$ – the order of the differential equation, it is possible to guarantee the boundedness by module of the oscillatory item of the solution [15].

However, the optimization of this process must be continued until the maximum deviation is minimized or until the maximum elastic forces of the system are minimized. The fundamental system for solving the equation (66) in the parameter space c_i $(i = 1, \ldots, n-2)$ is presented in the monograph [18] in the form of functions $x_0, x_1, \ldots, x_{n-1}$, expressing the system reaction to the defined initial values of the function and its derivatives up to $(n-1)$ - inclusively. Thus

$$x = \sum_{k=0}^{n-1} x_k. \tag{69}$$

Let the fundamental solution system form n-dimensional linear normed space X, i. e. $x_k \in X$ $(k = 0, \ldots, n-1)$ with the norm

$$\|x\| = \sum_{k=0}^{n-1} |x_k|. \tag{70}$$

As the module of functions is variable in time, the norm (70) also varies. Suppose that there is an absolute maximum of the norm, that is,

$$\max_k \max_{x \in X} \|x\| = \max_k \max_{x \in X} \sum_{k=0}^{n-1} |x_k|. \tag{71}$$

Assuming also that

$$\sup_{x \in X} |x_k| = \max_k \max_{x \in X} |x_k|, \tag{72}$$

and

$$\sum_{k=0}^{n-1} \max_k \max_{x \in X} |x_k| \geq \max_k \max_{x \in X} \sum_{k=0}^{n-1} |x_k|, \tag{73}$$

we will minimize the value of the form:

$$\|\bar{x}\| \geq \|x\|, \tag{74}$$

here

$$\|\bar{x}\| = \sum_{k=0}^{n-1} \max_{k} \max_{x \in X} |x_k| \quad . \tag{75}$$

The value (75) corresponds to the norm (70) under the most unfavorable conditions of motion in accordance with the so-called principle of unfavorable collinearity.

If $x_k \quad (k = 0, \ldots, n-1)$ – are the elastic forces of machinery links while transient is taking place, then the definition

$$\min_{c_i} \|\bar{x}\| = \min_{c_i} \sum_{k=0}^{n-1} \max_{k} \max_{x \in X} |x_k| \tag{76}$$

means the optimal problem solution, which ensures the lowest amplification factor.

Parameters (68) by which the system is optimized are proper fractions. The fractional denominator grows considerably with the increase in the index i, which makes it practically feasible to optimize them using PC. It should be noted that almost always it is possible to determine such a region of the form:

$$c_i \in C_p, \quad (i = 1, \ldots, n-2), \tag{77}$$

which satisfies the condition (76).

4. Discussion

The solution of the problem of optimal modes in the machinery dynamics consists in the following:

1. The most advantageous machinery dynamic mode is determined by the conditions of the technological process, which would ensure its highest productivity, the lowest energy consumption and other optimal technical and economic indicators. This regime corresponds to the motion of the unit as a whole, that is, to the variation in the quasi-cyclic coordinates.

2. The vector of external forces applied to the machinery is reduced to the initial conditions of its motion according to the formula (32); homogeneous differential equations are considered further. The fundamental system of their solutions (69) depends on the initial conditions of motion generated by external systems. If the initial conditions are represented in the form of a row matrix of the form

$$x_0^T = (x(0), \dot{x}(0), \ldots, x^{(n-1)}(0)), \tag{78}$$

and the fundamental system of solutions, corresponding to the ordinary initial conditions, in the form of a matrix-column

$$\tilde{x} = \begin{pmatrix} x_{(0)} \\ x_{(1)} \\ \ldots \\ x_{(n-1)} \end{pmatrix}, \tag{79}$$

then

$$x = x_0^T \tilde{x} \, . \tag{80}$$

3. In real machinery, there are always reasons generating internal friction, and, consequently, energy dissipation. By turning parameters (45) with odd indexes into zero, one can intensify the process attenuation and thereby eliminate the possible accumulation of perturbations, for example, in the case of repeatedly short-time technological modes.

4. The choice of the most advantageous parameters $c_i \quad (i = 1, \ldots, n-2)$, connected by means of the formula (67) with the coefficients of the differential equation (64), will lead to the lowest values of the amplification factor of the elastic links.

5. Conclusions

Considering the row matrix (78) as the coordinates of the n-dimensional Euclidean vector, that is, $x_0^T \in E^n$, and $\tilde{x} \in E^n$ – as an alternating vector of the same space, we can state that the scalar product (80) is generated by the vector (78). If we fix x, that is the vector (79), then we can form the norm of the vector x_0^T, which generates the scalar product (80) or the bilinear function of x_0^T and $\tilde{x}$. In this case, it is assumed that the norm of the vector x_0^T is the maximum of the values (80) on the ordinary sphere $\|\tilde{x}\| \le 1$, i.e.: $\left\|x_0^T\right\| = \max\limits_{\|\tilde{x}\|\le 1} x_0^T \tilde{x}$.

As the vector items x_0^T depend on external forces and system parameters, one must strive to choose their lowest values, which correspond, in general, to the smallest forces. However, the desire to define small forces should not worsen the technical and economic performance of the machinery.

According to the generalized input principle, the optimization of the power mode can be achieved by the variation of machinery design parameters. In this case, the maximum value of external forces and their duration, especially the small, creating a large dynamic effect, become less noticeable.

References

1. Vinogradov, O. *Fundamentals of kinematics and dynamics of machines and mechanisms*; Boca Raton, FL: CRC Press, 2000; 304 p.
2. Myszka, D.H. *Machines & Mechanisms: Applied Kinematic Analysis,* 4th ed; Prentice Hall, 2011; 376 p.
3. Uicker, J.; Pennock, G.; Shigley, J. *Theory of Machines and Mechanisms,* 4th ed.; Oxford University Press, New York, 2010; 926 p.
4. Chironis, N.; Sclater, N. *Mechanisms and Mechanical Drives Sourcebook,* 4th ed.; McGraw-Hill Book Company, New York, 2007; 495 p.
5. Waldron, K.J.; Kinzel, G.L. *Kinematics, Dynamics, and Design of Machinery*; John Wiley & Sons, New York, 1999; 668 p.
6. Braha, D. and Maimon, O. The design process: properties, paradigms, and structure. *IEEE Transactions on Systems, Man, and Cybernetics - Systems and Humans*, 1997, 27; pp. 146-166.
7. Zeigler, B.P.; Praehofer, H.; Kim, T.G. *Theory of Modeling and Simulation: Integrating Discrete Event and Continuous Complex Dynamic Systems*, 2nd ed; Academic Press, 2000; 510 p.
8. Histand, M.B. and Alciatore, D.G. *Introduction to Mechatronics and Measurement Systems*. Boston: Mc Graw-Hill, 1998; 553 p.
9. Triengo, M.J.L.; Bos, A.M. Modeling the Dynamics and Kinematics of Mechanical Systems with Multibond Graphs, *Journal of the Franklin Institute*, 1985, *319*, 37-50.
10. Piyush K. Bhandari; Ayan Sengupta. Dynamic Analysis Of Machine Foundation, *International Journal Of Innovative Research In Science, Engineering And Technology* 2014, *3*, 169-176.
11. Dulau M.; Oltean St.-E.; Duka A.-V. Modeling and Simulation of the Operation of a Mechanical System which is Affected by Uncertainties, *Procedia Technology,* 2016, 22, 662-669.
12. Steindl, A.; Troger, H. Methods for dimension reduction and their application in nonlinear dynamics, *International Journal of Solids and Structures* 2001, *38*, 2131-2147.
13. Luo, A.C.J. and Xing, S.Y. Symmetric and asymmetric period-1 motions in a periodically forced, time-delayed, hardening Duffing oscillator, *Nonlinear Dynamics* 2016, 85, 1141-1186.
14. Friswell, M.I.; Penny, J.E.T., and Garvey, S.D. The application of the IRS and balanced realization methods to obtain reduced models of structures with local nonlinearities, *Journal of Sound and Vibration* 1996, *196*, 453-468.
15. Kordt, M. and Lusebrink, H. Nonlinear order reduction of structural dynamic aircraft models, Aerospace Science and Technology 2001, 5, 55-68.
16. Fey, R.H.B., van Campen, D.H., and de Kraker, A. Long term structural dynamics of mechanical system with local nonlinearities, *ASME Journal of Vibration and Acoustic*s 1996, *118*, 147-163.
17. Lurie A.I. *Analytical Mechanics*; Springer: Berlin/New York, 2002; 387 p.
18. Golubentsev A.N. *Integral methods in dynamics*; Moscow, 1967; 273 p.
19. Wiggins, S. *Introduction to applied nonlinear dynamical systems and chaos*; Springer-Verlag: New York, 1990; 864 p.
20. Hu, H.Y.; Wang, Z.H. *Dynamics of Controlled Mechanical Systems with Delayed Feedback*; Springer: Berlin, 2002; 294 p.