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MATHEMATICAL MODEL OF OPERATIVE ESTIMATION OF VISCOSITY OF A PULP IN A BALL MILL AT GRINDING AN ORE

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Summary. Of the known mathematical models for determining the viscosity of dispersion media, the most reasonable is the mathematical model of G.S. Khodakov, but it is not suitable for the operative calculation of the parameter and for controlling the process. The proposed basic mathematical model reduces to establishing the distance between particles of solid particles with respect to its density, size and pulp density, suitable for rapid estimation of the parameter. The basic mathematical model on the basis of the found value of the distance between solid particles and its parameters allows us to quickly estimate the viscosity of the pulp. The approach of determining the model parameters is substantiated.

Key words: ball mill, ore, pulp viscosity, operational evaluation, mathematical model, parameters.

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Statement of the problem. Iron ore concentrate, obtained from poor iron ores by enriching them, has long become the raw material base of ferrous metallurgy. Significant overruns of electricity and materials during the grinding of the initial ore, especially in the first stage, increase the cost of domestic iron ore concentrate and thereby reduce its competitiveness and its products in the international market. One of the ways to improve the situation is to improve the automation of these processes, which is often hampered by the lack of any information tools, including the system for rapid assessment of pulp viscosity in a ball mill, which grinds the original ore in a closed cycle with a mechanical single-spiral classifier. These problems are addressed in government documents, it is reflected in the scientific topics of the Central Ukrainian National Technical University, where the themes are being implemented: «A computerized system for identifying loading ball mill grinding ore when controlling» (0109U007939), «Optimization of the productivity of ball mills for ore and finished product with minimal energy and material overruns» (0115U003942). Considering that this publication is aimed at solving part of the mentioned task, its topic should be recognized as relevant.

Analysis of the available investigations. The viscosity of pulp is one of the most important technological parameters of a ball mill. Evaluation of the viscosity of the pulp involved scientists and practitioners relatively little, because it affects many factors and maintain it at a certain level is a difficult task. Most attention was paid to the viscosity of suspensions. A large number of mathematical models have been proposed to determine the viscosity of suspensions under certain conditions with varying accuracy, for example [1 – 11].

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The most reasonable at the present time is the mathematical model of G.S. Khodakova [12], which has the form

\[
\frac{\mu}{\mu_0} = \frac{k}{1 - \left[1.5(1 - \varphi_0)^{0.5} + 1 + \Delta \varphi_0\right]},
\]

(1)

where \(k\) – parameter characterizing the waviness of the layers of the dispersion medium; \(\varphi_0\) – the actual volume concentration of dispersed phase particles; \(\Delta\) – the relative volume of the free dispersion medium; \(\mu_0\) – viscosity of the dispersion medium. In the case of a pulp, water acts as a dispersion medium.

At low concentrations of the dispersion phase \((\varphi_0 \leq 0.1)\) parameter \(k\) in (1) is equal to unity, at close packing of solid particles \((\varphi_0 \geq 0.5)\) \(k = 5\). In the concentration range of solid \(\varphi_0\) from 0.1 to 0.5, the regularity of the change in the undulation factor \(k\) of the suspensions has not been studied [13]. The second determining parameter of the dependence (1) \(\Delta\) depends on many factors – on the granulometric composition of the solid, on its packing density, on the interaction of media, on the solubility of solid particles and other. The specific value of \(\Delta\) for certain conditions is determined empirically [13]. So, now the perfect model of the viscosity of suspensions can not be applied to the operational management of the parameter.

**The Objective of the work.** Development of a mathematical model for the operational estimation of pulp viscosity in a ball mill operating in a closed cycle with a mechanical single-spiral classifier with grinding of the initial ore, based on the basic mathematical model for determining the distance between solid particles in dispersion media.

**Statement of the problem.** To achieve this goal, it is necessary to solve the following tasks:

- estimate the environment in which the basic mathematical model for determining the distance between solid particles in a mixture of ground ore-water can be adequately used;
- to study the dependence of the distance between solid particles on the parameters of the basic mathematical model obtained on the basis of the experiment;
- to study the relationship between the distance between solid particles and the viscosity of magnetite pulp;
- to obtain a mathematical model of pulp viscosity in a ball mill when grinding ore and an equation for determining its parameters;
- to check the mathematical model for the evaluation of pulp viscosity in a ball mill.

**Exposition of the main material of the study.** For the rapid assessment of pulp viscosity, a basic mathematical model for determining the distance between solid particles

\[
l = d_{CH} \left[\frac{\pi}{6} \left(\delta_T - \delta_B\right) \right] - 1,
\]

(2)

where \(d_{CH}\) – diameter of a spherical solid particle; \(\delta_T\) – density of solid; \(\delta_B\) – density of water; \(\gamma\) – pulp density.

The mathematical model (2) is derived from the considerations that the pulp, like the suspension, should be considered presented as two phases, from which the dispersion medium determines their rheological properties, and the dispersion phase is the state of the dispersion medium. This corresponds to G.S. Khodakov’s hypothesis that the viscosity of suspensions depends only on the viscosity of the dispersion medium, that is, water, but related to the effective total thickness of its free layer \(\Delta\). Here, the relative volume of the liquid that is not attached to the solid liquid \(\Delta\) depends on the concentration of \(\varphi_0\) and the size of the \(d_{CH}\) particles of the solid phase [13]. Under certain conditions, the dependence between the
parameters $\varphi_0$ and $\Delta$ is unambiguous [12], which made it possible to carry out a transition to another, more convenient parameter for estimating the viscosity, for example, the distance $l$ between solid particles, which is achieved in model (2). Consequently, it does not contradict the hypothesis of G.S. Khodakov. Taking into account the almost ideal mixing of the material in the ball mill drum, it is possible to assume that the solid particles in the two-phase medium are almost uniformly distributed. The assumption made in the derivation of the mathematical model (2) on the spherical shape and the same particle size of the solid in the pulp is a generally accepted technique for simplifying theoretical calculations and, therefore, does not distort the essence of the phenomenon that is being considered. The same method was used in [12]. In a ball mill the pulp is represented by a number of classes of coarseness, however in the concentrating industry the solid is often characterized by a weighted average size, which allows us to consider it as a medium with the same solid size in the first approximation. This constant size in different situations can take some or other constant values.

The distance $l$ between solid particles in (2) is a measure of the viscosity of the pulp, since the larger it is, the greater the relative volume of the non-bonded solid dispersion medium (water) that determines the viscosity of the pulp [12]. The value of $l$, on the other hand, is affected by the concentration of solid (pulp density $\gamma$ at a certain density of solid $\delta_r$) and its size $d_{CH}$. These are direct parameters that directly affect the distance between solid particles. However, the viscosity of the pulp is affected by other factors, such as the use of surfactants, the presence of soluble clay substances, finely divided ore particles-sludges, the formation of aggregates of solid particles, inter-aggregate interaction, the presence of chemicals, mechanical effects and other. When grinding the initial ore in the first stages, no surfactants are used. Since the ores are crystalline, clay-like water-soluble substances are not created in mills and chemical substances are not released. Therefore, these factors do not influence the viscosity of the pulp.

The concrete deposit of poor iron ores contains several technological types of ores that differ significantly in properties. Now it is proved expedient and possible to provide separate grinding of technological types of ores in the conditions of a particular deposit [14]. Under these conditions, the properties of the processed raw materials are maintained unchanged, and, consequently, the properties of the pulp. Here it is connected with a solid, the proportion of water will have a constant thickness and practically will not affect the interlayer of the «free» liquid between the solid, and as a consequence, the viscosity of the pulp. In the conditions of grinding one type of ore under a certain state of the parameters of a ball mill, the shape of the ore grains will not change, which will not affect the viscosity of the pulp. The fine particles of the crystalline structure that are created by grinding the ore do not exert a strong influence on the viscosity of the pulp [15]. They tend to create in the pulp aggregates, the strength and structure of which is determined by the physico-chemical properties of solid particles. In the conditions of a ball mill, such aggregates are not considered by the mathematical model (2), they can not exist, because they are destroyed by mechanical influences, vibration and the actions of moving balls, which together provide a uniform arrangement of the solid. Thus, the mathematical model (2) ensures the adequacy of the model and the real process with respect to the distance $l$ between solid particles. However, taking into account the influence of crystalline slurries, the characteristics of the size of the solid on the viscosity of the pulp, the basic mathematical model (2) in practical use must be «adapted» to the specific technological conditions and the material that is ground.
The approach of «adaptation» of the basic automatic model (2) to specific technological conditions will be considered with the example of magnetite with the density of 5000 kg/m³ of various sizes. The dependences of the viscosity of magnetite pulp on its density at different fineness are shown experimentally in Fig. 1. It can be seen from Fig. 1 that the viscosity of the pulp depends strongly on its density and the size of the solid. At any size of the solid, a situation occurs when a slight increase in pulp density leads to a rapid increase in its viscosity. Consider the effect on this effect of the size and density of the pulp, using their experimental values and the basic mathematical model for determining the distance between solid particles (2), considering them to be spherical with the same size.

The dependence of the distance between the particles of the solid density of magnetite pulp for different average particle sizes is shown in Fig. 2. From Fig. 2 that the distance between solid particles, assuming them to be spherical in the same size, decreases in a non-linear dependence in real pulp with increasing pulp density. With the same value of pulp density, the distance between solid particles is determined by their size. Large particles of solid in the pulp are removed from each other for a greater distance, which remains significant even at significant densities of the medium. Small solid particles are at much smaller distances, which become small at high pulp densities. The dependence of the distance between solid particles on their diameter at different densities, obtained from the experimental data using the dependence 2, is shown in Fig. 3. It can be seen from this that the distance between solid particles does not remain constant when their size changes. That is, the larger the solid size, the greater the distance between its particles. The sensitivity of the distance between solid particles decreases with increasing pulp density. That is, in dense pulps to influence the distance between particles of solid is much more difficult compared to sparse dispersion medium. So, the state of the pulp is characterized quite effectively by the intermediate parameter \( l \) – the distance between solid particles, which in turn is determined by technological parameters – pulp density \( \gamma \), solid size \( d_{CH} \) and its \( \delta_T \) density. However, the parameter \( l \) of the pulp must be «linked» to its viscosity.
Figure 2. Dependences of the distance between particles of solid density of magnetite pulp at different average magnite size: 1 – \(d_{CH} = 16 \, \mu m\); 2 – \(d_{CH} = 26 \, \mu m\); 3 – \(d_{CH} = 38 \, \mu m\); 4 – \(d_{CH} = 52 \, \mu m\)

Figure 3. Dependences of the distance between solid particles and their diameter for different pulp densities: 1 – \(\gamma = 1200 \, kg/m^3\); 2 – \(\gamma = 1400 \, kg/m^3\); 3 – \(\gamma = 1600 \, kg/m^3\); 4 – \(\gamma = 1800 \, kg/m^3\); 5 – \(\gamma = 2000 \, kg/m^3\); 6 – \(\gamma = 2200 \, kg/m^3\)

Using the basic mathematical model (2) and the experimentally obtained data (Fig. 1), we find the relationship between the viscosity of magnetite pulp and the distance between solid particles, which is shown in Fig. 4 in the form of graphs. It can be seen from the graphs in Fig. 4 that they are of the same type, but essentially depend on the size of the solid. Increasing the size of the solid reduces their bulge, which ensures that the pulp viscosity is much higher at certain values of the distance between the solid particles. Thus, there is a functional dependence between the viscosity of magnetite pulp and the distance between solid particles in it at certain sizes.

Figure 4. Dependences of the viscosity of magnetite pulp on the distance between solid particles at different sizes: 1 – \(d_{CH} = 16 \, \mu m\); 2 – \(d_{CH} = 26 \, \mu m\); 3 – \(d_{CH} = 38 \, \mu m\); 4 – \(d_{CH} = 52 \, \mu m\)

The results of computer approximation of the experimentally obtained dependencies in the MatLAB software package are shown in Fig. 5, from which it can be seen that the
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Experimental dependencies are described with high accuracy by analytic expressions. Depending on the size of the solid, they can be represented as a group of equations

\[ \mu(l) = 0.2526l^{-3.847} + 120.9 \times 10^{-6} \text{ N·s/m}^2 \text{ at } d_{CH} = 16 \mu m, \]  
\[ \mu(l) = 0.2190l^{-3.560} + 153 \times 10^{-6} \text{ N·s/m}^2 \text{ at } d_{CH} = 26 \mu m, \]  
\[ \mu(l) = 0.1888l^{-3.209} + 106.6 \times 10^{-6} \text{ N·s/m}^2 \text{ at } d_{CH} = 38 \mu m, \]  
\[ \mu(l) = 0.1717l^{-2.834} + 137.2 \times 10^{-6} \text{ N·s/m}^2 \text{ at } d_{CH} = 52 \mu m, \]

where \( \mu(l) \) – viscosity of magnetite pulp, which depends on the value of the distance between particles of solid \( l \).

It can be seen from equations (3) – (6) that they can be given in a general form

\[ \mu(l) = A \cdot l^{-B} + C, \]

where \( A, B, C \) – the parameters of the equation that depend on the diameter of the spherical solid particles.

**Figure 5.** Results of computer approximation of the dependence of the viscosity of magnetite pulp on the distance between solid particles at their size: 16 \( \mu m \) (a), 26 \( \mu m \) (b), 38 \( \mu m \) (c), 52 \( \mu m \) (d) \( x \) – this \( l \mu m; y \) – this \( \mu(l) \) \( \text{N·s/m}^2 \).
Figure 6. Dependence of parameters of the model of viscosity change of magnetite pulp on the size of solid: a – \( A \); b – \( B \); c – \( C \)

Equation (7) is a mathematical model of the operational estimation of pulp viscosity in a ball mill, which allows the «basic» model (2) to be adapted to the technological conditions for grinding ore of a certain type. It actually translates the parameter \( l \) into the viscosity of the pulp, taking into account its features. Specific values of \( A \), \( B \) and \( C \), in addition, take into account the properties of the pulp, which are not described by equation (2) of the basic mathematical model. In each case, the pulp must be determined experimentally.

Equations (3) – (6) show that there is a relationship between the values of their parameters and the size of the solid. Dependences of \( A \), \( B \) and \( C \) on the particle size of solid particles, obtained on the basis of equations (3) – (6), are shown in Fig. 6. The graphs of functions and equations of approximation of experimental data obtained with the help of a personal computer in the Microsoft Excel environment are also shown here. Equations have the following form

\[
A = 51 \cdot 10^{-6} d_{CH}^2 - 58,04 \cdot 10^{-4} d_{CH} + 0,335832 , \, \text{N} \cdot \text{s/m}^2 , \tag{8}
\]

\[
B = -2,82 \cdot 10^{-2} d_{CH} + 4,2936 , \tag{9}
\]
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\[ C = -0.11 \cdot 10^{-6} d_{CH}^2 + 8.12 \cdot 10^{-6} d_{cu} + 19.25 \cdot 10^{-6}, \text{ N\cdot s/m}^2. \] (10)

They allow you to «adapt» the basic mathematical model of determining the viscosity of the pulp to process conditions and material. For example, for a given density of magnetite pulp, \( \gamma = 2300 \text{ kg/m}^3 \), the solid is ground to a size \( d_{CH} = 45 \mu m \). It is necessary to determine the viscosity of the pulp.

We find the distance \( l \) between the particles of the solid with respect to the basic mathematical model (2). It will be at \( \gamma = 2300 \text{ kg/m}^3 \) and \( \delta_T = 5000 \text{ kg/m}^3 \) is equal to \( l = 7.74 \mu m \). We make use of the complete mathematical model of the viscosity of the pulp (7), for which we use the equations (8) – (10) to determine its parameters. At \( d_{CH} = 45 \cdot 10^6 \text{ m} \) the parameters are equal to \( A = 0.177 \text{ N\cdot s/m}^2, B = 3.025, C = 0.00016 \text{ N\cdot s/m}^2 \). By the equation (7) we determine the viscosity of magnetite pulp. It will be equal to \( \mu(l) = 0.000525 \text{ N\cdot s/m}^2 \).

When controlling the viscosity of the pulp in a ball mill, it is set to grind a particular technological variety of ore, so its parameters will not change. The ball mill will have the optimal ball load in terms of the volume and structure of the balls size, which are automatically maintained. The ratio of ore/water or pulp density is set at the mill inlet and is automatically maintained. Under such conditions, for a certain weight-average particle size, its solidity and grain shape are quite definite, the viscosity properties are unchanged. With such material, within the limits of a possible change in the weighted average particle size, it is necessary to carry out an experiment with obtaining results similar to those shown in Fig. 1. These results will be sustainable and will not change in the process of work. Further, by computer approximation, we obtain the equation for the dependence of the viscosity of the pulp on the distance between solid particles at different grain sizes, and by a second approximation, the dependences of the parameters \( A, B \) and \( C \) of the weighted average particle size of the solid. From the equations obtained for a certain solidity \( dh \), which can be measured or determined, we «adapt» the basic mathematical model (2) to the measurement conditions, determining, at the achieved \( d_{CH} \), the operational value of the viscosity of the pulp. Thus, it is possible to quickly assess the viscosity of the pulp in the process of grinding the ore by ball mills operating in a closed cycle with a mechanical single-spiral classifier, according to the previously established parameters of the mathematical model.

Conclusions. So, in the work the problem of finding a mathematical model for the operative estimation of pulp viscosity during ore grinding in ball mills operating in a closed cycle with a mechanical single-spiral classifier has been solved.

The scientific novelty of the obtained results consists in the fact that for the first time a mathematical model of the operative estimation of pulp viscosity was obtained for grinding ore in ball mills of a closed cycle in the form of an algebraic equation, including the known weighted average particle size, its density and pulp density and pre-established parameters characteristic for the crushed product and dependent on the average size of the solid, which allows improving the operational characteristics of the process unit.

The practical significance of the results obtained is that the proposed mathematical model by rapid assessment of the viscosity of the pulp in a ball mill can reduce the energy, ball and lining overruns when grinding ore in the first stages of iron ore beneficiation plants.

The prospect of further research in this direction is the development of approaches to the effective determination of the parameters of a mathematical model for different technological types of ores of a particular deposit.
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Список використаної літератури
МАТЕМАТИЧНА МОДЕЛЬ ОПЕРАТИВНОГО ОЦІНЮВАННЯ В’ЯЗКОСТІ ПУЛЬПИ У КУЛЬОВОМУ МЛИНІ ПРИ ПОДРІБНЕННІ РУДИ

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Резюме. Вітчизняний залізорудний концентрат втрачає в міжнародній конкуренції тим, що має більшу собівартість у наслідок підвищених витрат електроенергії й матеріалів при подрібненні руди, що значною мірою визначається відсутністю засобів оперативного оцінювання в’язкості пульпи у кульовому млині першої стадії. Розроблено математичну модель оперативного оцінювання в’язкості пульпи у кульовому млині, що працює в замкненому циклі з механічним односпіральним класифікатором. Пульпа у кульовому млині, особливо при подрібненні конкретного технологічного різнотипу руди створює ідеальні умови для оперативного оцінювання її в’язкості. Модель використовує параметр, що характеризує відстань між частками твердого, яка залежить від його густини, крупністі й густини пульпи. Потім в’язкість пульпи, що працює в замкненому циклі з механічним односпіральним класифікатором, визначається на основі інтегрування функції μ = f(γ), де μ – в’язкість пульпи, γ – густина пульпи при різних крупностях твердого за відомою або виміряною крупністю твердого.

Уперше розв’язано задачу знаходження математичної моделі для оперативного оцінювання в’язкості пульпи при подрібненні руди в кульових млинах. Перспективою подальших досліджень у даному напрямі є розроблення підходів ефективного визначення параметрів математичної моделі для різних технологічних різнотипів руд конкретного родовища.

Ключові слова: кульовий млин, руда, в’язкість пульпи, оперативне оцінювання, математична модель, параметри.

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