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# NUMERICAL PROCEDURE BASED ON BASIS AND CORRECTION SOLUTIONS FOR AXIAL STRESS CALCULATION IN PIPELINES PASSING THROUGH ZONES OF MINE SUBSIDENCE

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***Summary.*** *Mine subsidence can pose a considerable threat to pipeline integrity. There are three constituents which quantitatively determine the distribution of strains along the pipeline – the function of ground displacement along the pipeline axis; the physical law of soil-pipe interaction due to their relative displacements; and the pipe wall deformation response to axial stress. All three of them are usually well understood but there are still a small number of successful examples of prediction of stresses in such pipelines due to lack of effective algorithms of their accounting for. So here we develop the effective procedure for axial strain and displacement calculation based on notions of basic and correction solutions. The basic solution is algebraically corrected after each iteration step for correction solution, which obtained by numerically efficient transfer matrix method. The role of basic one is very narrow here: first it determines the particular type of law of soil-pipe interaction; second, the resulting solution is considered to be correct when basic and correction solutions coincide. The effectiveness of the algorithm application is shown on number of real examples.*

***Key words:*** *main pipeline, mining production, ground movement, axial stresses.*

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**Introduction.** Ground displacements resulting from landslides and subsidence can pose a threat to pipeline integrity and therefore require a considerable attention from pipeline operators. Three key topics need to be accounted for when assessing and mitigating their severity [1]:

− Identifying areas where landslide or subsidence hazards are possible, assess their scope and intensity with respect to environmental conditions.

− Identifying experimental and numerical approaches for assessing pipeline response to expected ground movements.

− Identifying appropriate risk mitigation measures and evaluating their effectiveness.

The most common case is deformation of ground in form of strike-slip and normal faults, and a lot of analytical and experimental works are devoted to their analysis [2]. The relative transverse displacement between pipeline and ground leads to the soil-pipeline interaction in transverse direction thus inducing the bending stress in pipe. The large transverse displacement can increase the length of deformed curved line of pipeline as comparing with initial straight line and may lead to secondary geometrically nonlinear tension effect (cable-like behavior of pipeline) and soil-pipeline interaction in longitudinal direction have to be considered too [3, 4].

Mine subsidence poses a special kind of geohazard for pipelines. As it stated in [1] «in terms of land area affected, underground mining accounts for about 20 percent of the total land subsidence in the United States, and most of this fraction is associated with underground mining for coal» [1, p. 67]. In Ukraine, for example, there are at least 12 transit gas pipeline sections suffering from mine subsidence influence, spreading from one to several kilometers. The main peculiarities of the ground deformation and their effect on pipelines are very clear described in ASCE document [5, p. 43]. The vertical displacement of ground prevails, but their effect on pipeline usually is negligible. More serious threat to pipeline arises from horizontal (in pipeline axial direction) soil displacement. The similar emphasis was made in book [6].

These displacements are usually very good understood in practice, supported by a big statistic of real observations. There exist normative documents, established practice and computer programs [7 – 10], which in most cases give reliable prediction of soil deformations and their development with time depending on geometrical characteristic of mine seam and the mechanical properties of ground. The relative displacements between the initially coinciding points of pipe and soil determine the value of soil resistance, which again is very well understood and is given in various standards, including such subtle effects as the trench geometry, humidity of soil, etc [5, 11].

Understanding of soil deformation, characteristics of soil resistance together with laws of pipe wall deformation due to axial forces, in principle, can provide a scientific basis for accurate prediction of pipeline deformation. Nevertheless, there are very few examples in literature [12] where calculated stresses due to mine subsidence were experimentally verified. Furthermore, in recently published paper [13] on prediction and monitoring the stresses in mine subsidence area the quite different pessimistic conclusions were made. It was stated that: «Pipe stresses should be monitoring using strain gauges or other means. A stress analysis may not predict pipe stresses in a subsidence zone with adequate accuracy».

At first glance, the direct measurement seems more preferable as compared with calculation. But there are still a few very serious arguments in favor of calculation. First, in any case the calculations are needed to understand, in principle, the severity of mine influence, the zones and places of gages installation [13]. Second, to our experience, the soil resistance can induce stresses ranged from *3 – 4 MPa* per meter for pipe of *6 mm* thickness up to *5 – 6 MPa/m* for *4 mm* thickness. So, with *50 meters* distance between gages, the stresses can be obtained with *25*6 = 150 MPa* accuracy, which is can hardly to be accepted; but the reduction of distance may be too expensive. Third, the results of monitoring can not, in principle, forecast the rate and sigh of stress development and it is hard to establish the stress threshold at which the measures should be implemented (but how fast and in what extent is also unclear), while the theoretical analysis can be able to formulate the mitigation strategy long before the time of coal extraction.

So the goal of this work is creation of effective (fast and accurate) algorithm and procedure for stress calculation for monotonic change of axial displacement.

The most difficulty in predicting the pipeline deformation is strong nonlinearity of physical law of soil-pipeline interaction. Usually the distributed force of interaction is presented as three parts piece-wise function of relative displacement of ground, $u_g$, with pipeline $u_p$, Fig. 4. The most problem in solution that we do not know in advance which state of interaction (I, II or III-d) is realized in given point.

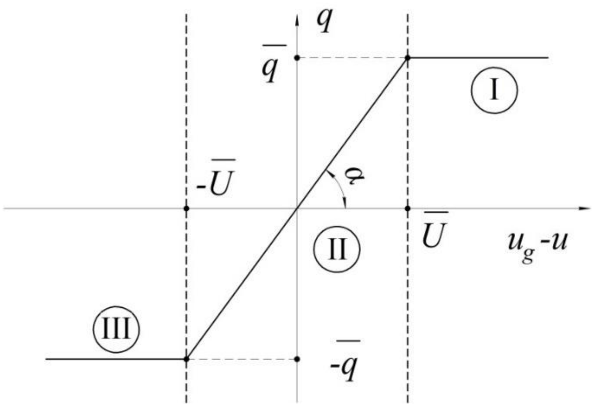



**Figure 4.** Typical 3 states law of pipe-soil interaction

The analytical methods can be very effective if we know the general patterns (usually simplest one) of ground deformation and can guess the general pattern of distribution of states of deformation (I, II or III) along the pipeline. The simplest example is the pipeline which is in zone of unstable slope, where usually the sequence of states is well understood. Then, from equilibrium conditions the boundaries between different states of deformation can be established, and general solution can be obtained [14]. Further development of this analytical model was proposed in [15, 16], where the plastic deformation of pipeline were additionally taken into account.

The numerical methods, of course, are more popular in literature, because they are not restricted to a typical ground displacement pattern. The solution is achieved by dividing the pipeline on a big number of elementary sections, where the distributed forces of interaction are the same, and applying the step by step incremental procedure for applied ground displacement. At each step the incremental pipe displacement is determined for the adopted here tangent stiffness for each pipeline section. The tangent stiffness of the soil-pipe interface is then updated as a function of the soil-pipe relative displacement [17, 18].

The general drawback of incremental procedures for highly nonlinear law of interactions (like one on Fig. 4) is that to converge they may require a prohibitory large number of iterations [19]. So, in this work we suggest the original iteration method for approaching the correct solution based on three distinct features.

The notions of Basic and Correction solutions are used [20]. No incremental procedure for forces or for tangent stiffness is envisaged. The correction solution is always calculated for ultimate system of loading. The basic solution on each iteration, $i$, is corrected based on difference between the corrected solution and previous basic one multiplied on the dynamic coefficient of motion $dk$.

The coefficient $dk$ is adjusted at each iteration based on value and sign of maximal difference between basic and correction solutions.

The general elementary solution for each small piece of pipeline is given in form convenient for transfer matrix method, TMM, application, which significantly reduce the number of unknowns (ideally, for any long pipeline we have only matrix with 4 independent unknowns). Other our examples of TMM applications for various 1D tasks are given in [21, 22].

## 1. Numerical Procedure based on Basic and Correction Solutions

### *1.1. Governing equations and their solution*

Consider only the case of elastic deformation of pipeline. This is explained by requirement of Norms [23] which are now active in Ukraine. Besides it will be not difficult to enhance the procedure with accounting for the nonlinear dependence between the stresses and strains. For straight pipeline the system of governing equations is trivial one:

$$\frac{dN(x)}{dx} = q_t(x); \qquad \frac{du}{dx} = -\frac{N(x)}{EF}, \tag{1}$$

where $N$ is axial force in pipeline and $u$ is axial displacement of points of pipeline both directed along the $x$ axis of pipeline, $E$ is elasticity module, $F$ is cross sectional area $F \approx 2\pi Rt$, where $R$ is the radius, $t$ is wall thickness. The distributed force $q_t(x)$, according to physical law shown on Fig. 4, depends on state of interaction and relative displacement $\overline{U}$

$$\overline{U} = u_g - u \tag{2}$$

where $u$ and $u_g$ – are the pipeline and ground displacements, accordingly. So, for three states of interaction we have

$$q_t(x) = \pm \pi D_н c_{X0} \overline{U} = const \tag{3 a}$$

for I-st and III-rd (plastic) states of interaction, where $D_н$ is outer diameter and $c_{X0};\overline{U}$ constants widely given in various documents [5, 11], and sigh «+» is for state I, and «–» is for state III. And

$$q_t(x) = \pi D_н c_X \overline{U} \tag{3 b}$$

for II-nd (elastic) state, where $c_X$ is another constant. The system (1) and (3) allows the very easy solution, which is widely presented in literature. Suppose the slightly complicated form of ground displacement on II state of interaction

$$u_g(x) = \mu_0 + \mu_1 x \tag{4}$$

We write the general solution for pipeline displacement in form convenient for TMM application

$$u(x) = u_0 ch(\beta x) - \frac{N_0}{\beta EF} sh(\beta x) + \mu_0 (1 - ch(\beta x)) + \mu_1 \left( x - \frac{1}{\beta} sh(\beta x) \right) \tag{5 a}$$

$$N(x) = -u_0 EF \beta sh(\beta x) + N_0 ch(\beta x) + \mu_0 EF \beta sh(\beta x) + \mu_1 EF (ch(\beta x) - 1) \tag{5 b}$$

for II state, where

$$\beta = \sqrt{\frac{\pi D_н C_x}{EF}} . \tag{5 c}$$

And

$$u(x) = u_0 - \frac{N_0}{EF} x - \left( \pm \frac{\overline{q} x^2}{2EF} \right); \qquad N(x) = N_0 \pm \overline{q} x. \tag{6}$$

for state III the «+» sign is taken and sigh «–» is for I one. The general solutions should be supplemented by boundary conditions on Left (L) side and Right (R) side of pipeline [see, for example, 20], for case of fully elastic interaction beyond the chosen analyzed pipeline (this means that pipeline section has to be long enough to provide it)

$$N_L = -u_L \beta EF , \quad N_R = u_R \beta EF . \tag{7}$$

*1.2. Algorithm*

Algorithm at the given stage of history is main novelty of the paper; it is based on notions of basic and correction solutions and consists in following:

1. The soil displacement is given for each point, $u_s$ .

2. Pipeline is broken on N small pieces and the notion of basic displacements are introduce for each point on each i-iteration $\overline{u_{\text{b, i}}}$. Before calculation we take that $\overline{u_{\text{b,0}}} = u_s$.

3. State of interaction is chosen based on given basic sdisplacement of pipe on previous iteration and the ultimate characteristic on interaction (Fig. 4).

4. Based on interaction type we writh the correct solution for differential equations which rtelate displacement and forces at beginning and at the end of pipe pieces. Boundary conditions are also taken into account.

5. Thus we obtain the matrix, the solution of which gives us the complete correction solution.

6. Then we perform adjustment of basic solution on i-iteration taking into account the completete displacements $\overline{u_{\text{k, i}}}$ on this i-iteration step

$$\overline{u_{\text{b, i}}} = \overline{u_{\text{b, i-1}}} + k\left(\overline{u_{\text{k, i}}} - \overline{u_{\text{b, i-1}}}\right), \tag{8}$$

a) where $k$ is dynamic coefficient of movement which is satisfied for following condition:

$$A = k \cdot \max \forall x \left\{\overline{u_{\text{p, i}}(x)} - \overline{u_{\text{b, i-1}}(x)}\right\} < 3\overline{U}; \tag{9}$$

b) if the sign of extremum on previous iteration has been changed then we decrease it in two times

$$A = \frac{A}{2}; \tag{10}$$

c) if the sign did not changed then we can encrease again this value A, but not more than in 1.3 times, so

$$A = 1.3A. \tag{11}$$

The restriction $A < 3\overline{U}$ should be kept.

**2. Calculation and measurement of stresses on real pipeline**

Consider the effect of axial soil displacements at mine production on the stress state of a real gas pipeline. In Fig. 1 shows the relative location of the coal seam, its zone of influence and the gas pipeline DN 300. To model the axial deformation of pipeline we take the following physical characteristics of soil (clay loam) and pipe diameter present in Table 1. In Fig. 2 in graphical form the predicted values of axial displacement of soil as a result of mine production, which are taken as input data for calculation, are presented.

**Table 1**

Physical characteristics of soil (clay loam) and pipe diameter

| Parameter name | Value |
|---|---|
| Specific weight | $\gamma_{гр} = 17000\ N/m^3$ |
| Specific cohesion of soil | $c_{soil} = 28kPa$ |
| Angle of internal friction | $\varphi_{soil} = 22°\ \deg ree$ |
| Outer diameter of piper | $D_{out} = 0.325m$ |
| Generalized coefficient of tangential resistance of soil | $C_x = 3MPa/m$ |

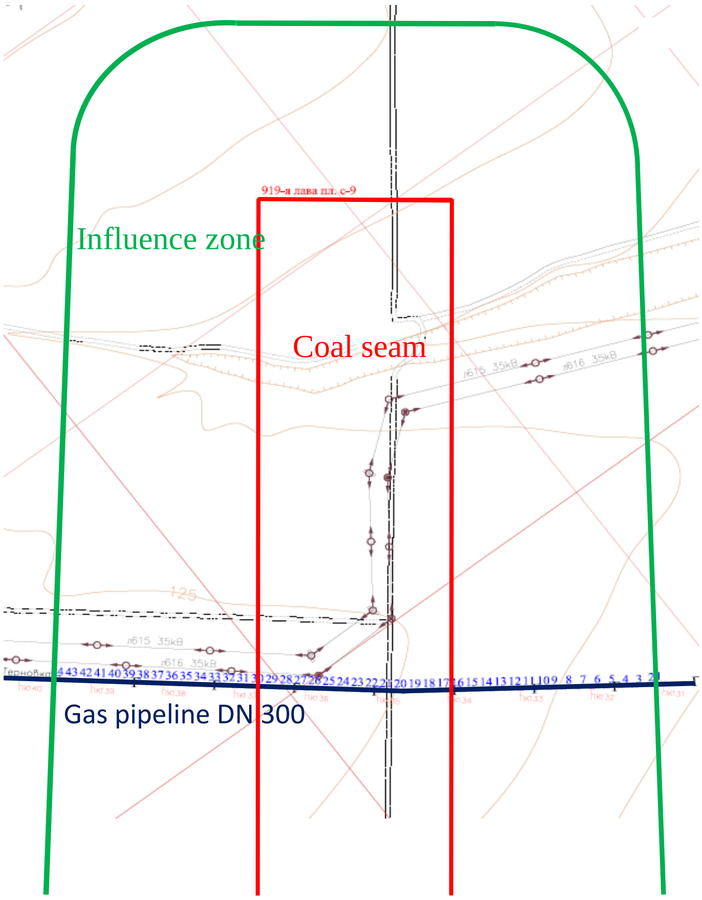



**Figure 1.** The location of the gas pipeline in zone of influence of mine production

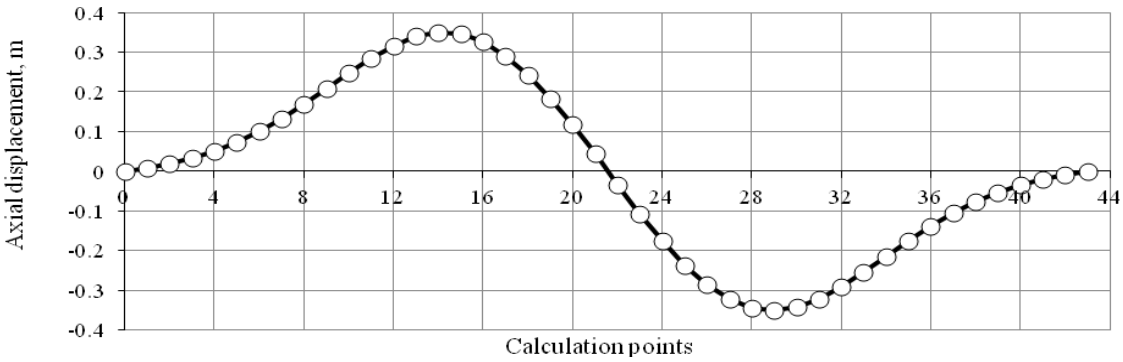



**Figure 2.** Input data for calculation

In Fig. 3 shows the results of the calculation of the initial axial displacements of the pipeline (bold line). Initial stress distribution is presented in Fig. 4 (bold line). According to [5, 6, 23] for this pipeline DN 300 with a wall thickness of *6 mm* (yield strength *300 MPa*, strength limit *470 MPa*), the permissible tensile stresses are *210 MPa*, compression is *300 MPa*. In this way, we have exceeded the actual tensions of their permissible values (see bold line in Fig. 4). To reduce the high tension, measures have been proposed that reduce the forces of interaction between the pipeline and the ground. Thus, in the places of maximum interaction power a gas pipeline was excavated with subsequent filling of excavated sites. Scheme of excavation is presented in Fig 5. In this case, the characteristics of the ground of backflood are taken into account [24].

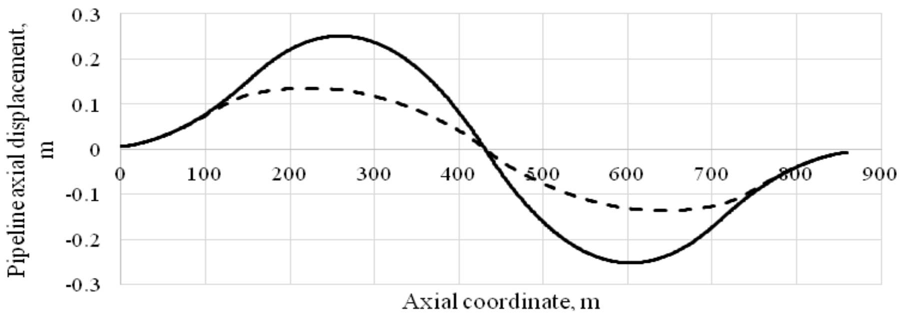



**Figure 3.** Results of calculations of gas pipeline displacement

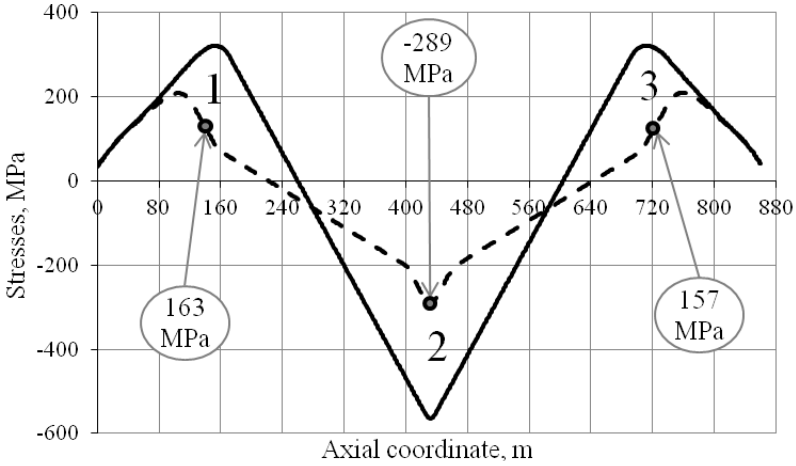



**Figure 4.** The results of calculation: bold line – for axial pristine soil, dotted line – for real disturbed soil. Circles show the maximal measured stress in Points 1, 2 and 3

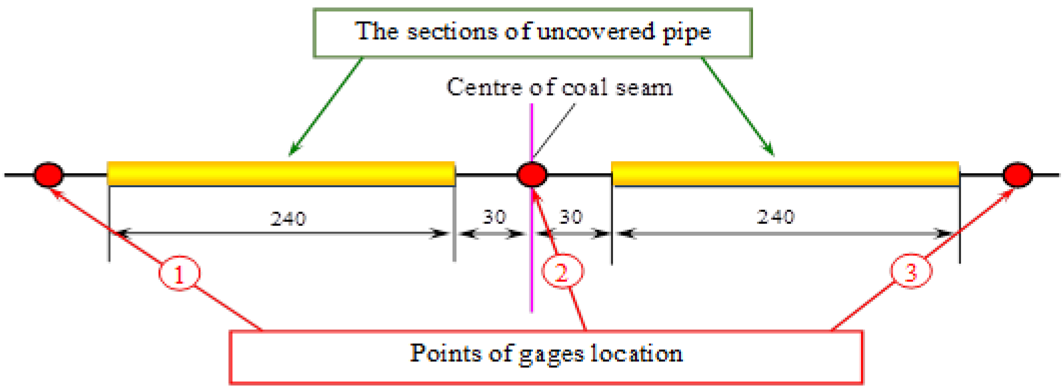
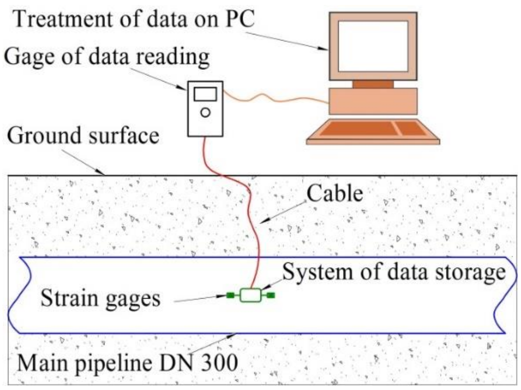
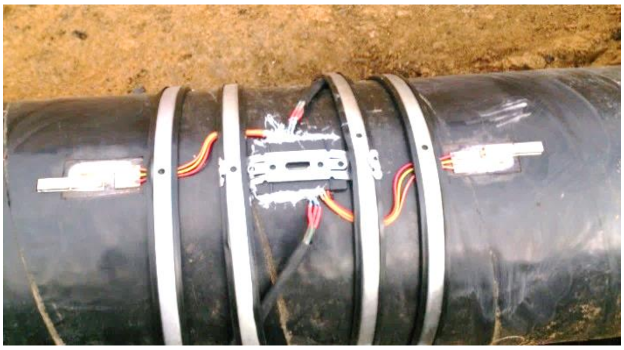



**Figure 5.** The general scheme of the mitigation measures and the gages placement

In Fig. 3 dotted line represents the estimated displacement of the pipeline. Distribution of stresses after implementation of measures with backflow is presented in Fig. 4 (dotted line).

To verify the calculation procedure and to assess the mitigation effectiveness for expected high stresses, the system of stress monitoring was developed and implemented in few points of pipeline section. The strain gage configurations are based on the concept of a Wheatstone bridge. The principle of strain measuring, storage and transmission as well as the general view of system are presented on Fig. 6. With aim of preventing of water admission, after the system mounting on pipe surface, it is isolated by polyurethane sealant. Before the eventual filling out the pit by soil, the system is additionally wrapped by dense film, after what the ends of cables of data transmission are provided to the ground surface.



a) the general scheme of system b) strain gages

**Figure 6.** The system of strain monitoring

The strain gages were placed where the highest values of stresses with accounting for the mitigation measures are predicted. The latter ones suppose performing the excavations of two relatively long pipeline sections (each of *240 m*) at summer. These sections were subsequently filled out by more light soil to allow the safe operation of pipeline during the winter months. The scheme of placement of strain monitoring system is shown on Fig. 5. So, two gages were located in zones of maximal tension stresses and one – in the zone of compressive stresses which coincides with the center of mine.

The active soil movement process in the zone of pipeline (for perpendicular coal extraction direction) is starting off when the tip of mine approaches to *20 – 10 meters* near the pipeline axis. According to the overage rate of mine extraction process (approximately *5 – 6 meters* per day) the active process of soil subsidence lasts for 2,5 – 3 months. During this time the soil displacements acquire 80 – 90% from the maximum values, and lately the process of soil movement proceeds far more slowly. So, the gages were placed and switched on accordingly to the stages of soil movement process.

The results of strains (they are recalculated into elastic stresses) measurements with time within 200 days period and with 2 hours interval for all three gages are presented on Fig. 7. The results of stress monitoring reflect the general tendency of soil subsidence process. The asymmetry in results for gages 1 and 3 is theoretically unclear but in practice it can easily be explained by small differences in local properties, distances, surface relief, etc. For all three gages the stresses attain their extremal values within 100 days and lately, instead of slowly increasing due to continuation of soil subsidence process, begin to decrease slowly. We can attribute this to relaxation of soil-pipe force of interaction with time, which requires additional experimental and theoretical study.

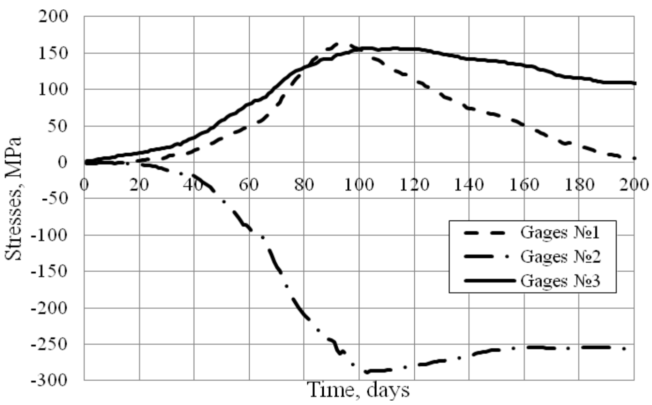



**Figure 7.** The results of stress (strain) monitoring for three gages

The results for (elastic) stress calculation for maximal predicted soil displacement are given on Fig. 4. Bold line shows the predicted stress for case of absence of any mitigation measures (for pristine soil), while dotted line gives the distribution of calculated stresses for real case of loosened soil (excavated and then filled in) at some particular sections of pipeline (Fig. 5). For comparison here are presented the maximal stresses in points 1, 2 and 3 taken from graphs of Fig. 7. In general, we can state the good correspondence of measured and calculated stresses.

**Conclusions.** The effective algorithm for axial strain and displacement calculation based on notions of basic and correction solutions is proposed. The basic solution is algebraically corrected after each iteration step for correction solution proportionally to dynamic coefficient of motion, the value of which decrease or may grow depending on divergence or convergence of results.

For cases whem loading grows proportionally (simple loading), the calculations are able to predict the stresses very fast and accurately. The latter is confirmed by experimentally observed results for real mine subsidence.

The results of monitoring reveal the practical significance of relaxation of soil-pipe force of interaction with time, the phenomenon which requires additional experimental and theoretical study.

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## ЧИСЕЛЬНА ПРОЦЕДУРА НА ОСНОВІ БАЗОВИХ ТА КОРЕКЦІЙНИХ РІШЕНЬ ДЛЯ РОЗРАХУНКУ ОСЬОВИХ НАПРУЖЕНЬ У ТРУБОПРОВОДАХ, ЩО ПРОХОДЯТЬ ЧЕРЕЗ ЗОНИ ШАХТНИХ ВИРОБОК

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***Резюме.*** *Шахтні виробки можуть бути значною загрозою цілісності трубопроводів. Існує три складові, які кількісно визначають розподіл напружень уздовж трубопроводу – функцію переміщення землі по осі трубопроводу; фізичний закон взаємодії ґрунту і труби через їх відносні зсуви; реакцію деформації стінки труби на осьові напруження. Всі три з них, як правило, добре зрозумілі, але мало існує успішних прикладів прогнозування напружень у таких трубопроводах через брак ефективних алгоритмів їх урахування. Створено ефективну процедуру розрахунку осьових деформацій та переміщень на основі понять базових та корекційних рішень. Основне рішення алгебраїчно корегується після кожного кроку ітерації для рішення корекції, що отримано методом чисельно-ефективного матричного розкладу. Ефективність застосування алгоритму показано на реальному прикладі.*

***Ключові слова:*** *магістральний трубопровід, шахтний виробіток, зсув ґрунту, осьові напруження.*

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