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Ternopil Ivan Pul'uj National Technical University

Department of Technical
Mechanics and Agricultural Machinery

THEORY OF MACHINES AND MECHANISMS

GUIDANCE BOOK
for course projecting

for full-time and part-time students of the specialty
133 «Industrial Machinery Engineering»

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Developed in accordance with the curriculum and is intended for full-time and part-time students of the specialty 133 «Industrial Machinery Engineering».

UDC 621.01

CONTENTS

INTRODUCTION

Theory of machines and mechanisms together with theoretical mechanics, strength of materials and machine parts is one of the basic general engineering subjects ensuring the necessary theoretical training of mechanical engineers.

Theory of machines and mechanisms is a study of general methods of investigating the properties of machines and mechanisms and their schemes design.

Analysis of mechanism consists in the study of its structural, kinematic and dynamic properties; the synthesis of the mechanism lies in designing a scheme of a new mechanism according to its properties.

I. GENERAL INSTRUCTIONS

Course project in Theory of Machines and Mechanisms (TMM) is practical application of received theoretical knowledge, the first independent work of students on complex designing and research of interconnected mechanisms, which are components of machines and devices.

The course project (work) covers the main parts of the course. In the course of its implementation students develop skills in the design of hinge-rod, cam, gear and other mechanisms. The project introduces the methodology for choosing and evaluating various schemes and key parameters of mechanisms alongside with their kinematic, dynamic and force calculations.

II. VOLUME AND CONTENT OF THE COURSE PROJECT

The course project consists of two sheets (course paper of one sheet) of A1 format (594 mm × 841 mm), and a calculation and explanatory note in the volume of 20 ... 25 pages of A4 format.

The course project (work) is carried out in such order:

Sheet 1. Structural and kinematic analysis of the mechanism.

Sheet 2. Force analysis of the mechanism.

The course work is done on one sheet of A1 format (594 mm ×841 mm). Calculation and explanatory note will be 15 ... 20 pages of A4 format.

Sheet 1. Structural and kinematic analysis of mechanism

1.1. Do structural analysis of a given scheme of the mechanism.
1.2. Determine the extreme positions of the driven link, to which the force of the production resistance F_{PR}. is applied. Determine the working and idle motion of the mechanism.
1.3. Having taken one of the extreme positions of the mechanism for zero (the beginning of the working course), draw *n* positions (the number of positions is given by the instructor).
1.5. Draw the velocity and acceleration plans for two chosen positions of the mechanism (working and idle motion).
1.6. Determine the angular velocities and angular accelerations of all links of the mechanism for two positions and show their directions on the kinematic scheme of the mechanism.
By the method of diagrams, study the kinematics of link point (draw the diagrams of rote, speeds and accelerations), to which the force of production resistance is applied.
1.7. Compare the numerical values of velocities and accelerations obtained by the method of plans and the method of kinematic diagrams, taking as a basis the plans of speed and acceleration. Relative error should not exceed 5%.
1.8. Calculation results tabulate in the drawing.

Sheet 2. Force analysis of mechanism

2.1. Determine the forces of weight, the forces of inertia, and the moments of inertia forces of all links for one position of the mechanism (working motion).
2.2. By the method of plans of forces, make the force calculation of the mechanism for one position (working motion).
2.3. Determine the reactions in all kinematic pairs of the mechanism. Tabulate the values of reactions in the drawing.
2.4. Determine the balancing force F_{bal} from the plan of forces.
2.5. By Zhukovsky «Firm lever» method, determine the balancing force for one position (working motion), taking into account the forces of weight, inertia and production resistance.
2.6. Compare the values of the balancing forces obtained by the method of the plans and Zhukovsky method. Relative error should not exceed 5 %.

III. REQUIREMENTS FOR EXECUTING OF COURSE WORKS (PROJECTS)

Course work (explanatory note, henceforth EN) are executed in one copy in accordance with the requirements of **DSTU 3008: 2015 «Information and documentation. Reports in the field of science and technology. Structure and rules of execution».** The language of the note is English or Ukrainian, the style is academic, clear, without spelling and syntax mistakes, the sequence is logical.

The text of the note should be written on one side of sheets of white paper of A4 format (297 mm × 210 mm) with the frame and main inscription (angular stamp, forms 2 and 2a) according to GOST 2.104-2006. The text of the note is placed with the following dimensions of the magins: from the left side – not less than 25 mm, from the right – not less than 15 mm, at the top – not less than 25 mm, from below – not less than 25 mm. The distance from the frame (corner stamp) of form 2 or 2a to the text of the text at the beginning and the end of the lines (left and right) should be not less than 5 mm, from the top of the frame – 10 mm, below the corner stamp – 10 mm.

When writing text using a computer (word processing software compatible with Word for Windows version 7.0 or later), **the general formatting recommendations must be followed:**

– main font, including headings, – Times New Roman, 14 points, normal (without bold, italic, and underscore), only black color, alignment in width;

– the main line spacing is 1.5 (without the use of any intervals before and after paragraphs and spaces of lines in the text);

– in multi-line names of items (sub-items), inscriptions under figures and table headings, within them inter-line space is 1,0;

– for inscriptions under figures and computer software 12 point font size and one line spacing are possible;

– within tables the line spacing is 1,0, font at any size (but not less than 7 points);

– within figures (illustrations) the line spacing is 1,0, font at any size (but not less than 7 points);

– paragraph space is the same throughout the text («new line») – 1,25 mm (five symbols).

Errors and graphic inaccuracies can be corrected by pasting, scratching or painting with white paint, followed by making corrected text.

Not more than two corrections per page are allowed.

Damaged, untidy pages of text documents, incompletely deleted traces of the previous text are **not allowed.**

IV. CONTENT OF CALCULATION AND EXPLANATORY NOTE

Calculation part has to include:

1. Sheet 1. Structural and kinematic analysis of mechanism

1.1. Structural analysis of the mechanism.
1.2. Determination of the mechanism's positions.
1.3. Drawing the velocity diagrams for two given positions of the mechanism.
1.4. Determination of angular velocity of links.
1.5. Drawing the acceleration diagrams for two given positions of the mechanism.
1.6. Determination of angular accelerations of the mechanism links.
1.7. Drawing kinematic diagrams.
1.8. Matching the results.

2. Sheet 2. Force analysis of mechanism

2.1. Determination of links weight forces.
2.2. Determination of inertia forces of the links.
2.3. Determination the inertia moments of links.
2.4. Determination of moments of inertia forces of links.
2.5. Reduction of forces moments of links inertia forces to inertia forces
2.6. Force calculation of group 4-5.
2.7. Force calculation of group 2-3.
2.8. Force calculation of the driving link. Determination of balancing force.
2.9. Determining the values of reactions.
2.10. Determination of balancing force by M.Ye. Zhukovsky method.
2.11. Matching the results.

V. EXAMPLE OF CARRYING OUT THE COURSE PROJECT (WORK)

SHEET 1
STRUCTURAL AND KINEMATIC ANALYSIS OF MECHANISM

1.1. Structural analysis of the mechanism

For a given mechanism (Figure 1*a*), we study the principle of its work, determine the types of movement of each link and their relative motion. Indicate the links of the mechanism with numbers, driving link of the mechanism is indicated with number 1 (Figure 1*a*).

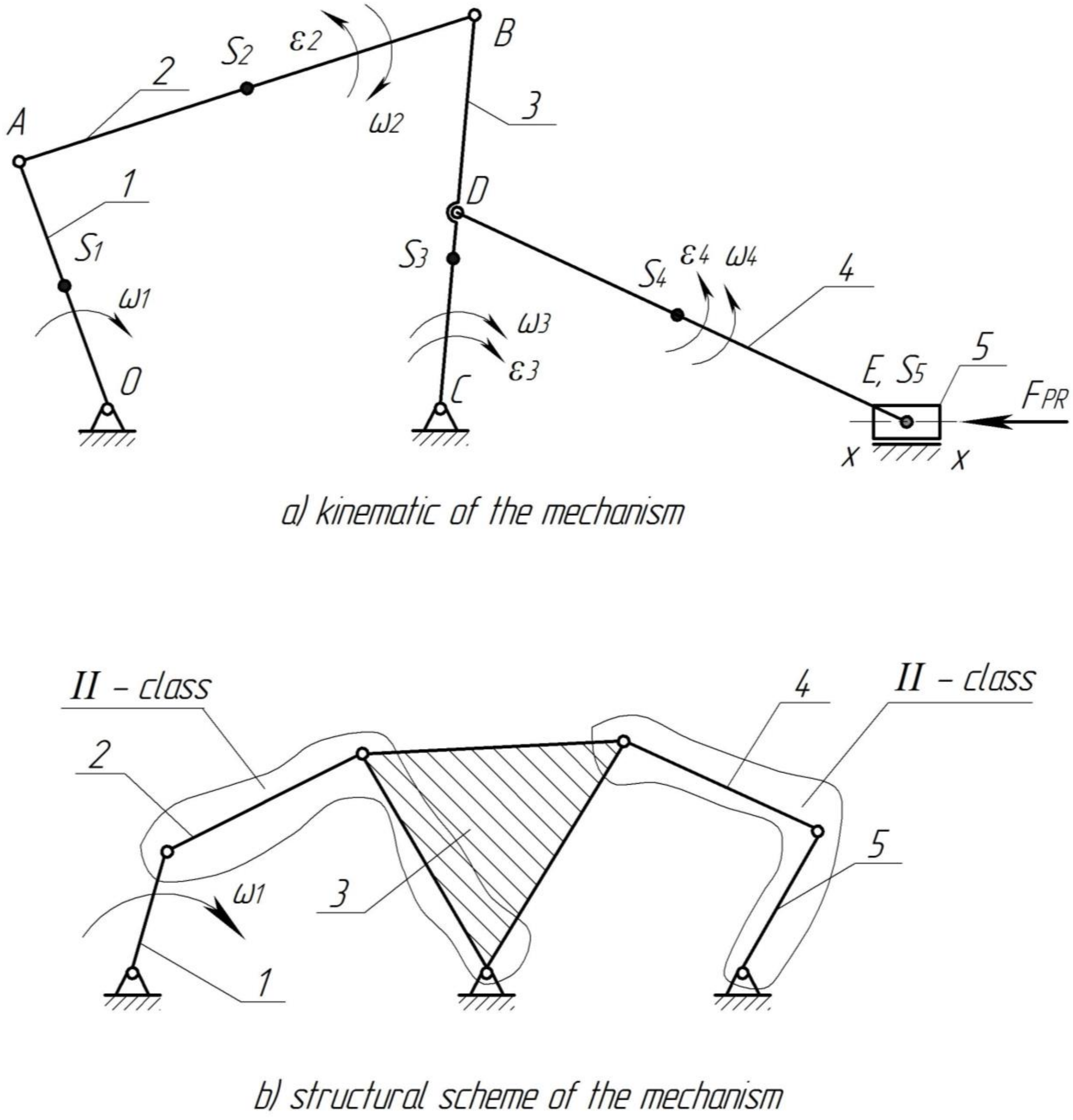


Figure 1 – Kinematic and structural scheme of the mechanism

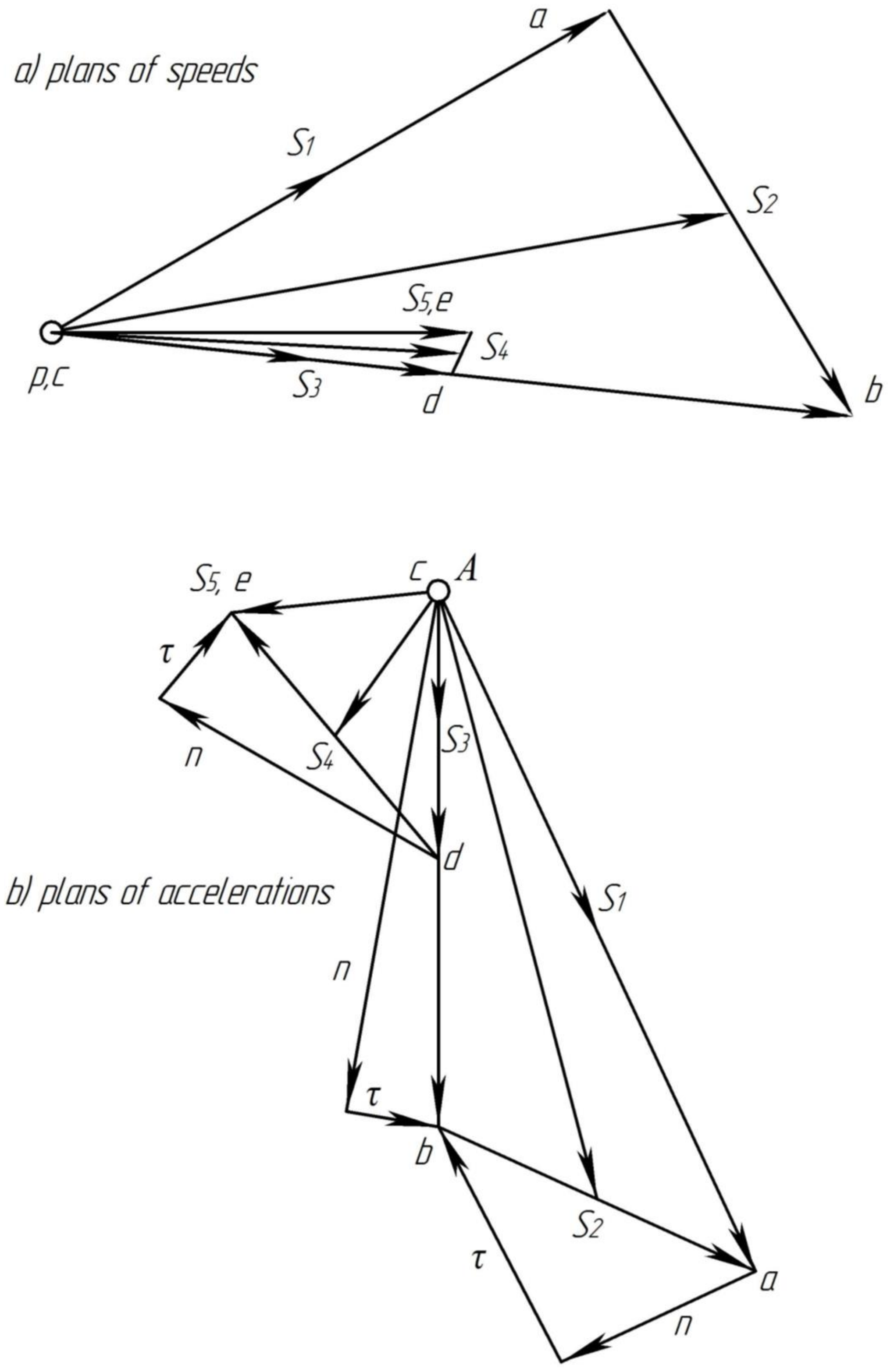


Figure 2 – Plans of speeds and accelerations for one of the mechanism positions

By the Chebyshev formula determine the degree of mobility of the mechanism

$$W = 3n - 2p_5 - p_4, \qquad (1)$$

where n – number of movable links (Figure 1*b*);
p_5 – number of fifth class kinematic pairs (Figure 1*b*);
p_4 – number of fourth class kinematic pairs.

For the given mechanism:

$$n = 5;\ p_5 = 7;\ p_4 = 0.$$

Then

$$W = 3 \cdot 5 - 2 \cdot 7 - 0 = 1.$$

Structure chart of the mechanism is drawn on Figure 1*b*.

The mechanism consists of 2 groups of Assura II classes and an initial mechanism of the I class. In general, the mechanism belongs to the second class.

1.2. Determination of the mechanism's positions

The kinematic scheme of the mechanism is performed for *n* positions (the number of positions of the crank is given by a course project supervisor). The extreme left position of the driven link is taken as a zero position (slider 5, point *E*, Figure 1*a*). To study the mechanism, we take one position from the working stroke of the mechanism, and the second – from idle.

The scale of kinematic diagram of mechanism μ_l, m/mm, is determined as relation of the link length in meters l_{OA} to the length of the interval OA in millimeters that indicates this link on the drawing:

$$\mu_l = \frac{l_{OA}}{OA}. \qquad (2)$$

1.3. Drawing the velocity diagrams

The angular velocity of the crank, s^{-1}, is determined by the formula:

$$\omega_1 = \frac{\pi \cdot n_1}{30}, \qquad (3)$$

where n_1 – frequency of the crank rotation (driving link), RPM, given to design tasks.

The velocity of point A, m/s, of the crank is determined by the formula:

$$V_A = \omega_1 \cdot l_{OA}. \tag{4}$$

Choose the scale of the velocity diagrams, μ_V, $\frac{\text{m/s}}{\text{mm}}$, by formula:

$$\mu_V = \frac{V_A}{pa}, \tag{5}$$

where pa – interval of the velocity diagram, mm, that corresponds the velocity vector of point A.

The length of the vector pa should be not less than 100 ... 150 mm. From the pole p draw the velocity vector of the point A perpendicular to the crank OA (Figure 2*a*).

The velocity of a point B is determined from the vector equations:

$$\begin{cases} \overline{V}_B = \overline{V}_A + \overline{V}_{BA}; \\ \overline{V}_B = \overline{V}_C + \overline{V}_{BC}. \end{cases}$$

A relative velocity vector $\overline{V}_{BA}$ is drawn perpendicular to the link AB from the point a. The velocity of point C is zero and on velocity diagram it coincides with the pole p. The velocity vector $\overline{V}_{BC}$ directs perpendicularly to the links BC from the point C (the intervals ab and bc on the velocity diagram, see Figure 2*a*). At the crossing of these segments we obtain point b.

Velocity of point D is determined by the similarity theorem

$$\frac{pd}{pb} = \frac{CD}{CB},$$

hence

$$pd = pb \cdot \frac{CD}{CB},$$

where pd, pb – intervals in mm from velocity diagrams;

CD, CB – intervals in mm from kinematic diagram.

Velocity of point E determine from the vector equations:

$$\begin{cases} \overline{V}_E = \overline{V}_D + \overline{V}_{ED}; \\ \overline{V}_E = \overline{V}_X + \overline{V}_{EX}. \end{cases}$$

The vector of relative velocity $\overline{V}_{ED}$ is drawn perpendicularly to the link ED from the point *d*. The velocity of the directs $x - x$ is zero, $V_X = 0$, coincides with the pole *p*. The velocity vector $\overline{V}_{EX}$ we plot parallel to the directs $x - x$ from the pole *p*. At the intersection of vectors $\overline{V}_{ED}$ and $\overline{V}_{EX}$ we obtain point *e*. The velocity of the point *E* is the vector of *pe* (see Figure 2*a*).

The velocity of the links' points of the masses centers S_1, S_2, S_3, S_4 and point D are determined by the method of geometric similarity.

The velocity diagram is shown on figure 2*a*.

The values of absolute and relative velocity of the mechanism points, m/s, are determined by formula:

$$V_i = l_{Vi} \cdot \mu_V, \tag{6}$$

where l_{Vi} – interval of the velocity diagram in mm that corresponds velocity to be found;

μ_V – scale of the velocity diagram.

Value of the velocities of all points of the mechanism is put in a table on sheet 1 (graphic part of the project or work, ***Appendix A***).

1.4. Determination of angular velocity of links

The angular velocities of the mechanism links s^{-1} are determined by the formula

$$\omega = \frac{V_{relat}}{l_{link}}, \tag{7}$$

where V_{relat} – relative velocity of the mechanism links points, m/s;

l_{link} – length of the mechanism links, m.

For the given mechanism:

$$\omega_2 = \frac{V_{BA}}{l_{BA}}; \qquad \omega_3 = \frac{V_{BC}}{l_{BC}}; \qquad \omega_4 = \frac{V_{ED}}{l_{ED}}.$$

Directions of angular velocities vectors depend on directions of relative linear velocities vectors.

On the kinematic diagram of the mechanism (see Figure 1*a* and sheet 1, ***Appendix A***), indicate the direction of the angular velocities.

1.5. Drawing the acceleration diagrams

Acceleration of the crank point A (leading link), m/s^2, is determined by the formula:

$$a_A = a_A^n = \omega_1^2 \cdot l_{OA} \quad . \tag{8}$$

The scale of acceleration μ_a, $\frac{\mathrm{m/s^2}}{\mathrm{mm}}$, is determined by formula:

$$\mu_a = \frac{a_A}{\pi a} \quad , \tag{9}$$

where πa – a interval of the acceleration diagram corresponding to the acceleration of the point A, $\pi a \approx 100...200$ мм (Figure 2*b*).

On the acceleration diagram, plot the section πa that corresponds to acceleration a_A. Direct the acceleration vector a_A to the center of rotation (center of the crank) (see Figure 2*b*).

Acceleration of point B is determined from vector equations:

$$\begin{cases} \overline{a_B} = \overline{a_A} + \overline{a_{BA}^n} + \overline{a_{BA}^\tau}; \\ \overline{a_B} = \overline{a_C} + \overline{a_{DC}^n} + \overline{a_{BC}^\tau}. \end{cases}$$

Acceleration $a_C = 0$. The values of normal components a_{BA}^n and a_{DC}^n of and accelerations are determined by the formulas:

$$a_{BA}^n = \frac{V_{BA}^2}{l_{BA}}; \qquad a_{BC}^n = \frac{V_{BC}^n}{l_{BC}}.$$

Determine the values of the segments $\left|a_{BA}^n\right|$ and $\left|a_{BC}^n\right|$, mm, corresponding to the normal components of accelerations:

$$\left|a_{BA}^{n}\right| = \frac{a_{BA}^{n}}{\mu_a}; \qquad \left|a_{BC}^{n}\right| = \frac{a_{BC}^{n}}{\mu_a}.$$

Vector of a normal component acceleration $\overline{a_{BA}^{n}}$ is plotted on the acceleration diagram from the point *a*, parallel to the link *AB* and is directed to the center of rotation, i.e. from p. *B* to p. *A* (see Figure 2*b*).

Vector of normal acceleration $\overline{a_{BC}^{n}}$ is plotted on the plan of acceleration from the pole, point *p*, parallel to the link *BC* and directed to the center of rotation, that is, from p. *B* to p. *C* (see Figure 2*b*).

Vectors of tangent components of acceleration $\overline{a_{BA}^{\tau}}$ and $\overline{a_{BC}^{\tau}}$ are drawn perpendicular from the end of the vectors $\overline{a_{BA}^{n}}$, $\overline{a_{BC}^{n}}$.

Acceleration of point *B* will be obtained at the intersection of the vectors $\overline{a_{BA}^{\tau}}$, $\overline{a_{BC}^{\tau}}$.

Acceleration of p. *D* is determined from the similarity theorem (see Figures 1*a* and 2*b*):

$$\frac{\pi d}{\pi b} = \frac{CD}{CB},$$

hence

$$\pi d = \frac{CD}{CB} \cdot \pi b,$$

where CD, CB – sections of the kinematic diagram (length of the mechanism links);

πb, πd – sections of the accelerations diagram, mm.

Acceleration of the point E is determined from the vector equations:

$$\begin{cases} \overline{a_E} = \overline{a_D} + \overline{a_{ED}^{n}} + \overline{a_{ED}^{\tau}}; \\ \overline{a_E} = \overline{a_X} + \overline{a_{EX}^{r}}, \end{cases}$$

where a_{EX}^{r} – relative component of acceleration of p. *E* in relation to directing $x - x$.

$a_X = 0$ – absolute acceleration of shears.

Acceleration vector $\overline{a_{EX}^{r}}$ on the acceleration diagram is directed parallel to $x - x$.

The value of the normal component of acceleration a_{CD}^n and its section is determined the same way as for p. B:

$$a_{ED}^n = \frac{V_{ED}^2}{l_{ED}}; \qquad \left|a_{ED}^n\right| = \frac{a_{ED}^n}{\mu_a}.$$

Acceleration of the links mass centers (Figure 1*a*) of points S_1, S_2, S_3, S_4 is determined by the method of geometric similarity.

The acceleration diagram for one of the positions of the mechanism is shown in Figure 2*b*.

Values of accelerations of each point of the mechanism a_i, m/s^2, are determined from the acceleration diagram by the formula:

$$a_i = l_{ai} \cdot \mu_a, \qquad (10)$$

where l_{ai} – section of the acceleration diagram in mm, corresponding acceleration to be determined;

μ_a – scale of accelerations diagram.

The results of calculations of accelerations of all points and tangent components are entered in the table in the drawing (Sheet 1, ***Appendix A***).

1.6. Determination of the angular accelerations

Angular accelerations of the mechanism links, 1/s^2, are determined by formula:

$$\varepsilon = \frac{a_{relat}^\tau}{l_{link}}, \qquad (11)$$

where a_{relat}^τ – tangent component of accelerations;

l_{link} – link length.

For the given mechanism:

$$\varepsilon_1 = 0; \quad \varepsilon_2 = \frac{a_{BA}^\tau}{l_{BA}}; \quad \varepsilon_3 = \frac{a_{BC}^\tau}{l_{BC}}; \quad \varepsilon_4 = \frac{a_{ED}^\tau}{l_{ED}}.$$

The direction of angular accelerations vectors depends on the directions of the vectors of tangent acceleration components.

On the kinematic diagram of the mechanism, we indicate the directions of the angular accelerations (see Figure 1*a*, sheet 1, ***Appendix A***).

1.7. Drawing kinematic diagrams

Kinematic diagrams (graphs) are used to determine the speed and acceleration of one of the points of the mechanism (point E) for the full turn of the crank.

Draw a graph of driven link displacement $S_E = S_E(t)$, (Figure 3*a*) to which the force of the production resistance F_{BO} is applied.

Sections $1-1'$, $2-2'$, $3-3'$ and others (see Figure 3*a*) show the displacement of slider *E* from the edge (zero) point.

Having joined points $0-1'-2'-3'$ and others we get the graph of point *E* displacement (see Figure 3*a*).

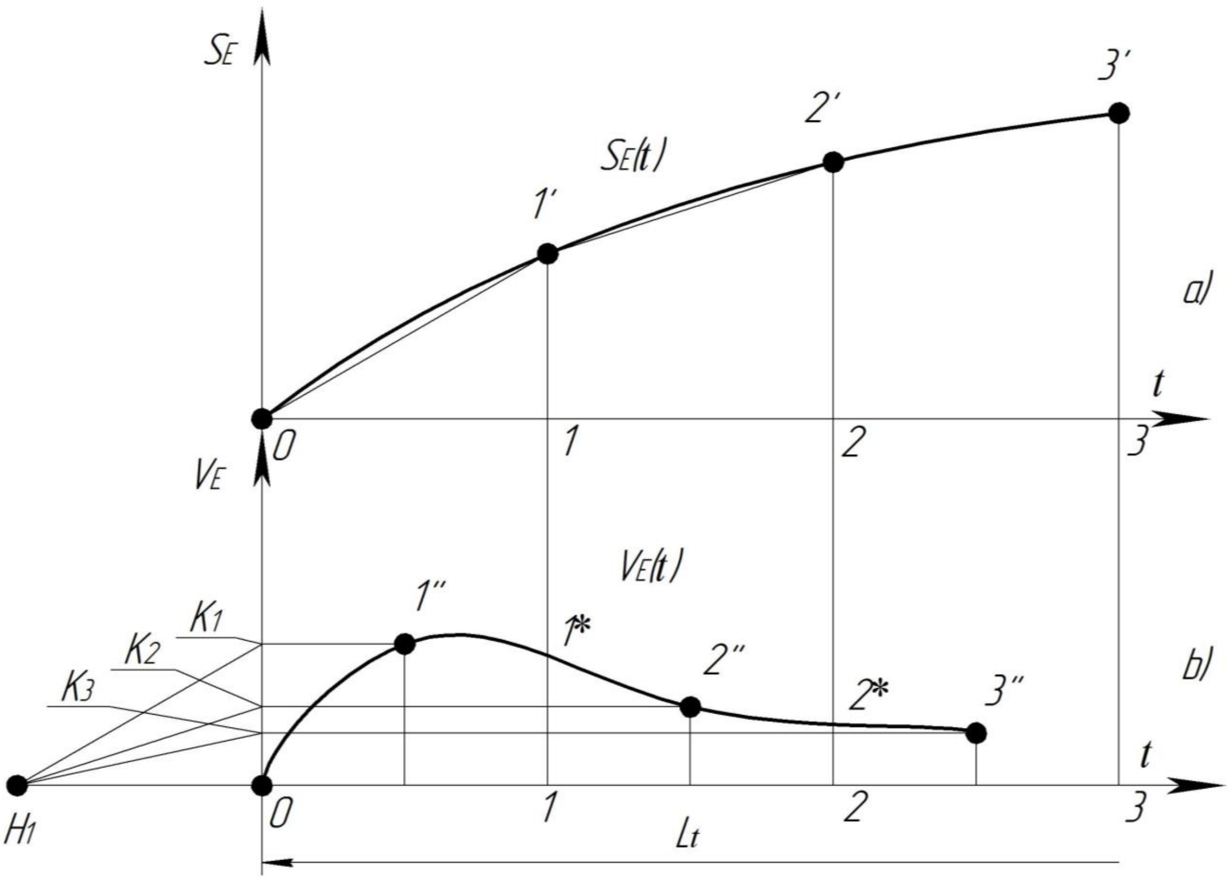


Figure 3 – Graph of displacement of p. E
and drawing the diagram of velocity

Diagram of velocity of p. E is drawn in such sequence:

a) use the graph of displacements of point E: $S_E = S_E(t)$ drawn on a scale $\mu_S = \mu_l$;

b) join points $0-1'$; $1'-2'$; $2'-3'$ etc. with straight lines;

c) plot the pole OH_1 (take 30…50 mm);

d) from the pole H_1 draw a straight line H_1K_1 parallel to the straight line $01'$; from point K_1 draw a horizontal straight line $K_1 1''$ to the middle of the interval 01; similarly, by drawing a straight line H_1K_2 parallel to $1'2'$ find point $2''$ in the middle of the interval 1-2;

e) having connected points 0, $1''$, $2''$, $3''$ etc. with a smooth curve we get a velocity chart $V_E = V_E(t)$.

Similarly, we draw the acceleration diagram of point E having graphically differentiated the velocity diagram.

The scales of the kinematic diagrams are determined by the formulas:

– time scale, $\frac{\text{s}}{\text{mm}}$;

$$\mu_t = \frac{60}{n_1 \cdot L_t},$$

where n_1 – crank revolutions per minute;

L_t – length of the segment in mm, which indicates time;

– velocity scale, $\frac{\text{m/s}}{\text{mm}}$;

$$\mu_V = \frac{\mu_S}{\mu_t \cdot OH_1},$$

where OH_1 – pole distance in mm, (take 30...50 mm);

– acceleration scale, $\frac{\text{m/s}^2}{\text{mm}}$;

$$\mu_a = \frac{\mu_V}{\mu_t \cdot OH_2},$$

where OH_2 – pole distance in mm, (take 30...50 mm).

Velocity of p. E for any position of the mechanismis determined using the speed diagram (see Figure 4*b*). The velocity is determined by the formula:

$$V_E = \left|1-1^*\right| \cdot \mu_V ,$$

where $\left|1-1^*\right|$ – section of the velocity diagram for the 1-st position in mm (see Figure 3).

1.8. Matching the results

Relative errors of velocities and accelerations of point E for the investigated positions obtained by diagrams methods are determined by the formulas:

$$\Delta_V = \frac{V_{plan} - V_{diagrams}}{V_{plan}} \cdot 100\% ; \qquad (12)$$

$$\Delta_a = \frac{a_{plan} - a_{diagrams}}{a_{plan}} \cdot 100\% . \qquad (13)$$

Relative error has not to exceed 5 %.

SHEET 2
FORCE (KINETOSTATIC) ANALYSIS OF MECHANISM

The force calculation of the mechanism is made to determine the reactions in kinematic pairs, as well as the balancing force.

When solving problems of kinetostatic mechanisms the following forces are taken into account:

– weight of each link G_i;

– inertia forces of each link $F_{IN.i}$;

– the strength of the productive resistance F_{PR}.

The forces of friction in kinematic pairs are not taken into account.

Force calculation is performed separately for each group of Assur. Calculation begins with the group which has been attached last in the process of mechanism formation and ends with the calculation of the leading link.

2.1. Determination of links weight forces

The force of weight G_i, N, of each link is determined by the formula:

$$G_i = m_i \cdot g, \tag{14}$$

where m_i – mass of i-th link, kg;

g – acceleration of gravity, $g = 9{,}81$ m/s^2.

Apply the links weight forces on kinematic model of the mechanism (Figure 4).

2.2. Determination of inertia forces of the links

Inertia forces of links $F_{IN.i}$, N, are determined by the formula:

$$\overline{F}_{IN.i} = -m_i \cdot \overline{a}_{si}, \tag{15}$$

where m_i – mass of i-th link, kg;

a_{si} – acceleration of centre of mass of this link, m/s^2.

The direction of inertia the force $\overline{F}_{IN.i}$ is opposite to the direction of the acceleration vector (see Figure 2*b*, Figure 4).

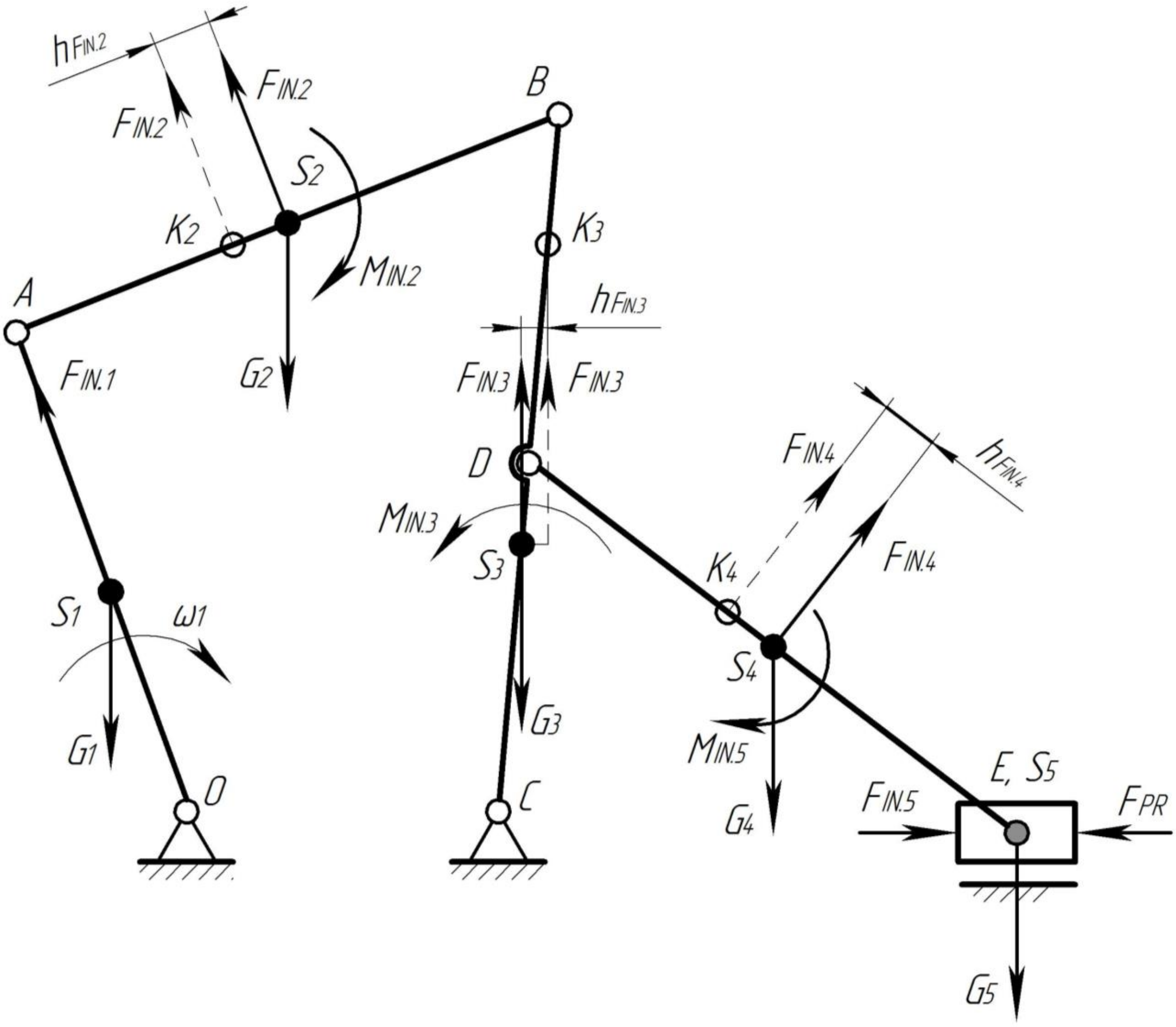


Figure 4 – Mapping (schematization) of power factors applied to the links of mechanism

2.3. Determination the inertia moments of links

Mass moments of links inertia J_{si}, $\mathrm{k}g \cdot \mathrm{m}^2$, in relation to the mass centre is determined by formula:

$$J_{si} = k \cdot m_i \cdot l_i^2, \qquad (16)$$

where m_i – mass of i-th link, kg;
l_i – length of i-th link, m;
k – coefficient, which depends on the location of the center of masses.

When placing the center of mass:
– on the middle of the link length, $k = 1/12$;
– on a third of the link length, $k = 0{,}175$.

2.4. Determination of moments of inertia forces

Moments of inertia forces, $\overline{M}_{IN.i}$, N·m, are determined by the formula:

$$\overline{M}_{IN.i} = -J_{si} \cdot \overline{\varepsilon}_i, \quad (17)$$

where J_{si} – moment of inertia of *i*-th link in relation to the mass centre, $kg \cdot m^2$;

$\overline{\varepsilon}_i$ – vector of angular acceleration of the link (see Sheet 1).

Direction of moment of inertia forces $\overline{M}_{IN.i}$, is opposite to the direction of the vector of corresponding angular acceleration of the link (see Figure 4).

2.5. Reduction of moments of inertia and inertia forces to inertia forces

At power calculation, it is convenient to reduce moments of inertia forces $M_{IN.i}$ and also of inertia forces F_{INi} to inertia forces.

Bringing arms, mm, are determined by the formula:

$$h_{F_{IN.i}} = \frac{M_{IN.i}}{F_{IN.i}}, \quad (18)$$

where $M_{IN.i}$ – moment of inertia force of *i*-th link, N·m;

$F_{IN.i}$ – inertia force of *i*-th link, N.

Bringing arms which are plotted on the kinematic diagram of the mechanism are translated into sections of the kinematic diagram scale, that is, into mm, (see Figure 4):

$$\left| h_{F_{IN.i}} \right| = \frac{h_{F_{IN.i}}}{\mu_l}, \quad (19)$$

where μ_l – scale of kinematic diagram, m/mm.

Reduction of moments of inertia forces to the inertia forces is carried out as follows:

– determine the bringing arms by formulas (18) and (19);

– the inertia force of $F_{IN.i}$ of the *i*-th link is shifted from the center of mass to the distance $h_{F_{IN.i}}$ so that this force has created the moment of inertia relative to the center of mass $M_{IN.i}$ (Figures 4, 5).

In the following force calculations of the Assur groups, the moments of inertia forces of the links are replaced by inertia forces of the links displaced from the centers of mass to the segments $h_{F_{IN.i}}$ (see Figures 4, 5).

2.6. Force calculation of group 4-5

Separately draw the Assura group 4-5 with the forces applied to it: weight G_4, inertia $F_{IN.4}$ productive (useful) resistance F_{PR} (Figure 5).

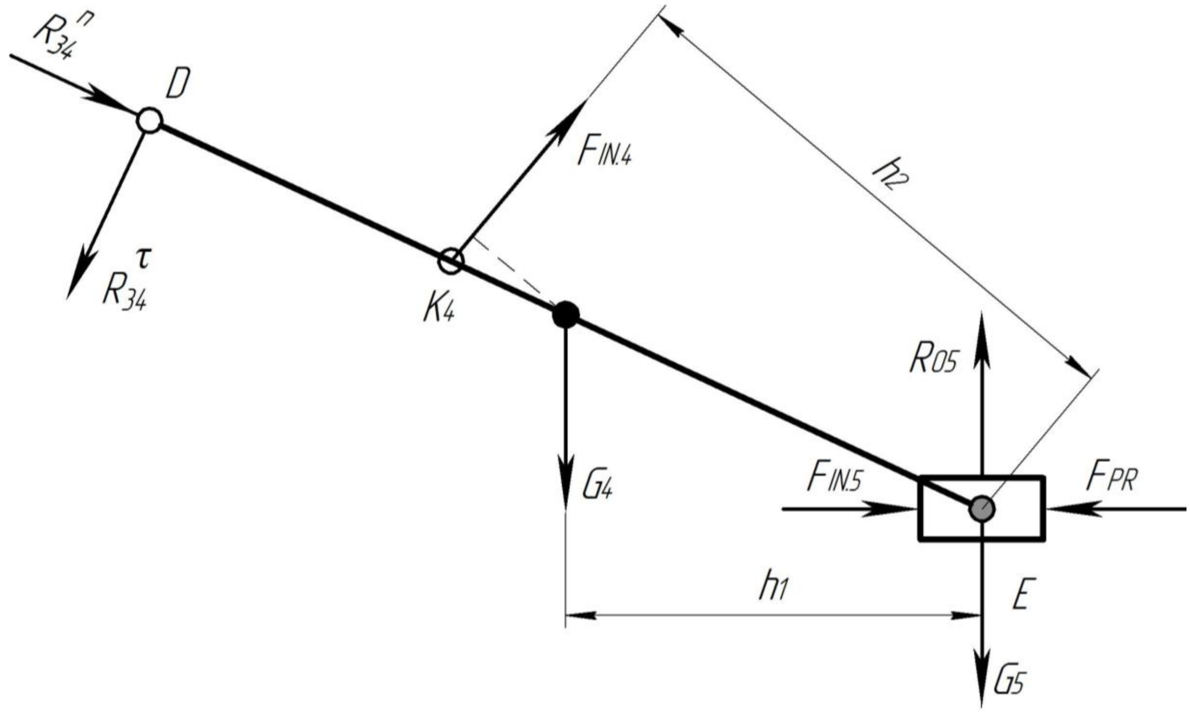


Figure 5 – Force calculation of the Assur group 4-5

In the hinge D there is a reaction force R_{34} which is resolved into two components: normal R_{34}^n, directed along the link 4 and tangential R_{34}^τ, directed perpendicular to link 4.

Determine the tangential component of the reaction R_{34}^τ using the static equation

$$\sum M_E = 0; \qquad G_4 \cdot h_1 - F_{IH.4} \cdot h_2 + R_{34}^\tau \cdot ED = 0,$$

hence

$$R_{34}^\tau = \frac{F_{IN.4} \cdot h_2 - G_4 \cdot h_1}{ED},$$

here h_1 – arm of weight force G_4 in relation to point E.

h_2 – arm of inertia force $F_{IN.4}$ in relation to point E.

If R_{34}^{τ} is obtained with minus sign «–», then its direction has to be changed to the opposite.

To draw a plan of forces choose a scale, μ_F, N/mm, using the formula:

$$\mu_F = \frac{F_{PR}}{|F_{PR}|}, \tag{20}$$

where F_{PR} – value of productive resistance force, N;

$|F_{PR}|$ – section of the force plan, mm, which shows the strength of the production resistance. It is recommended to take:

$$|F_{PR}| = (150...300) \text{ mm.}$$

To determine the values and directions R_{34}^{n} and R_{05} draw the plan of forces. Start writing vector equation from force with unknown magnitude R_{34}^{n} applied to the link 4. Record all the known forces that belong to Assur group 4-5. Finish drawing the plan of forces with the force of unknown magnitude R_{05}.

$$\overline{R_{34}^{n}} + \overline{R_{34}^{\tau}} + \overline{F_{IN.4}} + \overline{G_4} + \overline{F_{IN.5}} + \overline{G_5} + \overline{F_{PR}} + \overline{R_{05}} = 0,$$

where $\overline{R_{34}^{n}}$, $\overline{R_{05}}$ – vectors of forces known only by the direction.

Plan of forces of the of Assura 4-5 group (Figure 6) is drawn in the following sequence:

– determine the sections of each known force in mm by the formula

$$|h_{F.i}| = \frac{F_i}{\mu_F};$$

– taking point O as the coordinate origin (see Figure 6), plot the known values of forces in sections that coincide with their vectors

– complete the force polygon at point P (plan of forces, see Figure 6) by drawing the sections of forces unknown by value, but known by the direction till they cross.

From the plan of forces (see Figure 6) determine the reactions in the kinematic pairs R_{05}, R_{34}^{n} and R_{34}. The reaction in the kinematic pair D is indicated R_{34}. This is the force of pressure of link 3 on link 4.

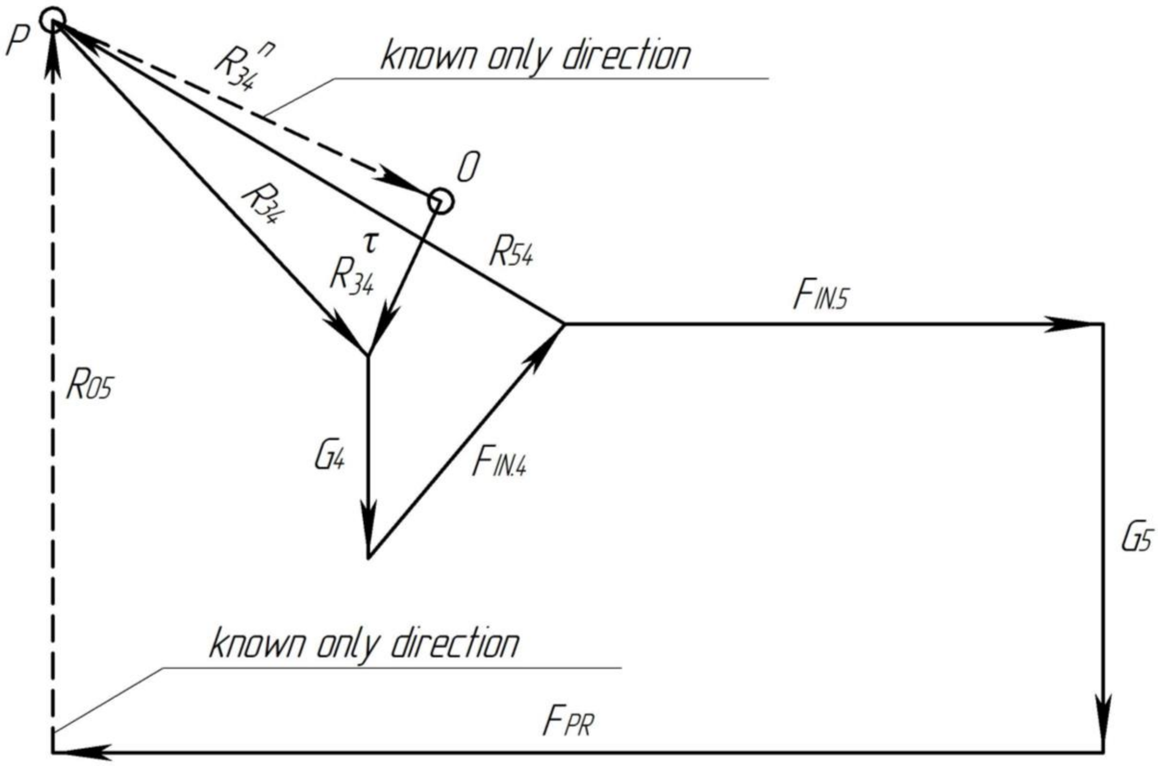


Figure 6 – Plan of forces of Assur group 4-5

To determine the reaction R_{54} in kinematic pair between 4 and 5 links, use the plan of forces (see Figure 6).

Considering the equilibrium of one link, such as link 4, we can write:

$$\overline{R_{34}^n} + \overline{R_{34}^\tau} + \overline{G_4} + \overline{F_{IN.4}} + \overline{R_{54}} = 0;$$

hence, determine $\overline{R_{54}}$.

The values of reaction forces in *N* are determined by the formula:

$$R_i = |R_i| \cdot \mu_F,$$

where $|R_i|$ – the section of the plan of forces in mm corresponding to the force to be determined.

2.7. Force calculation of link 2-3

Force calculation of the Assur group 2-3 (Figure 7) is performed similarly to the calculation of the Assur group 4-5.

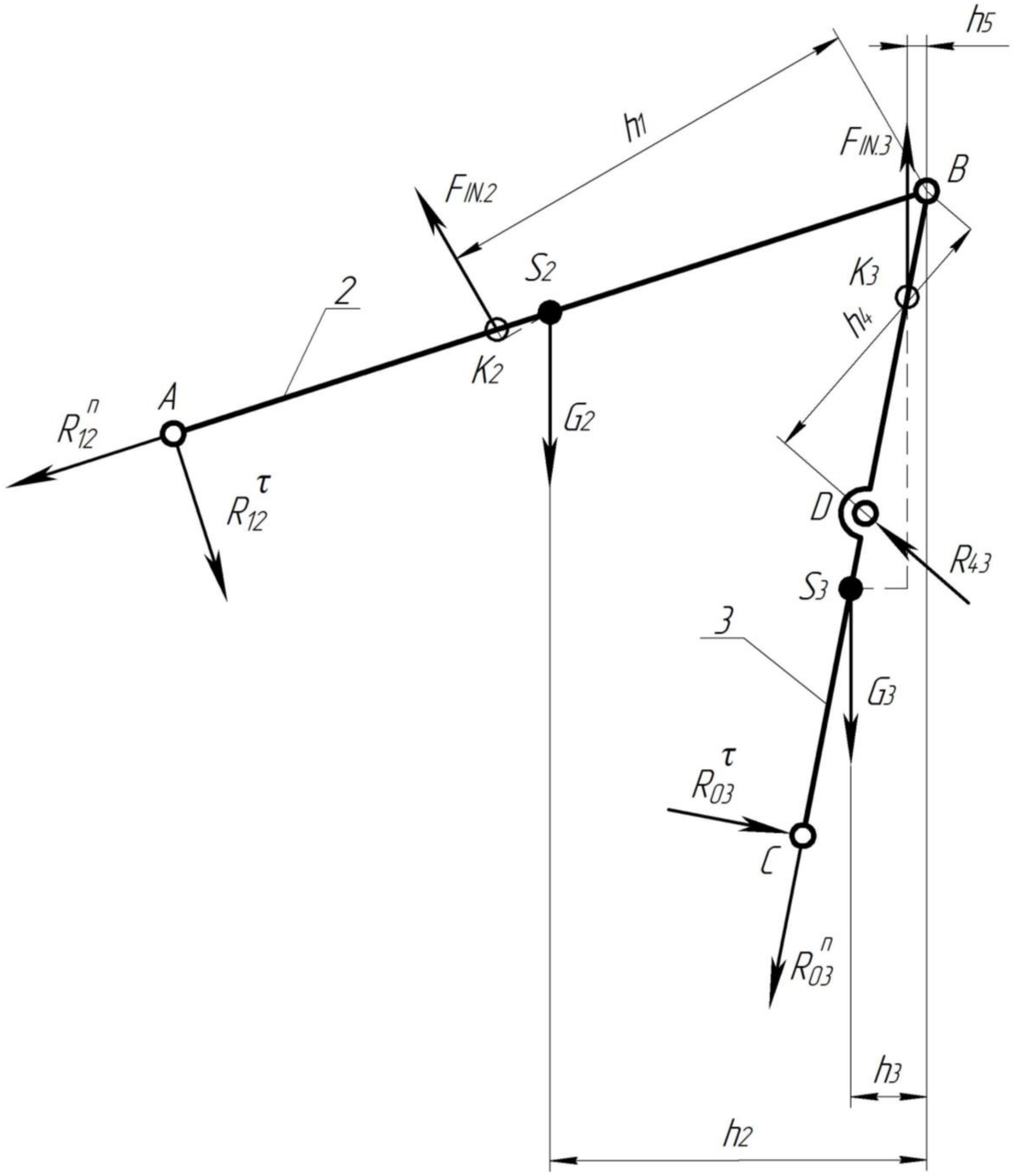


Figure 7 – Force calculation of the Assur group 2-3

To links 2 and 3 apply:

– weight forces G_2 and G_3;

– inertia forces $F_{IN.2}$ and $F_{IN.3}$;

– reaction force of link 4 onto link 3 – R_{43}.

The value of force R_{43} is taken from the calculation of Assur group 4-5, this force is R_{34}. The direction of the force reaction R_{43} is opposite to the direction R_{34}.

In kinematic pairs *A* and *C* there appear reaction forces R_{12} (hinge *A*), R_{03} (hinge *C*). The reaction forces R_{12} and R_{03} are decomposed into two components:

– normal R_{12}^{n}, R_{03}^{n} (directed along links 2 and 3);

– tangent R_{12}^{τ}, R_{03}^{τ} (directed perpendicular to links 2 and 3).

Components of the reactions R_{12}^{τ} and R_{03}^{τ} are determined from equilibrium equations:

$$\sum M_B = 0, \quad R_{12}^{\tau} \cdot AB - F_{IN.2} \cdot h_1 + G_2 \cdot h_2 = 0;$$

$$\sum M_B = 0, \quad R_{03}^{\tau} \cdot CB + G_3 \cdot h_3 - R_{43} \cdot h_4 - F_{IN.3} \cdot h_5 = 0.$$

Plan of forces of Assur group 2-3 is similar to the plan of forces of Assur group 4-5.

The components of reactions forces R_{12}^{n} and R_{03}^{n} are determined from the plan of forces using the equations of forces balance of Assur group 2-3:

$$\overline{R_{12}^{n}} + \overline{R_{12}^{\tau}} + \overline{F_{IN.2}} + \overline{G_2} + \overline{F_{IN.3}} + \overline{G_3} + \overline{R_{43}} + \overline{R_{03}^{\tau}} + \overline{R_{03}^{n}} = 0,$$

where R_{12}^{n}, R_{03}^{n} – vectors of forces, known only by the direction.

The value of the reaction R_{23} is determined similarly to the reaction R_{45}.

2.8. Force calculation of the driving link

Draw the driving link OA (Figure 8), apply:

– weight force G_1;

– inertia forces $F_{IN.1}$ directed along link 1;

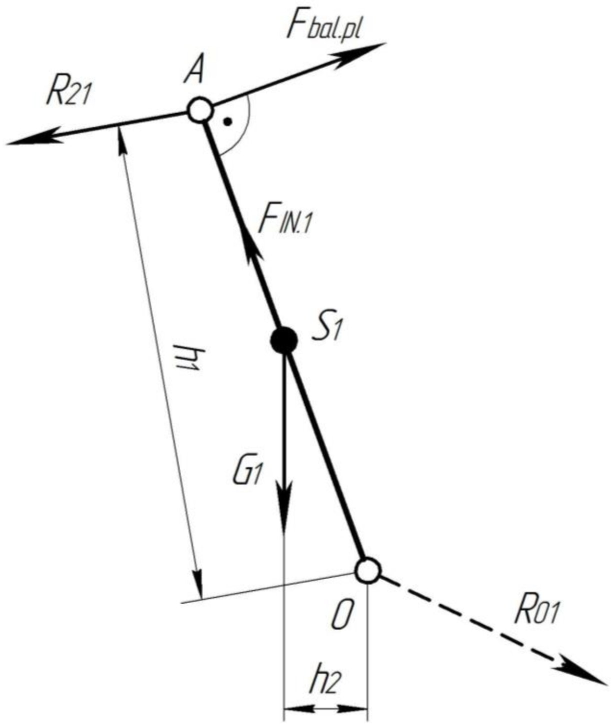
– reaction force R_{21} (taken from the Assur plan of forces 2-3);

– balancing force $F_{bal.pl}$, applied at point *A*, perpendicular to link *OA*.

Balancing force, $F_{bal.pl}$, is determined from the equation:

$$\sum M_O = 0; \quad F_{bal.pl} \cdot OA - R_{21} \cdot h_1 - G_1 \cdot h_2 = 0,$$

where h_1 – arm of the reaction force R_{21} in relation to point O;
h_2 – arm of weight force G in relation to point O.



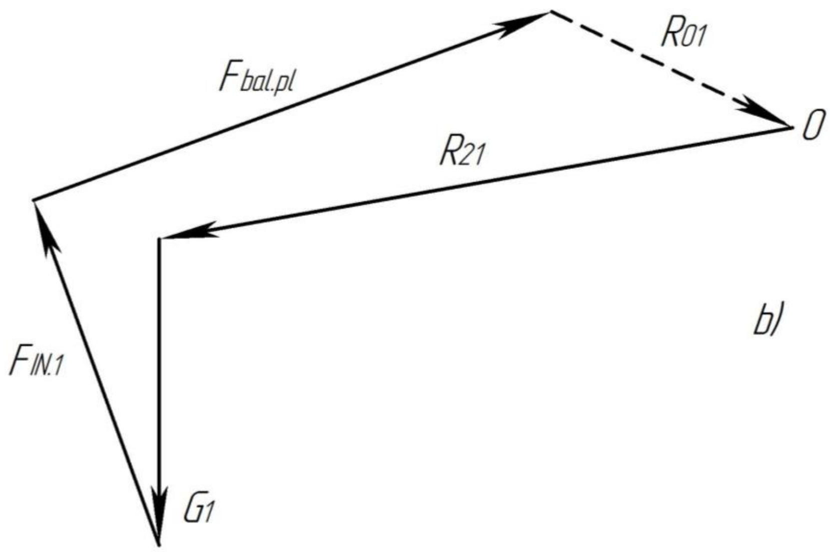


Figure 8 –Force calculation of driving link

Reaction in the kinematic pair O is determined from the plan of forces (Figure 8*b*), using the vector equation of equilibrium:

$$\overline{G_1} + \overline{F_{IN.1}} + \overline{R_{21}} + \overline{F_{bal.pl}} + \overline{R_{01}} = 0.$$

Using this, find the value and direction of the reaction force $\overline{R_{01}}$.

2.9. Determination of balancing force by M.Ye. Zhukovsky «Firm lever» method

To determine the balancing force $F_{bal.z}$, using M.Ye. Zhukovsky «firm lever» method draw the velocity diagram turned by $90°$ (Figure 9) to which we apply:

– gravity force G_i at points S_i;

– inertia force $F_{IN.i}$ at points K_i;

– force of productive resistance F_{PR} at the point of its application, point *e*;

– balancing force $F_{bal.z}$ at the point *a* perpendicular to *pa*.

Formulate equations of equilibrium of the moments relative to the pole *p* (see Figure 9).

$$\sum M_p = 0; \quad F_{bal.z} \cdot pa - G_1 \cdot h_1 - G_2 \cdot h_2 + G_3 \cdot h_3 + G_4 \cdot h_4 - \\ - F_{IN.2} \cdot h_6 - F_{IN.3} \cdot h_7 + F_{IN.4} \cdot h_8 + F_{IN.5} \cdot h_5 - F_{PR} \cdot h_5 = 0.$$

Using this, determine the value $F_{bal.z}$ (Sheet 2, ***Appendix B***).

2.10. Matching the results

Match the values of balance forces obtained by the method of force calculation of force plans and M.Ye. Zhukovsky method of «Firm lever».

Relative error is determined by the formula:

$$\Delta_F = \frac{F_{bal.z} - F_{bal.pl.}}{F_{bal.z}} \cdot 100\%. \tag{21}$$

Error has not to exceed 5 %.

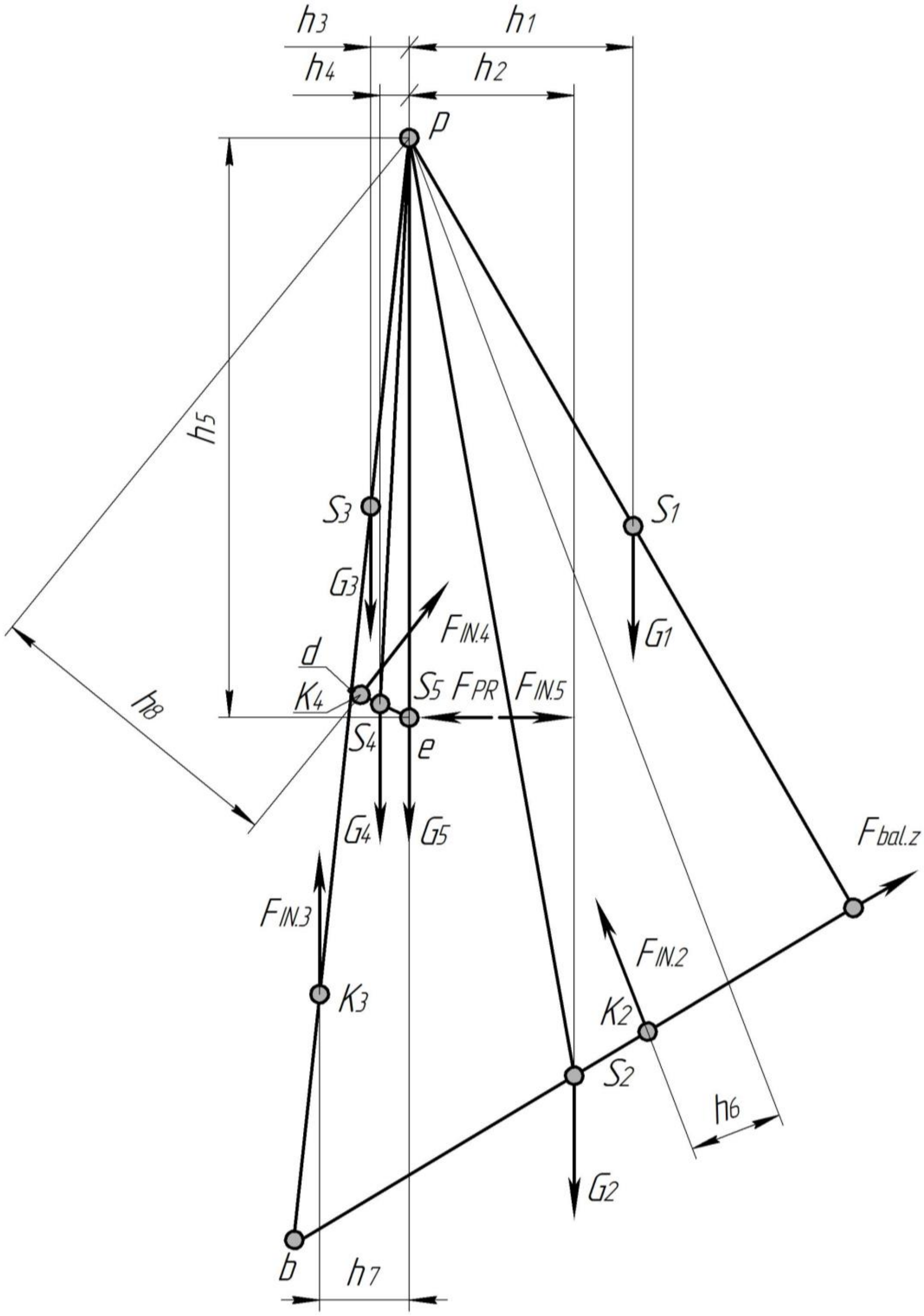
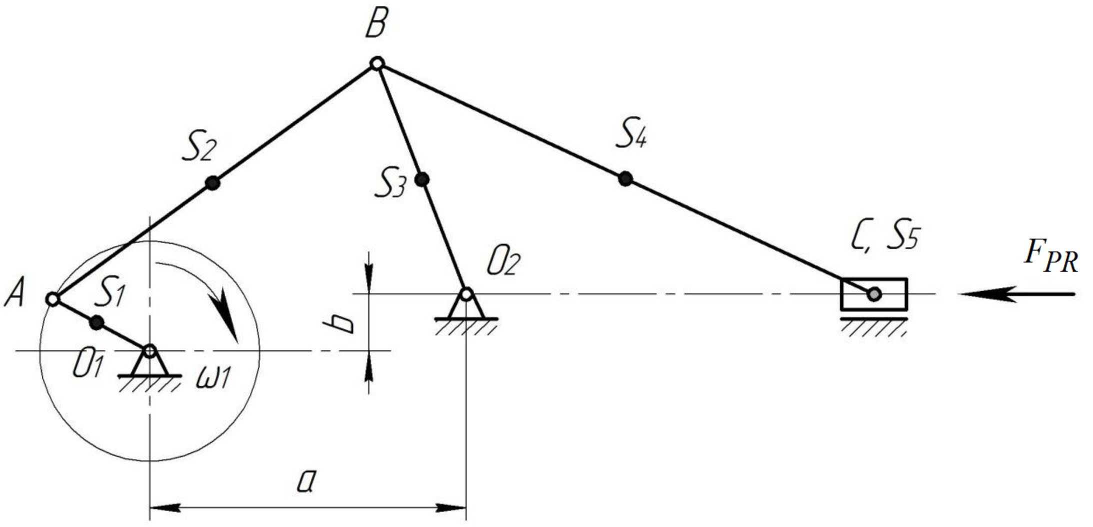


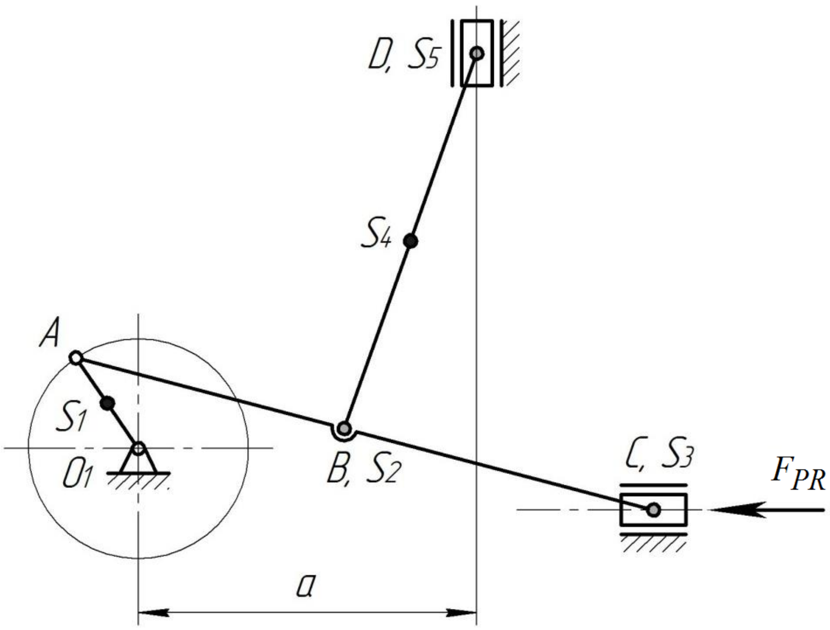
Figure 9 – M.Y. Zhukovsky's «Firm lever»

Task 1



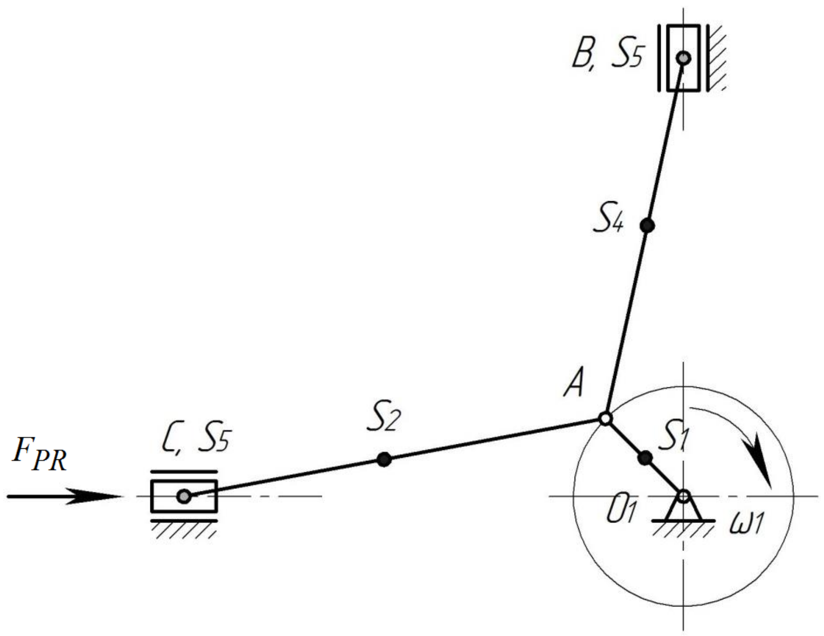
Output data	Version				
	1	2	3	4	5
a, mm	700	750	800	850	900
b, mm	70	80	90	100	110
L_{O_1A}, mm	200	210	250	270	300
L_{O_2B}, mm	350	400	450	500	550
L_{AB}, mm	800	850	850	900	1000
L_{BC}, mm	850	1000	1100	1200	1300
n_1, RPM	100	95	90	80	70
m_1, kg	15	20	24	28	32
m_2, kg	50	60	65	70	80
m_3, kg	30	40	50	60	70
m_4, kg	80	100	110	120	130
m_5, kg	80	90	100	110	120
F_{PR}, kN	9,0	9,5	9,7	9,8	9,4

Task 2



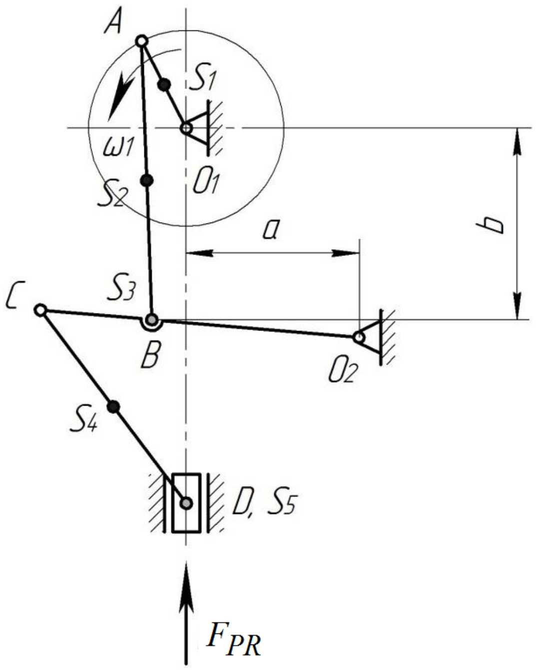
Output data	Version				
	1	2	3	4	5
a, mm	500	550	570	600	520
L_{O_1A}, mm	200	25	220	280	230
L_{AC}, mm	750	780	800	950	920
L_{BD}, mm	650	680	700	820	750
n_1, RPM	95	85	75	65	70
m_1, kg	18	19	21	24	20
m_2, kg	70	72	60	55	65
m_3, kg	18	20	30	35	28
m_4, kg	35	45	40	60	45
m_5, kg	32	40	34	20	24
F_{PR}, kN	5,2	6,2	5,5	6,0	5,4

Task 3



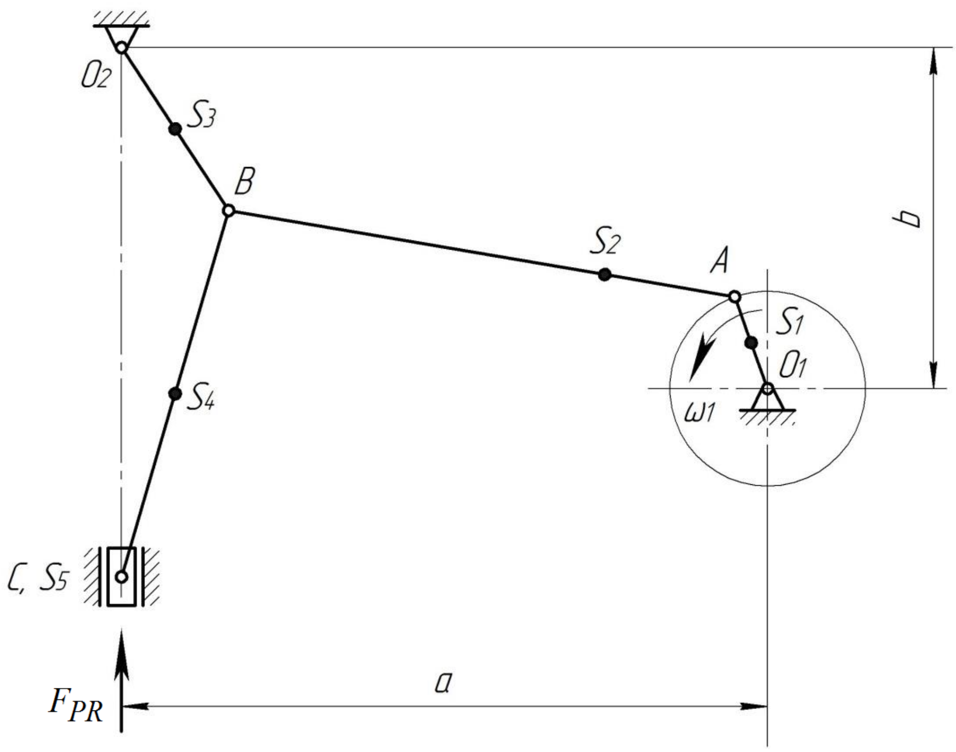
Output data	Version				
	1	2	3	4	5
L_{O_1A}, mm	200	180	160	150	140
L_{AB}, mm	500	480	450	440	400
n_1, RPM	120	130	140	150	160
m_1, kg	9	10	11	12	13
$m_2 = m_4$, kg	40	42	38	44	45
$m_3 = m_5$, kg	10	9	8	11	12
F_{PR}, kN	6,0	5,5	4,8	6,2	5,2

Task 4



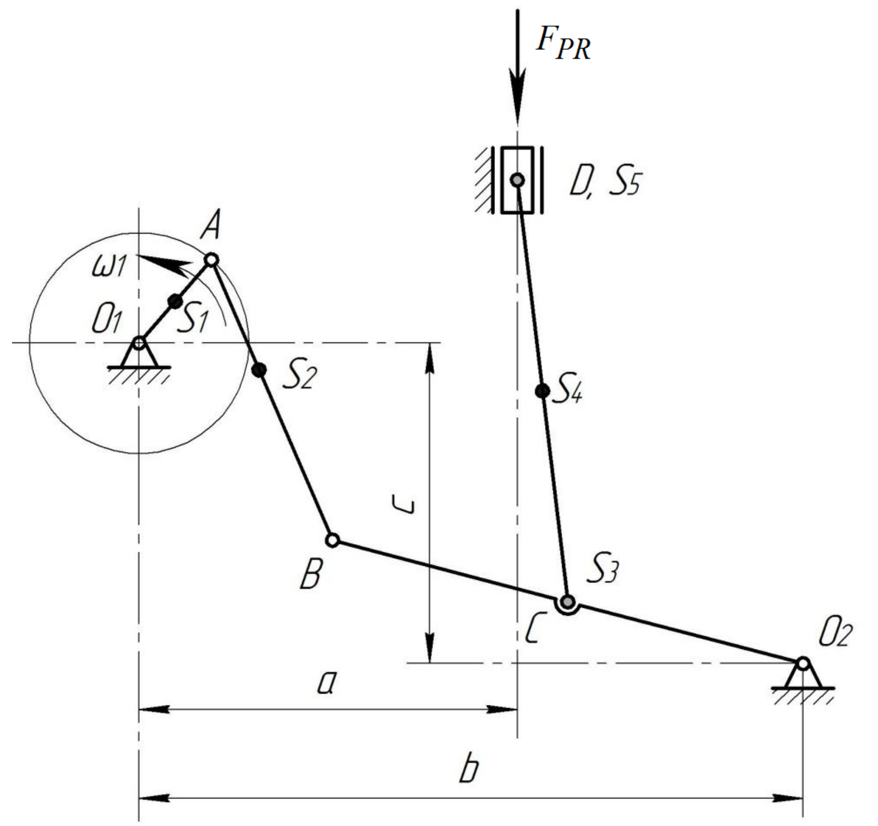
Output data	Version				
	1	2	3	4	5
a, mm	300	400	450	550	600
b, mm	400	500	600	700	750
L_{O_1A}, mm	200	220	250	270	300
L_{AB}, mm	500	600	700	800	900
L_{O_2C}, mm	450	550	650	700	850
L_{CD}, mm	600	700	800	900	1000
n_1, RPM	95	85	110	75	80
m_1, kg	25	27	30	35	40
m_2, kg	60	63	70	68	75
m_3, kg	55	60	70	67	75
m_4, kg	40	45	50	55	60
m_5, kg	100	90	105	110	120
F_{PR}, kN	9,0	9,5	9,1	9,8	9,9

Task 5



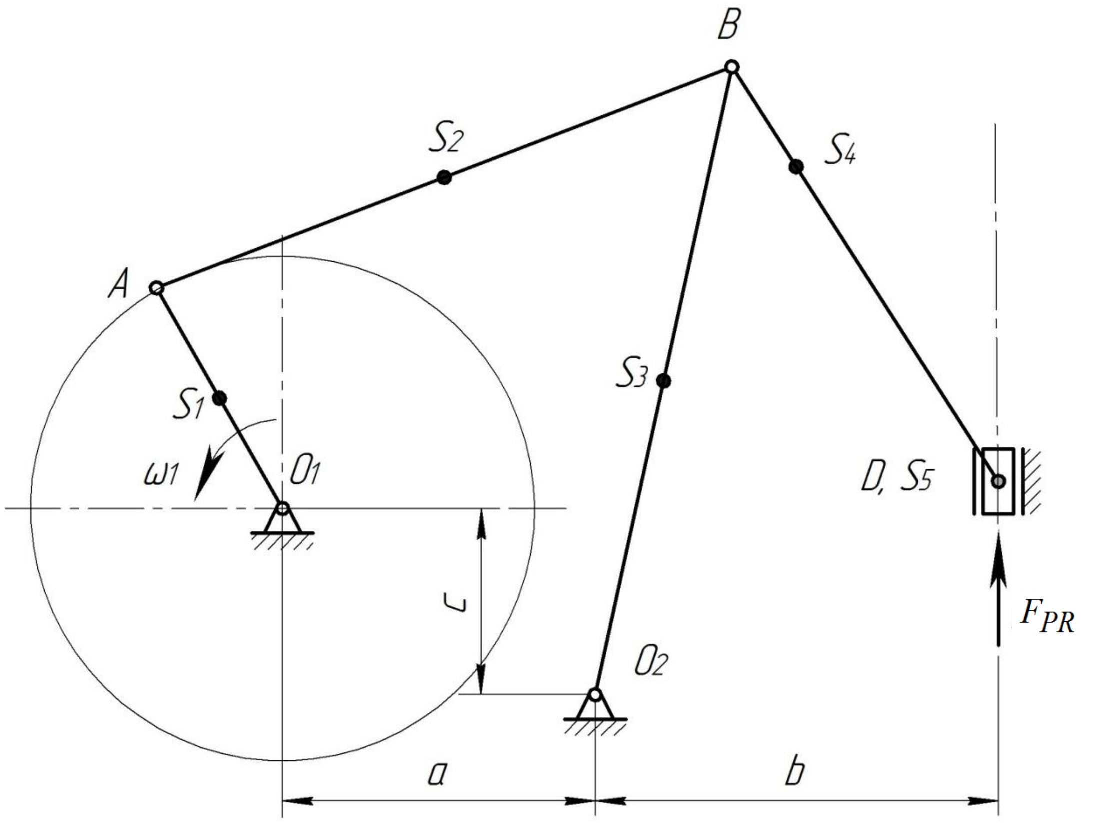
Output data	Version				
	1	2	3	4	5
a, mm	600	650	700	750	800
b, mm	400	450	500	550	600
L_{O_1A}, mm	150	170	190	220	250
L_{AB}, mm	to accept				
L_{O_2B}, mm	300	320	350	370	400
L_{BC}, mm	450	500	550	600	650
n_1, RPM	100	110	90	95	80
m_1, kg	11,0	10,5	10,0	8,0	12,0
m_2, kg	40	45	50	50	60
m_3, kg	20	22	24	20	30
m_4, kg	35	40	45	50	55
m_5, kg	60	70	80	90	100
F_{PR}, kN	8,6	9,2	9,6	9,2	9,6

Task 6



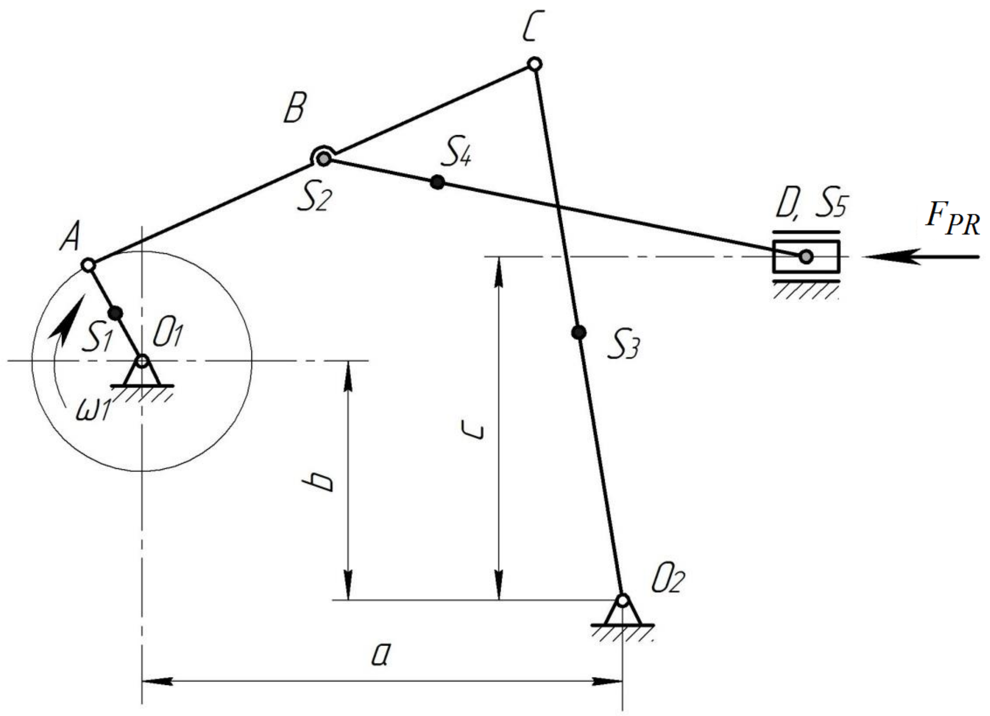
Output data	Version				
	1	2	3	4	5
a, mm	to accept				
b, mm	to accept				
c, mm	200	220	240	260	280
L_{O_1A}, mm	140	160	180	230	250
L_{AB}, mm	380	400	420	450	480
L_{CD}, mm	400	420	460	500	520
L_{O_2B}, mm	600	620	680	700	680
n_1, RPM	70	80	90	100	110
m_1, kg	12	14	16	18	20
m_2, kg	30	35	40	45	50
m_3, kg	65	70	70	75	75
m_4, kg	35	40	45	45	50
m_5, kg	20	24	28	30	35
F_{PR}, kN	6,0	6,2	6,4	6,6	6,8

Task 7



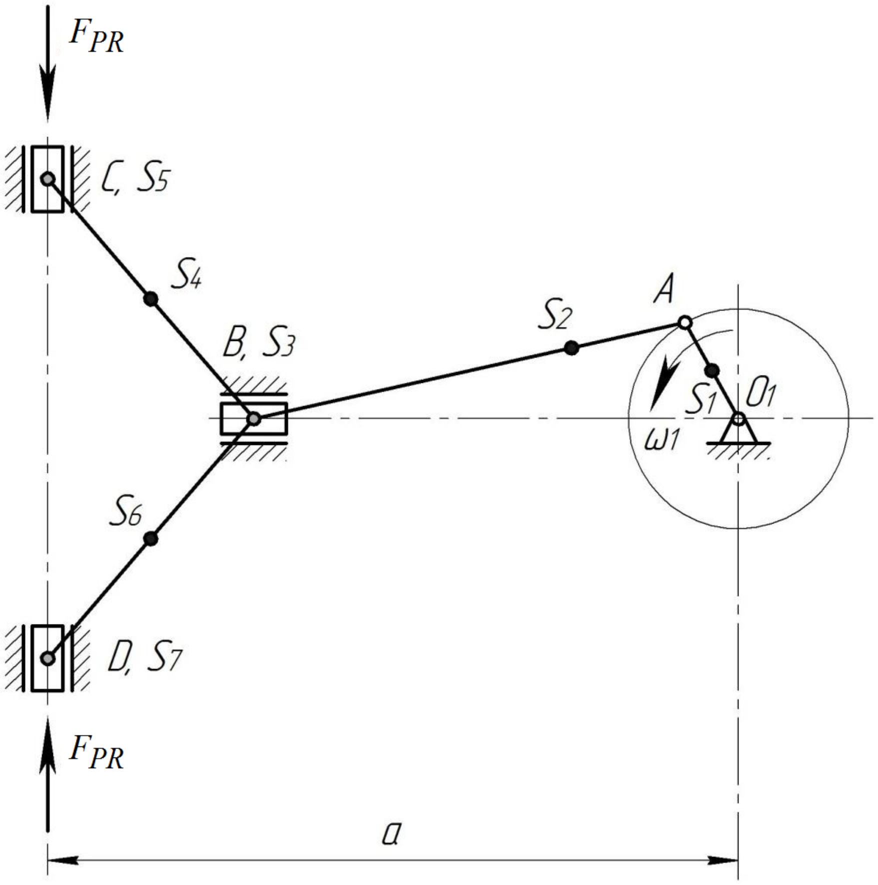
Output data	Version				
	1	2	3	4	5
a, mm	to accept				
b, mm	200	220	240	260	300
c, mm	150	160	180	200	220
L_{O_1A}, mm	180	200	160	190	210
L_{AB}, mm	600	650	700	680	720
L_{O_2B}, mm	500	520	540	580	620
L_{BD}, mm	450	480	500	500	480
n_1, RPM	60	50	55	60	55
m_1, kg	8	10	12	14	16
m_2, kg	42	44	44	42	40
m_3, kg	30	32	32	36	36
m_4, kg	37	30	35	38	38
m_5, kg	20	25	30	35	40
F_{PR}, kN	6,5	6,7	6,9	7,1	7,3

Task 8



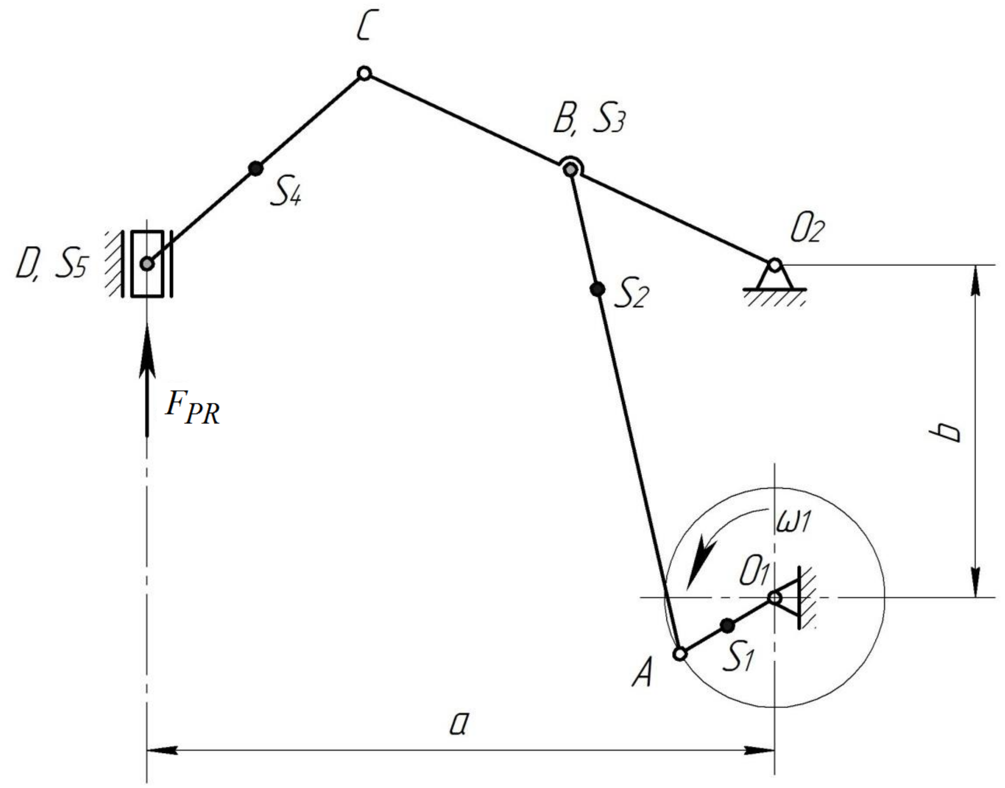
Output data	Version				
	1	2	3	4	5
a, mm	to accept				
b, mm	300	280	320	340	260
L_{O_1A}, mm	180	190	200	210	220
L_{AC}, mm	550	580	600	520	550
L_{O_2C}, mm	600	620	640	680	700
L_{BD}, mm	600	580	620	610	600
c, mm	450	480	500	500	550
n_1, RPM	60	65	70	75	80
m_1, kg	20	22	25	27	30
m_2, kg	45	47	50	48	52
m_3, kg	60	62	55	58	60
m_4, kg	50	52	54	56	58
m_5, kg	15	18	21	25	30
F_{PR}, kN	6,4	6,6	6,8	7,0	7,2

Task 9



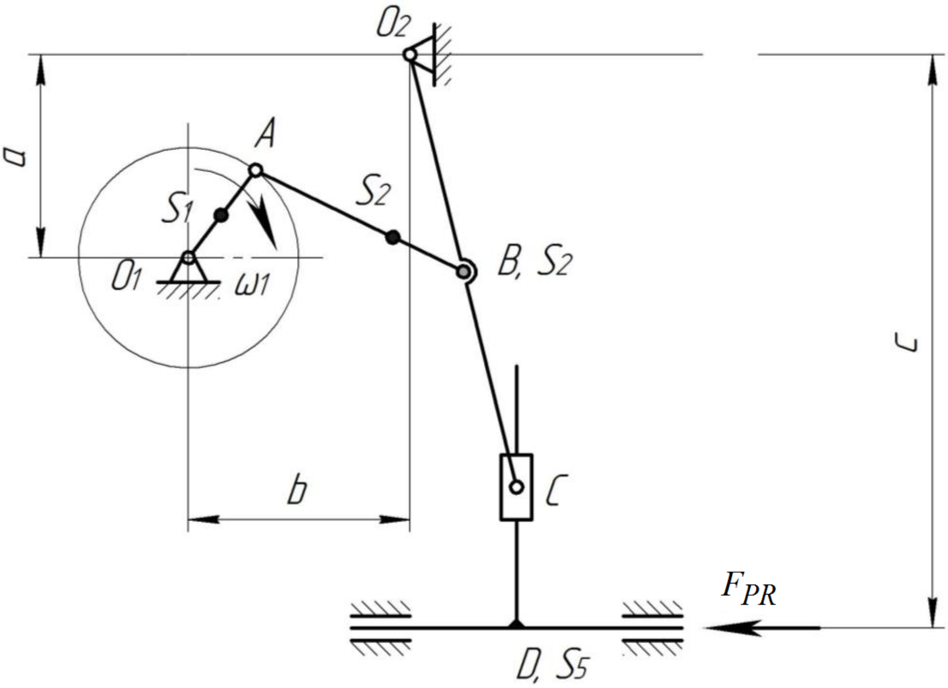
Output data	Version				
	1	2	3	4	5
a, mm	900	950	1000	1050	1100
L_{O_1A}, mm	140	150	180	200	220
L_{AB}, mm	550	600	750	800	900
$L_{BC} = L_{BD}$, mm	500	600	750	750	800
n_1, RPM	100	90	80	70	60
m_1, kg	10	12	14	16	18
m_2, kg	30	35	40	45	50
m_3, kg	15	18	21	24	27
$m_4 = m_6$, kg	40	50	60	60	60
$m_5 = m_7$, kg	12	16	20	24	28
F_{PR}, kN	6,0	6,5	7,0	7,5	8,0

Task 10



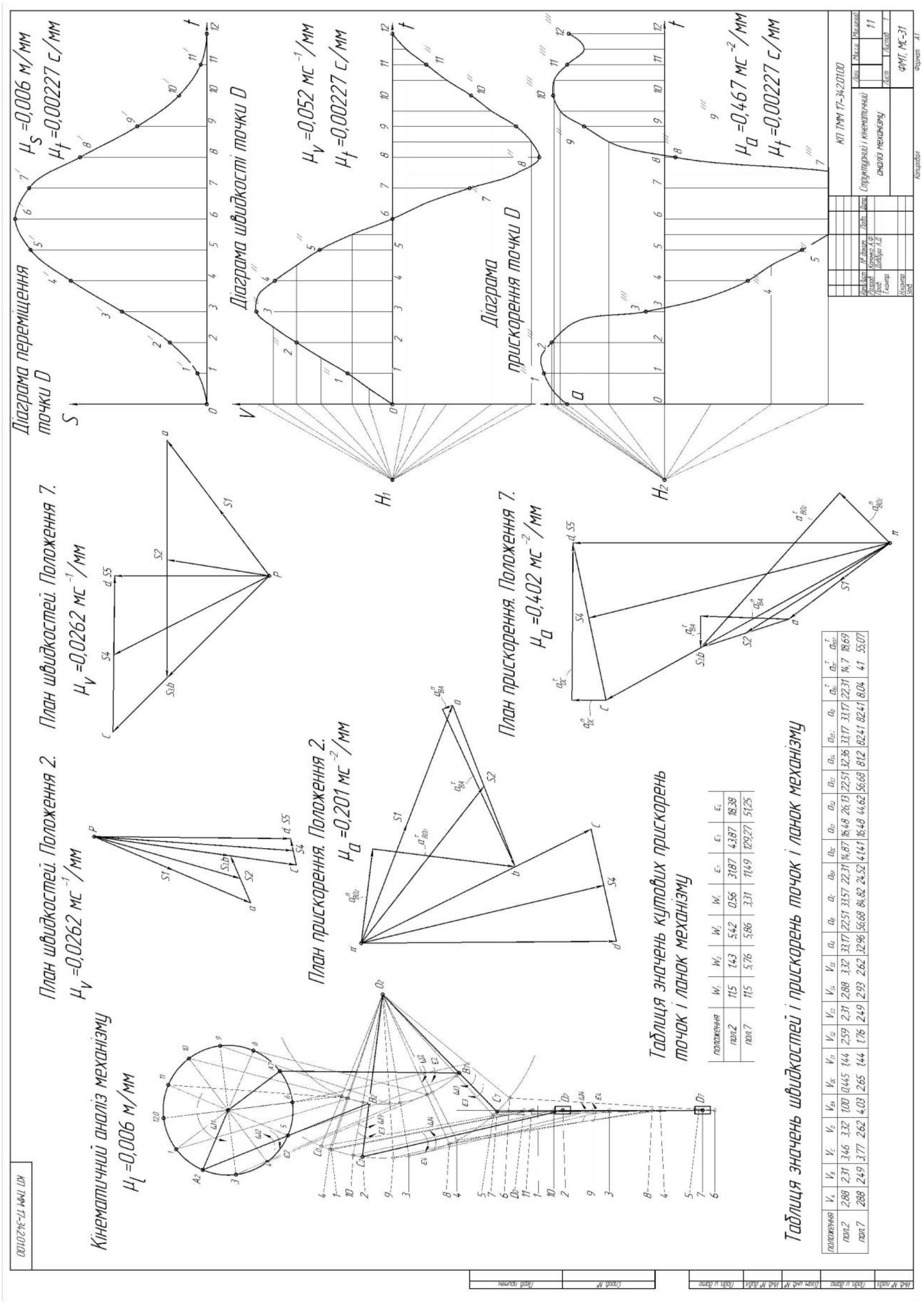
Output data	Version				
	1	2	3	4	5
a, mm	440	500	600	700	800
b, mm	700	750	800	900	1000
L_{O_1A}, mm	150	180	210	240	270
L_{AB}, mm	780	870	960	1080	1140
L_{O_2C}, mm	400	450	550	650	750
L_{CD}, mm	300	400	45	500	500
n_1, RPM	100	110	110	90	80
m_1, kg	20	25	28	32	35
m_2, kg	80	100	120	140	160
m_3, kg	40	50	55	60	65
m_4, kg	35	38	42	45	50
m_5, kg	10	15	20	25	28
F_{PR}, kN	4,0	4,5	5,0	5,5	6,0

Task 11



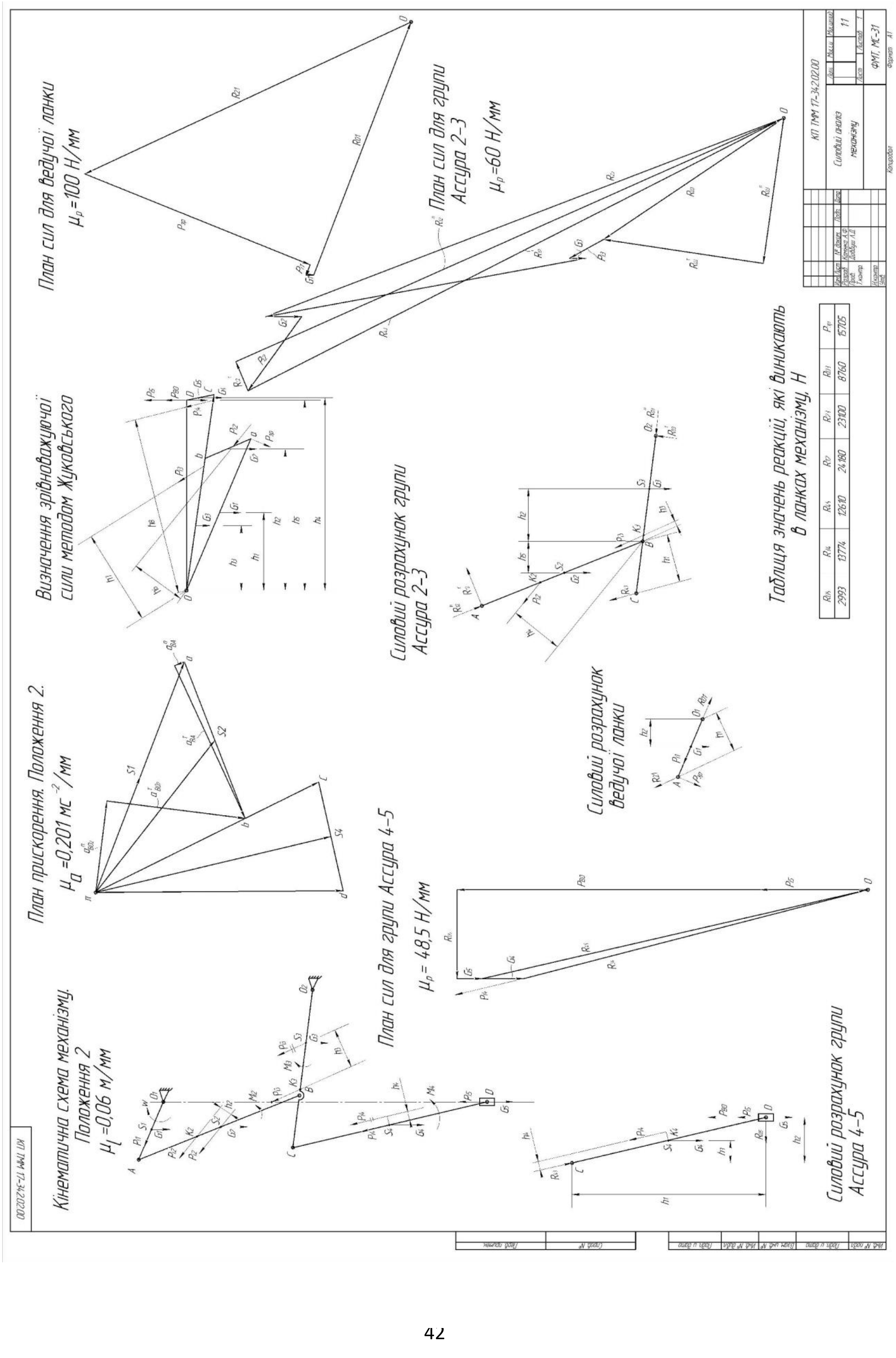
Output data	Version				
	1	2	3	4	5
a, mm	160	180	200	230	250
b, mm	200	220	240	280	320
c, mm	500	550	600	650	700
L_{O_1A}, mm	100	120	140	160	180
L_{AB}, mm	300	350	400	450	500
L_{O_2A}, mm	200	220	240	260	280
L_{O_2C}, mm	400	44	480	520	560
n_1, RPM	110	100	90	80	70
m_1, kg	10	12	14	16	18
m_2, kg	24	28	32	36	40
m_3, kg	28	31	34	37	40
m_4, kg	20	20	20	20	20
m_5, kg	0	0	0	0	0
F_{PR}, kN	6,0	6,6	7,2	7,8	8,4

APPENDIX A



Діаграма переміщення точки D
μ_S =0,006 м/мм
μ_t =0,00227 с/мм
Діаграма швидкості точки D
μ_V =0,052 мс$^{-1}$/мм
μ_t =0,00227 с/мм
Діаграма прискорення точки D
μ_a =0,467 мс$^{-2}$/мм
μ_t =0,00227 с/мм
План швидкостей. Положення 7.
μ_V =0,0262 мс$^{-1}$/мм
План прискорення. Положення 7.
μ_a =0,402 мс$^{-2}$/мм
План швидкостей. Положення 2.
μ_V =0,0262 мс$^{-1}$/мм
План прискорення. Положення 2.
μ_a =0,201 мс$^{-2}$/мм
Кінематичний аналіз механізму
μ_l =0,006 м/мм
Таблиця значень кутових прискорень точок і ланок механізму
Таблиця значень швидкостей і прискорень точок і ланок механізму
КП ТММ 17-34.2.01.00
Структурний і кінематичний аналіз механізму
ФМТ, МС-31

APPENDIX B


План сил для ведучої ланки
План сил для групи Ассура 2–3
Визначення зрівноважуючої сили методом Жуковського
План прискорення. Положення 2.
Силовий розрахунок групи Ассура 2–3
Таблиця значень реакцій, які виникають в ланках механізму, Н
Кінематична схема механізму. Положення 2
План сил для групи Ассура 4–5
Силовий розрахунок ведучої ланки
Силовий розрахунок групи Ассура 4–5
Силовий аналіз механізму

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