Вісник Тернопільського національного технічного університету
Scientific Journal of the Ternopil National Technical University
2018, № 2 (90)
ISSN 2522-4433. Web: ***visnyk.tntu.edu.ua***

UDC 539.3

THE METHOD OF DYNAMIC STRESSES DETERMINATION OF MEDIA WITH TUNNEL CAVITIES

Olena Mikulich; Vasyl' Shvabjuk

Lutsk National Technical University, Lutsk, Ukraine

Summary. *The method of studying the dynamic stress state of elastic media with tunnel cavities under the non-stationary load as the system of concentrated forces, which are applied in the internal points of media, is developed. On the basis of the application of the Fourier transformation of time variable and the modification of the method of boundary integral equations for the case of the first exterior problem, the integral equations are written. The integral equations kernels are of the Cauchy type. In the paper for determination of hoop and radial stresses the analytic representations have been constructed. Applying the proposed in the paper method high accuracy of calculations in the study of transient processes is provided. On the basis of the proposed method the dynamic stress state of elastic media with tunnel cavities of different cross-sections under the impulse concentrated load is investigated.*
Key words: *non-stationary problem, tunnel cavity, concentrated force.*

Received 02.04.2018

Statement of the problem. Strength calculation of objects located in the areas of extreme seismic activity must be carried out taking into account the effect of the transitional processes of dynamic load. That is why application of the complete spatial-time picture of the stress state of the object, caused by the nonstationary load in the solid bodies, makes possible to provide its reliability.

Solving of non-stationary dynamic problems for the solid fracture bodies with the cavity-like defects is one of the most complicated in mechanics, as it needs to apply time transformations along with the methods of the fracture mechanics. That is why the problem of investigation of the dynamic stress state of solid bodies, softened by the cavities of the random constant cross-section, is the pressing problem from both the theoretical and practical points of view.

Analysis of the available results of investigations. To investigate the procedure of the dynamic processes in the limited bodies with the cavities-like defects and cracks in the papers by Chen G.M., Shahani A.R., Zhang J.Y., etc. the direct numerical methods of the finite differences and finite elements, were used. The advantage of such methods is that of the possibility to be used for the bodies with the random boundary and the defects of the random shape. Besides, the main disadvantages of the direct numerical methods are caused by the need of quantification of the motion equations in the whole body area, which being under the quick – changing load, requires the narrowing of the quantification net in order to provide the accuracy of calculations.

The papers by Guz O.M., Zozulya V.V., Kubenko V.D. [1] are devoted to the analytical solutions of some types of dynamic problems. Here, taking advantage of the Laplace time transformation and the series method, the analytical solution of the problem on the investigation of the effect of the axis-symmetric non-stationary load in the continuous elastic plate with the circular hole, was obtained. But due to this method, the analytical solutions are possible only for the case of the circular holes or cylinder cavities, which make the engineering needs less effective.

Corresponding author: Olena Mikulich; e-mail: shypra@ukr.net

In the paper [2] the method of investigation of the dynamic stress state of the multi-connected media with the tunnel cavities of random cross-sections, based on the Fourier transformation according to the time and the modification of the boundary integral equations method, was developed.

The objective of the paper is to develop the analytical-numerical method for the investigation of the dynamic stress state of bodies with the tunnel cavities under the non-stationary load as the system of concentrated forces applied in the internal points of the medium, which would make possible to investigate the stress distributions along the cavity boundary and to study the distribution of the radial stresses in the medium.

Statement of the task. Let us analyse homogeneous isotropic medium with the Young's module E and the Poisson's ratio ν, which has a tunnel cavity of the constant cross-section. In the case of the plane deformation let us mark the boundary counter of the cavity in the plane x_1Ox_2 as L. The area occupied by the body is marked as $Ox_1x_2x_3$, in the center of the body weight, having directed the axis Ox_3 along the axis of the cavity (Fig. 1).

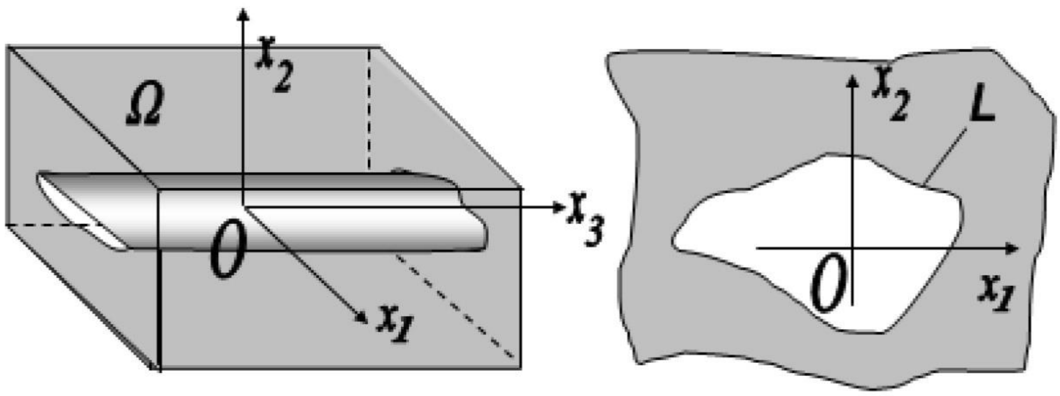

Figure 1. Model of elastic medium and cross-section of medium

Let us study the distribution of the dynamic stresses on the cavity boundary and the radial stresses in the medium under the system of concentrated impulse forces $\mathbf{P}_j = \left(P_{1j},\ P_{2j}\right)$ applied in the points $\left(a_{1j};\ a_{2j}\right)$.

Analysis of the available information. Having applied the Fourier transformation [3]

$$\widehat{f}(x,\,\omega) = \int\limits_{-\infty}^{\infty} f(x,\,t)e^{-i\omega t}dt,$$

to the motion equations of the classic theory of elasticity [4], we will obtain the equations:

$$\frac{\partial\hat{\sigma}_{mj}}{\partial x_m} + \hat{b}_j + \omega^2\rho\hat{u}_j = 0\,, \tag{1}$$

which are similar to those of the stable oscillations with the cyclic frequency ω [5]. Here $\hat{\sigma}_{mj}$, $\hat{b}_j$, $\hat{u}_j$ – the Fourier description of stresses, mass forces and displacements.

To investigate the non-stationary process in the media with the cavities the method of boundary integral equations is modified basing on the mutual application of the main principle of the weighted residual approach [6], the method of boundary elements [7] and the collocation method [6].

For the first main task the boundary conditions in the area of the Fourier representations are presented as follows [7]:

$$\hat{p}_j\Big|_L = \overline{\hat{p}}_j, \tag{2}$$

where $\hat{p}_j = \hat{\sigma}_{mj} n_m$ – the Fourier representation of the known functions on the boundary, $\vec{n}$ – the normal vector.

According to the weighted residual approach [6], taking into account the motion equations (1) and the boundary conditions (2), we can present:

$$\int_\Omega \left(\frac{\partial \hat{\sigma}_{mj}}{\partial x_m} + \hat{b}_j + \omega^2 \rho \hat{u}_j \right) u_j^* d\Omega = \int_L (\hat{p}_j - \overline{\hat{p}}_j) u_j^* dL, \tag{3}$$

where u_j^* – the fundamental solution of equations (1). Having integrated twice in parts the first component of equations (3) and having used the collocation method for the first main task, we can present potential representation for the displacements as follows:

$$\hat{u}_k = \int_L p_j \cdot U_{kj}^* \, dL + \int_\Omega \hat{b}_j \cdot U_{kj}^* \, d\Omega, \tag{4}$$

where $\hat{u}_k$ – displacement representation in the *k*-direction, U_{kj}^* – the fundamental functions, which correspond to the displacement in the *k*-direction, caused by the unit forces in the *j*-direction, $p_j = \overline{\hat{p}}_j - \hat{p}_j$ – unknown potential functions, *k, j =1, 2.*

The expressions for representation of functions U_{ij}^* are chosen taking into account the Zommerfield conditions [5] as follows [2]:

$$U_{ij}^* = \frac{1}{2\pi\mu} \left(K_0(\kappa_2 r)\delta_{ij} + \frac{\partial_i \partial_j}{\kappa_2^2} \left(K_0(\kappa_1 r) - K_0(\kappa_2 r) \right) \right), \tag{5}$$

where $\kappa_1 = i\omega / c_l$, $\kappa_2 = i\omega / c_\tau$ – wave numbers, $K_0(r)$ – modified Bessel's function of the third type zero order (or the McDonald's function), c_l, c_τ – the rates of wave expansion and shear, $c_l = \sqrt{(\lambda + 2\mu)/\rho}$, $c_\tau = \sqrt{\mu/\rho}$, λ, μ – the Lame's constants, $r = \sqrt{(x_1 - x_1^0)^2 + (x_2 - x_2^0)^2}$ – distance.

Having substituted the representations [4] in the formulas for finding stresses [4] taking into account (5), we will obtain the integral dependences:

$$\tilde{\sigma}_n = \int_L f_j\left(\mathbf{x}, \mathbf{x}^0\right) p_j ds; \quad \tilde{\tau}_{sn} = \int_L g_j\left(\mathbf{x}, \mathbf{x}^0\right) p_j ds \, ,$$

where $f_j,\ g_j,\ j=1,\ 2$ – the known functions [2]. Having separated the irregular components in the sub-integral functions and having applied the Plemell-Sokhotsky formulas [4] during the boundary transition, we will obtain the system of integral equations for finding unknown values on the boundary of the functions $p_1,\ p_2$:

$$\begin{aligned}
&\frac{1}{2}\operatorname{Re} q+\mathbf{v.p.}\int_L\left(f_1\left(\mathbf{x},\ \mathbf{x}^0\right)q d\zeta+f_2\left(\mathbf{x},\ \mathbf{x}^0\right)\bar{q} d\bar{\zeta}\right)=-\int_\Omega\left(f_1\left(\mathbf{x},\ \mathbf{x}^b\right)b+f_2\left(\mathbf{x},\ \mathbf{x}^b\right)\bar{b}\right)d\Omega;\\
&\frac{1}{2}\operatorname{Im} q+\mathbf{v.p.}\int_L\left(g_1\left(\mathbf{x},\ \mathbf{x}^0\right)q d\zeta+g_2\left(\mathbf{x},\ \mathbf{x}^0\right)\bar{q} d\bar{\zeta}\right)=-\int_\Omega\left(g_1\left(\mathbf{x},\ \mathbf{x}^b\right)b+g_2\left(\mathbf{x},\ \mathbf{x}^b\right)\bar{b}\right)d\Omega;
\end{aligned} \tag{6}$$

where $b=b_1+ib_2$, $q=i\cdot pdL/d\zeta$, $p=p_1+ip_2$ – the unknown function. Here the integrals will be treated as those of the main value (meaning)

In the case, when the dynamic stress state of the medium is caused by the system of the concentrated forces $P_j=P_j^{(1)}+iP_j^{(2)}$ applied in the internal points (a_{1j},a_{2j}), $j=\overline{1,\ J}$, the vector of the volume forces can be presented as follows:

$$b_1=\sum_{j=1}^{N}P_{1j}\delta(x_1-a_{1j})\delta(x_2-a_{2j}),\ \ b_2=\sum_{j=1}^{N}P_{2j}\delta(x_1-a_{1j})\delta(x_2-a_{2j}).$$

The system of integral equations (6) will be solved numerically basing on the approach [8], which is based on the method of the mechanical quadratures. Here, for the integrals with the Cauchy type kernels, the specified quadrature formulas are used [9]. Application of the algorithm [8] makes it possible to build the system of the linear algebraic equation for finding unknown values on the functions boundary.

Having substituted the potential representation [4] in the formulas for finding the hoop and radial stresses [4], we can present:

$$\begin{aligned}
&\sigma_\theta=\frac{\vartheta}{4}\operatorname{Re} q+\mathbf{v.\check{\delta}.}\int_L\left(h_1\left(\tilde{\mathbf{o}},\ \mathbf{x}^0\right)q d\zeta+h_2\left(\tilde{\mathbf{o}},\ \mathbf{x}^0\right)\bar{q} d\bar{\zeta}\right)+\int_\Omega\left(h_1\left(\tilde{\mathbf{o}},\ \mathbf{x}^b\right)b+h_2\left(\tilde{\mathbf{o}},\ \mathbf{x}^b\right)\bar{b}\right)d\Omega;\\
&\sigma_r=\int_L\left(\underline{f}_1\left(\mathbf{x},\ \mathbf{x}^0\right)q d\zeta+\underline{f}_2\left(\mathbf{x},\ \mathbf{x}^0\right)\bar{q} d\bar{\zeta}\right)+\int_\Omega\left(\underline{f}_1\left(\mathbf{x},\ \mathbf{x}^b\right)b+\underline{f}_2\left(\mathbf{x},\ \mathbf{x}^b\right)\bar{b}\right)d\Omega;
\end{aligned} \tag{7}$$

where $h_j,\ \underline{f}_j$ – the known functions [10], $\mathbf{x}^b=\left(a_{1j};\ a_{2j}\right)$ – the points cross-section coordinates, in which the concentrated dynamic forces are applied.

To find the originals obtained, basing on the formulas of stresses [7], the discrete Fourier transformation is used, which under the numerical calculations is realized due to the rapid discrete Fourier transformation basing on the Kulli-Tuki algorithm [11] at $K=2^k$, where k – the whole positive number.

The results of investigations. Taking advantage of the developed method, let us investigate the distribution of the dynamic hoop stress on the cavity boundary with the circular, half-circular and arched cross-sections. During the numerical calculations it was assumed, that the concentrated dynamic forces are applied in the points $(\pm1.5a;\ 0)$, where a – the radius of the circular hole. The origin of coordinates was located in the weight center of the cavity cross-section.

Under the numerical calculations the change of the concentrated forces intensity was assumed to be a weak impact impulse [12]:

$$f_*(\tau) = p_* \tau^{n_*} e^{-\alpha_* \tau}, \ \ \tau > 0;$$

where p_*, n_*, α_* – constants, $\tau = c_l t / a$ – dimensionless time parameter. The determination of stresses were performed for the interval of the dimensionless time parameter $\tau \in [0; 8]$ at the impulse duration $\tau_* = 2$.

The numerical calculation were performed for 80 nodal points on the boundary of the circular cross-section and 120 points for the non-circular one. The calculations of the dynamic stresses were performed for the medium of ρ=7,8·10^3 *kg/м*^{3} density, the Young's modulus E=2,0·10^5 *MPa* and the Poisson's ratio ν=0,27.

In Fig. 1 the results of the numerical calculations of the relative hoop stresses $\bar{\sigma}_\theta$ on the boundary of the tunnel cavity of the circular (Fig. 1, a), half- circular (Fig. 1, b) and arched (Fig. 1, c) cross-sections are presented. Here $\bar{\sigma}_\theta = \sigma_\theta / \sigma_0$, where $\sigma_0 = 1$ *ГПа*.

Time snapshots of the hoop stresses for the case of the half-circular and arched cross-sections are presented in Fig. 2, a and Fig. 2, b at the values of the dimensionless time parameter $\tau = 0.5;\ \tau = 0.75;\ \tau = 1$ correspondingly

During stresses calculation for the description of the cavities boundary shapes those of half-circle and the arch one, the dependences as the series were used for the comformal depiction of the circle on the given area [5].

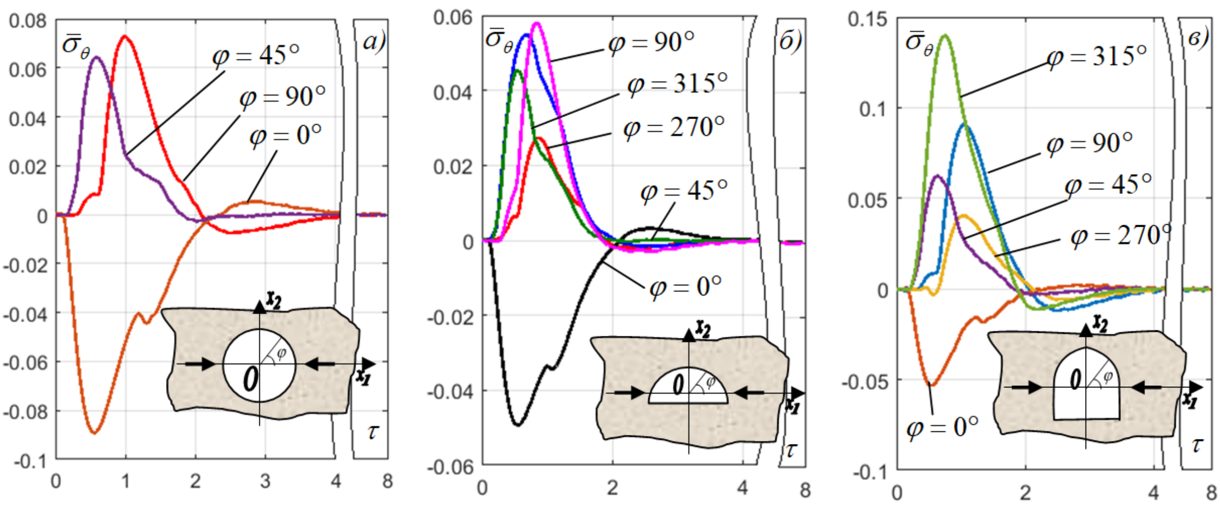

Figure 2. The distribution of relative hoop stresses on the boundary of circular, half-circular and arched tunnel cavities

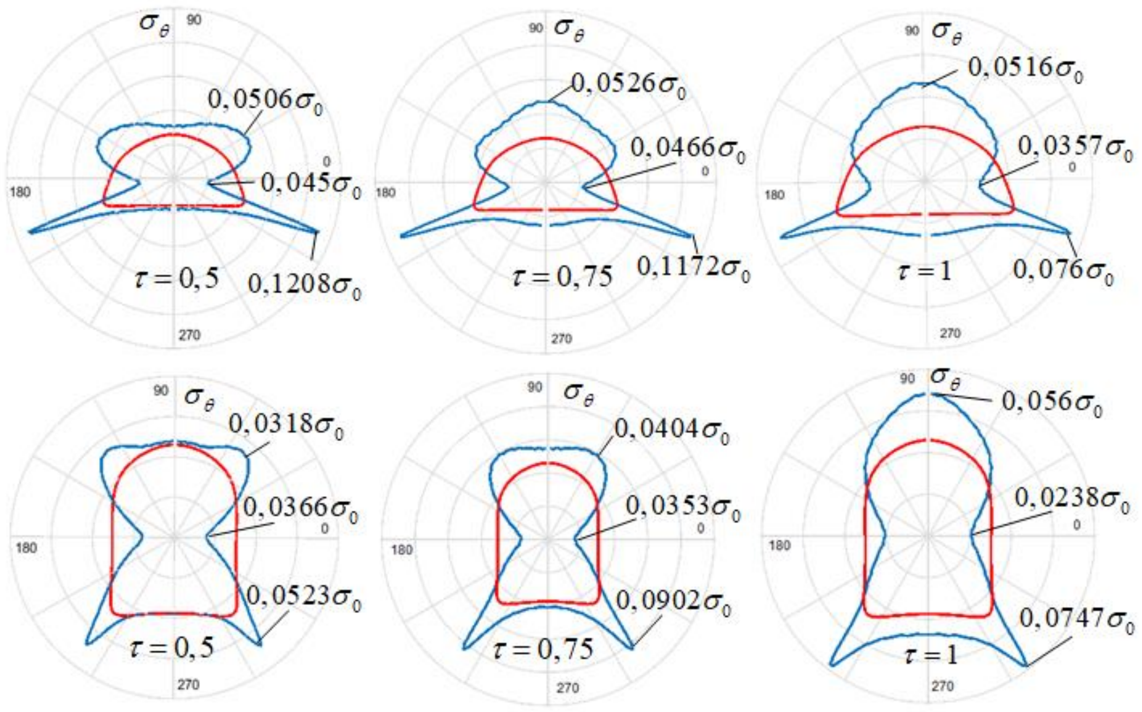

Figure 3. Fixed time snapshots of hoop stresses on the boundary of half-circular and arched cavities

The values of the relative radial stresses in the cross-sections being in distance of $\delta = 2a$; $4a$; $6a$; $10a$ from the center of the circular cross-section cavity on the axis Ox_1 and Ox_2, are presented in Fig. 3, a and Fig. 3, b, correspondingly. During the numerical calculations it was assumed, that $\overline{\sigma}_r = \sigma_r / \sigma_0$.

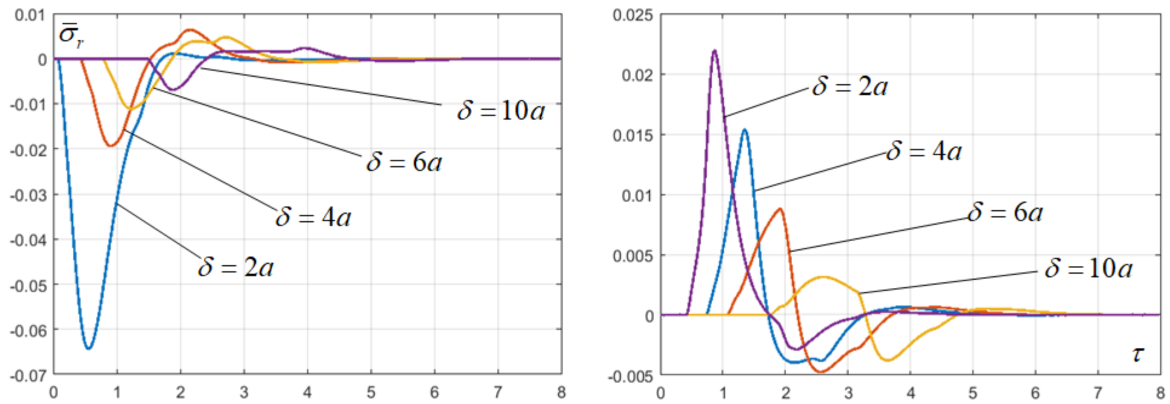

Figure 4. The distribution of relative radial stresses in the internal points of medium with tunnel cavities of circular cross-section

It is seen from the Fig. 3, that the obtained calculation results of the circular and radial stresses correspond well to the main principles of the wave mechanics: the stress values equal zero up till the wave approaches the corresponding cross-section. This fact testifies the reliability of the developed method.

Fig. 1 and Fig. 2 demonstrate, that the hoop stresses on the boundary of the cavity depend greatly on the shape of the cavity cross-section. In the case of cross-sections possessing “angle” points, the maximum values of stresses originate in these points.

Conclutions. Taking advantage of the developed method, based on the mutual application of the Fourier transformation, weighted-residual approach, the method of boundary integral equations and collocations, will make possible to obtain the complete spatial-time picture of the stress state of media with the tunnel cavities of almost random cross-sections.

References

1. Kubenko V.D. Nonstationary contact of a rigid body with an elastic medium: Plane problem (Review), Int. Appl. Mech., vol. 48, no. 5, 2012, pp. 121 – 129.
2. Mikulich O., Shvabjuk V. Investigation of the impact of the shock waves on the dynamic stress state of the medium with the system of tunnel cavities, Digest of TNTU, no. 3 (87), 2017, pp. 7 – 15.
3. Salih M.S. Fourier Transform – Signal Processing and Physical Sciences, London, InTech, 2015, 222 p.
4. Bozidarnyk V.V., Sulym G.T. Elements of theory of elasticity, Ljviv, Svit. 1994, 560 pp. [In Ukrainian].
5. Savin G.M. The distribution of the stresses near the holes, Kyiv, Naukova dumka, 1968, 888 p. [In Russian].
6. Hatami M. Weighted Residual Methods. Principles, Modifications and Applications, London, Elsevier, 2017, 384 p.
7. Banerjee P.K. Boundary element method in engineering science, McGraw Hill College, New York, 1994, 496 p.
8. V. Shvabyuk, H. Sulym, O. Mikulich Stress state of plate with incisions under the action of oscillating concentrated forces, Acta Mechanica et Automatica, vol. 9, no. 3, 2015, pp. 140 – 144.
9. Panasyuk V.V., Savruk M. P., Nazarchuk Z.T. The method of singular integral equations for two-dimensional diffraction problems, Kyiv, Naukova dumka, 1984, 346 p. [In Russian].
10. O.A. Mikulich, V.I. Shab’yuk Interaction of weak shock waves with rectangular meshes in plate, Odes’kyi Poliethnichnyi Universytet. PRASTI, vol. 2 (49), 2016, pp. 104 – 110.
11. T.V. Sidorova, T.V. Zykova, K.V. Safonov About modification of one-dimensional Fast Fourier transform on algorithm of Cooley-Tukey, Vestnik SibGAU, vol. 16, no. 2, 2015, pp. 360 – 363. [In Russian].
12. Ia. Pasternak, H. Sulym, R. Pasternak Dynamic stress concentration at thin elastic inclusions under the antiplane deformation, Physic and mathematic modeling and information technology, vol. 18, 2013, pp. 157 – 164 [In Ukrainian].

Список використаної літератури

1. Kubenko, V.D. Nonstationary contact of a rigid body with an elastic medium: Plane problem (Review). [Text] / V.D. Kubenko // Int. Appl. Mech. – 2012. – vol. 48, №. 5. – Pp. 121 – 129.
2. Mikulich, O., Investigation of the impact of the shock waves on the dynamic stress state of the medium with the system of tunnel cavities [Text] / O. Mikulich, V. Shvabjuk // Digest of TNTU. – Тернопіль, 2017. – № 3 (87). – С. 7 – 15.
3. Salih, M.S. Fourier Transform-Signal Processing and Physical Sciences [Text] / M.S. Salih. – London: InTech, 2015. – 222 pp.
4. Божидарнік, В.В. Елементи теорії пружності [Текст] / В.В. Божидарнік, Г.Т. Сулим. – Л.: Світ, 1994. – 560 с.
5. Савин, Г.Н. Распредиление напряжений около отверстий [Текст] / Г.Н. Савин. – К. : Наукова думка, 1968. – 888 с.
6. Hatami, M. Weighted Residual Methods. Principles, Modifications and Applications [Text] / M. Hatami. – London : Elsevier, 2017. – 384 pp.
7. Banerjee, P.K. Boundary element method in engineering science [Text] / P.K. Banerjee – McGraw Hill College, New York, 1994. – 496 pp.
8. Shvabyuk, V Stress state of plate with incisions under the action of oscillating concentrated forces [Text] / V. Shvabyuk, H. Sulym, O. Mikulich // Acta Mechanica et Automatica. – 2015. – vol. 9, № 3. – Pp. 140 – 144.
9. Метод сингулярных интегральных уравнений в двумерных задачах дифракции [Текст] / В.В. Панасюк, М.П. Саврук, З.Т. Назарчук. – К. : Наукова думка, 1984. – 346 с.
10. Mikulich, O.A. Interaction of weak shock waves with rectangular meshes in plate [Text] / O.A. Mikulich, V.I. Shvabjuk // Odes’kyi Poliethnichnyi Universytet. PRASTI. – 2016. – № 2 (49). – С. 104 – 110.
11. Сидорова, Т.В. О модификации быстрого одномерного преобразования Фурье по алгоритму Кули–Тьюки. [Текст] / Т.В. Сидорова// Вестник СибГАУ. – 2015. – Т. 16, № 2. – С. 360 – 363.
12. Пастернак, Я. Концентрація динамічних напружень біля тонких пружних включень за умови антиплоскої деформації [Текст] / Я. Пастернак, Г. Сулим, Р. Пастернак // Фізико-математичне моделювання та інформаційні технології. – Львів. – 2013. – Вип. 18. – С. 157 – 164.

УДК 539.3

МЕТОД ВИЗНАЧЕННЯ ДИНАМІЧНИХ НАПРУЖЕНЬ У СЕРЕДОВИЩАХ З ТУНЕЛЬНИМИ ПОРОЖНИНАМИ

Олена Мікуліч; Василь Шваб'юк

Луцький національний технічний університет, Луцьк, Україна

Резюме. *Розроблено метод визначення динамічних напружень у пружних середовищах з тунельними порожнинами за дії нестаціонарного навантаження. Дослідження динамічного напруженого стану проведено для випадку, коли у внутрішніх точках середовища прикладена система зосереджених змінних у часі сил. На основі застосування перетворення Фур'є за часом, зважено-залишкового підходу, методу колокації та модифікації методу граничних інтегральних рівнянь для випадку першої основної задачі записано інтегральні рівняння, ядра яких мають особливості типу Коші. Застосування методу механічних квадратур за числового розрахунку дало можливість звести нестаціонарну задачу механіки деформівного твердого тіла до розв'язання системи лінійних рівнянь для визначення невідомих функцій. Визначення кільцевих та радіальних напружень проводено на основі побудованих у роботі аналітичних представлень, що забезпечує високу точність розрахунків за використання розробленого методу при дослідженні перехідних процесів. Розрахунок оригіналів динамічних напружень проведено на основі застосування оберненого дискретного перетворення Фур'є, що за числових розрахунків реалізовано згідно з алгоритмом Кулі-Тьюкі. Використовуючи запропонований у роботі метод, досліджено динамічний напружений стан пружних середовищ з тунельними порожнинами різних перерізів за дії імпульсного зосередженого навантаження. На основі числових розрахунків вивчено вплив форми перерізу порожнини на розподіл кільцевих напружень на її границі. Побудовано часові зрізи полів кільцевих динамічних напружень на границі півкругового та аркового перерізу. Розраховано значення динамічних радіальних напружень у внутрішніх перерізах середовища. Запропонований у роботі метод може бути використаний для дослідження перебігу нестаціонарних процесів у пружних середовищах.*

Ключові слова: *нестаціонарна задача, тунельна порожнина, зосереджена сила.*

Отримано 02.04.2018