UDC 539.375

ANALYSIS OF STRESS INTENSITY FACTORS OBTAINED WITH THE FEM FOR SURFACE SEMIELLIPTICAL CRACKS IN THE ZONES OF STRUCTURAL STRESS CONCENTRATORS

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Summary. In the given article the analysis of stress intensity factors (SIF) for semielliptical surface cracks in low-alloy steel using the finite element method was conducted. The results of the distribution of stress intensity factors along the front of the semielliptical surface cracks in the zones of structural stress concentrators are obtained.

Key words: semielliptical surface crack, finite element method, stress intensity factor, structural stress concentrators.

Received 21.06.2018

Statement of the problem. Over the past decades, the evaluation of the welds' resource has become an important element both in designing and in assessing durability of metal constructions under operation. Characteristic features of welded joints in terms of strength and durability are the heterogeneity of the physical and mechanical properties of the material in the welded joint zone, the appearance of residual welding stresses, the presence of high stress concentration associated with the structural features and possible technological defects, degradation of material properties during operation. [1] In most cases, the fatigue fracture of structures has clearly explicit local behavior and is initiated by originally existing defects, as well as by constructive and technological stress concentrators. An important issue is the determination of the stress-strain state in three-dimensional bodies with real cracks (in particular, surface ones). The fundamental role in the theory of cracks is played by the stress intensity factor (SIF). SIF describes the stress-strain state near crack tip, in particular along the contour of surface crack, which most often becomes the initiator of the fracture of structural elements [2, 3].

Problem issues in the evaluation of the durability of the construction with surface cracks are the complexity of calculating stress intensity factors for cracks developing in zones of stress concentrators. These difficulties are related to the peculiarities of changing the shape of the contour of the surface crack during cyclic loading, which depends on the level of heterogeneity of the stress-strain state in the area of the concentrators, as well as the stochasticity of the initiation of several surface cracks and their possible coalescence [1, 2].

Analysis of the recent investigations. The growth of a crack in the conditions of a macro-elastic stress state of a weld joint during cyclic loading is determined by stress intensity factors. Analytical methods for calculation of the SIF for surface cracks in structures are practically missing due to mathematical difficulties.

Three common approaches: the weight function method; the numerical method; and the hybrid method, have been used to calculate the SIF for the crack propagation simulation of welded joints [4].
The weight function method was used to determine the SIF of the welded joints through two steps [5, 6]. The first step was to determine the reference SIF solution for certain welded joints as the weight function solution. The second step was to obtain the stress field at the position of the fatigue crack in an uncracked body. The stress distribution of the uncracked body is usually calculated using the finite element method (FEM). When the weight function and the stress field are obtained, they can be used to determine the SIF for any arbitrary loading on the crack faces.

Numerical methods have been developed to determine the SIF of welded joints, such as the finite element method (FEM) [7], the extended finite element method (XFEM) [8, 9], and the boundary element method (BEM) [10, 11]. The most widely used numerical method for calculating the stress-strain state of the structural elements with cracks is finite element method. The accuracy of the computed SIF is dependent on many factors, including the type of elements; mesh quality; mesh refinement; integration schemes; and the shape of the welding around the crack front. These factors control the accuracy of the stress and displacement fields obtained from the numerical models of the welded joints. However, these numerical methods often rely on complex mesh design and element selection, making them time consuming and always require experimental validation.

The hybrid method is an empirical method [12], which has attracted increasing attention due to its desirable accuracy and rapid calculation of the SIF. Currently, Japanese Society of Steel Construction (JSSC), British Standards Institution (BSI) and the International Institute of Welding (IIW) fatigue document are the three major fatigue design specifications to provide the SIF formula for the calculation of the SIF for welded joints and are all derived from the hybrid method [4].

**Statement of the task.** Development of approaches for the modeling of semi-elliptical surface cracks in the zones of concentrators, and their influence on the stress intensity factors $K_I$ along the contour of cracks under tension.

**Main thesis.** Stress intensity factor $K_I$ in any part of the front semi-elliptical surface crack (Fig. 1) in a homogeneous field of stress can be calculated as [13, 14]:

$$K_I = \frac{\sigma \sqrt{\pi a}}{E(k)} \cdot F_s \left( \frac{a}{c}, \frac{c}{b}, \varphi \right),$$  

(1)

where $\sigma$ – nominal stress, MPa; $F_s \left( \frac{a}{c}, \frac{c}{b}, \varphi \right)$ – correction function that takes into account the shape of the crack and the effect of free surfaces; $a$ and $c$ – depth and half length of the crack, respectively; $t$ and $b$ – thickness and width of the sample respectively; $\varphi$ – the parametric angle, which determines the position of the points of the contour of the surface crack; $E(k)$ – complete elliptical integral of the second kind.
The distribution of stress intensity factors along the crack front is obtained using finite elements method (FEM). A specialized software package «ANSYS-Workbench – 16.1» is used [15]. Semielliptical surface crack is simulated in finite size plate under tensile stress $\sigma=200$ MPa according to the method presented in the work [16]. A three-dimensional sample model with a global elements mesh and model of the crack area with a local mesh are created (Figure 2). Mesh elements are tetrahedral. The sample of thickness $t=20$ mm made of low-alloy steel with yield stress limit $\sigma_T = 380$ MPa is modeled. Poisson's ratio at elastic deformation is $\nu = 0.3$.

As a result of calculations, the SIF values along the contour of surface cracks $\varphi = 0\ldots 2\pi$ identified at 49 points are obtained. In the figure 5 the results of calculation of the SIF are presented with next dimensions of the surface crack: $a = 3$ mm; $c = 4.3$ mm ($a / c \approx 0.7$).
To estimate the SIF in the area of concentrators, the modified equation (1) is applied introducing the correction function $M_K$, specifying the redistribution of stresses in the vicinity of the cracks near the concentrators [17]:

$$K_I = \frac{\sigma\sqrt{\pi a}}{E(k)} \cdot F_s \left( \frac{a}{c'}, \frac{a}{t}, \frac{c}{b}, \varphi \right) \cdot M_K, \quad (2)$$

where $M_K$ – correction function that takes into account the structural concentration of stresses. Structural concentration of stress integrally evaluates the stresses associated with the connection of elements of different geometry, as well as the stresses associated with the geometry of the weld. It should be noted that the structural stress concentration has to be evaluated at the design stage, in contrast to the technological factor that determines the statistics of the transition radius from one surface to another (for welded joints, the radius of the joining of the base metal seam). The search procedure for $M_K$ must be strictly regulated, since the value of this parameter depends on the method of extrapolation of local stresses, and when applying the finite element method (FEM) to determine the stresses depends on the size of the elements of the mesh approximation [18].

It should be noted that due to the fact that it is impossible to simulate the geometry of the weld because of its irregular shape and presence of defects, the problem is solved by introducing an effective rounding radius in the concentrator [17], which is about 1 mm for the steel. The correctness of this approach was confirmed by the results of numerous experimental studies of welded joints of various types. Similar approaches are considered in the work [18], which states that for manual and semi-automatic welding with low-alloy materials without additional measures the effective value of the rounding radius of the concentrator is 1.2 mm.

Taking into account the size of the radius of rounding, as well as the peculiarities of finite elements mesh generating along the contour of the surface crack, its model is located at a certain distance from the weld. In our case, it is 1.5 mm.

While generating finite-element mesh, it is necessary to take into account the high gradient of stresses and deformations at the tip of the semi-elliptical surface crack, as well as the gradient of stresses near the concentrator. In this regard, it is necessary to rationally construct a finite elements mesh of object. According to this, a global mesh was used in the area of small changes of stress and strain with a size of elements up to 10 mm and two dense meshes of tetrahedral elements – local along the contour of the surface crack to evaluate the SIF (with the element size of 0.1 mm) and the transitional local mesh in the structural stress concentrator. An additional local mesh is designed to correctly evaluate the effect of the geometry of the weld and weld joint on the SIF of surface crack located in the area of concentrator, as well as for the correct combination of elements of the local mesh in the vicinity of the crack tip with elements of the global mesh while approaching to the contour of surface crack.

An estimation of the influence of the size of the transitional mesh elements of 1 mm adopted by us in the proposed model and 0.25 mm recommended in [23] with the magnitude of the effective stress concentrator equal to 1 mm. The crack is modeled in a welded joint with a longitudinal rib (Fig. 4c).
The calculations of SIF that were carried out for both variants (Fig. 3) show that the divergence of values did not exceed 1% for any point of the contour of semi-elliptical surface crack. In further simulation, the transitional local mesh with elements size of 1 mm was selected, which significantly saved computational resources of the computer.

Figure 3. Calculation of SIF $K_1$ along the contour of semi-elliptical surface crack with the size of the transitional local mesh: a) 1mm; b) 0.25 mm

In the study of the influence of structural stress concentrators on the magnitude of the SIF of semi-elliptical surface cracks under tensile stress, models with cracks in the weld reinforcement zones (Fig. 4a), welded plate (Fig. 4b) and a joint with a longitudinal rib (Fig. 4c) were considered.
Figure 4. Finite elements models of semi-elliptical surface crack in the zones of: a) weld reinforcement; b) welded plate; c) joint with a longitudinal rib for one crack; d) joint with a longitudinal rib for two cracks
We considered models with a simple geometric form and two-dimensional stress state (butt joint tee joint, joint with a welded plate), in which a semi-elliptical surface crack was located along the concentrator, as well as models of more complex geometric forms and three-dimensional stress states (joint with the longitudinal rib) when crack extends beyond the border of the local concentrator.

As a test task, the simulation of semi-elliptical surface crack under tensile stress equal to $\sigma_n = 200$ MPa in the zone of influence of the roller, which is a structural stress concentrator, and the calculation of SIF $K_1$ along the contour of this crack, was considered. A semi-elliptical surface crack was simulated with the following parameters: $a = 3$ mm; $c = 4.3$ mm; $a/c \approx 0.7$. The thickness of the sample $t = 20$ mm; its width $b = 80$ mm; radius of roller $r = 10$ mm.
The comparison of the SIF $K_1$ values obtained by the FEM for the semi-elliptical surface crack located in the zone of influence of the roller and the results of the SIF $K_1$ obtained by the method [19] shows that their divergence does not exceed 0.5% for the deepest point of the crack and 3.6% for the surface points of the same crack. The above comparison indicates the adequacy of the proposed models for the estimation of the SIF $K_1$ along the front of the semi-elliptical surface crack in the zones of structural stress concentrators.

Comparison of the calculated results of the SIF $K_1$ along the surface of the semi-elliptical surface crack for the investigated cases is given in Figure 5 and Table 1. These results were compared with the values of the SIF for a similar crack located in a homogeneous field of stress, the finite-element model of which is shown in Figure 2.

### Table 1

The ratio of the SIF $K_1$ for the characteristic points of a surface crack in the zone of the concentrator and in a homogeneous field of stress

<table>
<thead>
<tr>
<th>№</th>
<th>Type of concentrator</th>
<th>For one crack</th>
<th>For two cracks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Homogeneous field of stress</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>Weld reinforcement</td>
<td>1.11</td>
<td>0.979</td>
</tr>
<tr>
<td>3</td>
<td>Joint with welded plate</td>
<td>1.18</td>
<td>1.03</td>
</tr>
<tr>
<td>4</td>
<td>Joint with a longitudinal rib</td>
<td>1.20</td>
<td>1.23</td>
</tr>
</tbody>
</table>
Analysis of stress intensity factors obtained with the fem for surface semielliptical cracks in the zones of structural stress concentrators

It should be noted that the change of the SIF $K_I$ coincided in all cases in terms of quality and is characterized by a significant increase for the surface points of the contour (points B and C): by 11% for the zone of structural influence of the roller on the crack; and by 18% for the zone of influence of the welded plate. At the same time the change of the SIF $K_I$ for the deepest point of the contour of the crack (point A) is insignificant (up to 4%). Note that for the designing of a model with an attached plate, the geometric parameters of the joint were as follows: the width of the sample $- b = 80$ mm; thickness $- t = 20$ mm; cathetus of the weld is equal to 6 mm. Nominal tensile stress $- \sigma_n = 200$ MPa.

Consider the case of simulation of a transverse surface crack in the zone of influence of local stresses from the attached longitudinal element to the plate under tension. Two models are considered: the symmetric location of two semi-elliptic surface cracks under the attached element (the distance between the cracks was 3 mm), as well as one crack located asymmetrically to the rib. The sample section is 120 mm x 20 mm. The rib was sized 80 mm x 40 mm x 10 mm. Surface crack with parameters $a = 3$ mm; $c = 4.3$ mm was located at a distance of 1.5 mm from the axis of the rib and located at a distance of 1.5 mm along the weld perpendicular to the longitudinal edge of the rib.

The analysis of the results of the SIF $K_I$ for the characteristic points of the surface crack located in the zone of the concentrator shows their maximum increase: by 47% for the surface point in the central region of the influence of the rib; by 23% for the deepest point and by 20% for the initial stages of the exit of the crack from the zone of influence of local stresses (Fig. 5). The mutual effect of two cracks located in the zone of concentrator is insignificant. SIF $K_I$ along the front of the crack increases, but does not exceed 2%, when the distance between cracks is 3 mm.

As it is shown, the structural stress concentrators influence the stress intensity factors $K_I$, significantly increasing their values along the contour of the surface crack. The intensity of the influence increases with the higher theoretical coefficients of stress concentration of the investigated elements of structures [1]. Due to the heterogeneity of the stress-strain state in the thickness of the samples, the redistribution of the values of SIF $K_I$ along the front of surface crack occurs: there is a significant increase of the SIF $K_I$ on the surface and much less at the deepest point.

Since the stress field in the investigated models under study is heterogeneous along the contour of surface crack, it is also important to know the trajectory of its development, in addition to the values of the SIF that describe the strain-stress state in the vicinity of the crack front [20]. In particular, for the fatigue semi-elliptical surface crack, the configuration of its contour is determined by changing the ratio of the semi axes of the crack.

Conducted experimental tests of samples with initiated surface cracks during cyclic tensile loading (Fig. 6) demonstrated significant differences in the shape of these cracks when they were developed in a homogeneous field of stress and in the zone of the weld joint in which the heterogeneous stress state is realized for linear concentrators (cracks are located along the weld) [21, 22]. It should be noted that when conducting experimental tests for the initiation of surface cracks in zones of welded joints, notches were made at distances of 1.3-1.75 mm from the edge of the weld. The notches with a depth of 2-2.3 mm were carried out with a thin disk milling cutter of a diameter of 27 mm and a thickness of 0.13 mm.
The analysis of the kinetics of the propagation of the surface crack (Fig. 6) with its artificial initiation indicates a smooth change in the ratio of the semi axes of semi-elliptical surface crack to the values of $a/c = 0.7...0.78$ (a crack in a homogeneous field of stress) and $a/c = 0.6 ... 0.65$ (development of a crack in the zone of a structural stress concentrator) at $a/t < 0.6$. With an increase in the depth of the semi-ellipse $a/t > 0.6$ a slight decrease in $a/c$ is observed, which is explained by the influence of the free surfaces of the sample.

The obtained experimental data on the durability of the semi-natural specimens modeling the welded tee joints with the semi-elliptical surface crack and their comparison with the development of a similar crack developed in a homogeneous field of stress indicate that at the same depth of cracks and their different configuration, the durability of the element may decrease to 20%. Similar data are obtained for the crack initiated at the edge of the weld seam of the welded plate [22]. For the case of natural initiation of the cracks in the fusion zone of the plate with base metal, when there is a multicellular initiation of the cracks and their coalescence, the durability of the element decreases by several times.

**Conclusions.** 1. Approaches to the simulation of semi-elliptical surface cracks in the zones of stress concentrators are developed. In the simulation a global mesh was used in the area of small changes in stresses and deformations and two condensed meshes from tetrahedral elements: a local along the contour of a surface crack to evaluate the SIF and the transitional local mesh in the region of a constructive stress concentrator. Modeling of weld joints of different types was carried out within the framework of the presented model as a geometric factor – weld reinforcement, which allowed to quantify the influence of the stress concentrator on the change of the SIF on the contour of the surface cracks.

2. It was established that the qualitative nature of the change of the SIF $K_I$ coincides for all the investigated cases and is characterized by a significant increase of $K_I$ for the surface points of the crack contour in the zone of influence of the structural concentrator on the crack. At the same time, the change of SIF $K_I$ for the deepest point of the crack contour is much
smaller, and for the model of the weld joint the SIF slightly decreases in comparison with the crack in a homogeneous field of stress.

3. This type of change of SIF $K_I$ in the zones of the concentrators leads to elongation of the contour of the crack and to the change of durability of elements with cracks, confirmed by the results of experimental studies.

References

Список використаної літератури


ISSN 2522-4433. Вісник ТНТУ, № 2 (90), 2018
АНАЛІЗ КОЕФІЦІЄНТІВ ІНТЕНСИВНОСТІ НАПРУЖЕНЬ, ОТРИМАНИХ МСЕ ДЛЯ ПОВЕРХНЕВИХ ПІВЕЛІПІТНИХ ТРІЩИН У ЗОНАХ КОНСТРУКТИВНИХ КОНЦЕНТРАТОРІВ НАПРУЖЕНЬ

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Резюме. Проблемними питаннями при оцінюванні живучості конструкції з поверхневими тріщинами є складності обчислення коефіцієнтів інтенсивності напружень (КІН) для тріщин, які розвиваються у зонах концентраторів напружень. Ці труднощі пов’язані з особливостями зміни форми контуру поверхневої тріщини в процесі цикличного навантаження, який залежить від рівня неоднорідності напружено-деформованого стану в області концентраторів.

Проведено аналіз існуючих методів визначення коефіцієнтів інтенсивності напружень у зонах концентраторів: методу вагових функцій, чисельного і гібридного методів.

Моделювання зварних з’єднань різних типів з тріщинами проводили у рамках представленої моделі як геометричного фактора – підсилення зварного шва, що дозволило методом скінчених елементів оцінити особливості впливу концентратора напружень на зміну КІН по контуру поверхневої тріщини. При цьому розглянуто особливості побудови сітки скінчених елементів, вплив розмірів сітки на отримані розрахункові значення КІН, порівняння отриманих результатів з відомими рішеннями.

При цикличних випробуваннях розтяжом отримано кінетику форми втомних поверхневих півеліпітних тріщин у зонах досліджуваних концентраторів напружень та, для порівняння, в однорідному полі напружень. Проаналізовано отримані форми тріщин з результатами моделних досліджень.

Ключові слова: поверхнева півеліпітна тріщина, метод скінчених елементів, коефіцієнт інтенсивності напружень, конструктивний концентратор напружень.

Отримано 21.06.2018