

UDC 517.9

CONTACT INTERACTION OF PUNCH WITH PRESTRESSED HALF-PLANE

Hryhorii Habrusiev; Oleh Panchuk; Borys Shelestovs'kyi

Ternopil Ivan Puluj National Technical University, Ternopil, Ukraine

Summary. The article deals with the problem on the pressure of the plane rigid punch of random configuration on prestressed thick plate being modeled by the half-plane. The function of distribution of the contact stresses under punch was found. The effect of the punch shape on the distribution of the contact stresses was analyzed. The obtained results are of importance for the construction engineering while building the pile trestles and other structural elements.

Key words: initial deformation, punch elastic half-plane contact stresses.

Received 11.06.2018

Statement of the problem. Calculation of the structural elements and machine parts strength is one of the most important stages while designing the structures and machines. Here the residual deformations being available, which are not taken into account, can cause the structures damage. Taking into account the residual stresses and deformations on the contact interactions of bodies is of special importance.

Analysis of the available results of investigations. The problem of the bodies interaction possessing the residual deformations has been of interest for many scientists, our national scientists in particular [1, 2]. But the plane problems were studied only for the punches with the rectilinear base or that of the random punch shape, but without taking into account the residual deformations. Generally, the statement of such problems requires the introduction of the non-linear theory of elasticity, but the initial deformations being great enough, its linearized option can be introduced. The effect of the residual deformations on the contact interaction of the punch with the prestressed half-plane and the layer is presented in the papers [3 – 6].

Statement of the task. Let the rigid punch with the curvilinear trapezium cross-section [Fig. 1]

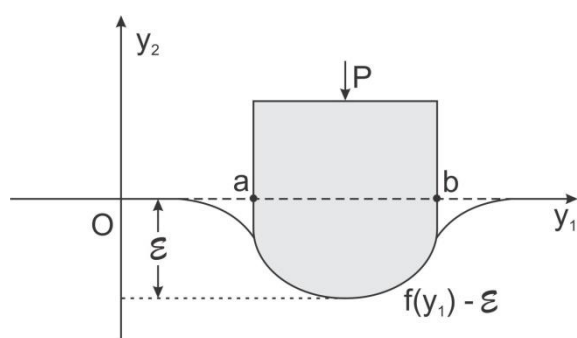


Figure 1. The scheme of the contact interaction

be pressed into the elastic half-plane with the initial stresses. The load, the main vector of which is marked as P , is applied to the upper side of the punch in such a way, that the half-plane under the punch is deformed in the reverse direction to the axis Oy_2 in the value ε . Let us consider, that the plane deformation specifies the half-plane in the initial state, that is, $\lambda_1 \equiv 1$ and the condition $S_{22}^0 = 0$ is satisfied.

The boundary conditions of the stated problem look like:

$$Q_{22}(y_1, 0) = 0, y_1 \notin [a, b]; \quad (1)$$

$$Q_{21}(y_1, 0) = 0, y_1 \notin [a, b]; \quad (2)$$

$$u_2(y_1, 0) = f(y_1) - \varepsilon; y_1 \in [a, b]; \quad (3)$$

$$Q_{21}(y_1, 0) = 0; y_1 \in [a, b], \quad (4)$$

where $Q_{22}(y_1, y_2)$ – normal stresses; $Q_{21}(y_1, y_2)$ – tangential stresses; $u_2(y_1, y_2)$ – displacements.

The function $f(y_1)$ describes the displacement of the elastic half-plane points in the area of its contact with the rigid punch.

Let us introduce the analytical functions $\omega_1(z)$ and $\omega_2(z)$ of the complex variable $z = y_1 + iy_2$ as the Cauchy integrals

$$\omega_1(z) = U_1 - iV_1 = \int_{-\infty}^{+\infty} Q_{22}(t, 0) \frac{dt}{t - z}; \quad \omega_2(z) = U_2 - iV_2 = \int_{-\infty}^{+\infty} Q_{21}(t, 0) \frac{dt}{t - z}.$$

Having used the Sokhotsky-Plemel formula to the last interrelation $z \rightarrow y_1 - i0$, we will obtain

$$U_1 - iV_1 = -i\pi Q_{22}(y_1, 0) + \int_{-\infty}^{+\infty} Q_{22}(t, 0) \frac{dt}{t - y_1};$$

$$U_2 - iV_2 = -i\pi Q_{21}(y_1, 0) + \int_{-\infty}^{+\infty} Q_{21}(t, 0) \frac{dt}{t - y_1}.$$

From here we will obtain:

$$V_1 = \pi Q_{22}(y_1, 0); \quad V_2 = \pi Q_{21}(y_1, 0);$$

$$U_1 = \int_{-\infty}^{+\infty} Q_{22}(t, 0) \frac{dt}{t - y_1}; \quad U_2 = \int_{-\infty}^{+\infty} Q_{21}(t, 0) \frac{dt}{t - y_1}. \quad (5)$$

The boundary condition (3) is presented as $\frac{\partial u_2}{\partial y_1} = \frac{\partial f}{\partial y_1}; y_1 \in [a, b]$.

Having used the expression for $\frac{\partial u_2}{\partial y_1}$ of the linearized theory of elasticity [1]

$$\frac{\partial u_2}{\partial y_1} = B_4^{(2)} Q_{21}(y_1, 0) + A_3^{(2)} \frac{1}{\pi} \int_{-\infty}^{+\infty} Q_{22}(t, 0) \frac{dt}{t - y_1},$$

where $A_3^{(2)}$ and $B_4^{(2)}$ – coefficients dependent on the field of the initial deformations and the interrelations (5), the boundary conditions of the problem (1) – (4) will be presented as follows:

$$V_1 = 0, y_1 \notin [a, b]; \quad V_2 = 0, y_1 \notin [a, b]; \quad \frac{B_4^{(2)}}{\pi} V_2 + \frac{A_3^{(2)}}{\pi} U_1 = \frac{\partial f}{\partial y_1}, y_1 \in [a, b]; \quad V_2 = 0; y_1 \in [a, b].$$

Thus, V_2 – the imaginable part of the function $\omega_2(z)$ equals zero on the whole axis Oy_1 . As this function is regular in the lower half-plane and behaves itself at infinity as cz^{-1} , it results from the mentioned above condition, that it must be equal to zero everywhere in the half-plane. Thus, $\omega_2(z) = 0$ and to find $\omega_1(z)$ we will have the following conditions:

$$V_1 = 0, y_1 \notin [a, b]; U_1 = \frac{\pi}{A_3^{(2)}} \frac{\partial f}{\partial y_1}; y_1 \in [a, b].$$

That is why, to find $\omega_1(z)$ we will obtain partial and the most simple case of the Rimman-Giblebert problem with the disruptive coefficients. As it is known, it deals with the construction of the complex function $\omega_1(z) = U_1 - iV_1$, which on the axis Oy_1 , satisfies the condition

$$a(y_1)U_1 + b(y_1)V_1 = F(y_1). \quad (6)$$

At $y_1 \in [a, b]$ we have $a(y_1) = 1$, $b(y_1) = 0$, at $y_1 \notin [a, b]$ – $a(y_1) = 0$, $b(y_1) = 1$, and the right part (6) was found as follows: $F(y_1) = 0, y_1 \notin [a, b]$;

$$F(y_1) = \frac{\pi}{A_3^{(2)}} \frac{\partial f}{\partial y_1}, y_1 \in [a, b].$$

To solve the next problem, let us use the method presented in [3].
The found solution is presented as follows

$$\omega_1(z) = \overline{\omega_1}(z) + \overline{\overline{\omega_1}}(z), \quad (7)$$

where $\overline{\overline{\omega_1}}(z) = U_0 + iV_0$ – the solution of the corresponding similar problem, that is, the function $\overline{\omega_1}(z)$ in the axis Oy_1 satisfies the condition

$$a(y_1)U_0 + b(y_1)V_0 = 0. \quad (8)$$

Let us analyze the function $w(y_1) = \operatorname{arctg} \frac{a(y_1)}{b(y_1)}$. The analytical function in the lower half-plane and uncontinuous small in the infinity, the real part of which in the axis being equal $w(y_1)$, can be presented as follows:

$$\Omega(z) = \Omega_1 - \Omega_2 i = -\frac{1}{\pi i} \int_{-\infty}^{+\infty} w(y_1) \frac{dy_1}{y_1 - z} = -\frac{1}{\pi i} \int_{-\infty}^{+\infty} \operatorname{arctg} \frac{a(y_1)}{b(y_1)} \frac{dy_1}{y_1 - z}.$$

That is, in the axis Oy_1 , approaching the point $z = y_1$ we will have below

$$\Omega_1(y_1) - \Omega_2(y_1)i = \operatorname{arctg} \frac{a(y_1)}{b(y_1)} - \Omega_2(y_1)i.$$

Let us rewrite the condition (8)

$$\begin{aligned} a(y_1)u_0 + b(y_1)v_0 &= \sqrt{a^2(y_1) + b^2(y_1)} \left[\frac{a(y_1)}{\sqrt{a^2(y_1) + b^2(y_1)}} u_0 + \frac{b(y_1)}{\sqrt{a^2(y_1) + b^2(y_1)}} v_0 \right] = \\ &= \sqrt{a^2(y_1) + b^2(y_1)} \left[\cos \left(\operatorname{arctg} \frac{a(y_1)}{b(y_1)} \right) u_0 + \sin \left(\operatorname{arctg} \frac{a(y_1)}{b(y_1)} \right) v_0 \right] = 0, \text{ or} \\ &\sqrt{a^2(y_1) + b^2(y_1)} \operatorname{Re} \left[e^{i\Omega_1(y_1)} w_0(y_1 - i0) \right] = 0. \end{aligned}$$

As the functions $a(y_1)$ and $b(y_1)$ are not transformed into zero simultaneously, then

$$\begin{aligned} \operatorname{Re} \left[e^{i\Omega_1(y_1)} w_0(y_1 - i0) \right] &= 0, \text{ or in other way} \\ \operatorname{Re} \left\{ \exp \left[-\frac{1}{\pi} \int_a^b \operatorname{arctg} \frac{a(y_1)}{b(y_1)} \frac{dy_1}{y_1 - z} \right] w_0(z) \right\} \Big|_{z=y_1-i0} &= 0. \end{aligned}$$

The last condition will be provided, if the expression, from which the real part is taken, will be equal to some function, which is of imaginable value in the real axis. As $w_0(z)$ must be analytical in the lower half-plane, such function can be chosen as follows:

$$\exp \left[-\frac{1}{\pi} \int_a^b \operatorname{arctg} \frac{a(y_1)}{b(y_1)} \frac{dy_1}{y_1 - z} \right] w_0(z) = \frac{P(z)}{Q(z)} i,$$

here

$$\frac{P(z)}{Q(z)} = \frac{P(z)}{\prod_n (z - \beta_n)},$$

where β_n – the real coefficients and the polynomial $P(z)$ is of real values in the axis Oy_1 . Thus, taking into account, that in the case of the contact interaction of the half-plane and one punch, the functions $a(y_1)$ and $b(y_1)$ have ruptures only in two points, for the function $w_0(z)$ we will have the following expression:

$$\overline{\overline{\omega_1}}(z) = iC \exp \left[\frac{1}{\pi} \int_{-\infty}^{+\infty} \operatorname{arctg} \frac{a(y_1)}{b(y_1)} \frac{dy_1}{y_1 - z} \right]. \quad (9)$$

To find the function $\omega_1(z) = u_1 - v_1 i$, which is the solution of the non-uniform problem,

let us find the real part of the function $\frac{i\overline{\omega_1}(z)}{\overline{\omega_0}(z)}$

Taking into account the conditions (8) and (6),

$$\operatorname{Re} \left\{ -\frac{i\omega_1(y_1 - i0)}{\omega_0(y_1 - i0)} \right\} = \frac{a(y_1)U_1 + b(y_1)V_1}{\sqrt{a^2(y_1) + b^2(y_1)}} \frac{1}{|\omega_0(y_1 - i0)|} = \frac{F(y_1)}{\sqrt{a^2(y_1) + b^2(y_1)}} \frac{1}{|\omega_0(y_1 - i0)|}.$$

That is why

$$\overline{\omega}_1(z) = \overline{\omega}_1(z) \left[-\frac{1}{\pi} \int_{+\infty}^{+\infty} \frac{F(y_1)}{\sqrt{a^2(y_1) + b^2(y_1)}} \frac{1}{|\overline{\omega}_1(y_1 - i0)|} \frac{dy_1}{y_1 - z} \right]. \quad (10)$$

And finally, basing on (9) and (10) we will have the expression

$$\omega_1(z) = \frac{1}{A_3^{(2)}} \frac{1}{\sqrt{(z-a)(b-z)}} \int_a^b \frac{\partial f}{\partial y_1} \sqrt{(t-a)(b-t)} \frac{dt}{t-z} + \frac{C}{\sqrt{(a-z)(b-z)}}. \quad (11)$$

To find the contact stresses, let us transfer in the last interrelation to the boundary at $z \rightarrow y_1 - i0$ and separate imaginable part:

$$Q_{22}(y_1, 0) = \frac{1}{A_2^{(3)}} \frac{1}{\sqrt{(y_1-a)(b-y_1)}} \int_a^b \frac{\partial f}{\partial y_1} \sqrt{(t-a)(b-t)} \frac{dt}{t-y_1} + \frac{C}{\sqrt{(y_1-a)(b-y_1)}}. \quad (12)$$

The constant C is found from the punch equilibrium condition, that is

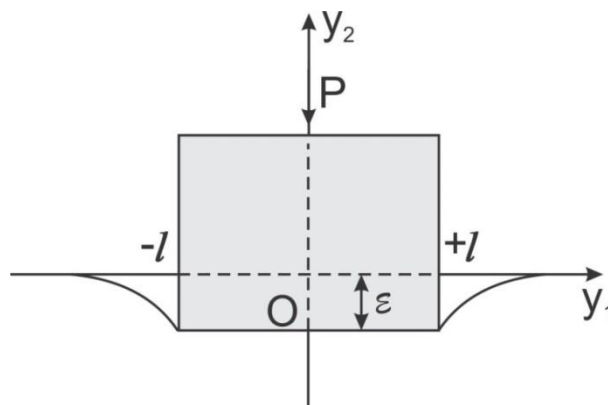
$$P = \int_{-l}^l Q_{22}(y_1, 0) dy_1.$$

Having used the interrelation (10), we can determine the distribution of the contact stresses under the punch of any configuration, and thus, to choose the optimal punch shape for every definite case. For the case of its rectilinear base (Fig 2) we will have:

$$f(y_1) = 0, \quad \frac{\partial f}{\partial y_1} = 0,$$

$$Q_{22}(y_1, 0) = \frac{C}{\sqrt{(y_1-l)(l-y_1)}},$$

Figure 2. The scheme of the contact interaction of the half-plane and the punch with the rectilinear base



$$P = \int_{-l}^l \frac{C}{\sqrt{(y_1+l)(l-y_1)}} dy_1, \quad C = \frac{P}{\pi} \cdot Q_{22}(y_1, 0) = \frac{P}{\pi \sqrt{l^2 - y_1^2}}.$$

In the case of the parabolic punch (Fig. 3) $f(y_1) = \frac{(y_1)^2}{2R}$, $\frac{df}{dy_1} = \frac{y_1}{R}$.

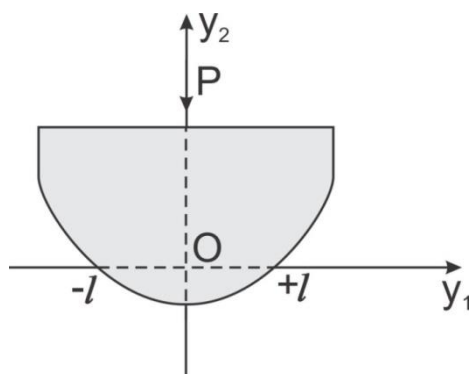


Figure 3. The scheme of contact interaction with the parabolic punch

$$Q_{22}(y_1, 0) = \frac{1}{A_2^{(3)}} \frac{1}{\sqrt{l^2 - y_1^2}} \int_{-l}^{+l} \frac{y_1}{R} \sqrt{l^2 - t^2} \frac{dt}{t - y_1} + \frac{C}{\sqrt{l^2 - y_1^2}}$$

$$Q_{22}(y_1, 0) = \frac{1}{A_2^{(3)}} \frac{e^2 - 2y_1^2}{R\sqrt{e^2 - y_1^2}} + \frac{P}{\pi\sqrt{l^2 - y_1^2}}.$$

In Fig. 4 σ^* – dimensionless value which describes the distribution of the contact stresses under the punch in the case of its rectilinear base, is presented. For the case of its parabolic base – in Fig. 5.

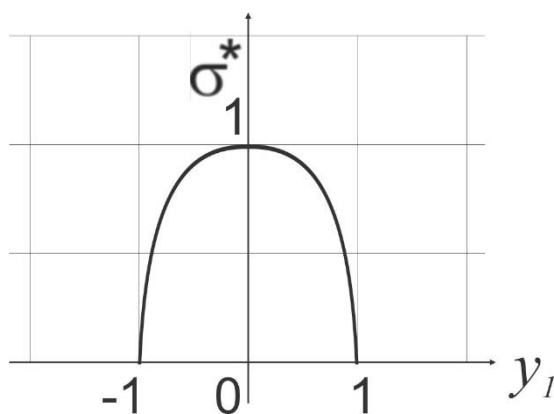


Figure 4. Distribution of contact stresses under the punch with the rectilinear base

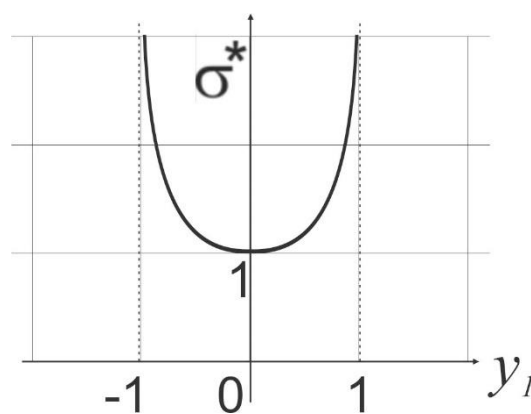


Figure 5. Distribution of contact stresses under the punch with the parabolic base

Conclusions. For the punch of random configuration the function of distribution of contact stresses was obtained. The examples of calculation the stresses for different punch shapes were presented. The dependence of the contact stresses on the shape of the rigid punch was shown. The developed approach and the obtained results make possible to make conclusions on the optimal punch shape in order to increase or decrease the contact stresses. The obtained results can be used while constructing the piles trestles or other structures in the construction engineering.

References

1. Huz A.N. Mekhanyka khrupkoho razrusheniya materyalov s nachalnymi napriazhenyamy, Huz A.N., K. : Nauk. dumka, 1983, 296 p. [In Russian].
2. Huz O.M., Babych S.Yu., Rudnytskyi V.B. Kontaktna vzaiemodiia pruzhnykh til z pochatkovyamy napruzheniamy; navch. posibnyk – K : Vyscha shkola, 1995, 304 p. [In Russian].
3. Shelestovs'kyi B.H. Zadacha Rimana-Hilberta dlia pivploshchyny, B.H. Shelestovs'kyi, O.I. Panchuk. Materialy XX naukovoï konferentsii TNTU im. I. Puluiua, 17 – 18 travnia 2017, TNTU, 2017, pp. 179 – 181 [In Ukrainian].

- 4 Habrusiev Hryhorii Valeriiovych, Habrusieva Iryna Yuriivna. Vplyv pochatkovykh deformatsii tovstoi plyty na yii kontaktnu vzaiemodiiu iz parabolichnym shtampom. Visnyk TNTU, 2017, vol. 85, no. 1, pp. 29 – 37 [In Ukrainian].
- 5 Panchuk Oleh Ihorovych, Habrusiev Hryhorii Valeriiovych, Shelestovs'kyi Borys Hryhorovych. Vplyv zalyshkovykh deformatsii na kontaktnu vzaiemodiiu zhorstkoho shtampa ta pruzhnoho pivprostoru. Visnyk TNTU, 2015, vol. 78, no. 2, pp. 93 – 100 [In Ukrainian].
- 6 Habrusieva Iryna, Panchuk Oleh, Shelestovs'kyi Borys. Contact interaction of the parabolic punch with preliminary stressed plate fixed on the rigid basis. Visnyk TNTU, 2016, vol. 82, no. 2, pp. 31 – 37.

Список використаної літератури

1. Гузь, А.Н. Механика хрупкого разрушения материалов с начальными напряжениями [Текст] / Гузь А.Н. – К. : Наук. думка, 1983. – 296 с.
2. Гузь, О.М. Контактна взаємодія пружних тіл з початковими напруженнями: навч. посібник [Текст] / О.М. Гузь, С.Ю. Бабич, В.Б. Рудницький. – К. : Вища школа, 1995. – 304 с.
3. Шелестовський, Б.Г. Задача Рімана-Гільберта для півплощини [Текст] / Б.Г. Шелестовський, О.І. Панчук // Матеріали XX наукової конференції ТНТУ ім. І. Пулюя, 17 – 18 травня 2017. – ТНТУ, 2017. – С. 179 – 181.
4. Габрусев, Г.В. Вплив початкових деформацій товстої плити на її контактну взаємодію із параболічним штампом [Текст] / Габрусев Григорій Валерійович, Габрусєва Ірина Юріївна // Вісник ТНТУ. – 2017. – Том 85. – № 1. – С. 29 – 37.
5. Панчук, О.І. Вплив залишкових деформацій на контактну взаємодію жорсткого штампа та пружного півпростору [Текст] / Олег Ігорович Панчук, Григорій Валерійович Габрусєв, Борис Григорович Шелестовський // Вісник ТНТУ. – Тернопіль : ТНТУ, 2015. – Том 78. – № 2. – С. 93 – 100.
6. Habrusieva, I. Contact interaction of the parabolic punch with preliminary stressed plate fixed on the rigid basis [Text] / Iryna Habrusieva, Oleh Panchuk, Borys Shelestovskiy // Вісник ТНТУ. – 2016 – Том 82. – № 2. – С. 31 – 37.

УДК 517.9

КОНТАКТНА ВЗАЄМОДІЯ ШТАМПА ТА ПОПЕРЕДНЬО НАПРУЖЕНОЇ ПІВПЛОЩИНИ

Григорій Габрусєв; Олег Панчук; Борис Шелестовський

*Тернопільський національний технічний університет імені Івана Пулюя,
Тернопіль, Україна*

Резюме. Розрахунок міцності структурних елементів та деталей механізмів є одним із найважливіших етапів у процесі проектування будівельних споруд і машин. При цьому в них майже завжди наявні залишкові деформації, неврахування яких призводить до руйнування конструкції. Особливо актуальним є врахування залишкових напружень і деформацій на контактну взаємодію тіл.

У статті розв'язано задачу про тиск плоского жорсткого штампа довільної конфігурації на попередньо напружену товсту плиту, що моделюється півплощиною. Знайдено функцію розподілу контактних напружень під штампом. Проаналізовано вплив форми штампа на розподіл контактних напружень. Результати роботи можуть бути використані під час проектування елементів споруд чи деталей машин.

Для розв'язання задачі використано методи теорії функцій комплексної змінної. Внаслідок застосування формули Сохоцького-Племеля отримуємо частковий випадок задачі Рімана-Гільберта з розривними коефіцієнтами. Для штампа довільної конфігурації отримано функцію розподілу контактних напружень, наведено приклади обчислення напружень для різних форм штампа. Показано залежність контактних напружень від форми жорсткого штампа. Побудовано графіки. Розвинений підхід та отримані результати дозволяють зробити висновок про оптимальну форму штампа для збільшення або зменшення контактних напружень. Отримані результати є актуальними у будівельній галузі в процесі створення опор чи інших структурних елементів.

Ключові слова: початкові деформації, пружна півплощина, штамп, контактні напруження.

Отримано 11.06.2018