Міністерство освіти і науки України
Тернопільський національний технічний університет
імені Івана Пулюя

навчально–методична література

кафедра
світлотехніки та
електротехніки

Electrical Engineering

PRACTICUM

Ternopil 2018

«Electrical Engineering. Practicum.»

Укладач: Н.А. Куземко. Тернопіль: ТНТУ, 2018.

Укладач: к.т.н., доц. Куземко Н.А.

Рецензент: д.т.н., проф. Лупенко А.М.

Редагування англійською мовою: к.п.н., доц. Кухарська В.Б.

Комп'ютерний набір: Куземко Н.А.

Затверджено на засіданні кафедри світлотехніки та електротехніки Тернопільського національного технічного університету імені Івана Пулюя.

Протокол № 4 від 22 листопада 2017 року

Схвалено й рекомендовано до друку методичною комісією факультету прикладних інформаційних систем та електроінженерії Тернопільського національного технічного університету імені Івана Пулюя.

Протокол № 4 від 24 листопада 2017 року

Посібник складено відповідно до робочих програм курсів "Електротехніка, електроніка та основи МПТ", "Електротехніка в будівництві", "Теорія електричних та магнітних кіл".

1.Electrical circuits calculations methods

Task 1. Draw the scheme (fig.1.1) according to your variant in table below. Write down the system of equations according to Kirchhoff's laws.

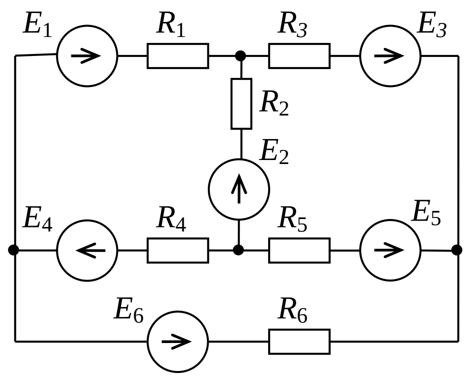


Fig.1.1

Task 2. Write down the system of equations according to the loop currents method and calculate the circuit using this method.

Task 3. Verify the calculations using the equation of power balance.

Task 4. Write down the system of equations according to the nodal potential method and write down the expressions of branches' currents.

Task 5. Define the external loop point potentials and draw the potential diagram for the external loop.

Var. №	E_1 *V*	E_2 *V*	E_3 *V*	E_4 *V*	E_5 *V*	E_6 *V*	R_1 Ω	R_2 Ω	R_3 Ω	R_4 Ω	R_5 Ω	R_6 Ω
00	12	15	18	–	–	–	3	4	5	6	7	8
01	–	15	18	21	–	–	4	5	6	7	8	3
02	–	–	18	21	24	–	5	6	7	8	3	4
03	–	–	–	21	24	27	6	7	8	3	4	5
04	15	18	21	–	–	–	7	8	3	4	5	6
05	–	18	21	24	–	–	8	3	4	5	6	7
06	–	–	21	24	27	–	3	4	5	6	7	8
07	–	–	–	24	27	12	4	5	6	7	8	3
08	18	21	24	–	–	–	5	6	7	8	3	4
09	–	21	24	27	–	–	6	7	8	3	4	5
10	–	–	24	27	12	–	7	8	3	4	5	6
11	–	-	-	27	12	15	8	3	4	5	6	7
12	21	24	27	–	–	–	3	4	5	6	7	8
13	–	24	27	12	–	–	4	5	6	7	8	3
14	–	–	27	12	15	–	5	6	7	8	3	4
15	–	–	–	12	15	18	6	7	8	3	4	5
16	24	27	12	–	–	–	7	8	3	4	5	6
17	–	27	12	15	–	–	8	3	4	5	6	7
18	–	–	12	15	18	-	3	4	5	6	7	8
19	–	–	–	15	18	21	4	5	6	7	8	3
20	27	12	15	–	–	–	5	6	7	8	3	4

Example for task 1. The graph for the circuit (fig.1.2) is shown at fig.1.3. There are six branches (unknown currents), $p=6$ and number of independent nodes is three at the circuit $q=3$. The currents' (I_1-I_6) directions are chosen arbitrarily. The nodes are noted as *N1*, *N2*, *N3*.

The equations according to the Kirchhoff's first law for the nodes *1-3* (if the current flows into the node it is assumed with "+", and out of the node with "-") are:

for *N1:* $-I_1 - I_3 - I_4 = 0$,

for *N2:* $+I_1 - I_2 - I_5 = 0$,

for *N3:* $+I_3 + I_2 + I_6 = 0$.

The equations according to the Kirchhoff's second law ($p-q=3$) for the loops,

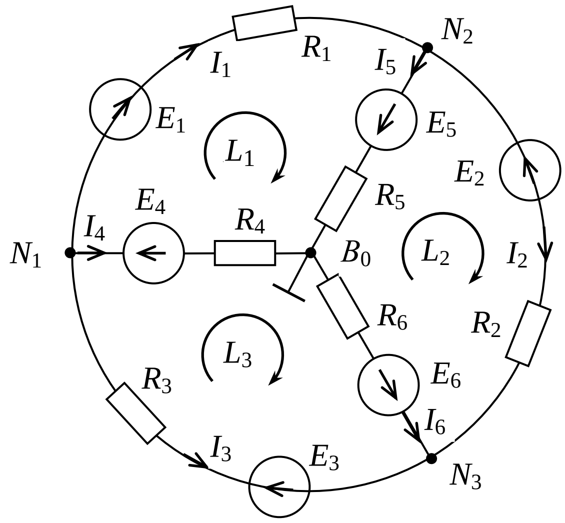


Fig 1.2

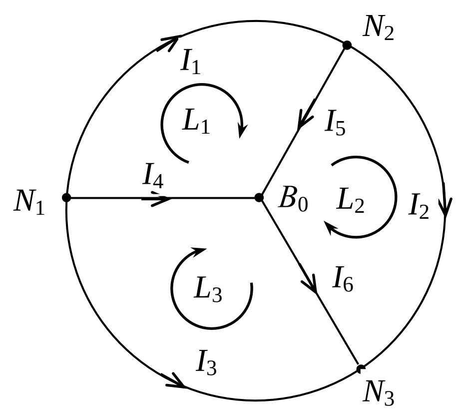


Fig.1.3

L_1, L_2, L_3 (the directions along the loops are chosen clockwise , if the directions of the bypass and the voltage or e.m.f. are the same, they are denominated with "+", if opposite with "-") are:

for *L1:* $+R_1 I_1 + R_5 I_5 - R_4 I_4 = +E_1 + E_5 + E_4$,

for *L2:* $+R_2 I_2 - R_5 I_5 - R_6 I_6 = -E_2 - E_5 - E_6$,

for *L3:* $+R_6 I_6 - R_3 I_3 + R_4 I_4 = +E_6 + E_3 - E_4$.

So, the equations system according to the *Kirchhoff's laws* is:

$$\begin{cases} -I_1 - I_3 - I_4 = 0 \\ +I_1 - I_2 - I_5 = 0 \\ +I_3 + I_2 + I_6 = 0 \\ +R_1 I_1 + R_5 I_5 - R_4 I_4 = +E_1 + E_5 + E_4 \\ +R_2 I_2 - R_5 I_5 - R_6 I_6 = -E_2 - E_5 - E_6 \\ +R_6 I_6 - R_3 I_3 + R_4 I_4 = +E_6 + E_3 - E_4 \end{cases}$$

Example for task 2 . For the circuit on fig.1.2 $E_1 = 10V$, $E_3 = 15V$, $E_6 = 20V$, $E_2 = E_4 = E_5 = 0V$, $R_1 = 10\Omega$, $R_2 = 6\Omega$, $R_3 = 5\Omega$, $R_4 = 7\Omega$, $R_5 = 8\Omega$, $R_6 = 5\Omega$.

The system of equations according to the loop currents method for the circuit with three independent loops is:

$$\begin{cases} +R_{11}I_{L1}-R_{12}I_{L2}-R_{13}I_{L3}=E_{L1} \\ -R_{21}I_{L1}+R_{22}I_{L2}-R_{23}I_{L3}=E_{L2}, \\ -R_{31}I_{L1}-R_{32}I_{L2}+R_{33}I_{L3}=E_{L3} \end{cases}$$

where $R_{11}=R_1+R_5+R_4=10+8+7=25\ \Omega$, $R_{22}=R_2+R_6+R_5=6+5+8=19\ \Omega$, $R_{33}=R_4+R_6+R_3=7+5+5=17\ \Omega$ - are the individual resistances of the loops, which are equal to the sum of all the resistances of the loop;

$R_{12}=R_{21}=R_5=8\ \Omega$, $R_{13}=R_{31}=R_4=7\Omega$, $R_{23}=R_{32}=R_6=5\Omega$ - are mutual resistances of the loops, i.e. the resistances of the branches which are mutual for the respective loops;

$E_{L1}=E_1+E_5+E_4=10+0+0=10\ V$, $E_{L2}=-E_2-E_5-E_6=0+0-20=-20\ V$, $E_{L3}=E_6+E_3-E_4\ =20+15+0=35\ V$ - are loops' e.m.f. These are equal to the algebraic sum of the electromotive forces alongside the loops.

When substituted in the equations system above, these values make:

$$\begin{cases} +25I_{L1}-8I_{L2}-7I_{L3}=10 \\ -8I_{L1}+19I_{L2}-5I_{L3}=-20. \\ -7I_{L1}-5I_{L2}+17I_{L3}=35 \end{cases}$$

When solved the values make: $I_{L1}=1.153A$, $I_{L2}=0.108A$, $I_{L3}=2.565A$.

Branches' currents can be defined from the loops' currents as: $I_1=I_{L1}=1.153A$, $I_2=I_{L2}=0.108A$, $I_3=-I_{L3}=-2.565A$, $I_4=I_{L3}-I_{L1}=2.565-1.153=1.412\,A$, $I_5=I_{L1}-I_{L2}=1.153-0.108=1.045A$, $I_6=I_{L3}-I_{L2}==2.565-0.108=2.457\,A$.

Example for task 3. The power balance equation is used to verify the calculations. For the circuit on fig.1.2, according to the calculations at example 2 the total power of the circuit's sources should be equal to the total power of the circuit's consumers $\Sigma P_R=\Sigma P_E$.

Thus, total power of the sources is:

$$\Sigma P_E=E_1I_1-E_3I_3+E_6I_6=10\cdot 1.153-15\cdot(-2.565)+20\cdot 2.457=99.145\ W.$$

When the e.m.f. and the current have the same directions the source power $P_E=EI$ is considered with "+", if opposite with "-".

The total power of the consumers makes:

$$\Sigma P_R=R_1I_1^2+R_2I_2^2+R_3I_3^2+R_4I_4^2+R_5I_5^2+R_6I_6^2=$$

$$=10\cdot(1.153)^2+6\cdot(0.108)^2+5\cdot(-2.565)^2+7\cdot(1.412)^2+8\cdot(1.045)^2+5\cdot(2.457)^2=99.13\ W$$

So, $\Sigma P_E\approx\Sigma P_R$.

Example for task 4. The system of equations according to the nodal potential method for the circuit with three independent nodes (fig.1.2) is:

$$\begin{cases} G_{11}\varphi_1 - G_{12}\,\varphi_2 - G_{13}\,\varphi_3 = J_1 \\ -G_{21}\,\varphi_1 + G_{22}\,\varphi_2 - G_{23}\,\varphi_3 = J_2 \\ -G_{31}\,\varphi_1 - G_{32}\,\varphi_2 + G_{33}\,\varphi_3 = J_3 \end{cases}.$$

Where $G_{11} = G_1 + G_3 + G_4$, $G_{22} = G_1 + G_2 + G_5$, $G_{33} = G_2 + G_3 + G_6$ - are the individual conductivities of the nodes. They are equal to the sum of the branch conductivities that are coming into the node;

$G_{12} = G_{21} = G_1$, $G_{23} = G_{32} = G_2$, $G_{13} = G_{31} = G_3$ - are the mutual conductivities of the nodes. They are equal to the conductivities of the branches that connect the respective nodes. Branches' conductivities are: $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, $G_4 = 1/R_4$, $G_5 = 1/R_5$, $G_6 = 1/R_6$.

Nodal potentials are φ_1, φ_2, φ_3 related to the node *0* with zero potential.

$$J_1 = -G_1\,E_1 + G_4\,E_4 + G_3\,E_3,\; J_2 = G_1\,E_1 - G_5\,E_5 + G_2\,E_2,$$

$J_3 = G_6\,E_6 - G_2\,E_2 - G_3\,E_3$ - the algebraic sum of the currents of currents' sources, that are flowing into the respective nodes. If the current J of the source flows into the node, it is marked with the sign "+", when it flows out – with the sign "-".

Branches' currents are defined this way: $\varphi_1 - R_4\,I_4 = E_4$, $I_4 = \dfrac{(\varphi_1 - E_4)}{R_4} = (\varphi_1 - E_4)\,G_4$,

$$\varphi_2 - R_5\,I_5 = -E_5,\; I_5 = \frac{(-\varphi_2 + E_5)}{R_5} = (-\varphi_2 + E_5)\,G_5,$$

$$\varphi_3 + R_6\,I_6 = E_6,\; I_6 = \frac{(E_6 - \varphi_3)}{R_6} = (E_6 - \varphi_3)\,G_6,$$

$$\varphi_1 - \varphi_2 - R_1\,I_1 = -E_1,\; I_1 = \frac{(\varphi_1 - \varphi_2 + E_1)}{R_1} = (\varphi_1 - \varphi_2 + E_1)G_1,$$

$$\varphi_2 - \varphi_3 - R_2\,I_2 = E_2,\; I_2 = \frac{(\varphi_2 - \varphi_3 - E_2)}{R_2} = (\varphi_2 - \varphi_3 - E_2)\,G_2,$$

$$\varphi_1 - \varphi_3 - R_3\,I_3 = E_3,\; I_3 = \frac{(\varphi_1 - \varphi_3 - E_3)}{R_3} = (\varphi_1 - \varphi_3 - E_3)\,G_3.$$

Example for task 5. The external loop of the circuit (fig.1.2) is shown at fig.1.4. The parameters of the circuit are: $E_1 = 10V$, $E_3 = 15V$, $E_6 = 20V$, $R_1 = 10\Omega$, $R_2 = 6\Omega$ $R_3 = 5\Omega$, $R_4 = 7\Omega$, $R_5 = 8\Omega$, $R_6 = 5\Omega$ and the branch currents are: $I_1 = 1.153A$, $I_2 = 0.108A$, $I_3 = -2.565A$ (example for task 2).

The loop points potentials are: $\varphi_1 = 0$, $\Sigma r_1 = 0$,

$$\varphi_a = \varphi_1 + E_1 = 10\,V,\; \Sigma r_a = 0,$$

$$\varphi_2 = \varphi_a - R_1\,I_1 = 10 - 10\cdot 1.153 = -1.53\,V,\; \Sigma r_2 = R_1 = 10\;\Omega,$$

$$\varphi_b = \varphi_2 - E_2 = -1.53 - 0 = -1.53\,V,\; \Sigma r_b = R_1 = 10\;\Omega,$$

$$\varphi_3 = \varphi_b - R_2\,I_2 = -1.53 - 6\cdot 0.108 = -2.178\;V,\; \Sigma r_3 = R_1 + R_2 = 10 + 6 = 16\;\Omega,$$

$$\varphi_c = \varphi_3 + E_3 = -2.178 + 15 = 12.822\ V,\ \Sigma r_3 = R_1 + R_2 = 10 + 6 = 16\ \Omega,$$

$$\varphi_1 = \varphi_c + R_3 I_3 = 12.822 + 5\cdot(-2.565) = 0\ V,\ \Sigma r_1 = R_1 + R_2 + R_3 = 10 + 6 + 5 = 21\ \Omega.$$

The potential diagram for the external loop is shown at fig.1.5.

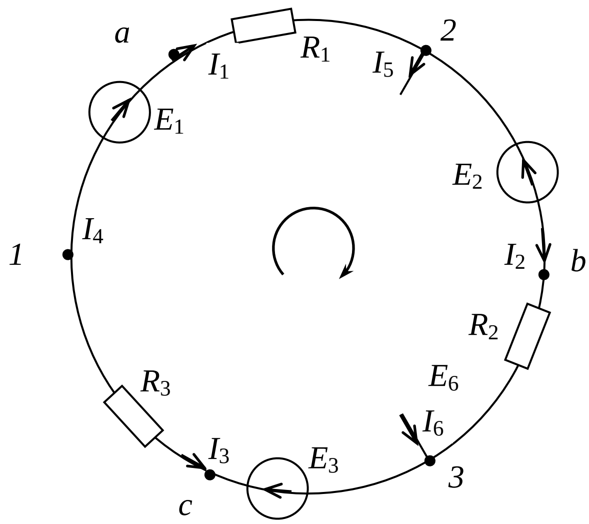


Fig 1.4

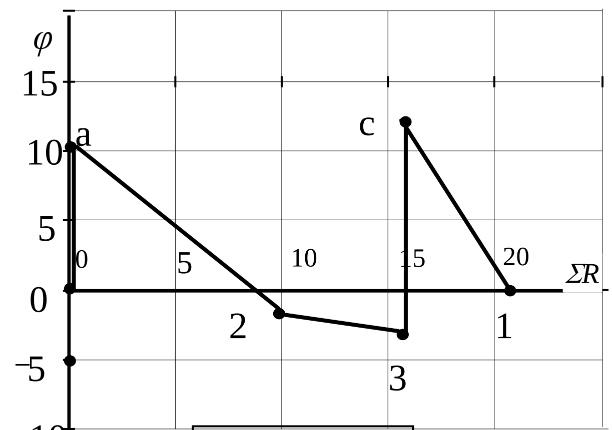


Fig.5

Task 6. Draw the scheme (fig.1.6). Define the branches' currents with the two nodes method and the operating mode of every source according to your variant in table below. Verify the calculations using the first *Kirchhoff's law*.

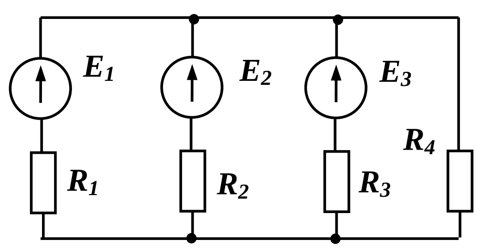


Fig.1.6

VAR	E_1	E_2	E_3	R_1	R_2	R_3	R_4
№	*V*	*V*	*V*	Ω	Ω	Ω	Ω
00	120	220	150	20	30	40	10
01	120	200	100	10	20	30	4
02	220	150	120	10	50	80	20
03	120	220	150	30	40	10	20
04	120	220	100	40	10	20	30
05	220	150	120	40	30	20	10
06	120	220	150	30	20	10	40
07	300	200	120	20	30	40	10
08	400	200	150	10	40	30	20
09	200	300	150	20	30	40	10
10	200	400	120	30	50	80	10

11	100	120	130	50	30	10	80
12	100	125	140	30	50	80	10
13	100	130	140	50	40	30	20
14	100	135	145	20	30	40	50
15	120	130	155	50	20	30	10
16	120	145	130	40	30	50	20
17	130	160	145	30	50	20	40
18	130	155	140	50	10	40	30
19	140	150	160	30	20	10	50
20	200	300	100	40	50	20	10

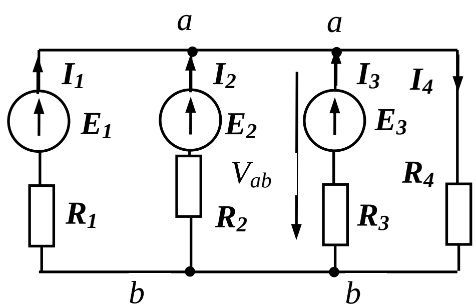


Fig.1.7

Example for task 6. The parameters of the circuit are: $E_1 = 10\,V$, $E_2 = 20\,V$, $E_3 = 30\,V$, $R_1 = 10\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 10\ \Omega$, $R_4 = 5\ \Omega$.

Choose arbitrarily the directions of branches' currents I_1, I_2, I_3, I_4 and the direction of the inter-node voltage V_{ab} (fig.1.7).

The branches' conductivities are:

$G_1 = 1/R_1 = 1/10 = 0.1\ Sm$, $G_2 = 1/R_2 = 1/5 = 0.2\ Sm$, $G_3 = 1/R_3 = 1/10 = 0.1\ Sm$, $G_4 = 1/R_4 = 1/5 = 0.2\ Sm$.

Calculate the inter-node voltage as:

$$V_{ab} = \frac{G_1E_1 + G_2E_2 + G_3E_3}{G_1 + G_2 + G_3 + G_4} = \frac{0.1\cdot 10 + 0.2\cdot 20 + 0.1\cdot 30}{0.1 + 0.2 + 0.1 + 0.2} = 13.33\,V.$$

For chosen positive currents' directions, their values are defined according to the second *Kirchhoff's law*.

$V_{ab} + R_1I_1 = E_1$, so $I_1 = (E_1 - V_{ab})/R_1 = (E_1 - V_{ab})G_1 = (10 - 13.33)\cdot 0.1 = -0.33\ A$.

Sign «-» means, that actual direction of the current is opposite to the chosen one.

$V_{ab} + R_2I_2 = E_2$, so $I_2 = (E_2 - V_{ab})G_2 = (20 - 13.33)\cdot 0.2 = 1.33\ A$.

Sign «+» means that actual direction of the current is the same as chosen one.

$V_{ab} + R_3I_3 = E_3$, so $I_3 = (E_3 - V_{ab})G_3 = (30 - 13.33)\cdot 0.1 = 1.67\ A$.

$V_{ab} = R_4I_4$, so $I_4 = V_{ab}/R_4 = V_{ab}\cdot G_4 = 13.33\cdot 0.2 = 2.67\ A$.

Verify the calculations by the first *Kirchhoff's law* for node a:

$I_1 + I_2 + I_3 - I_4 = 0$

so $-0.33 + 1.33 + 1.67 - 2.67 = 0$.

Define electrical sources operating modes. If the directions of the real branch current and e.m.f. are the same $P = EI > 0$, the source works as a generator, if not $P = EI < 0$ the source works as a consumer. Thus, the sources E_2, E_3 at fig.1.7 work as generators and E_1 as a consumer.

Task 7. Draw the scheme (fig.1.8) with values according to your variant in table below. Define the branches' currents by the superposition method**.**

Var. №	E_1 V	E_2 V	E_3 V	E_4 V	E_5 V	R_1 Ω	R_2 Ω	R_3 Ω	R_4 Ω	R_5 Ω
00	12	15	-	–	–	3	4	5	6	7
01	–	15	18	-	–	4	5	6	7	8
02	–	–	18	21	-	5	6	7	8	3
03	–	–	–	21	24	6	7	8	3	4
04	15	-	21	–	–	7	8	3	4	5
05	–	18	-	24	–	8	3	4	5	6
06	–	–	21	27	-	3	4	5	6	7
07	-	15	–	24	-	4	5	6	7	8
08	18	-	-	–	21	5	6	7	8	3
09	–	21	-	-	27	6	7	8	3	4
10	–	24	-	-	12	7	8	3	4	5
11	–	-	-	27	12	8	3	4	5	6
12	21	24	-	–	–	3	4	5	6	7
13	–	-	27	12	–	4	5	6	7	8
14	–	27	-	12	-	5	6	7	8	3
15	–	–	–	12	15	6	7	8	3	4
16	24	-	12	–	–	7	8	3	4	5
17	–	27	-	15	–	8	3	4	5	6
18	–	–	12	18	-	3	4	5	6	7
19	18	–	15	-	-	4	5	6	7	8
20	27	-	-	–	15	5	6	7	8	3

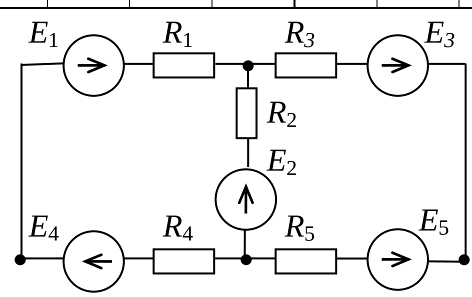


Fig.1.8

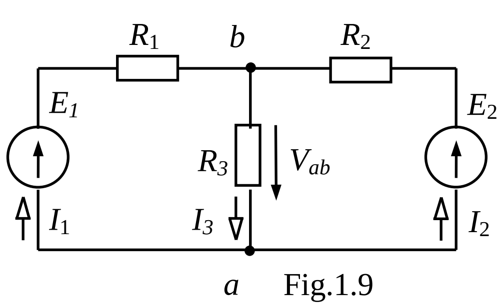


Fig.1.9

Example for task 7. For the circuit at fig.1.9 $R_1 = 5\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 10\ \Omega$, $E_1 = 20\ V$, $E_2 = 25\ V$.

The superposition principle means that every e.m.f. acts in the circuit independently. Thus, the calculation of one circuit (fig.1.9) with two sources, for example, can be reduced to the calculation of two circuits each with a single source (fig.1.10, 1.11).

Two partial circuits with partial currents should be calculated according to this principle. There is a single e.m.f. E_1 in the first partial circuit (fig.1.10): $R'_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{10 \cdot 15}{15+10} = 6\ \Omega$.

The total resistance of this scheme: $R' = R_1 + R'_{23} = 5 + 6 = 11\ \Omega$.

The partial current makes $I'_1 = E_1 / R' = 20/11 = 1.818\ A$, and voltage makes

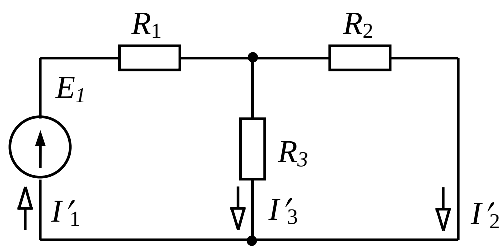


Fig.1.10

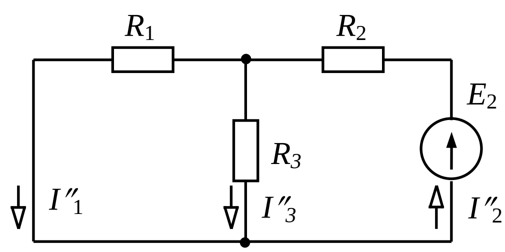


Fig.1.11

$V'_{ab} = I'_1 R'_{23} = 1.818 \cdot 6 = 10.91\ V$.

The partial branches' currents are: $I'_2 = V'_{ab} / R_2 = 10.91/15 = 0.727\,A$,

$I'_3 = V'_{ab} / R_3 = 10.91/10 = 0.091 A$.

There is a single e.m.f. E_2 in the second partial circuit (fig.1.11).

$$R''_{13} = \frac{R_1 R_3}{R_1 + R_3} = \frac{5 \cdot 10}{5+10} = 3.33\ \Omega.$$

The total resistance of this scheme is: $R'' = R_2 + R''_{13} = 15 + 3.33 = 18.33\,\Omega$.

The partial current makes $I''_2 = E_2 / R'' = 25/18.33 = 1.364\ A$, and voltage $V''_{ab} = I''_2 R''_{13} = 1.364 \cdot 3.33 = 4.54\ V$.

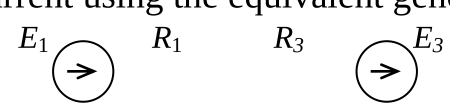
The partial branches' currents are: $I''_1 = V''_{ab} / R_1 = 4.54/5 = 0.909\,A$,

$I''_3 = V''_{ab} / R_3 = 4.54/10 = 0.454\,A$.

Therefore, the real branches' currents are defined as an algebraic sum of the respective partial currents (fig.1.9):

$$I_1 = I'_1 - I''_1 = 1{,}818 - 0{,}909 = 0{,}909\ A,$$
$$I_2 = I''_2 - I'_2 = 1.364 - 0.727 = 0.637 A,$$
$$I_3 = I''_3 + I'_3 = 0.091 + 0.454 = 0.545\ A.$$

Task 8. Draw the scheme (fig.1.12) according to your variant in table below. Define the second branch current using the equivalent generator method**.**



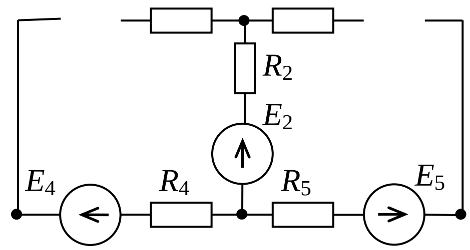


Fig.1.12

Var. №	E_1 V	E_2 V	E_3 V	E_4 V	E_5 V	R_1 Ω	R_2 Ω	R_3 Ω	R_4 Ω	R_5 Ω
00	12	-	15	–	–	3	4	5	6	7
01	–	-	18	-	27	4	5	6	7	8
02	–	–	18	21	-	5	6	7	8	3
03	–	–	–	21	24	6	7	8	3	4
04	15	-	21	–	–	7	8	3	4	5
05	–	-	15	24	–	8	3	4	5	6
06	–	–	21	-	27	3	4	5	6	7
07	15	–	–	24	-	4	5	6	7	8
08	18	-	-	–	21	5	6	7	8	3
09	25	-	-	-	27	6	7	8	3	4
10	–	–	24	-	12	7	8	3	4	5
11	–	-	-	27	12	8	3	4	5	6
12	21	-	-	18	–	3	4	5	6	7
13	–	-	27	12	–	4	5	6	7	8
14	–	–	27	12	-	5	6	7	8	3
15	–	–	–	12	15	6	7	8	3	4
16	24	-	12	–	–	7	8	3	4	5
17	–	-	-	15	21	8	3	4	5	6
18	–	–	12	-	18	3	4	5	6	7
19	18	–	–	15	-	4	5	6	7	8
20	27	-	-	–	15	5	6	7	8	3

Example for task 8. For the circuit at fig.1.13 $R_1 = 5\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 10\ \Omega$, $E_1 = 20\ V$, $E_3 = 25\ V$.

The branch with unknown current (I_2) is selected from the circuit on fig.1.13. An equivalent generator (fig.1.14) replaces the rest of the circuit. Its parameters are E_{eqv} - equivalent e.m.f., which is equal to the open circuit voltage on the clamps *ab* of an open second branch and R_{eqv} – equivalent resistance, which is equal to the input resistance of the circuit in respect to the open second branch clamps *ab* (the e.m.f. is shortened). The equivalent generator parameters (E_{eqv} and R_{eqv})

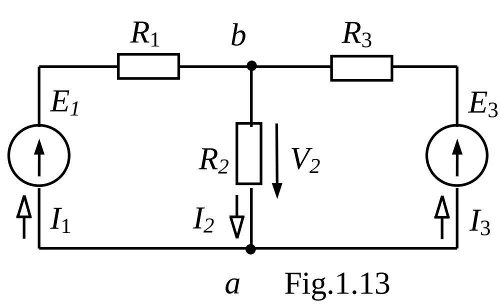


Fig.1.13

are to be calculated. For the circuit at fig.1.13 R_{eqv} is:

$$R_{eqv} = \frac{R_1 R_3}{R_1 + R_3} = \frac{5 \cdot 10}{5+10} = 1.333\,\Omega .$$

E_{eqv} (fig.1.15) is defined the following way: $V_{abOC} = E_{eqv} = E_1 - R_1 I$

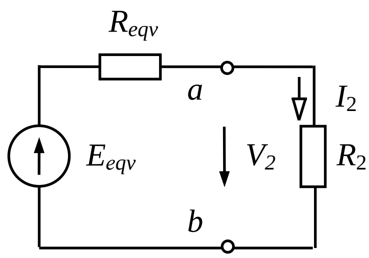


Fig.1.14

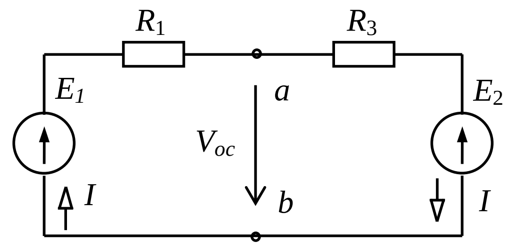


Fig.1.15

$= 20 - 5 \cdot (-0.33) = 21.65\ V$, where $I = \dfrac{E_1 - E_2}{R_1 + R_3} = \dfrac{20-25}{5+10} = -0.33\,A$. According to the fig.1.14 unknown current makes: $I_2 = E_{eqv} / (R_{eqv} + R_2) = \dfrac{21.65}{1{,}33 + 15} = 1.33\ A$.

Example 9. Define the non-linear element static and dynamic resistance at work point for $I_0 = 0.006\,A$, $V_0 = 4V$.

Static resistance (fig.1.16) is

$$R_{ST} = V_0 / I_0 = 4/0.006 = 666.67\,\Omega ,$$

Dynamic resistance (fig.1.16) is

$$R_D = \Delta V / \Delta I = 4/0.004 = 1000\,\Omega .$$

Example 10. For scheme at fig.1.17 $R_1 = 5\ \Omega$, $R_2 = 15\ \Omega$, $E_1 = 20\ V$, $E_2 = 25\ V$. Define the non-linear element R_3 current, if its volt-ampere relationship $V(I)$ is represented by the table.

I, A	1,0	2,0	3,0	4,0	5,0	6,0
V, B	13	17	18	18.5	19.5	20.0

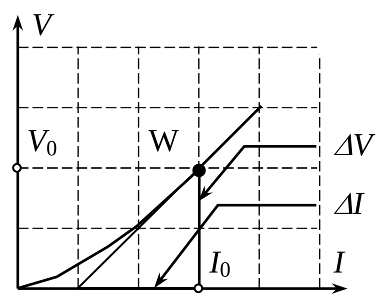


Fig. 1.16.

The equivalent generator method is used to solve this task. Firstly, the branch with non-linear resistance is selected. The rest of the scheme is represented as an active one-port network with the parameters $E_{EQV} = V_{OCab}$, R_{EQV} (fig.1.18).

The branch with non-linear resistance R_3 is disconnected and the input resistance R_{EQV} is defined as the one between the open

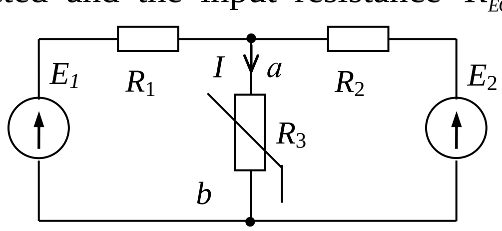


Fig. 1.17

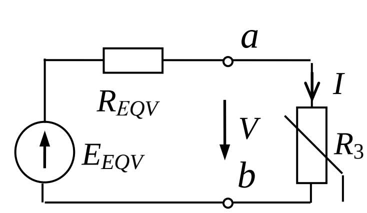


Fig. 1.18

clamps *a* and *b* (the sources are shortened): $R_{EQV} = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{5 \cdot 15}{5+15} = 3.75\Omega$.

The open circuit voltage is defined by two nodes method ($G_N = 1/R_N$) as

$$G_1 = 1/R_1 = 1/5 = 0.2\,Sm,\ G_2 = 1/R_2 = 1/15 = 0.067\,Sm.$$

$$E_{EQV} = V_{OCab} = \frac{G_1E_1 + G_2E_2}{G_1 + G_2} = \frac{0.2\cdot 20 + 0.067\cdot 25}{0.2 + 0.067} = 21.25\,V.$$

For scheme at fig.1.18 write down the equation according to the second Kirchhoff's law $E_{EQV} = R_{EQV} I + V_{ab}$, so $V(I) = V_{ab} = E_{EQV} - R_{EQV} I$. Solve this equation by graphic method (fig.1.19). The left part of this equation is a non-linear element VAC (curve) $V_{AB}(I)$. The right part of the equation is a line drawn across the points $(E_{EQV},0)$, $(0,I_{SC})$. By substituting the values E_{EQV}, $I_{SC} = E_{EQV}/R_{EQV}$, the points $(21.25,0)$ and ($0,5.67$) are obtained. The line is drawn across these points. It crosses the curve at the point with $V_A = 15V$, $I_A = 1.20\,A$, that defines a non-linear element working regime.

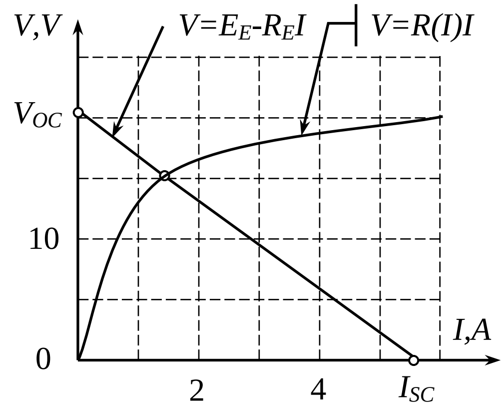
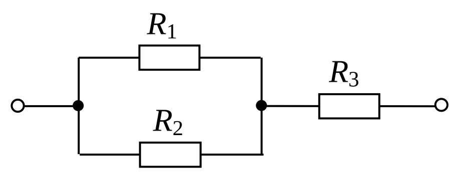


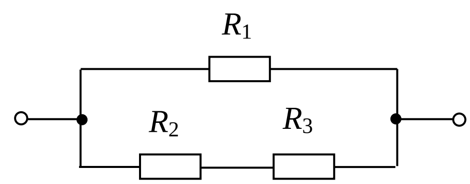
Fig. 1.19.

Tasks for individual work.

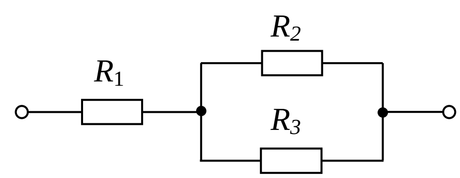
1. Current $I_{R2} = 2\ A$ and resistances $R_1 = 4\ \Omega$, $R_2 = 2\Omega$, $R_3 = 6\ \Omega$ are given for the circuit on the figure. Define the branches' currents, sub-circuits' voltages, input voltage.



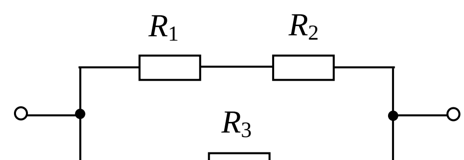
2. Voltage $V_{R2} = 2\ V$ and resistances $R_1 = 4\ \Omega$, $R_2 = 2\Omega$, $R_3 = 6\ \Omega$ are given for the circuit on the figure. Define the branches' currents, voltages across the elements and current of the unforked sub-circuit.



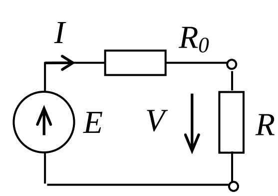
3. Current $I_{R2} = 2\ A$ and resistances $R_1 = 2\ \Omega$, $R_2 = 3\Omega$, $R_3 = 6\ \Omega$ are given for the circuit on the figure. Define the branches' currents, sub-circuits' voltages and input voltage.



4. Voltage $V_{R2} = 4\ V$ and resistances $R_1 = 2\ \Omega$, $R_2 = 4\Omega$, $R_3 = 6\ \Omega$ are given for the circuit on the figure. Define the branches' currents, voltages across the elements and the current of the unforked sub-circuit.

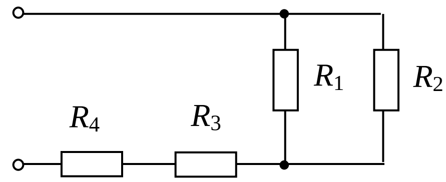


5. Define source internal resistance R_0 and electromotive force E, source's and consumer's powers if voltage is *V=190V*,



load resistance is $R=139\ \Omega$ and efficiency factor is $\eta=0.99$. Calculate the efficiency factor at the load equal to $R/10$.

6. Calculalate the current in the circle if the resistances are $R_1=165\Omega$, $R_2=197\Omega$, $R_3=198\Omega$, $R_4=193\Omega$ and the maximum power makes $P_3=7.4W$ when allocated to the resistor R_3 in the unforked circuit. Define the input voltage and sub-circuits' voltages, branches' currents, power of the circuit and the powers of the sub-circuits.



7. The consumer with resistance $R_C=10\ \Omega$ is connected to the clamps of the source with e.m.f. $E=40\ W$ and internal resistance $R_0=1\,\Omega$. Define the voltage across the source clamps and source efficiency factor.

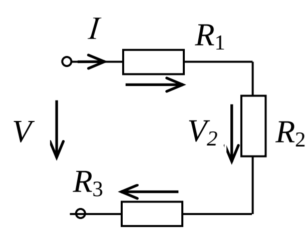
8. Define the voltage source parameters, if the parameters of current source are $J=5\ A$, $G_0=0.5\ Sm$. Draw the voltage source scheme and write down its equation.

9. Define the current source parameters, if the parameters of voltage source are $E=50\,V$, $R_0=5\,\Omega$. Draw the current source scheme and write down its equation.

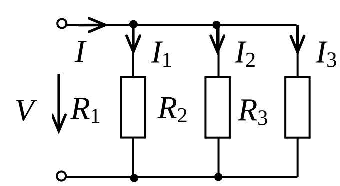
10. The voltage across the source clamps is $V=24V$. The load is $R=8\Omega$. In open circuit mode voltage at source clamps makes $V_{OC}=27V$. Define the source internal resistance.

11. In short circuit mode source current is $I_{SC}=48\,A$. When the source is connected to the resistive element $R=19.5\Omega$, the circuit current is $1.2\,A$. Define the source e.m.f. and its internal resistance.

12. The battery consists of three connected in series sources with the following parameters: $E=1.5V$, $R_0=0.5\Omega$. The battery power is $P=2.25W$. Define the load resistance, power and the voltage across the source clamps.



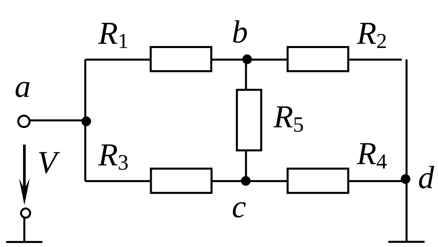
13. Define the resistance R_1 at the circuit with connected in series elements: $R_2=16\,Ом$, $R_3=24\,Ом$, $V_2=8\,V$, $I=2\,A$ and $V=60\,V$. Define the circuit total resistance.

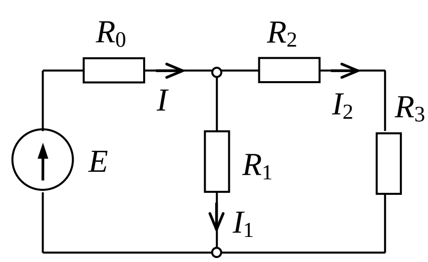


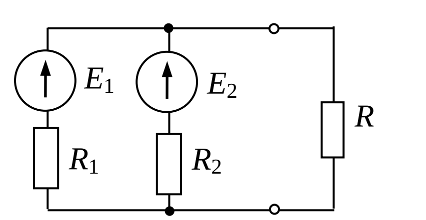
14. Define the conductivity of an element R_3 in the circuit with parallel connection of elements: $G_1=0.05\ Sm$, $G_2=0.1\ Sm$, $I_1=2\ A$ and $I=14\ A$. Define the circuit total conductivity.

15. Define the branches' currents and verify your calculations, if $R_1=R_3=R_5=9\Omega$, $R_2=3\,\Omega$, $R_4=9\,\Omega$ and $V=21V$.

16. The elements $R_1=16\ \Omega$, $R_2=1\ \Omega$ and $R_3=3\ \Omega$ are connected to the source with the parameters $E=40\ V$



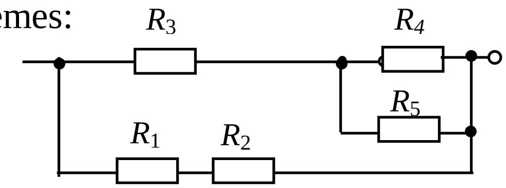


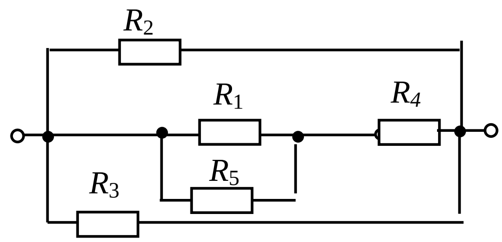


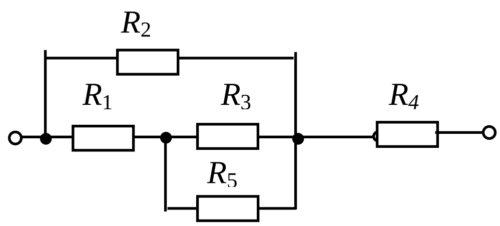
and $R_0 = 0{,}8\ \Omega$. Define the source voltage, branches' currents and voltages across the elements.

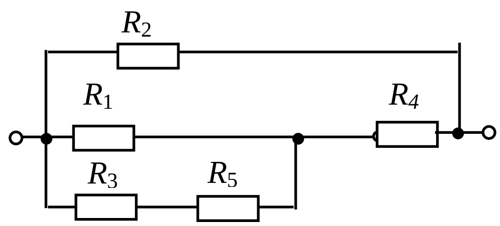
16. Define the load *R* power, if $E_1 = 200\,V$, $E_2 = 150\,V$, $R_1 = 20\,\Omega$, $R_2 = 30\,\Omega$. What should be the *R* value to reach the maximum power on it?

17. Write down the expressions of equivalent resistances for the following schemes:









2.Alternating current circuits calculation.

Task 1. The circuit of two connected in series coils and capacitor (fig.2.1) is powered by AC voltage source of frequency $f = 50\ Hz$. The parameters of the circuit are given in table below. Define the circuit current and the voltages across the coils. Define the resonance frequency of the circuit. Write down the current and voltages instantaneous values. Draw the vector diagram.

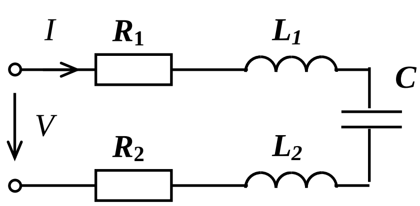


Fig.2.1

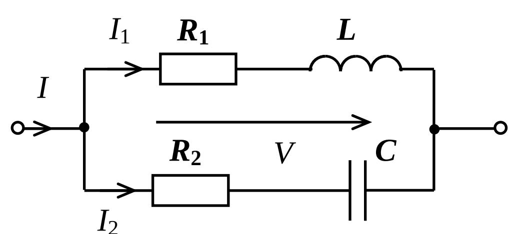


Fig.2.2

VAR	*V*	R_1	R_2	L_1	L_2	*C*	*L*
№	*V*	Ω	Ω	*H*	*H*	*μF*	*H*
00	50	10	20	0.096	0.032	320	0.096
01	120	12	22	0.0127	0.096	200	0.0127
02	100	22	15	0.019	0.016	500	0.019
03	110	10	22	0.016	0.0127	400	0.016
04	50	12	20	0.032	0.019	100	0.032
05	40	22	10	0.0127	0.032	320	0.0127
06	90	12	15	0.016	0.096	200	0.016
07	80	15	12	0.032	0.016	300	0.032
08	110	25	12	0.096	0.019	400	0.096
09	120	12	25	0.019	0.0127	630	0.019

10	50	20	10	0.096	0.032	700	0.096
11	70	22	12	0.0127	0.016	500	0.0127
12	60	15	22	0.019	0.096	680	0.019
13	110	10	20	0.016	0.019	750	0.016
14	50	12	22	0.032	0.0127	320	0.032
15	40	22	15	0.096	0.032	200	0.096
16	30	10	22	0.019	0.032	400	0.019
17	80	12	20	0.032	0.0127	500	0.032
18	110	15	12	0.0127	0.016	750	0.0127
19	120	25	12	0.016	0.096	680	0.016
20	70	12	25	0.096	0.0127	750	0.096

Example for task 1. The circuit (fig.2.3) has the following parameters: $V = 20V$, $f = 50\,Hz$ $R_1 = 5\Omega$, $R_2 = 10\ \Omega$, $L_1 = L_2 = 127\,mH$, $C = 318\ \mu F$. This circuit consists of two coils with parameters R_1, L_1, R_2, L_2 and capacitor C.

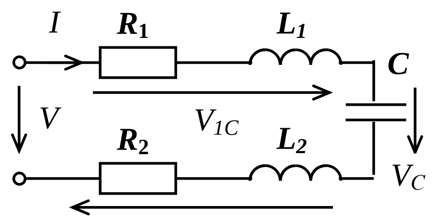


Fig.2.3

Angular frequency makes:

$$\omega = 2\pi f = 3 \cdot 3.14 \cdot 50 = 314\,rad/s.$$

Reactances elements are: $X_{L1} = \omega L_1 = 314 \cdot 0.0127 = 4\ \Omega$,

$X_{L2} = \omega L_2 = 314 \cdot 0.0127 = 4\ \Omega$,

$X_C = 1/(\omega C) = 1/(314 \cdot 0.000318) = 10\ \Omega$.

Circuit resistance is: $R = R_1 + R_2 = 5 + 10 = 15\Omega$.

Circuit reactance is: $X = X_{L1} + X_{L2} - X_C = 4 + 4 - 10 = -2\Omega$.

Circuit impedance is: $Z = \sqrt{R^2 + X^2} = \sqrt{15^2 + (-2)^2} = 15.13\Omega$.

Coils impedances are: $Z_{1C} = \sqrt{R_1^2 + X_{L1}^2} = \sqrt{5^2 + (4)^2} = 6.4\Omega$,

$Z_{2C} = \sqrt{R_2^2 + X_{L2}^2} = \sqrt{10^2 + (4)^2} = 10.77\Omega$.

Circuit current makes: $I = V/Z = 20/15.13 = 1.32\,A$.

Phase shift angle between input voltage and circuit current makes:

$$\varphi = arctg(X/R) = arctg(-2/15) = -6^\circ.$$

Input voltage initial phase is $\psi_V = 0^\circ$.

Circuit current I initial phase is defined from the expression $\varphi = \psi_V - \psi_I$:

$$\psi_I = \psi_V - \varphi = 6^\circ.$$

First coil voltage makes: $V_{1C} = Z_{1C} I = 6.4 \cdot 1.32 = 8.45\,V$.

Phase shift angle between this voltage and current I makes:

$$\varphi_1 = arctg(X_{L1}/R_1) = arctg(4/5) = 39^\circ.$$

Voltage V_{1C} initial phase is defined from the expression $\varphi_1 = \psi_{V1} - \psi_I$:

$$\psi_{V1} = \varphi_1 - \psi_I = 39^\circ - 6^\circ = 33^\circ.$$

Second coil voltage is: $V_{2C} = Z_{2C} I = 10.77 \cdot 1.32 = 14.22\,V.$

Phase shift angle between this voltage and current I is:

$$\varphi_2 = arctg(X_{L2}/R_2) = arctg(4/10) = 22^\circ.$$

Voltage V_{1C} initial phase is defined from the expression $\varphi_2 = \psi_{V2} - \psi_I$:

$$\psi_{V2} = \varphi_2 - \psi_I = 22^\circ - 6^\circ = 16^\circ.$$

Capacitor voltage is: $V_C = X_C I = 10 \cdot 1.32 = 13.2\,V.$

Phase shift angle between this voltage and current I is: $\varphi_C = -90^\circ$.

Voltage V_C initial phase is found from the expression $\varphi_C = \psi_{VC} - \psi_I$:

$$\psi_{VC} = \varphi_C - \psi_I = -90^\circ - 6^\circ = -96^\circ.$$

The resonance condition for this circuit is $X_L = X_C$, it means $(\omega_0 L_1 + \omega_0 L_2) = 1/(\omega_0 C)$, so resonance frequency makes

$$\omega_0 = 1/\sqrt{C(L_1 + L_2)} = 1/\sqrt{0.000318(0.0127 + 0.0127)} = 352\,rad/s.$$

Instantaneous values of voltages and current are:

$v(t) = V_m \sin(\omega t + \psi_V)$, $i(t) = I_m \sin(\omega t + \psi_I)$:

$v(t) = 20\sqrt{2}\sin(314t)$ B, $i(t) = 1.32\sqrt{2}\sin(314t + 6^\circ)$ V,

$v_{1COIL}(t) = 8.45\sqrt{2}\sin(314t + 33^\circ)$ B, $v_{2COIL}(t) = 14.22\sqrt{2}\sin(314t + 16^\circ)\,V$,

$v_C(t) = 13.2\sqrt{2}\sin(314t - 96^\circ)\,V$.

Task 2. The circuit, which consists of connected in parallel coil and capacitor (fig.2.2), is powered by AC voltage source with frequency $f = 50\,Hz$. The parameters of the circuit are given in table above. Define the circuit and branches currents, the phase shift angles. Define the resonance frequency of the circuit. Find the power balance for the circuit.

Example for task 2. Parameters for the circuit at fig.2.4 are: $V = 20V$, $R_1 = 5\Omega$, $R_2 = 10\Omega$, $L_2 = 12.7\,mH$, $C = 318\,\mu F$.

Angular frequency is $f = 50\,Hz$:

$$\omega = 2\pi f = 2 \cdot 3.14 \cdot 50 = 314\,rad/s.$$

Elements' reactances are: $X_L = \omega L = 314 \cdot 0.0127 = 4\,\Omega$, $X_C = 1/(\omega C) = 1/(314 \cdot 0.000318) = 10\,\Omega$.

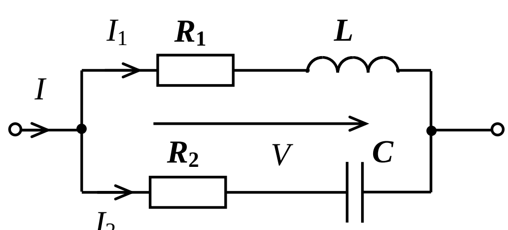


Fig.2.4

Branches' impedances are:

$$Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{5^2 + (4)^2} = 6.4\,\Omega.$$

$$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{10^2 + (-10)^2} = 14.14\,\Omega.$$

Branches' currents are: $I_1 = V/Z_1 = 20/6.4 = 3.13\,A$, $I_2 = V/Z_2 = 20/14.14 = 1.39\,A$.

Phase shift angles between the input voltage and the branches' currents are:
$\varphi_1 = arctg(X_L / R) = arctg(4/5) = 39^\circ$, $\varphi_2 = arctg(X_C / R) = arctg(-10/10) = -45^\circ$.

Input voltage initial phase is $\psi_V = 0^\circ$.

Currents I_1, I_2 initial phases are found from the expressions $\varphi_1 = \psi_V - \psi_{I1}$, $\varphi_2 = \psi_V - \psi_{I2}$: $\psi_{I1} = \psi_V - \varphi_1 = -39^\circ$, $\psi_{I2} = \psi_V - \varphi_2 = 45^\circ$.

The conductances and susceptances method can be used to define the total current. Serial connection is to be converted into parallel one. So, branches' conductances and susceptances are accordingly (fig.2.5):

$$G_1 = \frac{R_1}{Z_1^2} = \frac{5}{6.4^2} = 0.122\,Sm,\; B_1 = \frac{X_L}{Z_1^2} = \frac{4}{6.4^2} = 0.0976\,Sm.$$

$$G_2 = \frac{R_2}{Z_2^2} = \frac{10}{14.14^2} = 0.05\,Sm,\; B_2 = \frac{X_C}{Z_2^2} = \frac{10}{14.14^2} = 0.05\,Sm.$$

The circuit conductance and susceptance (fig.2.6) make accordingly:
$G = G_1 + G_2 = 0.122 + 0.05 = 0.172\ Sm$, $B = B_2 - B_1 = 0.05 - 0.0976 = -0.0476\ Sm$.
$B < 0$. Thus, susceptance has an inductive character.

Circuit admittance is: $Y = \sqrt{G^2 + B^2} = \sqrt{0.172^2 + (-0.0476)^2} = 0.178\,Sm.$

Total current makes: $I = VY = 20 \cdot 0.178 = 3.56A$.

Phase shift angle between input voltage and total current is:

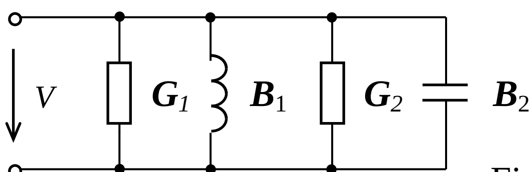


Fig.2.5

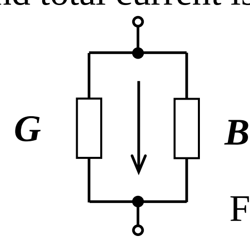


Fig.2.6

$\varphi = arctg(B/G) =\ arctg(0.0476/0.172) = 15^\circ$, if $\psi_V = 0^\circ$, then total current I initial phase can be found from the expression $\varphi = \psi_V - \psi_I$. Therefore, $\psi_I = \psi_V - \varphi = -15^\circ$.

The resonance condition for this circuit is $B_L = B_C$, or $B_1 = B_2$. It means that

$$\frac{\omega_0 L}{R_1^2 + (\omega_0 L)^2} = \frac{1/(\omega_0 C)}{R_2^2 + 1/(\omega_0 C)^2}.$$

From this expression ω_0 can be found.

Power balance equations are used to verify the obtained results.

Thus, consumers active and reactive powers are:

$$P_{cons} = R_1 I_1^2 + R_2 I_2^2 = 5 \cdot 3.13^2 + 10 \cdot 1.39^2 = 68.77W,$$

$$Q_{cons} = X_L I_1^2 - X_C I_2^2 = 4 \cdot 3.13^2 - 10 \cdot 1.39^2 = 18.43\,VAr.$$

Source active and reactive powers are:

$$P_{sour} = VI\cos\varphi = 20 \cdot 3.56\cos 15^\circ = 68.77W,$$

$$Q_{sour} = VI\sin\varphi = 20 \cdot 3.56\sin 15^\circ = 18.43VAr.$$

Thus, if $P_{cons} \approx P_{sour}$, $Q_{cons} \approx Q_{sour}$, power balance is true. Therefore, the circuit parameters have been defined correctly.

Task 3. Draw the scheme (fig.2.7) according to your variant at table below. Find out the circuit and branches' impedances for frequency $f = 50\,Hz$. Define the branches' currents and the voltages. Verify the calculations (balance of power equation). Draw the

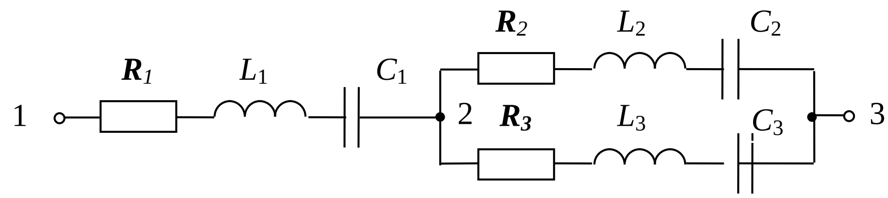


Fig.2.7

vector diagram of currents and voltages.

Example for task 3. For the circuit on fig.2.7 : $V = 70V$, $R_1 = 5\Omega$, $R_2 = 0\Omega$, $R_3 = 4\Omega$, $L_1 = 9mH$, $L_2 = 7mH$, $L_3 = 20mH$, $C_1 = C_3 = 0\,\mu F$, $C_2 = 281\,\mu F$.

The circuit according to this variant is shown at fig.2.8

Attention! Calculation is for frequency $f = 60\,Hz$.

Angular frequency, thus, is $\omega = 2\pi f = 2 \cdot 3.14 \cdot 60 = 377\ rad/s$.

Var №	*V* *V*	R_1 Ω	L_1 *MH*	C_1 *μF*	R_2 Ω	L_2 *MH*	C_2 *μF*	R_3 Ω	L_3 *MH*	C_3 *μF*
00	220	11	45		13	55		15		750
01	220	13	25		15		200	11	35	
02	220	15		300	11		240	13	55	
03	220		55		18	65		20		680
04	220	18	15		20		360		35	
05	220	18		820		35		18		270
06	220			750	20	50		24		330
07	220	22	65		24		150			220
08	220	24		360			270	22	45	
09	220	11		510	15		620	11	45	
10	220	13		180	15	20		11	50	
11	220	15	60		11			13		300
12	220	14	65		12		200	20	30	
13	220	18		680	20	20			50	
14	220	20	0	150		10		18		720
15	220	10		220	12		510	20	20	
16	220	22		430	24	25				270
17	220	24	30		15		270		55	330
18	220	11	35		13		360		15	150
19	220	13		470		30	200	11	15	
20	220	12	80	270	24	15		10		180

Reactances of inductive elements are $X_{Ln} = \omega L_n$:

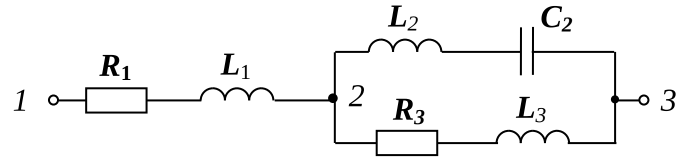


Fig. 2.8

$$X_{L1} = \omega L_1 = 377 \cdot 9 \cdot 10^{-3} = 3.39\,\Omega,\; X_{L2} = \omega L_2 = 377 \cdot 7 \cdot 10^{-3} = 2.64\,\Omega,$$

$$X_{L3} = \omega L_3 = 377 \cdot 20 \cdot 10^{-3} = 7.54\,\Omega.$$

Reactances of capacitive elements are $X_{Cn} = 1/\omega C_n$:

$$X_{C2} = 1/\omega C_2 = 1/377 \cdot 281 \cdot 10^{-6} = 9.44\ \Omega.$$

Branches' impedances make $\underline{Z}_n = R_n + jX_{Ln} - jX_n$:

$$\underline{Z}_1 = R_1 + jX_{L1} = 5 + j3.39 = 6{,}04e^{j34^\circ}\ \Omega,$$

$$\underline{Z}_2 = jX_{L2} - jX_{C2} = j(2.64 - 9.44) = -j6.8 = 6.8e^{-j90^\circ}\ \Omega,$$

$$\underline{Z}_3 = R_3 + jX_{L3} = 4 + j7.54 = 8.54e^{j62^\circ}\ \Omega.$$

Unforked part of the circuit impedance is:

$$\underline{Z}_{12} = \underline{Z}_1 = R_1 + jX_{L1} = 5 + j3.39 = 6{,}04e^{j34^\circ}\ \Omega.$$

Parallel connection impedance makes:

$$\underline{Z}_{23} = \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = \frac{6.8e^{-j90}8.54e^{j62}}{-j6.8 + 4 + j7.54} = \frac{58.05e^{-j28}}{4 + j0.74} = \frac{58.05e^{-j28}}{4.07e^{j10}} =$$

$$= 14.27e^{-j38} = (11.18 - j8.87)\ \Omega.$$

Circuit impedance is:

$$\underline{Z} = \underline{Z}_{12} + \underline{Z}_{23} = 5 + j3.39 + 11.18 - j8.87 = 16.18 - j5.47 = 17.08e^{-j19^\circ}\ \Omega.$$

Circuit current is: $\underline{I} = \underline{V}/\underline{Z} = 70/17.08e^{-j19^\circ} = 4.1e^{j19^\circ} = (3.88 + j1.31)A$.

Voltages across the parts of the circuit make:

$$\underline{V}_{12} = \underline{Z}_{12}\underline{I} = 6.04e^{j34^\circ} \cdot 4.10e^{j19^\circ} = 24.76e^{j53^\circ} = (14.95 + j19.74)V.$$

$$\underline{V}_{23} = \underline{Z}_{23}\underline{I} = 14.27e^{-j38^\circ} \cdot 4.10e^{j19^\circ} = 58.48e^{-j20^\circ} = (55.05 - j19.74)V.$$

According to the second Kirchhoff's law:

$$\underline{V} = \underline{V}_{12} + \underline{V}_{23} = (14.95 + j19.74) + (55.05 - j19.74) = 70V.$$

Branches' currents are:

$$\underline{I}_2 = \underline{V}_{23}/\underline{Z}_2 = 58.48e^{-j20^\circ}/6.8e^{-j90^\circ} = 8.6e^{j70^\circ} = (2.9 + j8.09)A,$$

$$\underline{I}_3 = \underline{V}_{23}/\underline{Z}_3 = 58.48e^{-j20^\circ}/8.54e^{j62^\circ} = 6.85e^{-j82^\circ} = (0.98 - j6.78)A.$$

According to the first Kirchhoff's law:

$$\underline{I} = \underline{I}_2 + \underline{I}_3 = (2.9 + j8.1) + (0.98 - j6.78) = 3.88 + j1.31A$$

Complex total power makes:

$$\underline{S} = \underline{V}\overset{*}{\underline{I}} = 70 \cdot 4.1e^{-j19^\circ} = 287e^{-j19^\circ} = (272 - j92)\,VA.$$

Consumers' active and reactive powers make:

$$P = R_1 I_1^2 + R_3 I_3^2 = 5 \cdot 4.1^2 + 4 \cdot 6.85^2 = 272 W.$$

$$Q = X_{L1} I_1^2 + (X_{L2} - X_{C2}) I_2^2 + X_{L3} I_3^2 =$$

$$= 3.39 \cdot 4.1^2 - 6.8 \cdot 8.6^2 + 7.54 \cdot 6.85^2 = -92 VAr.$$

Vector diagram.

Assumed voltages' vectors scale is $M_V = 15\ V/cm$, the voltage vectors' lengths are:

$$V = \frac{|V|}{M_V} = \frac{70}{15} = 4.7cm,\ V_{12} = \frac{|V_{12}|}{M_V} = \frac{24.76}{15} = 1.65cm,$$

$$V_{23} = \frac{|V_{23}|}{M_V} = \frac{58.48}{15} = 3.9cm.$$

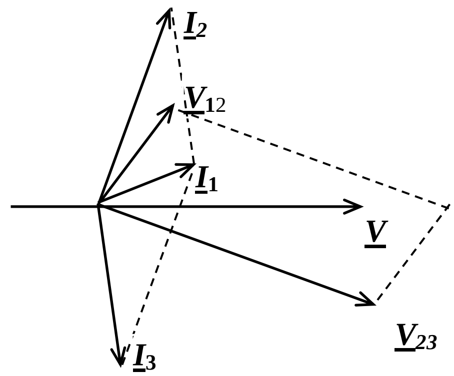


Fig.2.9

Voltage initial phases (the angles between axis X and the vector) are: $\psi_V = 0^\circ$, $\psi_{V12} = 53^\circ$, $\psi_{V23} = -20^\circ$.

Assumed currents' vectors scale is $M_I = 1.5\ A/cm$, the currents' vectors lengths are:

$$I_1 = \frac{|I_1|}{M_I} = \frac{4.1}{1.5} = 2.7cm,\ I_2 = \frac{|I_2|}{M_I} = \frac{8.6}{1.5} = 5.7cm,\ I_3 = \frac{|I_3|}{M_I} = \frac{6.85}{1.5} = 4.6cm.$$

Currents' initial phases (the angles between axis X and the vector) are: $\psi_{I1} = 19^\circ$,

$\psi_{I2} = 70^\circ$, $\psi_{I3} = -82^\circ$.

Example 4. The voltage $v = 12.56 \sin(314t + \pi/3)\ V$ is applied to inductivity $L = 0.02\ mH$. Write down the current instantaneous value. Draw the vector diagram for current and voltage effective values.

Inductivity reactance is: $X_L = \omega L = 314 \cdot 0.02 = 6.28 \Omega$.

Current amplitude is: $I_m = V_m / X_L = 12.56/6.28 = 2A$.

Phase shift angle for the element makes: $\varphi = \psi_V - \psi_I = \pi/2$.

Current initial phase is: $\psi_I = \psi_V - \varphi = \pi/3 - \pi/2 = -\pi/6$.

Current instantaneous value is: $i = I_m \sin(\omega t + \psi_I) = 2\sin(314t - \pi/6) A$.

The vector diagram is shown at fig.2.10.

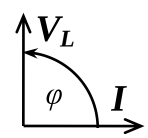


Fig.2.10

Example 5. The voltage $v = 141 \sin(314t - \pi/6)\ V$ is applied to capacitance $C = 320\ \mu F$. Write down the current instantaneous value. Draw the vector diagram for current and voltage effective values.

Capacitance reactance is: $X_C = 1/(\omega C) = 1/(314 \cdot 320 \cdot 10^{-6}) = 10 \Omega$.

Current amplitude is: $I_m = V_m / X_C = 141/10 = 14.1A$.

Phase shift angle for the element is: $\varphi = \psi_V - \psi_I = -\pi/2$.

Current initial phase is: $\psi_I = \psi_V - \varphi = -\pi/6 + \pi/2 = \pi/3$.

Current instantaneous value is: $i = I_m \sin(\omega t + \psi_I) = 14.1 \sin(314t + \pi/3)\ A$

The vector diagram is shown at fig.2.11.

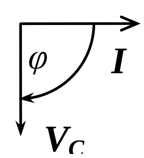


Fig.2.11

Example 6. The capacitor $C = 400\,\mu F$ and coil with parameters $R = 50\Omega$, $L = 0.3H$ are connected in series to the AC source. The voltage effective value is $V = 200V$, $f = 50\,Hz$. Define the circuit current, the voltages across the capacitor and coil, the circuit's active and reactive powers and the power factor.

Coil and capacitor reactances are: $X_L = \omega L = 314 \cdot 0.3 = 100\Omega$,

$$X_C = 1/(\omega C) = 1/(314 \cdot 400 \cdot 10^{-6}) = 8\Omega.$$

Circuit impedance is: $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (100 - 8)^2} = 105\Omega$.

Circuit current is: $I = V/Z = 200/105 = 1.9\,A$.

Coil impedance is: $Z_{COIL} = \sqrt{R^2 + X_L{}^2} = \sqrt{50^2 + 100^2} = 112\Omega$.

Coil voltage is: $V_{COIL} = IZ_{COIL} = 1.9 \cdot 112 = 213V$.

Capacitor voltage is $V_C = IX_C = 1.9 \cdot 8 = 15.2\,V$.

Elements reactive powers are: $Q_L = X_L I^2 = 100 \cdot 1{,}9^2 = 361VAr$,

$$Q_C = X_C I^2 = 8 \cdot 1{,}9^2 = 29\,VAr.$$

Circuit reactive power is: $Q = Q_L - Q_C = 361 - 29 = 332\,VAr$.

Circuit active power is: $P = RI^2 = 50 \cdot 1{,}9^2 = 180.5W$.

Power factor makes: $\cos\varphi = P/S = R/Z = 50/105 = 0.48$.

Example 7. The input voltage for the circuit with a parallel connection of the coil with parameters $R = 6\Omega$, $X_L = 12\,\Omega$ and capacitor with reactance $X_C = 6\Omega$ is $V = 20V$. Define the circuit current, the branches' currents and the phase shift angles.

The coil serial connection R, X_L is transformed into parallel one of G, B_L. Conductance and susceptances then are:

$$G = \frac{R}{(R^2 + X_L^2)} = \frac{6}{6^2 + 12^2} = 0.033\,Sm,$$

$$B_L = \frac{X_L}{(R^2 + X_L^2)} = \frac{12}{6^2 + 12^2} = 0.067\,Sm,$$

$$B_C = 1/X_C = 1/6 = 0.167\,Sm.$$

The circuit susceptance and admittance are:

$$B = B_C - B_L = 0.167 - 0.067 = 0.1Sm,$$

$$Y = \sqrt{G^2 + B^2} = \sqrt{0.033^2 + (0.1)^2} = 0.105Sm.$$

The coil admittance makes: $Y_{COIL} = \sqrt{G^2 + B_L^2} = \sqrt{0.033^2 + (0.067)^2} = 0.075Sm$.

The circuit current is: $I = YV = 0.105 \cdot 20 = 2.1A$.

The branches' currents are: $I_{COIL} = Y_{COIL}V = 0.075 \cdot 20 = 1{,}5\,A$,

$$I_C = B_C V = 0.167 \cdot 20 = 3{,}34\,A.$$

Phase shift angle between total current and input voltage is:

$$\varphi = arctg(B/G) = arctg(-0.1/0.033) = -72^\circ.$$

Phase shift angle between coil current and input voltage is:

$$\varphi_L = arctg(B_L / G) = arctg(0.067 / 0.033) = 64^{\circ}.$$

The circuit active power makes: $P = RI^2_{COIL} = 6 \cdot 1.5^2 = 13.5W$.

Example 8. The voltage $V = 220\,V$ is applied to the coil. The current is $I = 4\,A$, active power makes $P = 500\,W$ and $f = 50\,Hz$. Define the $\cos\varphi$ before and after the capacitors' battery $C = 400\,\mu F$ is connected in parallel to the coil.

The circuit resistance, impedance and reactance without capacitors make accordingly: $R = P / I^2 = 500/4^2 = 31.3\Omega$, $Z = V / I = 220/4 = 55\Omega$,

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{55^2 - 31.3^2} = 45.2\Omega.$$

Power factor is: $cos\,\varphi = R / Z = 31.3/55 = 0.57$.

When the capacitors' battery is connected, the circuit will have parallel connection of the coil and capacitors. Serial connection of X_L, R elements is to be transformed into parallel connection of B_L, G elements.

$$B_L = \frac{X_L}{Z^2} = \frac{45.2}{55^2} = 0{,}015\,Sm, \qquad G = \frac{R}{Z^2} = \frac{31.3}{55^2} = 0.01Sm.$$

The capacitors' susceptance makes: $B_C = \omega C = 314 \cdot 500 \cdot 10^{-6} = 0.0157\,Sm$.

The circuit susceptance is: $B = B_C - B_L = 0.0157 - 0.015 = 0.0007\,Sm$.

The circuit admittance makes: $Y = \sqrt{G^2 + B^2} = \sqrt{0.01^2 + 0.0007^2} = 0.01Sm$.

The power factor is: $cos\,\varphi = G / Y = 0.01/0.01 = 1$.

Example 9. Define the instantaneous values of currents i_1, i_2, i (fig.2.12), when the input voltage is $v = 35\sin 314t\,V$. Elements parameters are accordingly: $R_1 = 7\Omega$, $R_2 = 2\Omega$, $X_L = 3\,\Omega$ and $X_C = 5\,\Omega$.

This task is solved by using the vector diagrams method.

Branches' impedances are: $Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{7^2 + 3^2} = 7.62\Omega$,

$$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{2^2 + 5^2} = 5.39\Omega.$$

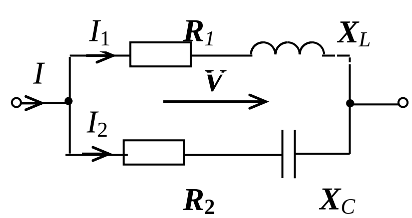


Fig.2.12

Branches' currents amplitudes are:

$$I_{m1} = V_m / Z_1 = 35/7.62 = 4.59\,\text{A},$$

$$I_{m2} = V_m / Z_2 = 35/5.39 = 6.49\,\text{A}.$$

Phase shift angles between V and branches' currents are: $\varphi_1 = arctg(\,X_L / R_1\,) = arctg(\,3/7\,) = 32^{\circ}$,

$$\varphi_2 = arctg\,(-X_C / R_2\,) = arctg(-5/2\,) = -68^{\circ},$$

Currents initial phases makes $\psi_I = \psi_V - \varphi$: $\psi_{I1} = 0 - 32^{\circ} = -32^{\circ}$, $\psi_{I2} = 68^{\circ}$.

The basic vector is a voltage vector $\overline{V}$. It is put along the axis X (fig. 2.13), the branches currents vectors are drawn at the angles $\varphi_1 = -32^{\circ}$ for $\overline{I}_{1m}$ and $\varphi_2 = 68^{\circ}$ for s $\overline{I}_{2m}$

Branches' currents instantaneous values are:

$$i_1 = 4.59\,sin(\,\omega t - 32^\circ\,)\,A, \qquad i_2 = 6.49\,sin(\,\omega t + 68^\circ\,)\,A.$$

The total current vector is the geometrical sum of branches' currents vectors: $\bar{I}_m = \bar{I}_{1m} + \bar{I}_{2m}$ (fig. 2.13):

$$I_m = \sqrt{I_{m1}^2 + I_{m2}^2 - 2I_{m1}I_{m2}\,cos(\,\varphi_1 - \varphi_2\,)} =$$
$$= \sqrt{4.59^2 + 6.49^2 - 2\cdot 4.59\cdot 6.49\,cos(\,32+68\,)} = 7.27\,A.$$

Total current effective value makes: $I = I_m / \sqrt{2} = 8.61/1.41 = 5.2\,A.$

Phase shift angle between input voltage and total current is:

$$\varphi = arctg\,\frac{I_{m1}\,sin\,\varphi_1 + I_{m2}\,sin\,\varphi_2}{I_{m1}\,cos\,\varphi_1 + I_{m2}\,cos\,\varphi_2} = arctg\,\frac{4.59\,sin\,32 + 6.49\,sin(\,-68\,)}{4.59\,cos\,32 + 6.49\,cos(\,-68\,)} = -30^\circ.$$

Circuit total current is: $i = 7.27\,sin(\,\omega t + 30^\circ\,)\,A.$

Example 10. Define the circuit current (fig. 2.12) at the input voltage effective value of $V = 25\,V$, $f = 50\,Hz$ and elements' corresponding parameters: $R_1 = 7\,\Omega$, $R_2 = 2\,\Omega$, $X_L = 3\,\Omega$, $X_C = 5\,\Omega$.

This task is solved by using the active and reactive constituents method.

Branches' impedances are: $Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{7^2 + 3^2} = 7.62\,\Omega$,

$$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{2^2 + 5^2} = 5.39\,\Omega.$$

Branches' currents effective values are:

$$I_1 = V / Z_1 = 25/7.62 = 3.28\,\mathrm{A}, \quad I_2 = V / Z_2 = 25/5.39 = 4.64\,\mathrm{A}.$$

Phase shift angles between the voltage and branches currents are:

$$\varphi_1 = arctg(\,X_L / R_1\,) = arctg(\,3/7\,) = 32^\circ,$$
$$\varphi_2 = arctg\,(\,-X_C / R_2\,) = arctg(\,-5/2\,) = -68^\circ.$$

The vector diagram for the circuit is shown at fig.2.13. The branches currents vectors are shown as I_1, I_2 together with their active and reactive constituents.

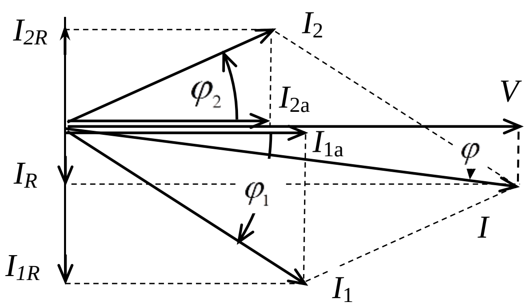


Fig.2.13

Active and reactive currents constituents are defined accordingly:

$$I_{a1} = I_1\cos\varphi_1 = 3.28\cos(32) = 2.78\,A$$
$$I_{a2} = I_2\,cos\,\varphi_2 = 4.64\,cos(\,-68\,) = 1.74\,A,$$
$$I_{R1} = I_1\sin\varphi_1 = 3.28\,sin\,32 = 1.74\,A,$$
$$I_{R2} = I_2\sin\varphi_2 = 4.64\sin(-68) = -4.3\,A.$$

Active and reactive total current constituents make accordingly:

$$I_a = I_{a1} + I_{a2} = 2.78 + 1.74 = 4.52\,A,$$
$$I_R = I_{R1} + I_{R2} = 1.74 - 4.3 = -2.526\,A.$$

The total current effective value is defined as geometrical sum of active and reactive constituents: $I = \sqrt{I_a^2 + I_R^2} = \sqrt{4.52^2 + 2.56^2} = 5.2\,A.$

Phase shift angle between total current and input voltage is:

$$\varphi = arctg(I_R / I_a) = arctg(-2.56/4.52) = -30^\circ.$$

The total current I initial phase is $\psi_I = -\varphi = 30^\circ$.

Example 11. Solve the previous task by using the symbolic method.

The branches' impedances are accordingly:

$$\underline{Z}_1 = R_1 + jX_L = 7 + j3 = 7.62e^{j32}\ \Omega,$$

$$\underline{Z}_2 = R_2 - jX_C = 2 - j5 = 5.39e^{-j68}\ \Omega.$$

Branches' complex currents are accordingly:

$$\underline{I}_1 = \underline{V}/\underline{Z}_1 = 25/7.62e^{j32} = 3.28e^{-j32} = 2.78 - j1.74A,$$

$$\underline{I}_2 = \underline{V}/\underline{Z}_2 = 25/5.39e^{-j68} = 4.64e^{j68} = 1.74 + j4.3A.$$

Complex total current is:

$$\underline{I} = \underline{I}_1 + \underline{I}_2 = 2.78 - j1.74 + 1.74 + j4.3 = 4.52 + j2.56 = 5.2e^{j30}\ A.$$

Tasks for individual work

1. Current effective value in the coil is 1*A*. Reactive and active powers are correspondingly *280 VAr* and *540 VA*. Define the impedance, the resistance and reactance of the coil.

2. The voltage across connected in parallel inductivity and resistor is *60 V*. The resistance and reactance are *10* Ω and *20* Ω. Define the currents in the circuit and the efficiency factor.

3. The voltage across connected in parallel inductivity, capacitance and resistor is 120 *V*, $R = 20\Omega$, $X_L = 20\Omega$ and $X_C = 10\Omega$. Define the currents in the circuit and the phase shift angle between input current and voltage.

4. The impedance of consumer is $\underline{Z} = (10 - j10)$ Ω, current effective value is 2*A* and its initial phase makes *π/3*. Write down the instantaneous value of the voltage.

5. Current effective value for connected in series resistor and capacitance is 2*A* and the impedance is 50 Ω. Define the dissipation factor of capacitor, if the voltage across the resistor is *40 V*.

6. Instantaneous value of the voltage of the circuit with the connected in series elements $R = 10\Omega$, $X_L = 20\Omega$ is $v(t) = 100\sin 314t\,B$. Write down the instantaneous value for the current.

7. Circuit meters indicate the following: *1A*, *200V*, *100W*. Define the resistance and reactance for connected in series elements.

8. Voltage across the coil is 220*V*. Current is 2*A*. Define the resistance and reactance, when the phase shift angle is *π/6*.

9. Instantaneous value of the circuit current with connected in series elements $R = 10\Omega$, $X_C = 20\Omega$ is $i(t) = 0.5\sin 314t\,A$. Write down the instantaneous value of the voltage.

10. Current effective value of connected in series resistive and capacitive elements is 1*A*. Active and reactive powers are accordingly $50W$ and $100VAr$. Define the resistance, reactance and impedance.

11. Effective value of the voltage across connected in parallel resistive $R = 10\Omega$

and capacitive $X_C = 20\Omega$ elements is 50*V*. Define the total current of the circuit and the phase shift angle.

12. The impedance of the element is $\underline{Z} = (10 + j20)\ \Omega$. Effective value of the voltage is $200V$. Phase shift angle is $\pi/3$. Write down the instantaneous value of the current.

13. Resonance circuit consists of the elements $R = 5\Omega$, $L = 20mH$, $C = 200\mu F$. Define the resonance frequency and the coil Q-factor.

14. Voltage and current instantaneous values are:

$$v(t) = 40\sin(314t + \pi/3)V,\ i(t) = 4\sin(314t - \pi/6)A.$$

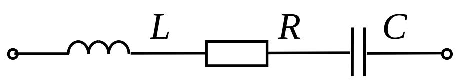
Write down the amplitude, the effective and average values of voltage and current, their initial phases and frequency. Determine the phase shift angle in grads and radians. Draw the vectors of voltage and current at vector diagram. Write down the voltage and the current as complex numbers.

15. Current instantaneous values of two parallel branches are:

$$i_1(t) = 1\sin(314t + \pi/3)A \text{ and } i_2(t) = 4\sin(314t - \pi/6)A.$$

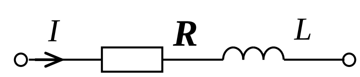
Find the total current in vector and complex form.

16. Define the circuit impedance and phase shift angle, if $R=15\Omega$, $L=32mH$

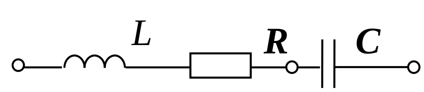


and $C=160\mu F$. Write down the impedance in complex form (both algebraic and exponential). Transform the connection of elements from in series into parallel. Define the conductance, the susceptance and the admittance. Write down the susceptance in complex form (both algebraic and exponential). Draw the vector diagram for the circuit.

17. Define the impedance, the input voltage, the phase shift angle, the active and reactive powers and the coil quality factor in a circuit at the given current $I=0.25A$, inductivity $L=1.2H$ and resistance $R=195\Omega$.



18. Define the input voltage, the coil voltage, the capacitor voltage, the active and reactive powers and the phase shift angles at the given current $I=200mA$, inductivity $L=1.2H$, resistance $R=195\Omega$ and capacitance $C=6.34\mu F$.



19. Define the total current, the coil current, the capacitor current, the active and reactive powers and the phase shift angles in the circuit at the given voltage $V=70V$, inductivity $L=1.2H$, resistance $R=195\Omega$ and capacitance $C=4.11\mu F$.