Міністерство освіти і науки України
Тернопільський національний технічний університет імені Івана Пулюя

*Кафедра
світлотехніки та
електротехніки*

Методичні вказівки
для іноземних студентів з дисципліни

«Electrical Engineering.

PRACTICUM»

Тернопіль
2018

УДК 621.3
Е50

Укладачі:
Куземко Н.А., канд. техн. наук, доцент.

Рецензенти:
Лупенко А.М., докт. техн. наук, професор.

Методичний посібник розглянуто й затверджено на засіданні
кафедри світлотехніки та електротехніки
Тернопільського національного технічного університету імені Івана Пулюя.
Протокол № 4 від 22 листопада 2017 р.

Схвалено та рекомендовано до друку на засіданні методичної комісії
факультету прикладних інформаційних систем та електроінженерії
Тернопільського національного технічного університету імені Івана Пулюя.
Протокол № 4 від 24 листопада 2017 р.

Е50 Elektrical Engineering. Practicum: методичні вказівки для іноземних студентів / Укладачі : Куземко Н.А. – Тернопіль : Тернопільський національний технічний університет імені Івана Пулюя, 2018. – 68 с.

УДК 621.3

Посібник складено відповідно до робочих програм курсів “Електротехніка, електроніка та основи МПТ“, “Електротехніка в будівництві”, “Теорія електричних та магнітних кіл”.

Редагування англійською мовою: канд. пед. наук, *Кухарська В.Б.*

Комп’ютерний набір: *Куземко Н.А.*

1. *Electrical circuits calculations methods*

Task 1. Draw the scheme (fig.1.1) according to your variant in table below. Write down the system of equations according to Kirchhoff's laws.

Task 2. Write down the system of equations according to the loop currents method and calculate the circuit using this method.

Task 3. Verify the calculations using the equation of power balance.

Task 4. Write down the system of equations according to the nodal potential method and write down the expressions of branches' currents.

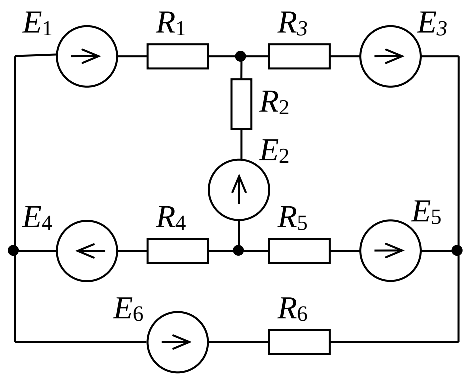


Fig.1.1

Task 5. Define the external loop point potentials and draw the potential diagram for the external loop.

Var. №	E_1 V	E_2 V	E_3 V	E_4 V	E_5 V	E_6 V	R_1 Ω	R_2 Ω	R_3 Ω	R_4 Ω	R_5 Ω	R_6 Ω
00	12	15	18	–	–	–	3	4	5	6	7	8
01	–	15	18	21	–	–	4	5	6	7	8	3
02	–	–	18	21	24	–	5	6	7	8	3	4
03	–	–	–	21	24	27	6	7	8	3	4	5
04	15	18	21	–	–	–	7	8	3	4	5	6
05	–	18	21	24	–	–	8	3	4	5	6	7
06	–	–	21	24	27	–	3	4	5	6	7	8
07	–	–	–	24	27	12	4	5	6	7	8	3
08	18	21	24	–	–	–	5	6	7	8	3	4
09	–	21	24	27	–	–	6	7	8	3	4	5
10	–	–	24	27	12	–	7	8	3	4	5	6
11	–	-	-	27	12	15	8	3	4	5	6	7
12	21	24	27	–	–	–	3	4	5	6	7	8
13	–	24	27	12	–	–	4	5	6	7	8	3
14	–	–	27	12	15	–	5	6	7	8	3	4
15	–	–	–	12	15	18	6	7	8	3	4	5
16	24	27	12	–	–	–	7	8	3	4	5	6
17	–	27	12	15	–	–	8	3	4	5	6	7
18	–	–	12	15	18	-	3	4	5	6	7	8
19	–	–	–	15	18	21	4	5	6	7	8	3
20	27	12	15	–	–	–	5	6	7	8	3	4

Example for task 1. The graph for the circuit (fig.1.2) is shown at fig.1.3. There are six branches (unknown currents), $p=6$ and number of independent nodes is three at the circuit $q=3$. The currents' (I_1-I_6) directions are chosen arbitrarily. The nodes are noted as *N1*, *N2*, *N3*.

The equations according to the Kirchhoff's first law for the nodes *1-3* (if the current flows into the node it is assumed with "+", and out of the node with "-") are:

for *N1:* $-I_1 - I_3 - I_4 = 0$,

for *N2:* $+I_1 - I_2 - I_5 = 0$,

for *N3:* $+I_3 + I_2 + I_6 = 0$.

The equations according to the Kirchhoff's second law ($p - q = 3$) for the loops,

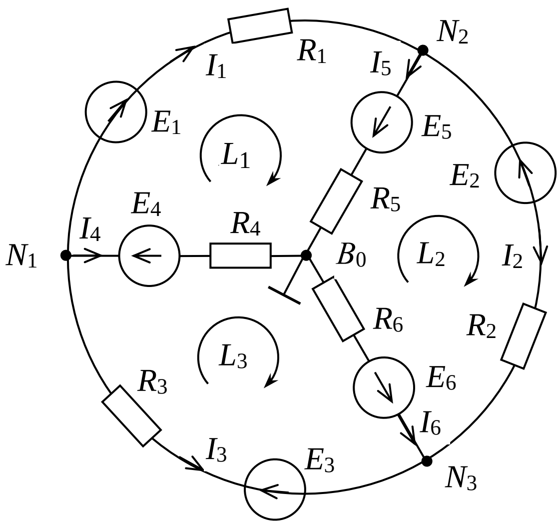


Fig 1.2

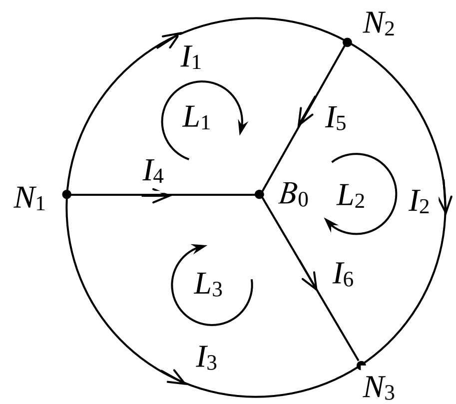


Fig.1.3

L_1, L_2, L_3 (the directions along the loops are chosen clockwise , if the directions of the bypass and the voltage or e.m.f. are the same, they are denominated with "+", if opposite with "-") are:

for *L1:* $+R_1 I_1 + R_5 I_5 - R_4 I_4 = +E_1 + E_5 + E_4$,

for *L2:* $+R_2 I_2 - R_5 I_5 - R_6 I_6 = -E_2 - E_5 - E_6$,

for *L3:* $+R_6 I_6 - R_3 I_3 + R_4 I_4 = +E_6 + E_3 - E_4$.

So, the equations system according to the *Kirchhoff's laws* is:

$$\begin{cases} -I_1 - I_3 - I_4 = 0 \\ +I_1 - I_2 - I_5 = 0 \\ +I_3 + I_2 + I_6 = 0 \\ +R_1 I_1 + R_5 I_5 - R_4 I_4 = +E_1 + E_5 + E_4 \\ +R_2 I_2 - R_5 I_5 - R_6 I_6 = -E_2 - E_5 - E_6 \\ +R_6 I_6 - R_3 I_3 + R_4 I_4 = +E_6 + E_3 - E_4 \end{cases}$$

Example for task 2 . For the circuit on fig.1.2 $E_1 = 10V$, $E_3 = 15V$, $E_6 = 20V$, $E_2 = E_4 = E_5 = 0V$, $R_1 = 10\,\Omega$, $R_2 = 6\,\Omega$, $R_3 = 5\,\Omega$, $R_4 = 7\,\Omega$, $R_5 = 8\,\Omega$, $R_6 = 5\,\Omega$.

The system of equations according to the loop currents method for the circuit with three independent loops is:

$$\begin{cases} +R_{11}I_{L1} - R_{12}I_{L2} - R_{13}I_{L3} = E_{L1} \\ -R_{21}I_{L1} + R_{22}I_{L2} - R_{23}I_{L3} = E_{L2} \\ -R_{31}I_{L1} - R_{32}I_{L2} + R_{33}I_{L3} = E_{L3} \end{cases},$$

where $R_{11} = R_1 + R_5 + R_4 = 10+8+7 = 25\ \Omega$, $R_{22} = R_2 + R_6 + R_5 = 6+5+8 = 19\ \Omega$, $R_{33} = R_4 + R_6 + R_3 = 7+5+5 = 17\ \Omega$ – are the individual resistances of the loops, which are equal to the sum of all the resistances of the loop;

$R_{12} = R_{21} = R_5 = 8\ \Omega$, $R_{13} = R_{31} = R_4 = 7\Omega$, $R_{23} = R_{32} = R_6 = 5\Omega$ – are mutual resistances of the loops, i.e. the resistances of the branches which are mutual for the respective loops;

$E_{L1} = E_1 + E_5 + E_4 = 10+0+0 = 10\ V$, $E_{L2} = -E_2 - E_5 - E_6 = 0+0-20 = -20\ V$, $E_{L3} = E_6 + E_3 - E_4 = 20+15+0 = 35\ V$ – are loops' e.m.f. These are equal to the algebraic sum of the electromotive forces alongside the loops.

When substituted in the equations system above, these values make:

$$\begin{cases} +25I_{L1} - 8I_{L2} - 7I_{L3} = 10 \\ -8I_{L1} + 19I_{L2} - 5I_{L3} = -20 \\ -7I_{L1} - 5I_{L2} + 17I_{L3} = 35 \end{cases}.$$

When solved the values make: $I_{L1} = 1.153\,A$, $I_{L2} = 0.108\,A$, $I_{L3} = 2.565\,A$.

Branches' currents can be defined from the loops' currents as: $I_1 = I_{L1} = 1.153A$, $I_2 = I_{L2} = 0.108\,A$, $I_3 = -I_{L3} = -2.565\,A$, $I_4 = I_{L3} - I_{L1} = 2.565 - 1.153 = 1.412\,A$, $I_5 = I_{L1} - I_{L2} = 1.153 - 0.108 = 1.045\,A$, $I_6 = I_{L3} - I_{L2} = = 2.565 - 0.108 = 2.457\,A$.

Example for task 3. The power balance equation is used to verify the calculations. For the circuit on fig.1.2, according to the calculations at example 2 the total power of the circuit's sources should be equal to the total power of the circuit's consumers $\Sigma P_R = \Sigma P_E$.

Thus, total power of the sources is:

$$\Sigma P_E = E_1 I_1 - E_3 I_3 + E_6 I_6 = 10 \cdot 1.153 - 15 \cdot (-2.565) + 20 \cdot 2.457 = 99.145\ W.$$

When the e.m.f. and the current have the same directions the source power $P_E = EI$ is considered with "+", if opposite with "-".

The total power of the consumers makes:

$$\Sigma P_R = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 + R_4 I_4^2 + R_5 I_5^2 + R_6 I_6^2 =$$

$$= 10 \cdot (1.153)^2 + 6 \cdot (0.108)^2 + 5 \cdot (-2.565)^2 + 7 \cdot (1.412)^2 + 8 \cdot (1.045)^2 + 5 \cdot (2.457)^2 = 99.13\ W$$

So, $\Sigma P_E \approx \Sigma P_R$.

Example for task 4. The system of equations according to the nodal potential method for the circuit with three independent nodes (fig.1.2) is:

$$\begin{cases} G_{11}\varphi_1 - G_{12}\,\varphi_2 - G_{13}\,\varphi_3 = J_1 \\ -G_{21}\,\varphi_1 + G_{22}\,\varphi_2 - G_{23}\,\varphi_3 = J_2 \\ -G_{31}\,\varphi_1 - G_{32}\,\varphi_2 + G_{33}\,\varphi_3 = J_3 \end{cases}.$$

Where $G_{11} = G_1 + G_3 + G_4$, $G_{22} = G_1 + G_2 + G_5$, $G_{33} = G_2 + G_3 + G_6$ – are the individual conductivities of the nodes. They are equal to the sum of the branch conductivities that are coming into the node;

$G_{12} = G_{21} = G_1$, $G_{23} = G_{32} = G_2$, $G_{13} = G_{31} = G_3$ – are the mutual conductivities of the nodes. They are equal to the conductivities of the branches that connect the respective nodes. Branches' conductivities are: $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, $G_4 = 1/R_4$, $G_5 = 1/R_5$, $G_6 = 1/R_6$.

Nodal potentials are φ_1, φ_2, φ_3 related to the node *0* with zero potential.

$$J_1 = -G_1\,E_1 + G_4\,E_4 + G_3\,E_3,\ J_2 = G_1\,E_1 - G_5\,E_5 + G_2\,E_2,$$

$J_3 = G_6\,E_6 - G_2\,E_2 - G_3\,E_3$ – the algebraic sum of the currents of currents' sources, that are flowing into the respective nodes. If the current J of the source flows into the node, it is marked with the sign "+", when it flows out – with the sign "-".

Branches' currents are defined this way: $\varphi_1 - R_4\,I_4 = E_4$, $I_4 = \dfrac{(\varphi_1 - E_4)}{R_4} = (\varphi_1 - E_4)\,G_4$,

$$\varphi_2 - R_5\,I_5 = -E_5,\ I_5 = \frac{(-\varphi_2 + E_5)}{R_5} = (-\varphi_2 + E_5)\,G_5,$$

$$\varphi_3 + R_6\,I_6 = E_6,\ I_6 = \frac{(E_6 - \varphi_3)}{R_6} = (E_6 - \varphi_3)\,G_6,$$

$$\varphi_1 - \varphi_2 - R_1\,I_1 = -E_1,\ I_1 = \frac{(\varphi_1 - \varphi_2 + E_1)}{R_1} = (\varphi_1 - \varphi_2 + E_1)G_1,$$

$$\varphi_2 - \varphi_3 - R_2\,I_2 = E_2,\ I_2 = \frac{(\varphi_2 - \varphi_3 - E_2)}{R_2} = (\varphi_2 - \varphi_3 - E_2)\,G_2,$$

$$\varphi_1 - \varphi_3 - R_3\,I_3 = E_3,\ I_3 = \frac{(\varphi_1 - \varphi_3 - E_3)}{R_3} = (\varphi_1 - \varphi_3 - E_3)\,G_3.$$

Example for task 5. The external loop of the circuit (fig. 1.2) is shown at fig. 1.4. The parameters of the circuit are: $E_1 = 10V$, $E_3 = 15V$, $E_6 = 20V$, $R_1 = 10\Omega$, $R_2 = 6\Omega$ $R_3 = 5\Omega$, $R_4 = 7\Omega$, $R_5 = 8\Omega$, $R_6 = 5\Omega$ and the branch currents are: $I_1 = 1.153\,A$, $I_2 = 0.108\,A$, $I_3 = -2.565\,A$ (example for task 2).

The loop points potentials are: $\varphi_1 = 0$, $\Sigma r_1 = 0$,

$$\varphi_a = \varphi_1 + E_1 = 10\ V,\ \Sigma r_a = 0,$$

$$\varphi_2 = \varphi_a - R_1\,I_1 = 10 - 10 \cdot 1.153 = -1.53\ V,\ \Sigma r_2 = R_1 = 10\ \Omega,$$

$$\varphi_b = \varphi_2 - E_2 = -1.53 - 0 = -1.53\ V,\ \Sigma r_b = R_1 = 10\ \Omega,$$

$$\varphi_3 = \varphi_b - R_2 I_2 = -1.53 - 6 \cdot 0.108 = -2.178 \ V,\ \Sigma r_3 = R_1 + R_2 = 10 + 6 = 16 \ \Omega,$$

$$\varphi_c = \varphi_3 + E_3 = -2.178 + 15 = 12.822 \ V,\ \Sigma r_3 = R_1 + R_2 = 10 + 6 = 16 \ \Omega,$$

$$\varphi_1 = \varphi_c + R_3 I_3 = 12.822 + 5 \cdot (-2.565) = 0 \ V,\ \Sigma r_1 = R_1 + R_2 + R_3 = 10 + 6 + 5 = 21 \ \Omega.$$

The potential diagram for the external loop is shown at fig.1.5.

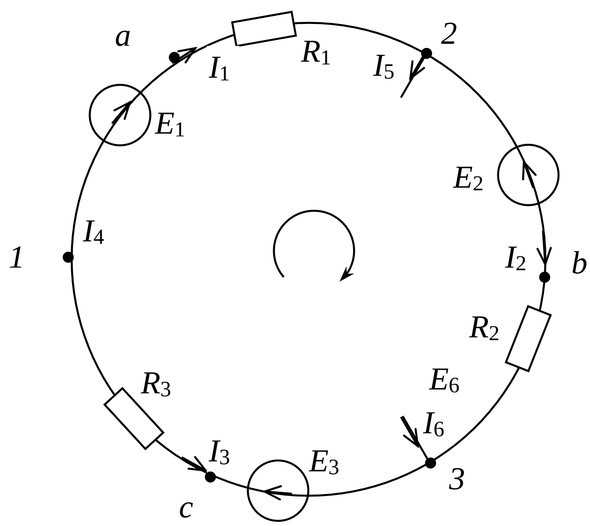


Fig. 1.4

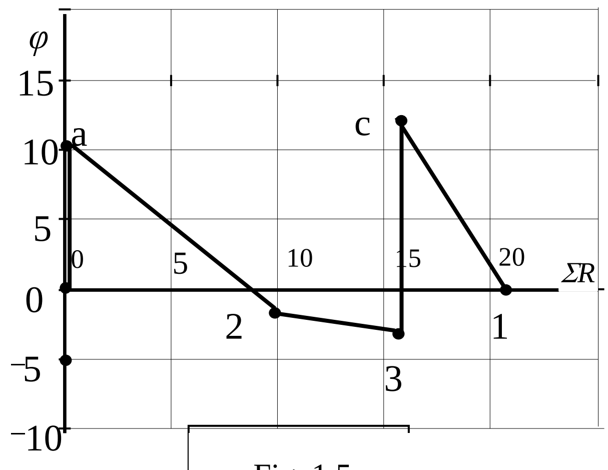


Fig. 1.5

Task 6. Draw the scheme (fig. 1.6). Define the branches' currents with the two nodes method and the operating mode of every source according to your variant in table below. Verify the calculations using the first *Kirchhoff's law*.

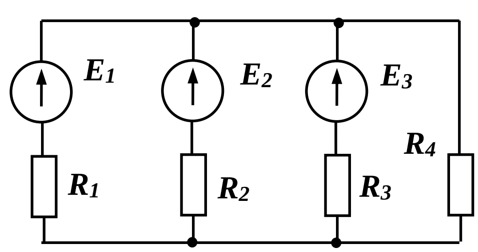


Fig. 1.6

VAR	E_1	E_2	E_3	R_1	R_2	R_3	R_4
№	*V*	*V*	*V*	Ω	Ω	Ω	Ω
00	120	220	150	20	30	40	10
01	120	200	100	10	20	30	4
02	220	150	120	10	50	80	20
03	120	220	150	30	40	10	20
04	120	220	100	40	10	20	30
05	220	150	120	40	30	20	10
06	120	220	150	30	20	10	40
07	300	200	120	20	30	40	10
08	400	200	150	10	40	30	20

09	200	300	150	20	30	40	10
10	200	400	120	30	50	80	10
11	100	120	130	50	30	10	80
12	100	125	140	30	50	80	10
13	100	130	140	50	40	30	20
14	100	135	145	20	30	40	50
15	120	130	155	50	20	30	10
16	120	145	130	40	30	50	20
17	130	160	145	30	50	20	40
18	130	155	140	50	10	40	30
19	140	150	160	30	20	10	50
20	200	300	100	40	50	20	10

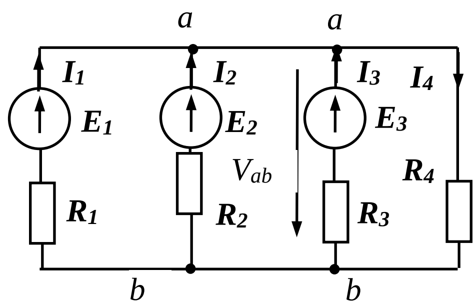


Fig. 1.7

Example for task 6. The parameters of the circuit are: $E_1 = 10\ V$, $E_2 = 20\ V$, $E_3 = 30\ V$, $R_1 = 10\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 10\ \Omega$, $R_4 = 5\ \Omega$.

Choose arbitrarily the directions of branches' currents I_1, I_2, I_3, I_4 and the direction of the inter-node voltage V_{ab} (fig.1.7).

The branches' conductivities are:

$G_1 = 1/R_1 = 1/10 = 0.1\ Sm$, $G_2 = 1/R_2 = 1/5 = 0.2\ Sm$, $G_3 = 1/R_3 = 1/10 = 0.1\ Sm$, $G_4 = 1/R_4 = 1/5 = 0.2\ Sm$.

Calculate the inter-node voltage as:

$$V_{ab} = \frac{G_1 E_1 + G_2 E_2 + G_3 E_3}{G_1 + G_2 + G_3 + G_4} = \frac{0.1 \cdot 10 + 0.2 \cdot 20 + 0.1 \cdot 30}{0.1 + 0.2 + 0.1 + 0.2} = 13.33\ V.$$

For chosen positive currents' directions, their values are defined according to the second *Kirchhoff's law*.

$V_{ab} + R_1 I_1 = E_1$, so $I_1 = (E_1 - V_{ab})/R_1 = (E_1 - V_{ab})G_1 = (10 - 13.33) \cdot 0.1 = -0.33\ A$.

Sign «-» means, that actual direction of the current is opposite to the chosen one.

$V_{ab} + R_2 I_2 = E_2$, so $I_2 = (E_2 - V_{ab})G_2 = (20 - 13.33) \cdot 0.2 = 1.33\ A$.

Sign «+» means that actual direction of the current is the same as chosen one.

$V_{ab} + R_3 I_3 = E_3$, so $I_3 = (E_3 - V_{ab})G_3 = (30 - 13.33) \cdot 0.1 = 1.67\ A$.

$V_{ab} = R_4 I_4$, so $I_4 = V_{ab}/R_4 = V_{ab} \cdot G_4 = 13.33 \cdot 0.2 = 2.67\ A$.

Verify the calculations by the first *Kirchhoff's law* for node a: $I_1 + I_2 + I_3 - I_4 = 0$ so $-0.33 + 1.33 + 1.67 - 2.67 = 0$.

Define electrical sources operating modes. If the directions of the real branch current and e.m.f. are the same $P = EI > 0$, the source works as a generator, if not

$P = EI < 0$ the source works as a consumer. Thus, the sources E_2, E_3 at fig. 1.7 work as generators and E_1 as a consumer.

Task 7. Draw the scheme (fig. 1.8) with values according to your variant in table below. Define the branches' currents by the superposition method**.**

Var. *№*	E_1 *V*	E_2 *V*	E_3 *V*	E_4 *V*	E_5 *V*	R_1 Ω	R_2 Ω	R_3 Ω	R_4 Ω	R_5 Ω
00	12	15	-	–	–	3	4	5	6	7
01	–	15	18	-	–	4	5	6	7	8
02	–	–	18	21	-	5	6	7	8	3
03	–	–	–	21	24	6	7	8	3	4
04	15	-	21	–	–	7	8	3	4	5
05	–	18	-	24	–	8	3	4	5	6
06	–	–	21	27	-	3	4	5	6	7
07	-	15	–	24	-	4	5	6	7	8
08	18	-	-	–	21	5	6	7	8	3
09	–	21	-	-	27	6	7	8	3	4
10	–	24	-	-	12	7	8	3	4	5
11	–	-	-	27	12	8	3	4	5	6
12	21	24	-	–	–	3	4	5	6	7
13	–	-	27	12	–	4	5	6	7	8
14	–	27	-	12	-	5	6	7	8	3
15	–	–	–	12	15	6	7	8	3	4
16	24	-	12	–	–	7	8	3	4	5
17	–	27	-	15	–	8	3	4	5	6
18	–	–	12	18	-	3	4	5	6	7
19	18	–	15	-	-	4	5	6	7	8
20	27	-	-	–	15	5	6	7	8	3

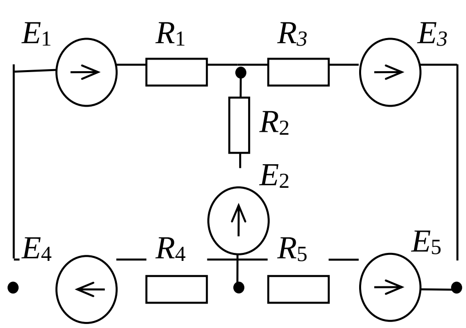


Fig.1.8

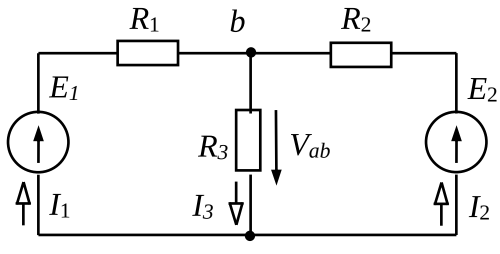


Fig. 1.9

Example for task 7. For the circuit at fig.1.9 $R_1 = 5\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 10\ \Omega$, $E_1 = 20\ V$, $E_2 = 25\ V$.

The superposition principle means that every e.m.f. acts in the circuit independently. Thus, the calculation of one circuit (fig. 1.9) with two sources, for example, can be reduced to the calculation of two circuits each with a single source (fig. 1.10, 1.11).

Two partial circuits with partial currents should be calculated according to this principle. There is a single e.m.f. E_1 in the first partial circuit (fig. 1.10): $R'_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{10 \cdot 15}{15+10} = 6\ \Omega$.

The total resistance of this scheme: $R' = R_1 + R'_{23} = 5 + 6 = 11\ \Omega$.

The partial current makes $I'_1 = E_1 / R' = 20/11 = 1.818\ A$, and voltage makes

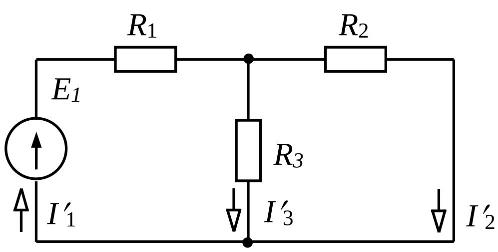


Fig. 1.10

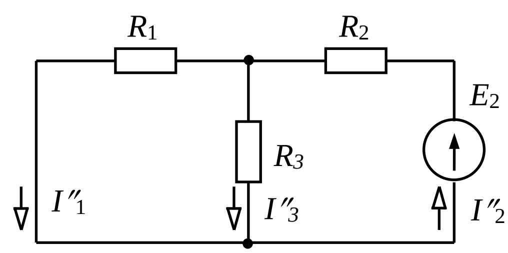


Fig. 1.11

$V'_{ab} = I'_1 R'_{23} = 1.818 \cdot 6 = 10.91\ V$.

The partial branches' currents are: $I'_2 = V'_{ab} / R_2 = 10.91/15 = 0.727\ A$,

$I'_3 = V'_{ab} / R_3 = 10.91/10 = 0.091\ A$.

There is a single e.m.f. E_2 in the second partial circuit (fig. 1.11).

$$R''_{13} = \frac{R_1 R_3}{R_1 + R_3} = \frac{5 \cdot 10}{5+10} = 3.33\ \Omega.$$

The total resistance of this scheme is: $R'' = R_2 + R''_{13} = 15 + 3.33 = 18.33\ \Omega$.

The partial current makes $I''_2 = E_2 / R'' = 25/18.33 = 1.364\ A$, and voltage $V''_{ab} = I''_2 R''_{13} = 1.364 \cdot 3.33 = 4.54\ V$.

The partial branches' currents are: $I''_1 = V''_{ab} / R_1 = 4.54/5 = 0.909\ A$,

$I''_3 = V''_{ab} / R_3 = 4.54/10 = 0.454\ A$.

Therefore, the real branches' currents are defined as an algebraic sum of the respective partial currents (fig.1.9):

$$I_1 = I'_1 - I''_1 = 1{,}818 - 0{,}909 = 0{,}909\ A,$$

$$I_2 = I''_2 - I'_2 = 1.364 - 0.727 = 0.637\ A,$$

$$I_3 = I_3'' + I_3' = 0.091 + 0.454 = 0.545\ A.$$

Task 8. Draw the scheme (fig. 1.12) according to your variant in table below. Define the second branch current using the equivalent generator method**.**

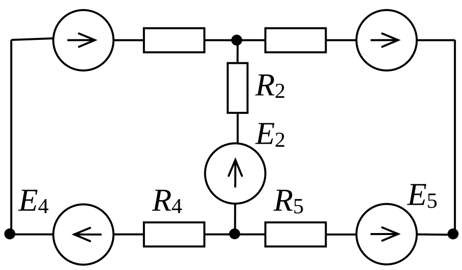


Fig.1.12

Var. №	E_1 V	E_2 V	E_3 V	E_4 V	E_5 V	R_1 Ω	R_2 Ω	R_3 Ω	R_4 Ω	R_5 Ω
00	12	-	15	–	–	3	4	5	6	7
01	–	-	18	-	27	4	5	6	7	8
02	–	–	18	21	-	5	6	7	8	3
03	–	–	–	21	24	6	7	8	3	4
04	15	-	21	–	–	7	8	3	4	5
05	–	-	15	24	–	8	3	4	5	6
06	–	–	21	-	27	3	4	5	6	7
07	15	–	–	24	-	4	5	6	7	8
08	18	-	-	–	21	5	6	7	8	3
09	25	-	-	-	27	6	7	8	3	4
10	–	–	24	-	12	7	8	3	4	5
11	–	-	-	27	12	8	3	4	5	6
12	21	-	-	18	–	3	4	5	6	7
13	–	-	27	12	–	4	5	6	7	8
14	–	–	27	12	-	5	6	7	8	3
15	–	–	–	12	15	6	7	8	3	4
16	24	-	12	–	–	7	8	3	4	5
17	–	-	-	15	21	8	3	4	5	6
18	–	–	12	-	18	3	4	5	6	7
19	18	–	–	15	-	4	5	6	7	8
20	27	-	-	–	15	5	6	7	8	3

Example for task 8. For the circuit at fig.1.13 $R_1 = 5\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 10\ \Omega$, $E_1 = 20\ V$, $E_3 = 25\ V$.

The branch with unknown current (I_2) is selected from the circuit on fig. 1.13. An equivalent generator (fig. 1.14) replaces the rest of the circuit. Its parameters are E_{eqv} – equivalent e.m.f., which is equal to the open circuit voltage on the clamps *ab* of an open second branch and R_{eqv} – equivalent resistance, which is equal to the input resistance of the circuit in respect to the open second branch clamps *ab* (the e.m.f. is shortened). The equivalent generator parameters (E_{eqv} and R_{eqv}) are to be calculated. For the circuit at fig.1.13 R_{eqv} is:

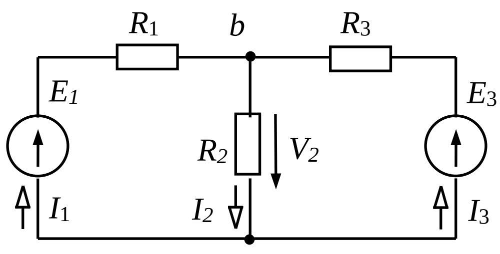


Fig. 1.13

$$R_{eqv} = \frac{R_1 R_3}{R_1 + R_3} = \frac{5 \cdot 10}{5+10} = 1.333\ \Omega.$$

E_{eqv} (fig. 1.15) is defined the following way: $V_{abOC} = E_{eqv} = E_1 - R_1 I$

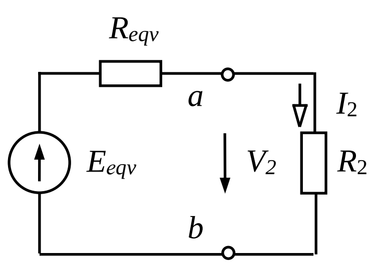


Fig. 1.14

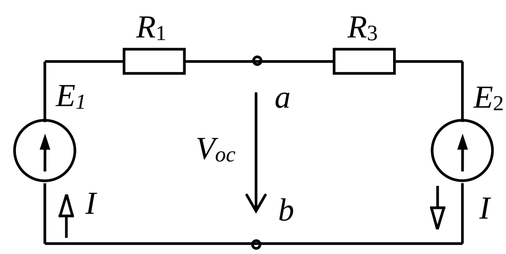


Fig. 1.15

$= 20 - 5 \cdot (-0.33) = 21.65\ V$, where $I = \frac{E_1 - E_2}{R_1 + R_3} = \frac{20-25}{5+10} = -0.33\,A$. According to the fig.1.14 unknown current makes: $I_2 = E_{eqv} / (R_{eqv} + R_2) = \frac{21.65}{1{,}33 + 15} = 1.33\ A$.

Example 9. Define the non-linear element static and dynamic resistance at work point for $I_0 = 0.006\,A$, $V_0 = 4V$.

Static resistance (fig. 1.16) is

$$R_{ST} = V_0 / I_0 = 4/0.006 = 666.67\,\Omega,$$

Dynamic resistance (fig. 1.16) is

$$R_D = \Delta V / \Delta I = 4/0.004 = 1000\,\Omega.$$

Example 10. For scheme at fig.1.17 $R_1 = 5\ \Omega$, $R_2 = 15\ \Omega$, $E_1 = 20\ V$, $E_2 = 25\ V$. Define the non-linear element R_3 current, if its volt-ampere relationship $V(I)$ is represented by the table.

I, A	1,0	2,0	3,0	4,0	5,0	6,0
V, B	13	17	18	18.5	19.5	20.0

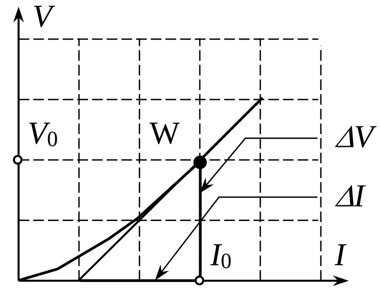


Fig. 1.16.

The equivalent generator method is used to solve this task. Firstly, the branch with non-linear resistance is selected. The rest of the scheme is represented as an active one-port network with the parameters $E_{EQV} = V_{OCab}$, R_{EQV} (fig. 1.18).

The branch with non-linear resistance R_3 is disconnected and the input resistance R_{EQV} is defined as the one between the open clamps

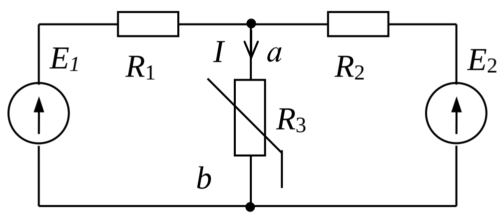


Fig. 1.17

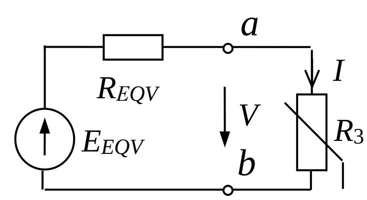


Fig. 1.18

a and *b* (the sources are shortened): $R_{EQV} = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{5 \cdot 15}{5+15} = 3.75\,\Omega$.

The open circuit voltage is defined by two nodes method ($G_N = 1/R_N$) as

$$G_1 = 1/R_1 = 1/5 = 0.2\,Sm,\ G_2 = 1/R_2 = 1/15 = 0.067\,Sm.$$

$$E_{EQV} = V_{OCab} = \frac{G_1 E_1 + G_2 E_2}{G_1 + G_2} = \frac{0.2 \cdot 20 + 0.067 \cdot 25}{0.2 + 0.067} = 21.25\ V.$$

For scheme at fig. 1.18 write down the equation according to the second Kirchhoff's law $E_{EQV} = R_{EQV} I + V_{ab}$, so $V(I) = V_{ab} = E_{EQV} - R_{EQV} I$. Solve this equation by graphic method (fig. 1.19). The left part of this equation is a non-linear element VAC (curve) $V_{AB}(I)$. The right part of the equation is a line drawn across the points $(E_{EQV},0)$, $(0,I_{SC})$. By substituting the values E_{EQV}, $I_{SC} = E_{EQV}/R_{EQV}$, the points $(21.25,0)$ and $(0,5.67)$ are obtained. The line is drawn across these points. It crosses the curve at the point with $V_A = 15V,\ I_A = 1.20\,A,$ that defines a non-linear element working regime.

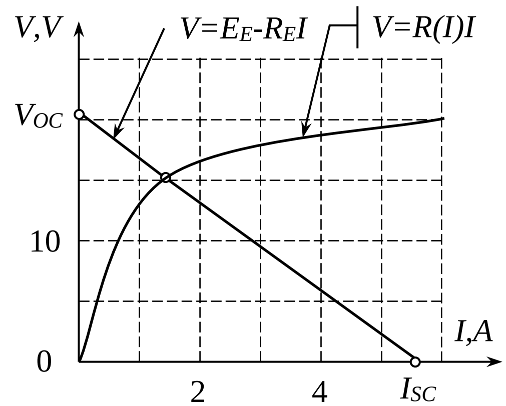
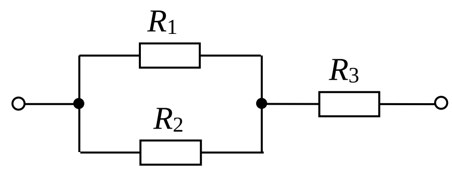


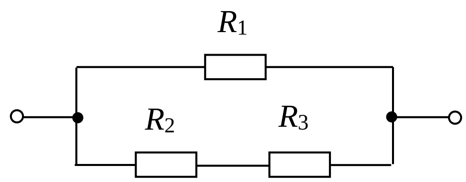
Fig. 1.19.

Tasks for individual work.

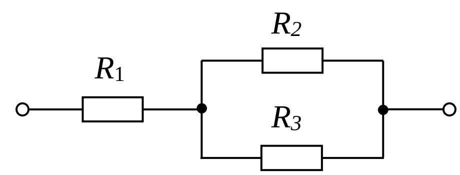
1. Current $I_{R2} = 2\ A$ and resistances $R_1 = 4\ \Omega$, $R_2 = 2\Omega$, $R_3 = 6\ \Omega$ are given for the circuit on the figure. Define the branches' currents, sub-circuits' voltages, input voltage.



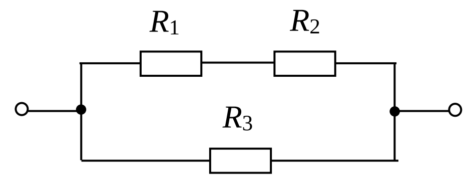
2. Voltage $V_{R2} = 2\ V$ and resistances $R_1 = 4\ \Omega$, $R_2 = 2\Omega$, $R_3 = 6\ \Omega$ are given for the circuit on the figure. Define the branches' currents, voltages across the elements and current of the unforked sub-circuit.



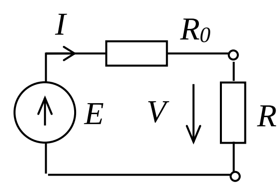
3. Current $I_{R2} = 2\ A$ and resistances $R_1 = 2\ \Omega$, $R_2 = 3\Omega$, $R_3 = 6\ \Omega$ are given for the circuit on the figure. Define the branches' currents, sub-circuits' voltages and input voltage.



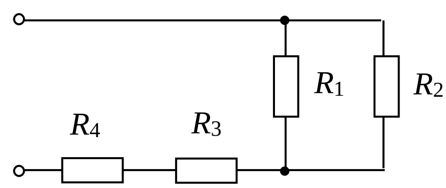
4. Voltage $V_{R2} = 4\ V$ and resistances $R_1 = 2\ \Omega$, $R_2 = 4\Omega$, $R_3 = 6\ \Omega$ are given for the circuit on the figure. Define the branches' currents, voltages across the elements and the current of the unforked sub-circuit.



5. Define source internal resistance R_0 and electromotive force E, source's and consumer's powers if voltage is $V=190V$, load resistance is $R=139\ \Omega$ and efficiency factor is $\eta=0.99$. Calculate the efficiency factor at the load equal to $R/10$.



6. Calculalate the current in the circle if the resistances are $R_1 = 165\Omega$, $R_2 = 197\Omega$, $R_3 = 198\Omega$, $R_4 = 193\Omega$ and the maximum power makes $P_3 = 7.4W$ when allocated to the resistor R_3 in the unforked circuit. Define the input voltage and sub-circuits' voltages, branches' currents, power of the circuit and the powers of the sub-circuits.



7. The consumer with resistance $R_C = 10\ \Omega$ is connected to the clamps of the source with e.m.f. $E = 40\ W$ and internal resistance $R_0 = 1\ \Omega$. Define the voltage across the source clamps and source efficiency factor.

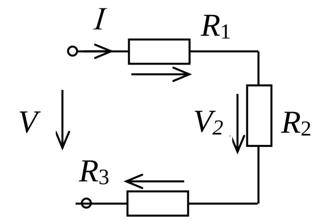
8. Define the voltage source parameters, if the parameters of current source are $J = 5\ A$, $G_0 = 0.5\ Sm$. Draw the voltage source scheme and write down its equation.

9. Define the current source parameters, if the parameters of voltage source are $E = 50\ V$, $R_0 = 5\ \Omega$. Draw the current source scheme and write down its equation.

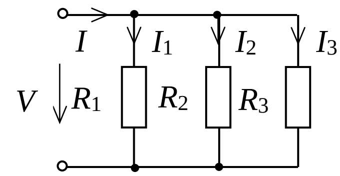
10. The voltage across the source clamps is $V = 24V$. The load is $R = 8\Omega$. In open circuit mode voltage at source clamps makes $V_{OC} = 27V$. Define the source internal resistance.

11. In short circuit mode source current is $I_{SC} = 48\,A$. When the source is connected to the resistive element $R = 19.5\Omega$, the circuit current is $1.2\,A$. Define the source e.m.f. and its internal resistance.

12. The battery consists of three connected in series sources with the following parameters: $E = 1.5V$, $R_0 = 0.5\Omega$. The battery power is $P = 2.25W$. Define the load resistance, power and the voltage across the source clamps.



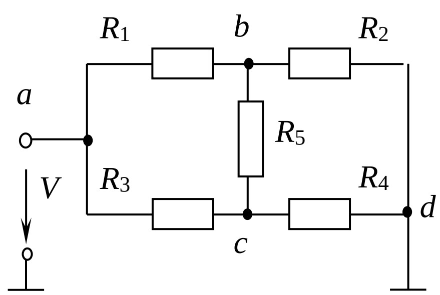
13. Define the resistance R_1 at the circuit with connected in series elements: $R_2 = 16\,Ом$, $R_3 = 24\,Ом$, $V_2 = 8\,V$, $I = 2\,A$ and $V = 60\,V$. Define the circuit total resistance.

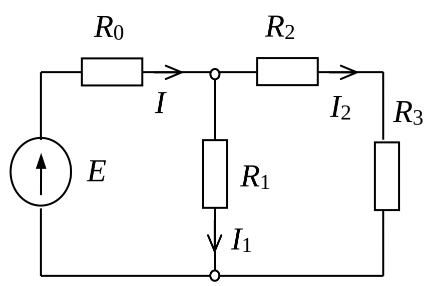


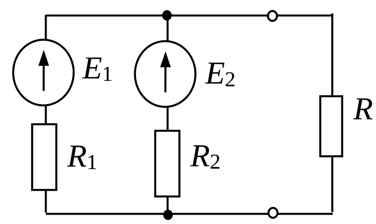
14. Define the conductivity of an element R_3 in the circuit with parallel connection of elements: $G_1 = 0.05\,Sm$, $G_2 = 0.1\,Sm$, $I_1 = 2\,A$ and $I = 14\,A$. Define the circuit total conductivity.

15. Define the branches' currents and verify your calculations, if $R_1 = R_3 = R_5 = 9\Omega$**,** $R_2 = 3\,\Omega$**,** $R_4 = 9\,\Omega$ and $V = 21V$**.**

16. The elements $R_1 = 16\,\Omega$, $R_2 = 1\,\Omega$ and $R_3 = 3\,\Omega$ are connected to the source with the parameters $E = 40\,V$ and $R_0 = 0{,}8\,\Omega$. Define the source voltage, branches'



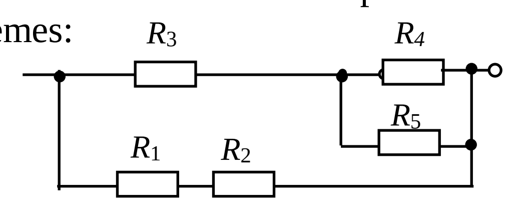


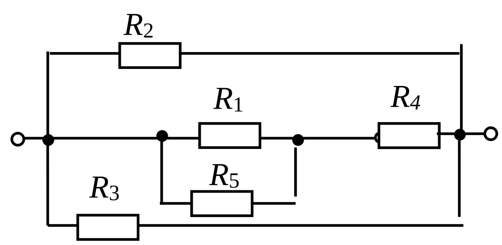


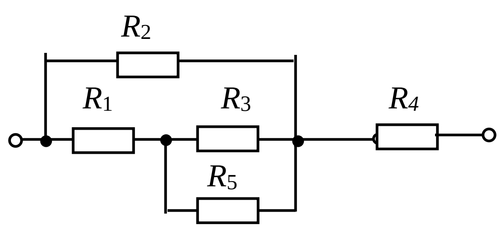
currents and voltages across the elements.

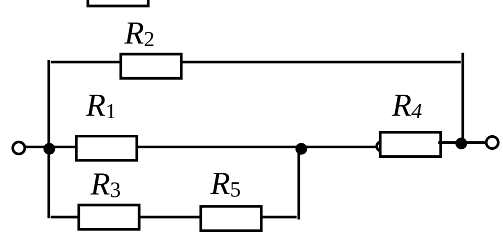
16. Define the load R power, if $E_1 = 200\,V$, $E_2 = 150\,V$, $R_1 = 20\,\Omega$, $R_2 = 30\,\Omega$. What should be the R value to reach the maximum power on it?

17. Write down the expressions of equivalent resistances for the following schemes:









2. Alternating current circuits calculation.

Task 1. The circuit of two connected in series coils and capacitor (fig. 2.1) is powered by AC voltage source of frequency $f = 50\ Hz$. The parameters of the circuit are given in table below. Define the circuit current and the voltages across the coils. Define the resonance frequency of the circuit. Write down the current and voltages instantaneous values. Draw the vector diagram.

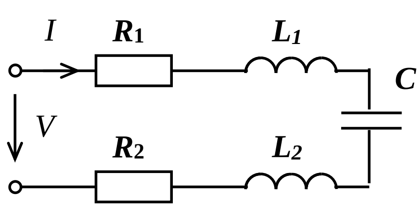


Fig. 2.1

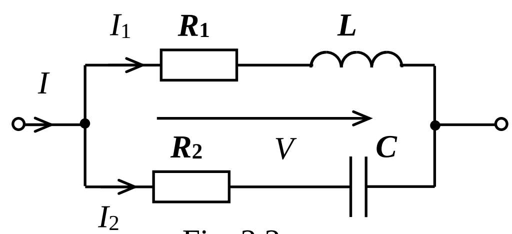


Fig. 2.2

VAR	*V*	R_1	R_2	L_1	L_2	*C*	*L*
№	*V*	Ω	Ω	*H*	*H*	μF	*H*
00	50	10	20	0.096	0.032	320	0.096
01	120	12	22	0.0127	0.096	200	0.0127
02	100	22	15	0.019	0.016	500	0.019
03	110	10	22	0.016	0.0127	400	0.016
04	50	12	20	0.032	0.019	100	0.032
05	40	22	10	0.0127	0.032	320	0.0127
06	90	12	15	0.016	0.096	200	0.016
07	80	15	12	0.032	0.016	300	0.032
08	110	25	12	0.096	0.019	400	0.096
09	120	12	25	0.019	0.0127	630	0.019
10	50	20	10	0.096	0.032	700	0.096
11	70	22	12	0.0127	0.016	500	0.0127
12	60	15	22	0.019	0.096	680	0.019
13	110	10	20	0.016	0.019	750	0.016
14	50	12	22	0.032	0.0127	320	0.032
15	40	22	15	0.096	0.032	200	0.096
16	30	10	22	0.019	0.032	400	0.019
17	80	12	20	0.032	0.0127	500	0.032
18	110	15	12	0.0127	0.016	750	0.0127
19	120	25	12	0.016	0.096	680	0.016
20	70	12	25	0.096	0.0127	750	0.096

Example for task 1. The circuit (fig. 2.3) has the following parameters: $V = 20V$, $f = 50\,Hz$ $R_1 = 5\Omega$, $R_2 = 10\ \Omega$, $L_1 = L_2 = 127\,mH$, $C = 318\ \mu F$. This circuit consists of two coils with parameters R_1, L_1, R_2, L_2 and capacitor C.

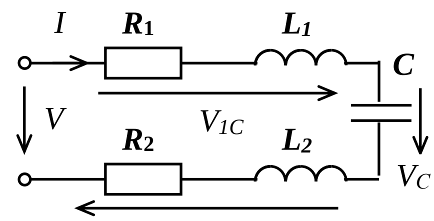


Fig. 2.3

Angular frequency makes:

$$\omega = 2\pi f = 3 \cdot 3.14 \cdot 50 = 314\,rad/s.$$

Reactances elements are: $X_{L1} = \omega L_1 = 314 \cdot 0.0127 = 4\ \Omega$,

$X_{L2} = \omega L_2 = 314 \cdot 0.0127 = 4\ \Omega$,

$X_C = 1/(\omega C) = 1/(314 \cdot 0.000318) = 10\ \Omega$.

Circuit resistance is: $R = R_1 + R_2 = 5 + 10 = 15\Omega$.

Circuit reactance is: $X = X_{L1} + X_{L2} - X_C = 4 + 4 - 10 = -2\Omega$.

Circuit impedance is: $Z = \sqrt{R^2 + X^2} = \sqrt{15^2 + (-2)^2} = 15.13\Omega$.

Coils impedances are: $Z_{1C} = \sqrt{R_1^2 + X_{L1}^2} = \sqrt{5^2 + (4)^2} = 6.4\,\Omega$,

$Z_{2C} = \sqrt{R_2^2 + X_{L2}^2} = \sqrt{10^2 + (4)^2} = 10.77\,\Omega$.

Circuit current makes: $I = V/Z = 20/15.13 = 1.32\,A$.

Phase shift angle between input voltage and circuit current makes:

$$\varphi = arctg\,(X/R) = arctg\,(-2/15) = -6^\circ.$$

Input voltage initial phase is $\psi_V = 0^\circ$.

Circuit current I initial phase is defined from the expression $\varphi = \psi_V - \psi_I$:

$$\psi_I = \psi_V - \varphi = 6^\circ.$$

First coil voltage makes: $V_{1C} = Z_{1C} I = 6.4 \cdot 1.32 = 8.45\,V$.

Phase shift angle between this voltage and current I makes:

$$\varphi_1 = arctg\,(X_{L1}/R_1) = arctg\,(4/5) = 39^\circ.$$

Voltage V_{1C} initial phase is defined from the expression $\varphi_1 = \psi_{V1} - \psi_I$:

$$\psi_{V1} = \varphi_1 - \psi_I = 39^\circ - 6^\circ = 33^\circ.$$

Second coil voltage is: $V_{2C} = Z_{2C} I = 10.77 \cdot 1.32 = 14.22\,V$.

Phase shift angle between this voltage and current I is:

$$\varphi_2 = arctg\,(X_{L2}/R_2) = arctg\,(4/10) = 22^\circ.$$

Voltage V_{1C} initial phase is defined from the expression $\varphi_2 = \psi_{V2} - \psi_I$:

$$\psi_{V2} = \varphi_2 - \psi_I = 22^\circ - 6^\circ = 16^\circ.$$

Capacitor voltage is: $V_C = X_C I = 10 \cdot 1.32 = 13.2\,V$.

Phase shift angle between this voltage and current I is: $\varphi_C = -90^\circ$.

Voltage V_C initial phase is found from the expression $\varphi_C = \psi_{VC} - \psi_I$:

$$\psi_{VC} = \varphi_C - \psi_I = -90^\circ - 6^\circ = -96^\circ.$$

The resonance condition for this circuit is $X_L = X_C$, it means $(\omega_0 L_1 + \omega_0 L_2) = 1/(\omega_0 C)$, so resonance frequency makes

$\omega_0 = 1/\sqrt{C(L_1 + L_2)} = 1/\sqrt{0.000318(0.0127 + 0.0127)} = 352\, rad/s$.

Instantaneous values of voltages and current are:

$v(t) = V_m \sin(\omega t + \psi_V)$, $i(t) = I_m \sin(\omega t + \psi_I)$:

$v(t) = 20\sqrt{2}\sin(314t)$ B, $i(t) = 1.32\sqrt{2}\sin(314t + 6^\circ)$ V,

$v_{1COIL}(t) = 8.45\sqrt{2}\sin(314t + 33^\circ)$ B, $v_{2COIL}(t) = 14.22\sqrt{2}\sin(314t + 16^\circ)\, V$,

$v_C(t) = 13.2\sqrt{2}\sin(314t - 96^\circ)\, V$.

Task 2. The circuit, which consists of connected in parallel coil and capacitor (fig. 2.2), is powered by AC voltage source with frequency $f = 50\, Hz$. The parameters of the circuit are given in table above. Define the circuit and branches currents, the phase shift angles. Define the resonance frequency of the circuit. Find the power balance for the circuit.

Example for task 2. Parameters for the circuit at fig. 2.4 are: $V = 20V$, $R_1 = 5\Omega$, $R_2 = 10\Omega$, $L_2 = 12.7\, mH$, $C = 318\, \mu F$.

Angular frequency is $f = 50\, Hz$:

$$\omega = 2\pi f = 2 \cdot 3.14 \cdot 50 = 314\, rad/s.$$

Elements' reactances are: $X_L = \omega L = 314 \cdot 0.0127 = 4\ \Omega$,

$X_C = 1/(\omega C) = 1/(314 \cdot 0.000318) = 10\ \Omega$.

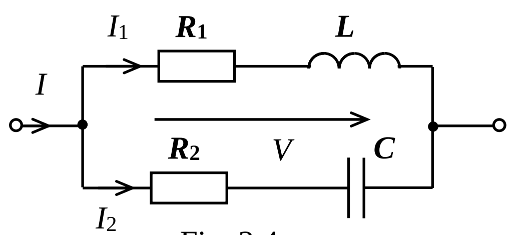


Fig. 2.4

Branches' impedances are:

$Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{5^2 + (4)^2} = 6.4\,\Omega$.

$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{10^2 + (-10)^2} = 14.14\,\Omega$.

Branches' currents are: $I_1 = V/Z_1 = 20/6.4 = 3.13A$, $I_2 = V/Z_2 = 20/14.14 = 1.39\, A$.

Phase shift angles between the input voltage and the branches' currents are:

$\varphi_1 = arctg(X_L/R) = arctg(4/5) = 39^\circ$, $\varphi_2 = arctg(X_C/R) = arctg(-10/10) = -45^\circ$.

Input voltage initial phase is $\psi_V = 0^\circ$.

Currents I_1, I_2 initial phases are found from the expressions $\varphi_1 = \psi_V - \psi_{I1}$, $\varphi_2 = \psi_V - \psi_{I2}$: $\psi_{I1} = \psi_V - \varphi_1 = -39^\circ$, $\psi_{I2} = \psi_V - \varphi_2 = 45^\circ$.

The conductances and susceptances method can be used to define the total current. Serial connection is to be converted into parallel one. So, branches' conductances and susceptances are accordingly (fig.2.5):

$$G_1 = \frac{R_1}{Z_1^2} = \frac{5}{6.4^2} = 0.122\ Sm,\ B_1 = \frac{X_L}{Z_1^2} = \frac{4}{6.4^2} = 0.0976\ Sm.$$

$$G_2 = \frac{R_2}{Z_2^2} = \frac{10}{14.14^2} = 0.05\,Sm,\ B_2 = \frac{X_C}{Z_2^2} = \frac{10}{14.14^2} = 0.05\,Sm.$$

The circuit conductance and susceptance (fig. 2.6) make accordingly:

$G = G_1 + G_2 = 0.122 + 0.05 = 0.172\ \ Sm,\ B = B_2 - B_1 = 0.05 - 0.0976 = -0.0476\ \ Sm.$

$B < 0$. Thus, susceptance has an inductive character.

Circuit admittance is: $Y = \sqrt{G^2 + B^2} = \sqrt{0.172^2 + (-0.0476)^2} = 0.178\,Sm.$

Total current makes: $I = VY = 20 \cdot 0.178 = 3.56\,A$.

Phase shift angle between input voltage and total current is:

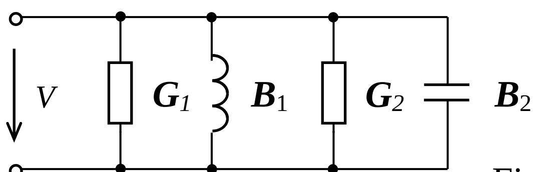


Fig. 2.5

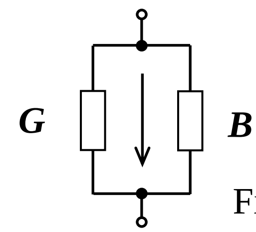


Fig. 2.6

$\varphi = arctg\,(B/G) = arctg\,(0.0476/0.172) = 15^\circ$, if $\psi_V = 0^\circ$, then total current I initial phase can be found from the expression $\varphi = \psi_V - \psi_I$. Therefore, $\psi_I = \psi_V - \varphi = -15^\circ$.

The resonance condition for this circuit is $B_L = B_C$, or $B_1 = B_2$. It means that

$$\frac{\omega_0 L}{R_1^2 + (\omega_0 L)^2} = \frac{1/(\omega_0 C)}{R_2^2 + 1/(\omega_0 C)^2}.$$

From this expression ω_0 can be found.

Power balance equations are used to verify the obtained results.

Thus, consumers active and reactive powers are:

$$P_{cons} = R_1 I_1^2 + R_2 I_2^2 = 5 \cdot 3.13^2 + 10 \cdot 1.39^2 = 68.77\,W,$$

$$Q_{cons} = X_L I_1^2 - X_C I_2^2 = 4 \cdot 3.13^2 - 10 \cdot 1.39^2 = 18.43\,VAr.$$

Source active and reactive powers are:

$$P_{sour} = VI\cos\varphi = 20 \cdot 3.56\cos 15^\bullet = 68.77\,W,$$

$$Q_{sour} = VI\sin\varphi = 20 \cdot 3.56\sin 15^\bullet = 18.43\,VAr.$$

Thus, if $P_{cons} \approx P_{sour}$, $Q_{cons} \approx Q_{sour}$, power balance is true. Therefore, the circuit parameters have been defined correctly.

Task 3. Draw the scheme (fig. 2.7) according to your variant at table below. Find out the circuit and branches' impedances for frequency $f = 50\,Hz$. Define the branches' currents and the voltages. Verify the calculations (balance of power equation). Draw the vector diagram of currents and voltages.

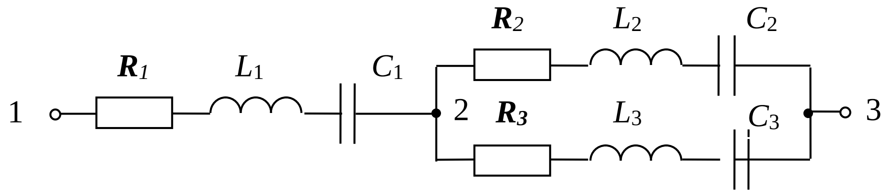


Fig. 2.7

Example for task 3. For the circuit on fig. 2.7: $V = 70V$, $R_1 = 5\Omega$, $R_2 = 0\Omega$, $R_3 = 4\Omega$, $L_1 = 9\,mH$, $L_2 = 7\,mH$, $L_3 = 20\,mH$, $C_1 = C_3 = 0\,\mu F$, $C_2 = 281\,\mu F$.

The circuit according to this variant is shown at fig. 2.8

Attention! Calculation is for frequency $f = 60\,Hz$.

Angular frequency, thus, is $\omega = 2\pi f = 2 \cdot 3.14 \cdot 60 = 377\ rad/s$.

Var №	*V* *V*	R_1 Ω	L_1 *MH*	C_1 μF	R_2 Ω	L_2 *MH*	C_2 μF	R_3 Ω	L_3 *MH*	C_3 μF
00	220	11	45		13	55		15		750
01	220	13	25		15		200	11	35	
02	220	15		300	11		240	13	55	
03	220		55		18	65		20		680
04	220	18	15		20		360		35	
05	220	18		820		35		18		270
06	220			750	20	50		24		330
07	220	22	65		24		150			220
08	220	24		360			270	22	45	
09	220	11		510	15		620	11	45	
10	220	13		180	15	20		11	50	
11	220	15	60		11			13		300
12	220	14	65		12		200	20	30	
13	220	18		680	20	20			50	
14	220	20	0	150		10		18		720
15	220	10		220	12		510	20	20	
16	220	22		430	24	25				270
17	220	24	30		15		270		55	330
18	220	11	35		13		360		15	150
19	220	13		470		30	200	11	15	
20	220	12	80	270	24	15		10		180

Reactances of inductive elements are $X_{Ln} = \omega L_n$:

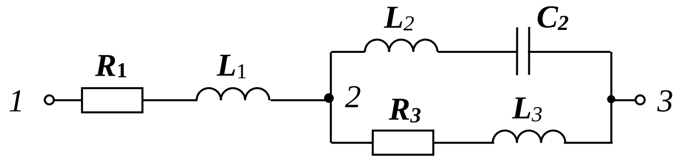


Fig. 2.8

$$X_{L1} = \omega L_1 = 377 \cdot 9 \cdot 10^{-3} = 3.39\,\Omega,\ X_{L2} = \omega L_2 = 377 \cdot 7 \cdot 10^{-3} = 2.64\,\Omega,$$

$$X_{L3} = \omega L_3 = 377 \cdot 20 \cdot 10^{-3} = 7.54\,\Omega.$$

Reactances of capacitive elements are $X_{Cn} = 1/\omega C_n$:

$$X_{C2} = 1/\omega C_2 = 1/377 \cdot 281 \cdot 10^{-6} = 9.44\ \Omega.$$

Branches' impedances make $\underline{Z}_n = R_n + jX_{Ln} - jX_n$:

$$\underline{Z}_1 = R_1 + jX_{L1} = 5 + j3.39 = 6{,}04e^{j34^\circ}\ \Omega,$$

$$\underline{Z}_2 = jX_{L2} - jX_{C2} = j(2.64 - 9.44) = -j6.8 = 6.8e^{-j90^\circ}\ \Omega,$$

$$\underline{Z}_3 = R_3 + jX_{L3} = 4 + j7.54 = 8.54e^{j62^\circ}\ \Omega.$$

Unforked part of the circuit impedance is:

$$\underline{Z}_{12} = \underline{Z}_1 = R_1 + jX_{L1} = 5 + j3.39 = 6{,}04e^{j34^\circ}\ \Omega.$$

Parallel connection impedance makes:

$$\underline{Z}_{23} = \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = \frac{6.8e^{-j90}8.54e^{j62}}{-j6.8 + 4 + j7.54} = \frac{58.05e^{-j28}}{4 + j0.74} = \frac{58.05e^{-j28}}{4.07e^{j10}} =$$

$$= 14.27e^{-j38} = (11.18 - j8.87)\ \Omega.$$

Circuit impedance is:

$$\underline{Z} = \underline{Z}_{12} + \underline{Z}_{23} = 5 + j3.39 + 11.18 - j8.87 = 16.18 - j5.47 = 17.08e^{-j19^\circ}\ \Omega.$$

Circuit current is: $\underline{I} = \underline{V}/\underline{Z} = 70/17.08e^{-j19^\circ} = 4.1e^{j19^\circ} = (3.88 + j1.31)A$.

Voltages across the parts of the circuit make:

$$\underline{V}_{12} = \underline{Z}_{12}\underline{I} = 6.04e^{j34^\circ} \cdot 4.10e^{j19^\circ} = 24.76e^{j53^\circ} = (14.95 + j19.74)V.$$

$$\underline{V}_{23} = \underline{Z}_{23}\underline{I} = 14.27e^{-j38^\circ} \cdot 4.10e^{j19^\circ} = 58.48e^{-j20^\circ} = (55.05 - j19.74)V.$$

According to the second Kirchhoff's law:

$$\underline{V} = \underline{V}_{12} + \underline{V}_{23} = (14.95 + j19.74) + (55.05 - j19.74) = 70V.$$

Branches' currents are:

$$\underline{I}_2 = \underline{V}_{23}/\underline{Z}_2 = 58.48e^{-j20^\circ}/6.8e^{-j90^\circ} = 8.6e^{j70^\circ} = (2.9 + j8.09)A,$$

$$\underline{I}_3 = \underline{V}_{23}/\underline{Z}_3 = 58.48e^{-j20^\circ}/8.54e^{j62^\circ} = 6.85e^{-j82^\circ} = (0.98 - j6.78)A.$$

According to the first Kirchhoff's law:

$$\underline{I} = \underline{I}_2 + \underline{I}_3 = (2.9 + j8.1) + (0.98 - j6.78) = 3.88 + j1.31\,A$$

Complex total power makes:

$$\underline{S} = \underline{V}\overset{*}{\underline{I}} = 70 \cdot 4.1e^{-j19^\circ} = 287e^{-j19^\circ} = (272 - j92)\,VA.$$

Consumers' active and reactive powers make:

$$P = R_1 I_1^2 + R_3 I_3^2 = 5 \cdot 4.1^2 + 4 \cdot 6.85^2 = 272\,W .$$

$$Q = X_{L1} I_1^2 + (X_{L2} - X_{C2}) I_2^2 + X_{L3} I_3^2 =$$

$$= 3.39 \cdot 4.1^2 - 6.8 \cdot 8.6^2 + 7.54 \cdot 6.85^2 = -92\,VAr .$$

Vector diagram.

Assumed voltages' vectors scale is $M_V = 15\ V/cm$, the voltage vectors' lengths are:

$$V = \frac{|V|}{M_V} = \frac{70}{15} = 4.7cm\ ,\ V_{12} = \frac{|V_{12}|}{M_V} = \frac{24.76}{15} = 1.65cm\ ,$$

$$V_{23} = \frac{|V_{23}|}{M_V} = \frac{58.48}{15} = 3.9cm\ .$$

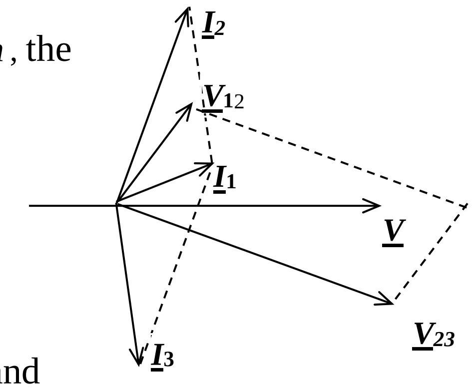


Fig. 2.9

Voltage initial phases (the angles between axis X and the vector) are: $\psi_V = 0^\circ$, $\psi_{V12} = 53^\circ$, $\psi_{V23} = -20^\circ$.

Assumed currents' vectors scale is $M_I = 1.5\ A/cm$, the currents' vectors lengths are:

$$I_1 = \frac{|I_1|}{M_I} = \frac{4.1}{1.5} = 2.7cm\ ,\ I_2 = \frac{|I_2|}{M_I} = \frac{8.6}{1.5} = 5.7cm\ ,\ I_3 = \frac{|I_3|}{M_I} = \frac{6.85}{1.5} = 4.6cm\ .$$

Currents' initial phases (the angles between axis X and the vector) are: $\psi_{I1} = 19^\circ$, $\psi_{I2} = 70^\circ$, $\psi_{I3} = -82^\circ$.

Example 4. The voltage $v = 12.56\sin(314t + \pi/3)\,V$ is applied to inductivity $L = 0.02\,mH$. Write down the current instantaneous value. Draw the vector diagram for current and voltage effective values.

Inductivity reactance is: $X_L = \omega L = 314 \cdot 0.02 = 6.28\,\Omega$.

Current amplitude is: $I_m = V_m / X_L = 12.56/6.28 = 2\,A$.

Phase shift angle for the element makes: $\varphi = \psi_V - \psi_I = \pi/2$.

Current initial phase is: $\psi_I = \psi_V - \varphi = \pi/3 - \pi/2 = -\pi/6$.

Current instantaneous value is: $i = I_m\, sin(\omega t + \psi_I) = 2\sin(314t - \pi/6)\,A$.

The vector diagram is shown at fig. 2.10.

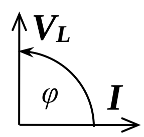


Fig. 2.10

Example 5. The voltage $v = 141\sin(314t - \pi/6)\ V$ is applied to capacitance $C = 320\,\mu F$. Write down the current instantaneous value. Draw the vector diagram for current and voltage effective values.

Capacitance reactance is: $X_C = 1/(\omega C) = 1/(314 \cdot 320 \cdot 10^{-6}) = 10\,\Omega$.

Current amplitude is: $I_m = V_m / X_C = 141/10 = 14.1A$.

Phase shift angle for the element is: $\varphi = \psi_V - \psi_I = -\pi/2$.

Current initial phase is: $\psi_I = \psi_V - \varphi = -\pi/6 + \pi/2 = \pi/3$.

Current instantaneous value is: $i = I_m\, sin(\omega t + \psi_I) = 14.1 sin(314t + \pi/3)\ A$

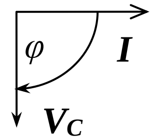


Fig. 2.11

The vector diagram is shown at fig. 2.11.

Example 6. The capacitor $C = 400\ \mu F$ and coil with parameters $R = 50\,\Omega$, $L = 0.3\,H$ are connected in series to the AC source. The voltage effective value is $V = 200\,V$, $f = 50\,Hz$. Define the circuit current, the voltages across the capacitor and coil, the circuit's active and reactive powers and the power factor.

Coil and capacitor reactances are: $X_L = \omega L = 314 \cdot 0.3 = 100\,\Omega$,

$$X_C = 1/(\omega C) = 1/(314 \cdot 400 \cdot 10^{-6}) = 8\,\Omega.$$

Circuit impedance is: $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (100-8)^2} = 105\,\Omega$.

Circuit current is: $I = V/Z = 200/105 = 1.9\,A$.

Coil impedance is: $Z_{COIL} = \sqrt{R^2 + X_L^{\ 2}} = \sqrt{50^2 + 100^2} = 112\,\Omega$.

Coil voltage is: $V_{COIL} = IZ_{COIL} = 1.9 \cdot 112 = 213\,V$.

Capacitor voltage is $V_C = IX_C = 1.9 \cdot 8 = 15.2\,V$.

Elements reactive powers are: $Q_L = X_L I^2 = 100 \cdot 1{,}9^2 = 361\,VAr$,

$$Q_C = X_C I^2 = 8 \cdot 1{,}9^2 = 29\ VAr.$$

Circuit reactive power is: $Q = Q_L - Q_C = 361 - 29 = 332\ VAr$.

Circuit active power is: $P = RI^2 = 50 \cdot 1{,}9^2 = 180.5\,W$.

Power factor makes: $\cos\varphi = P/S = R/Z = 50/105 = 0.48$.

Example 7. The input voltage for the circuit with a parallel connection of the coil with parameters $R = 6\,\Omega$, $X_L = 12\ \Omega$ and capacitor with reactance $X_C = 6\,\Omega$ is $V = 20\,V$. Define the circuit current, the branches' currents and the phase shift angles.

The coil serial connection R, X_L is transformed into parallel one of G, B_L. Conductance and susceptances then are:

$$G = \frac{R}{(R^2 + X_L^2)} = \frac{6}{6^2 + 12^2} = 0.033\ Sm,$$

$$B_L = \frac{X_L}{(R^2 + X_L^2)} = \frac{12}{6^2 + 12^2} = 0.067\ Sm,$$

$$B_C = 1/X_C = 1/6 = 0.167\ Sm.$$

The circuit susceptance and admittance are:

$$B = B_C - B_L = 0.167 - 0.067 = 0.1\,Sm,$$

$$Y = \sqrt{G^2 + B^2} = \sqrt{0.033^2 + (0.1)^2} = 0.105\,Sm.$$

The coil admittance makes: $Y_{COIL} = \sqrt{G^2 + B_L^2} = \sqrt{0.033^2 + (0.067)^2} = 0.075\,Sm$.

The circuit current is: $I = YV = 0.105 \cdot 20 = 2.1\,A$.

The branches' currents are: $I_{COIL} = Y_{COIL}V = 0.075 \cdot 20 = 1{,}5\,A$,

$$I_C = B_C V = 0.167 \cdot 20 = 3{,}34\,A.$$

Phase shift angle between total current and input voltage is:

$$\varphi = arctg\,(B/G) = arctg\,(-0.1/0.033) = -72^\circ.$$

Phase shift angle between coil current and input voltage is:

$$\varphi_L = arctg\,(B_L/G) = arctg\,(0.067/0.033) = 64^\circ.$$

The circuit active power makes: $P = RI^2_{COIL} = 6 \cdot 1.5^2 = 13.5W$.

Example 8. The voltage $V = 220\,V$ is applied to the coil. The current is $I = 4\,A$, active power makes $P = 500\,W$ and $f = 50\,Hz$. Define the $\cos\varphi$ before and after the capacitors' battery $C = 400\,\mu F$ is connected in parallel to the coil.

The circuit resistance, impedance and reactance without capacitors make accordingly: $R = P/I^2 = 500/4^2 = 31.3\,\Omega$, $Z = V/I = 220/4 = 55\,\Omega$,

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{55^2 - 31.3^2} = 45.2\,\Omega.$$

Power factor is: $\cos\varphi = R/Z = 31.3/55 = 0.57$.

When the capacitors' battery is connected, the circuit will have parallel connection of the coil and capacitors. Serial connection of X_L, R elements is to be transformed into parallel connection of B_L, G elements.

$$B_L = \frac{X_L}{Z^2} = \frac{45.2}{55^2} = 0{,}015\,Sm, \qquad G = \frac{R}{Z^2} = \frac{31.3}{55^2} = 0.01\,Sm.$$

The capacitors' susceptance makes: $B_C = \omega C = 314 \cdot 500 \cdot 10^{-6} = 0.0157\,Sm$.

The circuit susceptance is: $B = B_C - B_L = 0.0157 - 0.015 = 0.0007\,Sm$.

The circuit admittance makes: $Y = \sqrt{G^2 + B^2} = \sqrt{0.01^2 + 0.0007^2} = 0.01\,Sm$.

The power factor is: $\cos\varphi = G/Y = 0.01/0.01 = 1$.

Example 9. Define the instantaneous values of currents i_1, i_2, i (fig. 2.12), when the input voltage is $v = 35\sin 314t\,V$. Elements parameters are accordingly: $R_1 = 7\,\Omega$, $R_2 = 2\,\Omega$, $X_L = 3\,\Omega$ and $X_C = 5\,\Omega$.

This task is solved by using the vector diagrams method.

Branches' impedances are: $Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{7^2 + 3^2} = 7.62\,\Omega$,

$$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{2^2 + 5^2} = 5.39\,\Omega.$$

Branches' currents amplitudes are:

$$I_{m1} = V_m / Z_1 = 35/7.62 = 4.59\text{ A},$$

$$I_{m2} = V_m / Z_2 = 35/5.39 = 6.49\text{ A}.$$

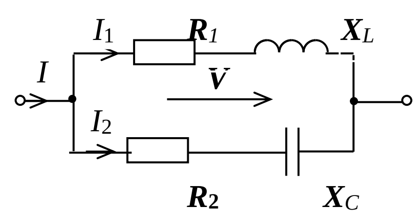


Fig. 2.12

Phase shift angles between V and branches' currents are: $\varphi_1 = arctg(X_L/R_1) = arctg(3/7) = 32^\circ$,

$$\varphi_2 = arctg\,(-X_C/R_2) = arctg(-5/2) = -68^\circ,$$

Currents initial phases makes $\psi_I = \psi_V - \varphi$: $\psi_{I1} = 0 - 32^\circ = -32^\circ$, $\psi_{I2} = 68^\circ$.

The basic vector is a voltage vector $\overline{V}$. It is put along the axis X (fig. 2.13), the branches currents vectors are drawn at the angles $\varphi_1 = -32^\circ$ for $\overline{I}_{1m}$ and $\varphi_2 = 68^\circ$ for s$\overline{I}_{2m}$

Branches' currents instantaneous values are:

$$i_1 = 4.59\, sin(\, \omega t - 32^\circ\,)\, A, \qquad i_2 = 6.49\, sin(\, \omega t + 68^\circ\,)\, A.$$

The total current vector is the geometrical sum of branches' currents vectors: $\bar{I}_m = \bar{I}_{1m} + \bar{I}_{2m}$ (fig. 2.13):

$$I_m = \sqrt{I_{m1}^2 + I_{m2}^2 - 2 I_{m1} I_{m2}\, cos(\, \varphi_1 - \varphi_2\,)} =$$

$$= \sqrt{4.59^2 + 6.49^2 - 2 \cdot 4.59 \cdot 6.49\, cos(\, 32 + 68\,)} = 7.27\, A.$$

Total current effective value makes: $I = I_m / \sqrt{2} = 8.61 / 1.41 = 5.2\, A$.

Phase shift angle between input voltage and total current is:

$$\varphi = arctg\, \frac{I_{m1}\, sin\, \varphi_1 + I_{m2}\, sin\, \varphi_2}{I_{m1}\, cos\, \varphi_1 + I_{m2}\, cos\, \varphi_2} = arctg\, \frac{4.59\, sin\, 32 + 6.49\, sin(\, -68\,)}{4.59\, cos\, 32 + 6.49\, cos(\, -68\,)} = -30^\circ.$$

Circuit total current is: $i = 7.27\, sin(\, \omega t + 30^\circ\,)\, A$.

Example 10. Define the circuit current (fig. 2.12) at the input voltage effective value of $V = 25\, V$, $f = 50\, Hz$ and elements' corresponding parameters: $R_1 = 7\, \Omega$, $R_2 = 2\, \Omega$, $X_L = 3\, \Omega$, $X_C = 5\, \Omega$.

This task is solved by using the active and reactive constituents method.

Branches' impedances are: $Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{7^2 + 3^2} = 7.62\, \Omega$,

$$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{2^2 + 5^2} = 5.39\, \Omega.$$

Branches' currents effective values are:

$$I_1 = V / Z_1 = 25/7.62 = 3.28\ \text{A}, \quad I_2 = V / Z_2 = 25/5.39 = 4.64\ \text{A}.$$

Phase shift angles between the voltage and branches currents are:

$$\varphi_1 = arctg(\, X_L / R_1\,) = arctg\,(\, 3/7\,) = 32^\circ,$$

$$\varphi_2 = arctg\,(\, -X_C / R_2\,) = arctg\,(\, -5/2\,) = -68^\circ.$$

The vector diagram for the circuit is shown at fig. 2.13. The branches currents vectors are shown as I_1, I_2 together with their active and reactive constituents.

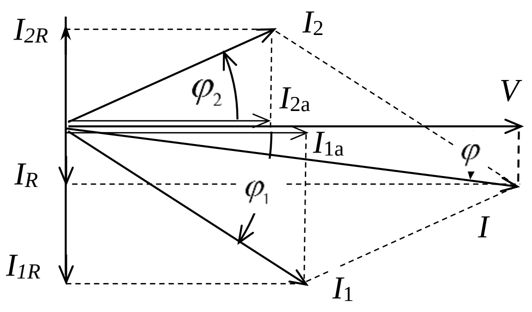


Fig. 2.13

Active and reactive currents constituents are defined accordingly:

$$I_{a1} = I_1 \cos \varphi_1 = 3.28 \cos(32) = 2.78\, A$$

$$I_{a2} = I_2\, cos\, \varphi_2 = 4.64\, cos(\, -68\,) = 1.74\, A,$$

$$I_{R1} = I_1 \sin\, \varphi_1 = 3.28\, sin\, 32 = 1.74\, A,$$

$$I_{R2} = I_2 \sin\, \varphi_2 = 4.64 \sin(\, -68) = -4.3\, A.$$

Active and reactive total current constituents make accordingly:

$$I_a = I_{a1} + I_{a2} = 2.78 + 1.74 = 4.52\, A,$$

$$I_R = I_{R1} + I_{R2} = 1.74 - 4.3 = -2.526\, A.$$

The total current effective value is defined as geometrical sum of active and reactive constituents: $I = \sqrt{I_a^2 + I_R^2} = \sqrt{4.52^2 + 2.56^2} = 5.2\, A$.

Phase shift angle between total current and input voltage is:

$$\varphi = arctg\,(I_R / I_a) = arctg\,(-2.56/4.52) = -30^\circ .$$

The total current I initial phase is $\psi_I = -\varphi = 30^\circ$.

Example 11. Solve the previous task by using the symbolic method.

The branches' impedances are accordingly:

$$\underline{Z}_1 = R_1 + jX_L = 7 + j3 = 7.62e^{j32}\ \Omega ,$$

$$\underline{Z}_2 = R_2 - jX_C = 2 - j5 = 5.39e^{-j68}\ \Omega .$$

Branches' complex currents are accordingly:

$$\underline{I}_1 = \underline{V} / \underline{Z}_1 = 25/7.62e^{j32} = 3.28e^{-j32} = 2.78 - j1.74\,A,$$

$$\underline{I}_2 = \underline{V} / \underline{Z}_2 = 25/5.39e^{-j68} = 4.64e^{j68} = 1.74 + j4.3\,A.$$

Complex total current is:

$$\underline{I} = \underline{I}_1 + \underline{I}_2 = 2.78 - j1.74 + 1.74 + j4.3 = 4.52 + j2.56 = 5.2e^{j30}\ A.$$

Tasks for individual work

1. Current effective value in the coil is 1*A*. Reactive and active powers are correspondingly *280 VAr* and *540 VA*. Define the impedance, the resistance and reactance of the coil.

2. The voltage across connected in parallel inductivity and resistor is *60 V*. The resistance and reactance are *10* **Ω** and *20* **Ω**. Define the currents in the circuit and the efficiency factor.

3. The voltage across connected in parallel inductivity, capacitance and resistor is 120 *V*, $R = 20\,\Omega$, $X_L = 20\,\Omega$ and $X_C = 10\,\Omega$. Define the currents in the circuit and the phase shift angle between input current and voltage.

4. The impedance of consumer is $\underline{Z} = (10 - j10)$ **Ω**, current effective value is 2*A* and its initial phase makes *π/3*. Write down the instantaneous value of the voltage.

5. Current effective value for connected in series resistor and capacitance is 2*A* and the impedance is 50 **Ω**. Define the dissipation factor of capacitor, if the voltage across the resistor is *40 V*.

6. Instantaneous value of the voltage of the circuit with the connected in series elements $R = 10\,\Omega$, $X_L = 20\,\Omega$ is $v(t) = 100\sin 314t\ B$. Write down the instantaneous value for the current.

7. Circuit meters indicate the following: *1A, 200V, 100W*. Define the resistance and reactance for connected in series elements.

8. Voltage across the coil is 220*V*. Current is 2*A*. Define the resistance and reactance, when the phase shift angle is *π/6*.

9. Instantaneous value of the circuit current with connected in series elements $R = 10\,\Omega$, $X_C = 20\,\Omega$ is $i(t) = 0.5\sin 314t\,A$. Write down the instantaneous value of the voltage.

10. Current effective value of connected in series resistive and capacitive elements is 1*A*. Active and reactive powers are accordingly $50W$ and $100VAr$. Define the resistance, reactance and impedance.

11. Effective value of the voltage across connected in parallel resistive $R = 10\,\Omega$ and capacitive $X_C = 20\,\Omega$ elements is 50*V*. Define the total current of the circuit and the phase shift angle.

12. The impedance of the element is $\underline{Z} = (10 + j20)$ **Ω**. Effective value of the voltage is $200V$. Phase shift angle is *π/3*. Write down the instantaneous value of the current.

13. Resonance circuit consists of the elements $R = 5\,\Omega$, $L = 20\,mH$, $C = 200\,\mu F$. Define the resonance frequency and the coil Q-factor.

14. Voltage and current instantaneous values are:

$$v(t) = 40\sin(314t + \pi/3)\,V,\ i(t) = 4\sin(314t - \pi/6)\,A.$$

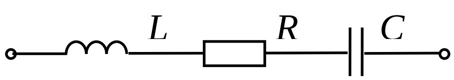
Write down the amplitude, the effective and average values of voltage and current, their initial phases and frequency. Determine the phase shift angle in grads and radians. Draw the vectors of voltage and current at vector diagram. Write down the voltage and the current as complex numbers.

15. Current instantaneous values of two parallel branches are:

$$i_1(t) = 1\sin(314t + \pi/3)\,A \text{ and } i_2(t) = 4\sin(314t - \pi/6)\,A.$$

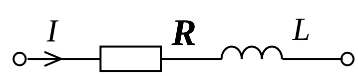
Find the total current in vector and complex form.

16. Define the circuit impedance and phase shift angle, if $R = 15\,\Omega$, $L = 32\,mH$ and

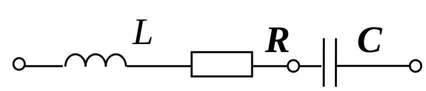


$C = 160\,\mu F$. Write down the impedance in complex form (both algebraic and exponential). Transform the connection of elements from in series into parallel. Define the conductance, the susceptance and the admittance. Write down the susceptance in complex form (both algebraic and exponential). Draw the vector diagram for the circuit.

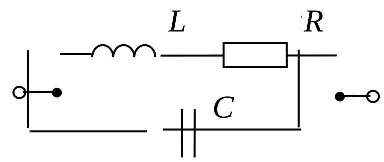
17. Define the impedance, the input voltage, the phase shift angle, the active and reactive powers and the coil quality factor in a circuit at the given current $I = 0.25\,A$, inductivity $L = 1.2\,H$ and resistance $R = 195\,\Omega$.



18. Define the input voltage, the coil voltage, the capacitor voltage, the active and reactive powers and the phase shift angles at the given current $I = 200\,mA$, inductivity $L = 1.2\,H$, resistance $R = 195\,\Omega$ and capacitance $C = 6.34\,\mu F$.



19. Define the total current, the coil current, the capacitor current, the active and reactive powers and the phase shift angles in the circuit at the given voltage $V = 70V$, inductivity $L = 1.2\,H$, resistance $R = 195\,\Omega$ and capacitance $C = 4.11\,\mu F$.



3. Three-phase circuits calculation.

Task 1. 1. Draw the scheme (fig.3.1) according to your variant at table below.

2. Calculate the phases impedances for frequency $f = 50\,Hz$.

3. Write out the phases' voltages in a complex form and calculate the complex phases' currents.

4.Define the neutral current and the circuit's complex total power.

5. Draw the vector diagram of currents and voltages.

6. Calculate the circuit when one of the phases is cut off.

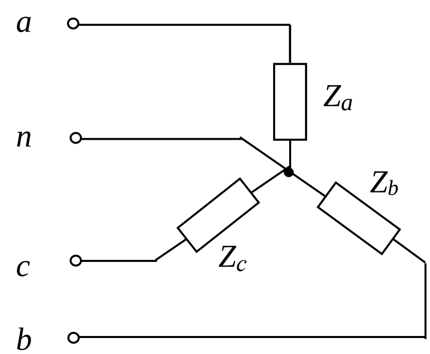


Fig. 3.1

		Z_a			Z_b			Z_c		
Var. №	V_{ph}, *V*	R_a, Ω	L_a, *mH*	C_a, *μF*	R_b, Ω	L_b, *mH*	C_b, *μF*	R_c, Ω	L_c, *mH*	C_c, *μF*
00	127	15	–	–	12	65	–	10	–	750
01	127	10	–	510	15	–	–	12	25	–
02	127	12	15	–	10	–	470	15	–	–
03	127	–	75	–	10	–	240	12	65	–
04	127	12	35	–	–	35	–	10	–	240
05	127	10	–	270	12	35	–	–	35	
06	127	–	–	150	12	45	–	10	–	680
07	127	10	–	390	–	–	130	12	35	–
08	127	12	65	–	10	–	360	–	–	300
09	127	14	–	–	11	25	–	11	–	820
10	127	11	–	910	14	–	–	11	60	–
11	127	11	20	–	18	–	240	14	–	–
12	127	–	50	–	11	–	150	12	35	–
13	127	11	20	–	–	30	–	11	–	510
14	127	9	–	270	10	45	–	–	70	–
15	127	–	–	180	9	35	–	12	–	–
16	127	11	–	750	–	–	330	13	50	–
17	127	11	40		12		680			430
18	127	13	–	–	10	10	–	12	–	910
19	127	12	–	160	13	–	–	10	15	–
20	127	10	20	–	12	–	560	13	–	–

Example for task 1. The circuit (fig. 3.1) has the following parameters: $V_{ph} = 127V$, $f = 50\,Hz$ $R_a = 18\,\Omega$, $R_b = 10\,\Omega$, $R_c = 20\,\Omega$, $L_a = L_c = 0\,mH$, $L_b = 25\,mH$, $C_a = 531\ \mu F$, $C_b = 0\ \mu F$, $C_c = 318\ \mu F$.

The circuit according to this variant is shown in fig. 3.2

Angular frequency makes: $\omega = 2\pi f = 2 \cdot 3.14 \cdot 50 = 314\ rad/s$.

Phases' reactances are: $jX_a = 1/(j\omega C_a) = 1/(j314 \cdot 531 \cdot 10^{-6}) = -j6 = 6e^{-j90}\Omega$,

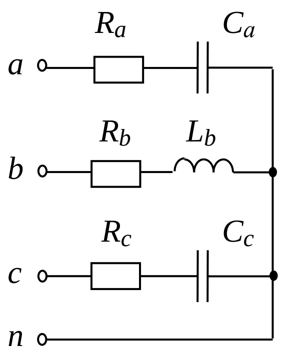


Fig. 3.2

$$jX_b = j\omega L_b = j314 \cdot 5 \cdot 10^{-3} = j8 = 8e^{j90}\Omega,$$

$$jX_c = 1/(j\omega C_c) = 1/(j314 \cdot 318 \cdot 10^{-6}) = -j10 = 10e^{-j90}\Omega.$$

Phases' impedances are:

$$\underline{Z}_a = R_a + jX_a = 18 - j6 = 18{,}97e^{-j18^\circ}\ \Omega,$$

$$\underline{Z}_b = R_b + jX_b = 10 + j8 = 12{,}81e^{j39^\circ}\ \Omega.$$

$$\underline{Z}_c = R_c + jX_c = 20 - j10 = 22{,}36e^{-j27^\circ}\ \Omega.$$

Phases' voltages are $V_{ph} = 127V$. In complex form they are accordingly:

$\underline{V}_a = 127\,e^{j0}\,V$, $\underline{V}_b = 127\,e^{-j120}\,V$, $\underline{V}_c = 127\,e^{j120}\,V$.

Phases' currents are $\underline{I}_{ph} = \underline{V}_{ph} / \underline{Z}_{ph}$:

$$\underline{I}_a = \underline{V}_a / \underline{Z}_a = 127\,e^{j0} / 18.97e^{-j18^\circ} = 6.69e^{j18^\circ} = (6.36 + j2.07)A,$$

$$\underline{I}_b = \underline{V}_b / \underline{Z}_b = 127\,e^{-j120} / 12.81e^{j39^\circ} = 9.91e^{-j159^\circ} = (-9.25 - j3.55)A,$$

$$\underline{I}_c = \underline{V}_c / \underline{Z}_c = 127\,e^{j120} / 22.36e^{-j27^\circ} = 5.68e^{j147^\circ} = (-4.76 + j3.09)A.$$

Neutral current makes: $\underline{I}_N = \underline{I}_a + \underline{I}_b + \underline{I}_c =$

$$= 6.36 + j2.07 - 9.25 - j3.55 - 4.76 + j3.09 = -7.65 + j1.61 = 7.81e^{j168}A.$$

Branches' complex total powers make:

$$\underline{S}_a = \underline{V}_a \overset{*}{\underline{I}}_a = 127\,e^{j0} \cdot 6.69e^{-j18^\circ} = 850e^{-j18^\circ} = (808 - j263)VA,$$

$$\underline{S}_b = \underline{V}_b \overset{*}{\underline{I}}_b = 127\,e^{-j120} \cdot 5.73e^{j159^\circ} = 1258e^{j39^\circ} = (978 + j792)VA,$$

$$\underline{S}_c = \underline{V}_c \overset{*}{\underline{I}}_c = 127\,e^{j120} \cdot 3.28e^{-j147^\circ} = 721e^{-j27^\circ} = (642 - j327)VA,$$

Circuit complex total power is: $\underline{S} = \underline{S}_a + \underline{S}_b + \underline{S}_c =$

$$= 808 - j263 + 978 + j792 + 642 - j327 = 2428 + j202VA.$$

When line A is cut off, phases' currents make: $\underline{I}_a = 0$,

$\underline{I}_b = \underline{I}_c = \underline{V}_{CB} / (\underline{Z}_b - \underline{Z}_c) = 127\sqrt{3}e^{j90} / (10 + j8 - 20 + j10) =$

$= 220e^{j90} / (-10 + j18) = 220e^{j90} / 20.59e^{j119} = 10.68e^{-j29}\ A.$

Vector diagram is at fig. 3.3.

Voltage vectors scale is $M_V = 30\ V/sm$. The voltage vectors lengths' are accordingly: $V = \frac{|V|}{M_V} = \frac{127}{30} = 4.2 sm$.

Voltages' initial phases (the angles between the axis X and the corresponding vectors) make correspondingly: $\psi_{VA} = 0^{\circ}$, $\psi_{VB} = -120^{\circ}$, $\psi_{VC} = 120^{\circ}$.

Current vectors scale is $M_I = 2\,A/sm$. The current vectors lengths' are correspondingly:

$$I_a = \frac{|I_a|}{M_I} = \frac{6.69}{2} = 3.34 sm,\ I_b = \frac{|I_b|}{M_I} = \frac{9.91}{2} = 4.95 sm,\ I_c = \frac{|I_c|}{M_I} = \frac{5.68}{2} = 2.84 sm,$$

$$I_N = \frac{|I_N|}{M_I} = \frac{7.81}{2} = 3.9 sm.$$

Currents' initial phases (the angles between the axis X and the corresponding vectors) are correspondingly: $\psi_{Ia} = 18^{\circ}$, $\psi_{Ib} = -159^{\circ}$, $\psi_{Ic} = 147^{\circ}$, $\psi_{IN} = 168^{\circ}$

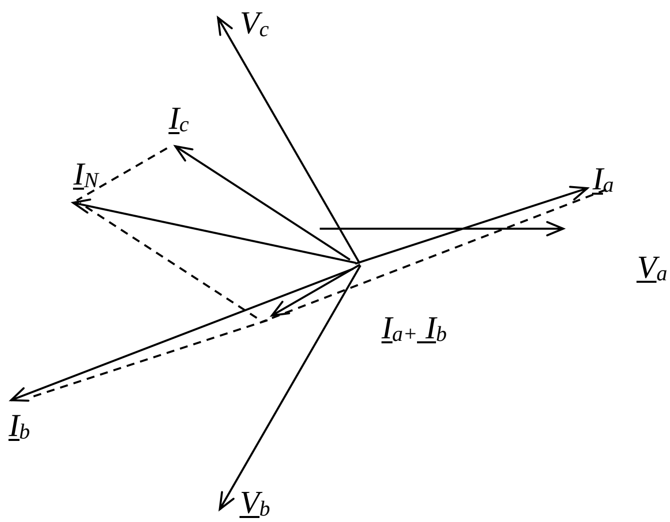


Fig. 3.3

Task 2. 1. Draw the scheme (fig. 3.4) according to your variant at table below.

2. Calculate the phases' impedances for frequency $f = 50\,Hz$.

3. Write out the phases' voltages in the complex form and calculate the complex phases' currents.

4. Define complex linear currents and the circuit complex total power.

5. Draw the vector diagram of currents and voltages.

6. Calculate the circuit when one of the phases is cut off.

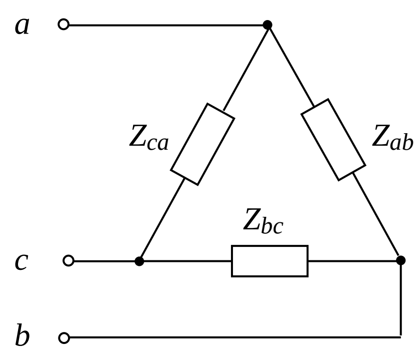
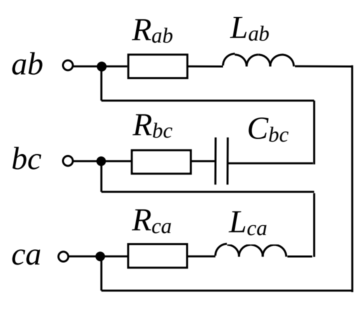


Fig. 3.4

		Z_{ab}			Z_{bc}			Z_{ca}		
Var. №	V_{ph} *V*	R_{ab}, *Ω*	L_{ab}, *mH*	C_{ab}, *μF*	R_{bc}, *Ω*	L_{bc}, *mH*	C_{bc}, *μF*	R_{ca}, *Ω*	L_{ca}, *mH*	C_{ca}, *μF*
1	127	–	–	150	8	35	–	13	–	910
2	127	14	–	820	12	40	–	10	30	–
3	127	10	55	–	–	70	–	10	–	270
4	127	–	90	–	12	–	240	10	75	–
5	127	10	20	–	12	–	560	13	–	–
6	127	12	–	160	13	–	–	10	15	–
7	127	13	–	–	10	10	–	12	–	910
8	127	11	40	–	12	–	680	–	–	430
9	127	11	–	750	–	–	330	13	50	–
10	127	–	–	180	9	35	–	12	–	–
11	127	9	–	270	10	45	–	–	70	–
12	127	11	20	–	–	30	–	11	–	510
13	127	–	50	–	11	–	150	12	35	–
14	127	11	20	–	18	–	240	14	–	–
15	127	11	–	910	14	–	–	11	60	–
16	127	14	–	–	11	25	–	11	–	820
17	127	12	65	–	10	–	360	–	–	300
18	127	10	–	390	–	–	130	12	35	–
19	127	–	–	150	12	45	–	10	–	680
20	127	10	–	270	12	35	–	–	35	–

Example for task 2. The circuit (fig. 3.4) has the following parameters: $V_{ph} = 127\,V$, $f = 50\,Hz$ $R_{ab} = 10\,\Omega$, $R_{bc} = 15\,\Omega$, $R_{ca} = 5\,\Omega$, $L_{ab} = 10\,mH$, $L_{bc} = 0\,mH$ $L_{ca} = 15\,mH$, $C_{ab} = C_{ca} = 0\ \mu F$, $C_{bc} = 270\ \mu F$.

Fig. 3.5



The circuit according to this variant is shown in fig. 3.5.

Angular frequency is $\omega = 2\pi f = 2 \cdot 3.14 \cdot 50 = 314\ rad/s$.

Phases' reactances are:

$$j X_{ab} = j\omega L_{ab} = = j314 \cdot 10 \cdot 10^{-3} = j3.14 = 3.14e^{j90}\Omega,$$

$$j X_{bc} = \frac{1}{j\omega C_{bc}} = \frac{1}{j314 \cdot 270 \cdot 10^{-6}} = -j11.79 = 11.79 \cdot e^{-j90^\circ}\,\Omega,$$

$$j X_{ca} = j\omega L_{ca} = j314 \cdot 15 \cdot 10^{-3} = j4.71 = 4.71e^{j90}\Omega.$$

Phases' impedances are: $\underline{Z}_{ab} = R_{ab} + j X_{ab} = 10 + j3.14 = 10.48e^{j17}\Omega$,

$$\underline{Z}_{bc} = R_{bc} + jX_{bc} = 15 - j11.79 = 19.08e^{-j38}\Omega,$$

$$\underline{Z}_{ca} = R_{ca} + jX_{ca} = 5 + j4.71 = 6.87e^{j43}\Omega.$$

Phases' and linear voltages are $V_{ph} = V_L = 127V$. In complex form they are:

$$\underline{V}_{ab} = 127e^{j0}V, \quad \underline{V}_{bc} = 127e^{-j120}V, \quad \underline{V}_{ca} = 127e^{j120}V.$$

Phases' currents are $\underline{I}_{ph} = \underline{V}_{ph} / \underline{Z}_{ph}$:

$$\underline{I}_{ab} = \underline{V}_{ab} / \underline{Z}_{ab} = 127 / 10.48e^{j17} = 12.12e^{-j17} = 11.59 - j3.54A,$$

$$\underline{I}_{bc} = \underline{V}_{bc} / \underline{Z}_{bc} = 127e^{-j120} / 19.08e^{-j38} = 6.66e^{-j82} = 0.93 - j6.6A,$$

$$\underline{I}_{ca} = \underline{V}_{ca} / \underline{Z}_{ca} = 127e^{j120} / 6.87e^{j43} = 18.49e^{j77} = 4.16 + j18.04A.$$

Linear currents in complex form are:

$$\underline{I}_A = \underline{I}_{ab} - \underline{I}_{ca} = 11.59 - j3.54 - 4.16 - j18.04 = 7.43 - j20.81 = 22.07e^{-j70}A,$$

$$\underline{I}_B = \underline{I}_{bc} - \underline{I}_{ab} = 0.93 - j6.6 - 11.59 + j3.54 = -10.66 - j3.06 = 11.08e^{-j164}A,$$

$$\underline{I}_C = \underline{I}_{ca} - \underline{I}_{bc} = 4.16 + j18.04 - 0.93 + j6.6 = 3.23 + j25 = 25.21e^{j83}A.$$

Branches' complex total powers make:

$$\underline{S}_{ab} = \underline{V}_{ab}\overset{*}{\underline{I}}_{ab} = 127 \cdot 12.12e^{j17} = 1539e^{j17} = (1472 + j450)VA,$$

$$\underline{S}_{bc} = \underline{V}_{bc}\overset{*}{\underline{I}}_{bc} = 127e^{-j120} \cdot 6.66e^{j82} = 845.8e^{-j38} = (666.5 - j520.7)VA,$$

$$\underline{S}_{ca} = \underline{V}_{ca}\overset{*}{\underline{I}}_{ca} = 127e^{j120} \cdot 18.49e^{-j77} = 2347e^{j43} = (1716 + j1600)VA.$$

Circuit complex total power is:

$$\underline{S} = \underline{S}_{ab} + \underline{S}_{bc} + \underline{S}_{ca} =$$

$$(1472 + j450) + (666.5 - j520.7) + (1716 + j1600) = (3854 + j1529) = 4146e^{j22}VA.$$

When the line A is cut off, phases' currents are:

$$\underline{I}_{ab} = \underline{I}_{ca} = \underline{V}_{CB} /(\underline{Z}_{ab} + \underline{Z}_{ca}) = -127e^{-j120} /(10 - j3{,}14 + 5 + j4{,}71) =$$

$$= -127e^{-j120} /(15 + j1{,}57) = 127e^{j60} / 15.08e^{j6} = 8.42e^{j54} = 4.95 + j6.81A.$$

$$\underline{I}_{bc} = \underline{V}_{BC} / \underline{Z}_{bc} = 127e^{-j120} /(19e^{-j38}) = 6.68e^{-j158} = -6.19 - j2.5A.$$

Linear currents are:

$$\underline{I}_A = 0, \ \underline{I}_C = -\underline{I}_B = \underline{I}_{ca} - \underline{I}_{bc} = 4.95 + j6.81 + 6.19 + j2.5 = 11.14 + j9.31A.$$

Vector diagram is at fig. 3.6.

Voltage vectors scale is $M_V = 20\ V/sm$. The voltage vectors lengths are accordingly:

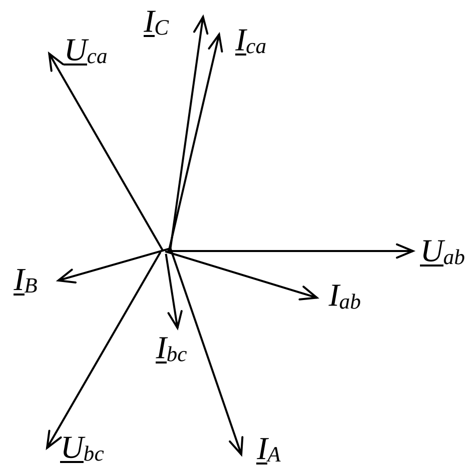


Fig. 3.6

$$V = \frac{|V|}{M_V} = \frac{127}{20} = 6.35\,cm$$

Voltages' initial phases (the angles between the axis X and the corresponding vectors) are:

$$\psi_{VBA} = 0^\circ,\ \psi_{V\,BC} = -120^\circ,\quad \psi_{V\,CA} = 120^\circ.$$

Current vectors scale is $M_I = 3A/sm$. The currents' vectors lengths are correspondingly:

$$I_{AB} = \frac{|I_{AB}|}{M_I} = \frac{12.12}{3} = 4.04\,cm,\ I_{BC} = \frac{|I_{BC}|}{M_I} = \frac{6.66}{3} = 2.22\,cm,\ I_{CA} = \frac{|I_{CA}|}{M_I} = \frac{18.49}{3} = 6.16\,cm,$$

$$I_A = \frac{|I_A|}{M_I} = \frac{22.07}{3} = 7.35\,\text{cm},\ I_B = \frac{|I_B|}{M_I} = \frac{11.08}{3} = 3.69\,cm,\ I_C = \frac{|I_C|}{M_I} = \frac{25.21}{3} = 8.27\,cm.$$

Currents' initial phases (the angles between the axis X and the corresponding vectors) are correspondingly: $\psi_{IA} = -71^\circ$, $\psi_{IB} = -164^\circ$, $\psi_{IC} = 83^\circ$, $\psi_{Iab} = -17^\circ$, $\psi_{Ibc} = -82^\circ$, $\psi_{Ica} = 77^\circ$.

Example 3. Three phase WYE network ($380/220\,V$) is connected to three resistances $R_a = 20\,\Omega$, $R_b = 40\,\Omega$, $R_c = 10\,\Omega$. Define the bias neutral voltage and the phases' currents.

The connection is WYE without neutral. Phases' voltages are correspondingly:

$$\underline{V}_A = V_{ph}e^{j0^\circ} = 220V,\ \underline{V}_B = V_{ph}e^{-j3\pi/2} = 220e^{-j120^\circ} = -110 - j190.5V,$$

$$\underline{V}_C = V_{ph}e^{j3\pi/2} = 220e^{j120^\circ} = -110 - j190.5V.$$

Phases' conductances are correspondingly: $G_a = 1/R_a = 1/20 = 0.05\ Sm$, $G_b = 1/R_b = 1/40 = 0.025\ Sm$, $G_c = 1/R_c = 1/10 = 0.01\ Sm$.

Bias neutral voltage is defined by the two nodes method:

$$\underline{V}_{nN} = \frac{\underline{V}_A G_a + \underline{V}_B G_b + \underline{V}_C G_c}{G_a + G_b + G_C} = \frac{220 \cdot 0.05 + 220 e^{-j120^\circ} \cdot 0.025 + 220 e^{j120^\circ} \cdot 0.01}{0.05 + 0.025 + 0.01} =$$

$$= 83.15 e^{j101^\circ} = -15.72 + j81.65\ V.$$

The second Kirchhoff's law defines consumer phases' voltages:

$$\underline{V}_a = \underline{V}_A - \underline{V}_{nN} = 220 + 15.72 - j81.65 = 235.72 - j81.65 = 249.46 e^{-j19} V,$$

$$\underline{V}_b = \underline{V}_B - \underline{V}_{nN} = -110 - j190.5 + 15.72 - j81.65 = -94.28 - j272.18 = 288.05 e^{-j109} V,$$

$$\underline{V}_c = \underline{V}_C - \underline{V}_{nN} = -110 + j190.5 + 15.72 - j81.65 = -94.28 + j108.88 = 144.03 e^{j131} V.$$

Phases' currents are defined: $\underline{I}_A = \underline{V}_a / R_a = 249.46 e^{-j19} / 20 = 12.47 e^{-j19} A$,

$$\underline{I}_B = \underline{V}_b / R_b = 288.05 e^{-j109} / 40 = 7.2 e^{-j109} A,$$

$$\underline{I}_C = \underline{V}_c / R_c = 144.03 e^{j131} / 10 = 14.4 e^{j131} A.$$

Tasks for individual work

1. Three inductive elements are the load of the three-phase circuit connected WYE and are $L_{ph} = 100\ mH$ each. Define the phase and the linear voltages, when the phase current is $I_{ph} = 0.7\ A$. Draw the vector diagram of the phases' voltages and currents.

2. Three capacitive elements are the load of the three-phase circuit connected DELTA and are $C_{ph} = 50\ \mu F$ each. Define the phases and the linear currents, when the phase voltage is $V_{ph} = 220 V$. Draw the vector diagram of phases' voltages and currents.

3. Three phase network ($380 / 220\ V$) is connected to the three reactances that are $X_{ph} = -50\ \Omega$ each. The voltage applied for each reactance is $220\ V$. Define the linear currents and the circuit total power. Draw the vector diagram of phases' voltages and currents.

4. Three phase network ($220 / 127\ V$) is connected to three reactances that are $X_{ph} = 50\ \Omega$ each. Voltage applied for each reactance is $220\ V$. Define the linear currents and the circuit reactive power. Draw the vector diagram of phases' voltages and currents.

5. Three resistive elements are the load of three-phase circuit connected WYE with neutral. They are $R_{ph} = 100\ \Omega$ each. Define the linear currents and the neutral current, when the linear voltage is $V_L = 220\ V$.

6. Load' currents of the three-phase circuit connected WYE with neutral are correspondingly: $10\ A$, $30\ A$, $5\ A$. Define the neutral current, when the load is of an active type.

7. Three-phase circuit total power is $S = 14\ kVA$ and reactive power is $Q = 9.5\ kVAr$. Define the load power factor.

8. Three phase DELTA network ($V_L = 220\ V$) is connected to three groups of lamps

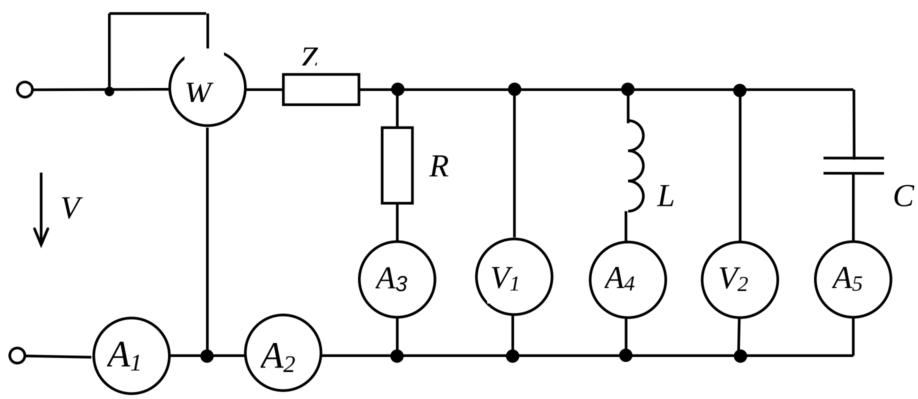


Fig. 4.1

with the same power. Consuming power is $P = 3.6\ kW$, every lamp power is $40\,kW$. Define the quantity of lamps for every phase.

9. Write down the phases' voltages in a complex form for the three-phase power source with linear voltage $V_L = 380\,V$. Three groups of consumers are connected to this source. The consumers phases' powers are correspondingly: $P_{ab} = 80\,W$, $P_{bc} = 100\,W$, $P_{ca} = 20\,W$ (fig. 3.7). Calculate the phases' resistances, the phase and the linear complex currents. Define the power of the circuit.

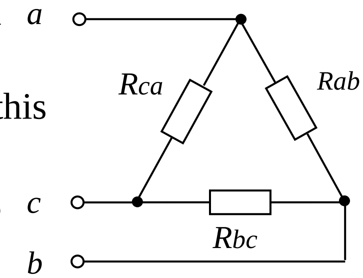


Fig. 3.7

10. Write down the phase voltages in a complex form for the three-phase power source with a linear voltage $V_L = 220\,V$. Calculate the phases' resistances, the phases' complex currents, the neutral complex current for the three-phase consumer. The consumers' phase powers are accordingly: $P_a = 100\,W$, $P_b = 80\,W$, $P_c = 50\,W$ (fig. 3.8).

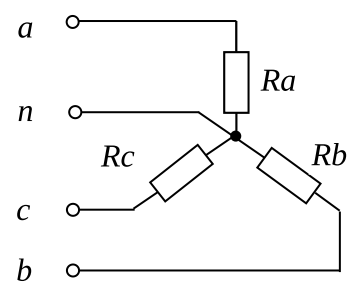


Fig. 3.8

4. Non-sinusoidal circuits calculation.

Task 1. Define the corresponding currents, voltages and power (Fig. 4.1), where A_1, V_1 – show the average values, A_2, A_3, A_4, A_5, V_2, W – show the effective values. The input non-sinusoidal voltage has a shape shown in fig.4.2. The circuit parameters are given in the table below.

Var	α_1	α_2	V_m, *V*	*Z*, Ω	*R*, Ω	X_L, Ω	X_C, Ω
01	π/4	π/4	18	10+j5	5	4	3
02	π/4	π/3	15	5+j5	6	5	4
03	π/4	π/6	18	3+j4	7	6	5
04	π/3	π/3	20	5-j5	8	7	6
05	π/3	π/4	21	10-j5	3	8	7
06	π/3	π/6	15	3-j4	4	3	8
07	π/6	π/6	20	4+j3	5	4	3
08	π/6	π/3	22	2+j4	6	5	4
09	π/6	π/4	24	4-j3	7	6	5
10	π/6	π/6	25	2-j4	8	4	6
11	π/3	π/3	22	7+j9	3	5	7
12	π/4	π/4	16	9-j7	4	3	8
13	0	π/6	21	9+j7	5	4	3
14	π/6	0	15	7-j9	6	5	4
15	0	π/3	27	6+j9	7	6	5
16	π/3	0	26	6-j9	8	7	6
17	0	π/4	25	9+j6	3	8	7
18	π/4	0	25	3+j3	4	3	8
19	0	π/2	27	9-j6	5	4	3
20	π/2	0	26	3-j3	6	5	4

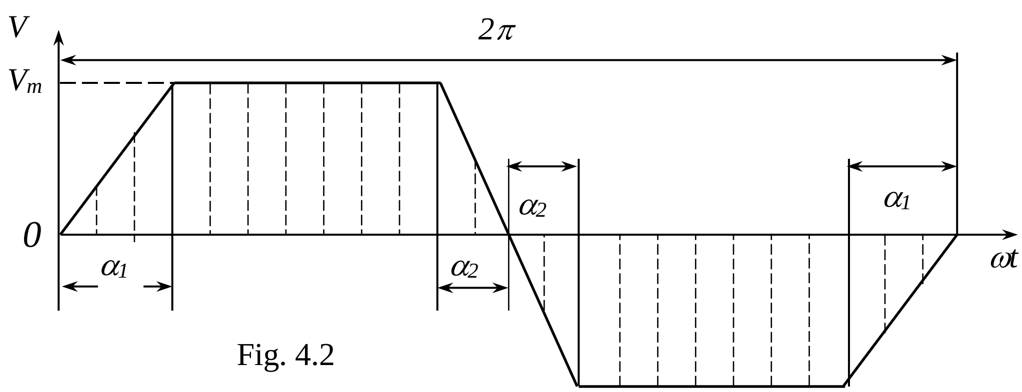


Fig. 4.2

The following are the parameters of the voltage shape and the circuit: $\alpha_1 = \pi/4$, $\alpha_2 = \pi/6$, $V_m = 250V$, $Z = (5 + j1)\Omega$, $R = 2\Omega$, $X_L = 6\Omega$, $X_C = 8\Omega$, $n = 24$.

Graphical method.

Graphical method of determination the Fourier series harmonics is based on the definite integral replacement with the finite number of intervals. For that purpose the function $f(x)$ with period 2π is divided by n equal intervals $\Delta x = 2\pi / n$ and the sum of the function's values replaces the integral.

Thus, DC component is represented as:

$$A_0 = \frac{1}{2\pi}\sum_{p=1}^{n} f_p(x)\Delta x = \frac{1}{2\pi}\sum_{p=1}^{n} f_p(x)2\pi/n \text{ or } A_0 = \frac{1}{n}\sum_{p=1}^{n} f_p(x),$$

where p is an incremental index from 1 to n,
$f_p(x)$ – is a function $f(x)$ value for $x=(p-0.5)\Delta x$, in the middle of a p – interval.

Sine constituent amplitude of k harmonic is:

$$B_k = 2\frac{1}{2\pi}\sum_{p=1}^{n} f_p(x)2\pi/(n)\cdot \sin_p kx \text{ or } A_k' = \frac{2}{n}\sum_{p=1}^{n} f_p(x)sin_p\, kx.$$

Cosine constituent amplitude of k harmonic is:

$$C_k = \frac{2}{n}\sum_{p=1}^{n} f_p(x)\cos_p kx,$$

where $sin_p\, kx$ and $cos_p\, kx$ are the functions' $sin\, k\, x$ and $cos\, k\, x$ values for $x=(p-0.5)\Delta x$, in the middle of a p – interval.

It is typical to divide the period into 24 or 18 intervals in calculations. Before calculations it is recommended to identify whether the function is relatively symmetrical across axises. The certain type of symmetry hints to the presence of particular harmonics.

Example for task 2. Since the voltage curve is symmetrical across the origin, then its Fourier series will look like: $f(\omega t) = B_1 \sin \omega t + B_2 \sin 2\omega t + ... + B_k \sin k\omega t$, where sine constituent amplitude of a k harmonic is:

$$B_k = \frac{2}{n}\sum_{p=1}^{n} f_p(\omega t)\sin_p k\omega t = \frac{4}{n}\sum_{p=1}^{n/2} f_p(\omega t)sin_p\, k\omega t\,.$$

Assumed the number of intervals $n = 24$ for three harmonics and the graph (fig. 4.2) of applied voltage is represented by the table:

№	ωt, deg	$f(\omega t)$ V	$sin\,\omega t$	$f(\omega t)$ $sin\,\omega t$
1	15	83,3	0.259	21,6
2	30	167	0.5	83,4
3	45	250	0.707	177
4	60	250	0.866	217
5	75	250	0.966	242
6	90	250	1,0	250
7	105	250	0.966	242
8	120	250	0.866	217
9	135	250	0.707	177
10	150	250	0.5	125
11	165	125	0.259	32,4
12	180	0	0	0

$$B_1 = \frac{4}{n}(21.6 + 83.4 + 177 + 217 + 242 + 217 + 177 + 125 + 32.4) = 298$$

№	$2\omega t$, deg	$f(\omega t)$ V	$\sin 2\omega t$	$f(\omega t)$ $\sin 2\omega t$
1	30	83,3	0.5	41,7
2	60	167	0.866	145
3	90	250	1,0	250
4	120	250	0.866	217
5	150	250	0.5	125
6	180	250	0	0
7	210	250	-0,5	-125
8	240	250	-0.866	-217
9	270	250	-1,0	-250
10	300	250	-0,866	-217
11	330	125	-0,5	-62,5
12	360	0	0	0

$$B_2 = \frac{4}{n}(41.7 + 145 + 250 + 217 + 125 - 125 - 217 - 250 - 217 - 62.5) = -15.5$$

№	$3\omega t$, deg	$f(\omega t)$ V	$\sin 3\omega t$	$f(\omega t)$ $\sin 3\omega t$
1	45	83,3	0,707	58,9
2	90	167	1,0	167
3	135	250	0,707	177
4	180	250	0	0
5	225	250	-0,707	-177
6	270	250	-1	-250
7	315	250	-0,707	-177
8	360	250	0	0
9	405	250	0.707	177
10	450	250	1,0	250
11	495	125	0,707	177
12	540	0	0	0

$$B_3 = \frac{4}{n}(58.9 + 167 + 177 - 177 - 250 - 177 + 177 + 250 + 177) = 67$$

If assumed three harmonics are enough, the following decomposition in Fourier series for input voltage will look like:

$$v(t) = 298 \sin \omega t - 15.5 \sin 2\omega t + 67 \sin 3\omega t \ .$$

Analitycal method.

$$f(\omega t)=\begin{cases}1000\,\dfrac{\omega t}{\pi}, & for\ \ 0\le \omega t\le \dfrac{\pi}{4},\\ 250, & for\ \ \dfrac{\pi}{4}\le \omega t\le \dfrac{5\pi}{6},\\ 1500\,(1-\dfrac{\omega t}{\pi}), & for\ \ \dfrac{5\pi}{6}\le \omega t\le \pi\end{cases},$$

$$B_k=\frac{2}{\pi}\int_0^{\pi} f(\omega t)\cdot \sin k\omega t\, d(\omega t),$$

$$B_k=\frac{2}{\pi}\int_0^{\pi/4}1000\,(\omega t/\pi)\cdot \sin k\omega t\, d(\omega t)+\frac{2}{\pi}\int_{\pi/4}^{5\pi/6}250\cdot \sin k\omega t\, d(\omega t)+,$$

$$+\frac{2}{\pi}\int_{5\pi/6}^{\pi}1500(1-\omega t/\pi)\cdot \sin k\omega t\, d(\omega t),$$

$$B_k=\frac{2}{\pi k}\left[\frac{1000}{\pi}\big(\sin(k\omega t)/k-\omega t\cdot\cos(k\omega t)\big)\right]\Bigg|_0^{\pi/4}-\frac{2}{\pi k}250\cos(k\omega t)\Bigg|_{\pi/4}^{5\pi/6}-$$

$$-\frac{2}{\pi k}1500\cos(k\omega t)\Bigg|_{5\pi/6}^{\pi}+\frac{2}{\pi k}\frac{1500}{\pi}\omega t\cdot\cos(k\omega t)\Bigg|_{5\pi/6}^{\pi}.$$

Having resolved the above equation, the amplitude of the first three harmonics are: $k=1$, $B_1=298$, $k=2$, $B_2=-15.5$, $k=3$, $B_3=67$. Therefore, Fourier series for input voltage will look like:

$$v(t)=298\sin\omega t-15.5\sin 2\omega t+67\sin 3\omega t.$$

The following input data are used for calculation of the first harmonic: $V_{m1}=298V$, $\underline{Z}_1=5+j1=5.1e^{j11}$ Ω, $R=2\Omega$. $jX_{L1}=j6=6e^{j90}$ Ω, $-jX_{C1}=-j8=8e^{-j90}$ Ω.

Thus, equivalent circuit impedance is:

$$\underline{Z}_{LC1}=\frac{jX_{L1}\cdot(-jX_{C1})}{jX_{L1}-jX_{C1}}=\frac{j6\cdot(-j8)}{j6-j8}=\frac{48}{-j2}=j24\Omega,$$

$$\underline{Z}_{RLC1}=\frac{R\cdot\underline{Z}_{LC1}}{R+\underline{Z}_{LC1}}=\frac{2\cdot(j24)}{2+j24}=\frac{j48}{2+j24}=\frac{48e^{j90}}{24{,}08e^{j85}}=2e^{j5}=1{,}99+j0{,}17\Omega,$$

$$\underline{Z}_{EQV1}=\underline{Z}_1+\underline{Z}_{RLC1}=5+j1+1{,}99+j0{,}17=6.99+j1.17=7{,}09e^{j9}\ \Omega,$$

Voltage amplitude is: $V_{m1}=298V$, therefore, total current amplitude makes: $\underline{I}_{m1}=\underline{V}_{m1}/\underline{Z}_{EQV1}=298/(7{,}09e^{j9})=42e^{-j9}=41{,}5-j6{,}57A$.

Impedance Z voltage is: $V_{mZ_1}=\underline{I}_{m1}\underline{Z}_1=42e^{-j9}\cdot 5.1e^{j11}=214e^{j2}=214+j7.5V$.

Parallel connection voltage is: $\underline{V}_{mRLC1}=\underline{V}_{m1}-\underline{V}_{mZ_1}$ $=298-214-j7.5=84-j7.5=84.3e^{-j5}V$.

Branches' currents make correspondingly: $\underline{I}_{mR1} = \underline{V}_{mRLC1} / R$
$= 84.3e^{-j5} / 2 = 42.2e^{-j5} = 42 - j3.68A$,
$\underline{I}_{mL1} = \underline{V}_{mRLC1} / jX_{L1} = 84.3e^{-j5} / 6e^{j90} = 14.05e^{-j95} = -1.22 - j14A$,
$\underline{I}_{mC1} = \underline{V}_{mRLC1} / (-jX_{C1}) == 84.3e^{-j5} / 8e^{-j90} = 10.5e^{j85} = 0.92 + j10.46A$.
Currents' instantaneous values make:
$i_1 = 42\sin(\omega t - 9^\circ)A$, $i_{R_1} = 42.2\sin(\omega t - 5^\circ)A$,
$i_{L_1} = 14{,}05\sin(\omega t - 95^\circ)A$, $i_{C_1} = 10{,}5\sin(\omega t + 85^\circ)A$.

The following input data are used for calculation of the second harmonic:
$V_{m2} = -15.5V$, $R = 2\Omega$, $\underline{Z}_2 = 5 + j2$ Ω, $jX_{L2} = j12 = 12e^{j90}$ Ω, $-jX_{C2} = -j4 = 4e^{-j90}$ Ω.

Equivalent circuit impedance is:

$$\underline{Z}_{LC2} = \frac{jX_{L2} \cdot (-jX_{C2})}{jX_{L2} - jX_{C2}} = \frac{j12 \cdot (-j4)}{j12 - j4} = \frac{48}{j8} = -j6 = 6e^{-j90}\,\Omega,$$

$$\underline{Z}_{RLC2} = \frac{R \cdot \underline{Z}_{LC2}}{R + \underline{Z}_{LC2}} = \frac{2 \cdot (-j6)}{2 - j6} = \frac{-j12}{2 - j6} = \frac{48e^{j90}}{6{,}32e^{-j72}} = 1.9e^{-j18} = 1{,}81 - j0{,}6\Omega,$$

$$\underline{Z}_{EQV2} = \underline{Z}_2 + \underline{Z}_{RLC2} = 5 + j2 + 1{,}81 - j0{,}6 = 6.81 + j1.4 = 6{,}95e^{j12}\ \Omega,$$

Voltage amplitude is: $V_{m2} = -15.5\,V$, therefore, total current amplitude makes:
$\underline{I}_{m2} = \underline{V}_{m2} / \underline{Z}_{EQV2} = 15.5e^{-j180} / (6.95e^{j12}) = 2.23e^{j168} = -2{,}18 + j0{,}46A$.

Impedance Z voltage is: $V_{mZ_2} = \underline{I}_{m2}\underline{Z}_2$
$= 2.23e^{j168} \cdot 5.4e^{j22} = 12.04e^{j190} = -11.86 - j2.1V$.

Parallel connection voltage is:

$$\underline{V}_{mRLC2} = \underline{V}_{m2} - \underline{V}_{mZ_2} = -15.5 - (-11.86 - j2.1) = -3.64 + j2.1 = 4.2e^{j150}V.$$

Branches' currents make correspondingly: $\underline{I}_{mR2} = \underline{V}_{mRLC2} / \underline{Z}_R = 4.2e^{j150} / 2 = 2.1^{j150}\ A$,
$\underline{I}_{mL2} = \underline{V}_{mRLC2} / \underline{X}_{L2} = 4.2e^{j150} / 12e^{j90} = 0.35e^{j60}\ A$,
$\underline{I}_{mC2} = \underline{V}_{mRLC2} / \underline{X}_{C2} = 4.2e^{j150} / 4e^{-j90} = 1.05e^{j-120}\ A$.

Currents instantaneous values are: $i_2 = 2{,}23\sin(\omega t - 0.8^\circ)A$,
$i_{R2} = 2{,}1\sin(\omega t + 150^\circ)A$, $i_{L2} = 0{,}35\sin(\omega t + 60^\circ)A$, $i_{C2} = 1{,}05\sin(\omega t - 120^\circ)A$.

The following input data are used for calculation of the third harmonic: $V_{m3} = 67V$
, $R = 2\Omega$, $\underline{Z}_3 = 5 + j3$ Ω, $jX_{L3} = j18 = 18e^{j90}$ Ω, $-jX_{C2} = -j2.7 = 2.7e^{-j90}$ Ω,

Equivalent circuit impedance is:

$$\underline{Z}_{LC3} = \frac{jX_{L3} \cdot (-jX_{C3})}{jX_{L3} - jX_{C3}} = \frac{j18 \cdot (-j2.7)}{j18 - j2.7} = \frac{48.6}{j15.3} = -j3.18 = 3.18e^{-j90}\,\Omega,$$

$$\underline{Z}_{RLC3} = \frac{R \cdot \underline{Z}_{LC3}}{R + \underline{Z}_{LC3}} = \frac{2 \cdot (-j3.18)}{2 - j3.18} = \frac{-j6.36}{2 - j3.18} = \frac{6.36e^{-j90}}{3.76e^{-j58}} = 1.69e^{-j32} = 1.43 - j0.9\Omega,$$

$$\underline{Z}_{EQV3} = \underline{Z}_3 + \underline{Z}_{RLC3} = 5 + j3 + 1.43 - j0.9 = 6.43 + j2.1 = 6.76e^{j18}\ \Omega,$$

Voltage amplitude is: $V_{m3} = 67\,V$, therefore, total current amplitude makes:

$\underline{I}_{m3} = \underline{V}_{m3} / \underline{Z}_{EQV3} = 67/(6.76e^{j18}) = 9.91e^{-j18} = 9.42 - j3.06\,A$.

Impedance Z voltage is:

$V_{mZ_3} = \underline{I}_{m3}\underline{Z}_3 = 9.91e^{-j18} \cdot 5.83e^{j31} = 57.78e^{j13} = 56.3 + j13\,V$.

Parallel connection voltage is:

$$\underline{V}_{mRLC3} = \underline{V}_{m3} - \underline{V}_{mZ_3} = 67 - (56.3 + j13) = 10.7 - j13 = 16.84e^{-j51}\,V .$$

Branches' currents make correspondingly: $\underline{I}_{mR3} = \underline{V}_{mRLC3} / \underline{Z}_R$ $= 16.84e^{-j51}/2 = 8.42^{-j51}\,A$,

$\underline{I}_{mL3} = \underline{V}_{mRLC3} / \underline{X}_{L3} = 16.84e^{-j51}/18e^{j90} = 0.94e^{-j141}\,A$,

$\underline{I}_{mC3} = \underline{V}_{mRLC3} / \underline{X}_{C3} = 16.84e^{-j51}/2.7e^{-j90} = 6.24e^{j39}\,A$.

Currents' instantaneous values make: $i_3 = 9.91\sin(\omega t - 18^\circ)\,A$, $i_{R3} = 8.42\sin(\omega t - 51^\circ)\,A$, $i_{L3} = 0{,}94\sin(\omega t - 141^\circ)\,A$, $i_{C3} = 6.24\sin(\omega t + 39^\circ)\,A$.

Active power (at $\varphi_k = \psi_{Vk} - \psi_{Ik}$) makes:

$$P = \frac{1}{2}(I_{m1}V_{m1}\cos\varphi_1 + I_{m2}V_{m2}\cos\varphi_2 + I_{m3}V_{m3}\cos\varphi_3) =$$

$$= \frac{1}{2}(42 \cdot 298\cos 9^\circ + 2.23 \cdot 15.5\cos 12^\circ + 9.91 \cdot 67\cos 18^\circ) = 6514\,W .$$

Reactive power is accordingly:

$$Q = \frac{1}{2}(I_{m1}V_{m1}\sin\varphi_1 + I_{m2}V_{m2}\sin\varphi_2 + I_{m3}V_{m3}\sin\varphi_3) =$$

$$= \frac{1}{2}(42 \cdot 298\sin 9^\circ + 2.23 \cdot 15.5\sin 12^\circ + 9.91 \cdot 67\sin 18^\circ) = 1085\,VAr .$$

Total power makes: $S = \sqrt{P^2 + Q^2} = \sqrt{6514^2 + 1085^2 =}6664\,VA$.

Average currents' values are correspondingly:

$$I_{AV} = \frac{2}{\pi}(42 + 2.23/2 + 9.91/3) = 29.55\,A,$$

$$I_{AVR} = \frac{2}{\pi}(42.2 + 2.1/2 + 8.42/3) = 29.32\,A,$$

$$I_{AVL} = \frac{2}{\pi}(14.05 + 0.35/2 + 0.94/3) = 9.25\,A,$$

$$I_{AVC} = \frac{2}{\pi}(10.5 + 1.05/2 + 6.24/3) = 8.34\,A.$$

Average voltage value is:

$$V_{AV} = = \frac{2}{\pi}(298 + 15.5/2 + 67/3) = 209\,V .$$

Currents' effective values are correspondingly:

$$I_1 = \sqrt{\frac{1}{2}(I_{m1}^2 + I_{m2}^2 + I_{m3}^2)} = 30.55\,A,\; I_R = \sqrt{\frac{1}{2}(I_{mR1}^2 + I_{mR2}^2 + I_{mR3}^2)} = 30.46\,A,$$

$$I_L = \sqrt{\frac{1}{2}(I_{mL1}^2 + I_{mL2}^2 + I_{mL3}^2)} = 9.96\,A,\ I_C = \sqrt{\frac{1}{2}(I_{mC1}^2 + I_{mC2}^2 + I_{mC3}^2)} = 8.86\,A.$$

Voltage effective value is:

$$V = \sqrt{\frac{1}{2}(V_{mRLC1}^2 + V_{mRLC2}^2 + V_{mRLC3}^2)} = 59.69V.$$

Task 2. The circuit shown on fig.4.3 is connected to the source of non-sine voltage of $v(t) = (100 + 100\sqrt{2}\sin\omega t + 76\sin 3\omega t)V$, where $\omega = 314\ rad/s$. The parameters of the circuit are: $R = 5\,\Omega$, $L = 31.9\ mH$, $C = 91\ \mu F$. Define the voltage, the current's effective value, write down the current instantaneous values and define the circuit's powers.

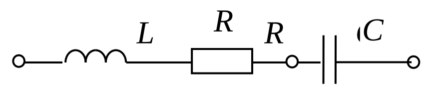


Fig. 4.3

The voltage instantaneous value is represented by constant component $V_0 = 100\ V$ and two harmonics the first $V_{1m} = 100\sqrt{2}\ V$ and the third $V_{3m} = 76\,V$ ones: $v(t) = (V_0 + V_{1m}\sin\omega t + V_{3m}\sin 3\omega t)\,V$.

The reactance of inductance for every component is:

$$k = 0 \quad X_L(0) = 0\cdot L = 0,\ X_L(1) = \omega L = 314\cdot 0.0319 = 10\,\Omega,$$

$$X_L(3) = 3\omega L = 3\cdot 314\cdot 0.0319 = 30\,\Omega.$$

The reactance of capacitance for every component is:

$$X_C(0) = 1/0\cdot C = \infty,\ X_C(1) = 1/\omega C = 1/(314\cdot 91\cdot 10^{-6}) = 35\ \Omega,$$

$$X_C(3) = 1/3\omega C = 1/(3\cdot 314\cdot 91\cdot 10^{-6}) = 11.66\ \Omega.$$

The circuit impedance for every component makes correspondingly:

$$Z(0) = \sqrt{R^2 + [X_L(0) - X_C(0)]^2} = \sqrt{5^2 + (0-\infty)^2} = \infty,$$

$$Z(1) = \sqrt{R^2 + [X_L(1) - X_C(1)]^2} = \sqrt{5^2 + (10-35)^2} = 25.5\,\Omega,$$

$$Z(3) = \sqrt{R^2 + [X_L(3) - X_C(3)]^2} = \sqrt{5^2 + (30-11.66)^2} = 19\ \Omega.$$

The voltage steady component is $V_0 = 100\ V$.

The voltage first harmonic amplitude and effective value are correspondingly:

$$V_{1m} = 100\sqrt{2} = 141\ V,\ V_1 = V_{1m}/\sqrt{2} = 100\ V.$$

The voltage third harmonic amplitude and effective value are:

$$V_{3m} = 76\ V,\ V_3 = V_{3m}/\sqrt{2} = 76/\sqrt{2} = 53.9\ V.$$

The current steady component is $I_0 = 0\ A$.

The current first harmonic amplitude and effective value are:

$$I_{1m} = V_{1m}/Z_1 = 141/25.5 = 5.53\,A,\ I_1 = I_{1m}/\sqrt{2} = 5.53/1.41 = 3.92\,A.$$

The current third harmonic amplitude and effective value are:

$$I_{3m} = V_{3m}/Z_3 = 76/19 = 4\,A,\ I_3 = I_{3m}/\sqrt{2} = 4/1.41 = 2.836\,A.$$

The power factor for the first harmonic makes:

$$cos\,\varphi_1 = R/Z_1 = 5/25.5 = 0.196\,,\ \varphi_1 = 79\,\text{deg}\,.$$

The impedance for the first harmonic is an active-capacitive one $(X_C > X_L)$. That means that the current fluctuates ahead of voltage by angle $\varphi_1 = 79\,\text{deg}$.

The power factor for the third harmonic is:

$$cos\,\varphi_3 = R/Z_3 = 5/19 = 0.263\,,\ \varphi_3 = 74\,\text{deg}\,.$$

The impedance for the third harmonic is an active-inductive one $(X_L > X_C)$,

The impedance for the first harmonic is an active-capacitive one $(X_C > X_L)$. The current lags behind the voltage fluctuations by angle $\varphi_3 = 74\,\text{deg}$.

The current instantaneous value is:

$$i = I_{1m}\,sin(\,\omega t + \varphi_1\,) + I_{3m}\,sin(\,3\omega t - \varphi_3\,) = 5.53\sin(\omega t + 79^\circ) + 4\sin(3\omega t - 74^\circ)\ \text{A}.$$

The input voltage effective value is:

$$V = \sqrt{V_0^2 + V_1^2 + V_3^2} = \sqrt{100^2 + 100^2 + 53.9^2} = 142.44\,V\,.$$

The current effective value makes:

$$I = \sqrt{I_0^2 + I_1^2 + I_3^2} = \sqrt{0 + 3.92^2 + 2.836^2} = 4.84\,A.$$

Active power is:

$$P = P_0 + P_1 + P_3 = V_0 I_0 + V_1 I_1 \cos\varphi_1 + V_3 I_3 \cos\varphi_3 =$$
$$= 0 + 100 \cdot 3.92 \cdot 0.196 + 53.9 \cdot 2.836 \cdot 0.263 = 117\,W\,.$$

The tasks for individual work.

1. Define the non-sine voltage and current's effective and average values out of their instantaneous values:
$v(t) = (200 + 200\sin\,\omega t + 10\sin\,3\omega t)V$, $i(t) = (2\sin\,\omega t + 1\sin\,2\omega t + 0.5\sin\,3\omega t)\,A$.

2. Define active, reactive and total powers for the circuit with the following instantaneous values of voltage and current:
$v(t) = (200 + 200\sin\,\omega t + 10\sin\,3\omega t)V$, $i(t) = (2\sin\,\omega t + 1\sin\,2\omega t + 0.5\sin\,3\omega t)\,A$.

3. Define the current instantaneous value for the R-element ($R = 100\,\Omega$) if the voltage instantaneous value is: $v(t) = (200 + 200\sin\,\omega t + 10\sin\,3\omega t)V$.

4. Define the voltage instantaneous value for the L-element ($L = 0.1H$) if the current instantaneous value is $i(t) = (2\sin\,\omega t + 1\sin\,2\omega t + 0.5\sin\,3\omega t)\,A$.

5. Define the voltage instantaneous value for the C-element ($C = 100\,\mu F$) if the current instantaneous value is $i(t) = (2\sin\,\omega t + 1\sin\,2\omega t + 0.5\sin\,3\omega t)\,A$.

6. Define the current instantaneous value for the coil with the parameters $R = 10\,\Omega$, $X_L = 30\,\Omega$ and voltage $v(t) = (100 + 141\sin\,\omega t)V$.

7. Define the voltage instantaneous value for the circuit with the parameters $R = 10\,\Omega$, $X_C = 30\,\Omega$ and current $i(t) = (2\sin\,\omega t + 1\sin\,3\omega t)\,A$.

5. Transient processes calculation

Task 1. Define the currents' instantaneous values $i\ (t)$, $i_2(t)$ and $i_3(t)$ at the

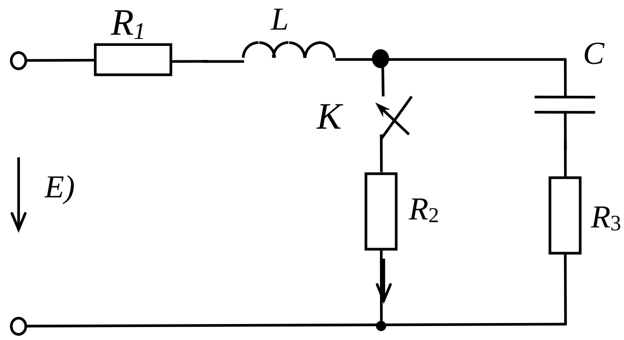


Fig. 5.1

circuit (fig. 5.1) when the key is switched on and *i(0)=0*. Draw the currents' $i\ (t)$, $i_2(t)$ and $i_3(t)$ lines. The parameters of the circuit are given in the table below.

Var. №	*Initial condition*	*The circuit parameters*				
		E,V	*f, Hz*	*R, Ω*	*L, H*	*C, μF*
01	$v_C(0)=10V$	10	50	50	0,1	10
02	$v_C(0)=20V$	15	100	75	0,2	20
03	$v_C(0)=30V$	20	50	100	0,3	30
04	$v_C(0)=40V$	25	100	150	0,4	40
05	$v_C(0)=50V$	30	50	200	0,5	50
06	$v_C(0)=10V$	35	100	250	0,6	60
07	$v_C(0)=20V$	40	50	300	0,7	70
08	$v_C(0)=30V$	45	100	50	0,8	80
09	$v_C(0)=40V$	50	50	75	0,9	90
10	$v_C(0)=50V$	55	100	100	0,9	10
11	$v_C(0)=10V$	60	50	150	0,8	20
12	$v_C(0)=20V$	10	100	200	0,7	30
13	$v_C(0)=30V$	15	50	250	0,6	40
14	$v_C(0)=40V$	20	100	300	0,5	50
15	$v_C(0)=50V$	25	50	50	0,4	60
16	$v_C(0)=10V$	30	100	75	0,3	70
17	$v_C(0)=20V$	35	50	100	0,2	80
18	$v_C(0)=30V$	40	100	150	0,1	90
19	$v_C(0)=40V$	45	50	200	0,2	30
20	$v_C(0)=50V$	50	100	250	0,3	40

The transient process in the circuit with connected in series RLC

The transient process occurs in a *RLC* link that is switched on to a DC source (fig. 5.2). According to the Kirchhoff's second law the differential equation for after commutation steady-state mode is: $v_L + v_C + v_R = V$. After the following substitution

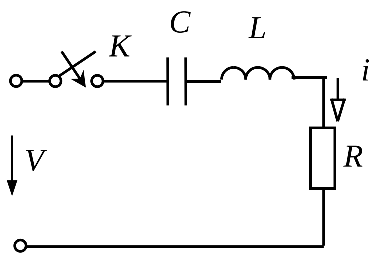


Fig. 5.2

$v_L = L\dfrac{di}{dt}$, $v_R = Ri$, $v_C = \dfrac{1}{C}\int idt$ the final equation is $L\dfrac{di}{dt} + \dfrac{1}{C}\int idt + Ri = V$. After differentiation the second order equation is: $L\dfrac{d^2i}{dt^2} + R\dfrac{di}{dt} + \dfrac{1}{C}i = 0$, Since $dV/dt = 0$, the equation is a homogeneous one. It's solution is $i(t) = i_T + i_{SS}$, where partial solution i_{SS} is equal to the current value when transient process is over $i_{SS} = 0$ (because $X_C = \infty$ for DC). So $i(t) = i_T$. The characteristic equation corresponding to this differential one is $Lp^2 + Rp + 1/C = 0$. Its roots are $p_{1,2} = -\dfrac{R}{2L} \pm \sqrt{\dfrac{R^2}{(2L)^2} - \dfrac{1}{(LC)^2}}$. After substitutions $\delta = R/2L$ and $\omega_0 = 1/(LC)$, the equation makes $p_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$. The character of transient process depends on circuit's parameters and the characteristic equation roots' characters. If they are real $\delta^2 - \omega_0^2 > 0$ and $p_1 \neq p_2$ the transient process is a capacitor aperiodic discharge. The capacitor is charging to the voltage V through the coil and resistor and then discharging to zero.

The general solution of homogeneous second order differential equation is $i_T = A_1 e^{p_1 t} + A_2 e^{p_2 t}$ (fig. 5.3). The constant of integration is defined from initial conditions. The initial condition for the current (according to the first commutation law) is: $i(0) = i(0\_) = 0$. Thus $i(0) = i_T(0) = A_1 + A_2 = 0$ and $A_1 = -A_2$. The second initial condition is defined for the derivative $\left.\dfrac{di}{dt}\right|_{t=0} = \dfrac{di(0)}{dt}$ from differential equation $L\dfrac{di}{dt} + v_C + Ri = V$. For the time moment $t = 0$: $L\dfrac{di(0)}{dt} + v_C(0) + Ri(0) = V$. Taking into account that $v_C(0) = 0$ and $i(0) = 0$, the following is true $L\dfrac{di(0)}{dt} = V$, $\dfrac{di(0)}{dt} = V/L$.

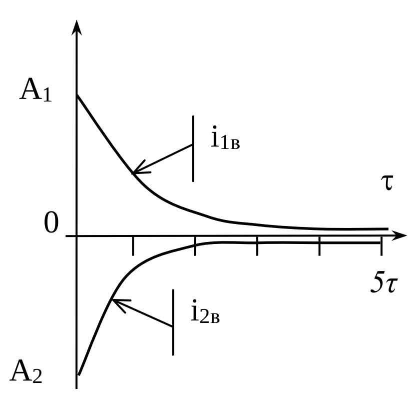


Fig. 5.3

The following equation is to be resolved to define the constant of integration:

$$\frac{di}{dt} = \frac{di_T}{dt} = \frac{d}{dt}(A_1 e^{p_1 t} + A_2 e^{p_2 t}) = A_1 p_1 e^{p_1 t} + A_2 p_2 e^{p_2 t}.$$

At the moment $t = 0$ $\dfrac{di(0)}{dt} = \dfrac{di_T(0)}{dt} = A_1 p_1 + A_2 p_2 = \dfrac{V}{L}$.

Taking into account that $A_1 = -A_2$ the following is true

$A_1 = -A_2 = V / L(p_2 - p_1), = V/(2L\sqrt{\delta^2 - \omega_0^2})$. Thus, the current is: $i = i_T = \dfrac{V}{2L\sqrt{\delta^2 - \omega_0^2}}(e^{p_1 t} - e^{p_2 t})$.

The transient current consists of two exponential constituents $i = i_T = A_1 e^{p_1 t} + A_2 e^{p_2 t} = i_{1в} + i_{2в}$.

If the characteristic equation roots are real $\delta^2 - \omega_0^2 = 0$ and equal $p_1 = p_2$, the transient process is the capacitor aperiodic boundary discharge $p_1 = p_2 = p = -\delta$.

The general solution of homogeneous second order differential equation is $i_T = (A_1 + A_2 t)e^{pt}$. From initial conditions $i(0) = 0$ and $\dfrac{di(0)}{dt} = V / L$ the constant of integration is defined as $i(0) = i_T(0) = A_1 = 0$. Taking into account that $\dfrac{di_T}{dt} = A_2 e^{pt} + A_2 pte^{pt}$, the following is true $\dfrac{di_T(o)}{dt} = A_2 + A_2 p = \dfrac{V}{L}$, $A_2 = \dfrac{V}{L(1+p)}$. Thus, $i = i_T = \dfrac{V}{L(1-\delta)} te^{-\delta t}$.

If the characteristic equation roots are complex-conjugates $\delta^2 - \omega_0^2 < 0$, the transient process is a damped oscillatory process and the capacitor periodically discharges to the coil. The characteristic equation roots are

$$p_{1,2} = -\delta \pm j\sqrt{\omega_0^2 - \delta^2},$$

After the substitution $\sqrt{\omega_0^2 - \delta^2} = \omega_T$, the roots are $p_{1,2} = -\delta \pm j\omega_T$

The general solution of homogeneous second order differential equation for complex-conjugates roots is $i = i_T = Ae^{-\delta t}\sin(\omega_T t + \phi)$. From initial conditions $i(0) = 0$ and $\dfrac{di(0)}{dt} = V / L$ the constant of integration A and angle ϕ are defined. $i(0) = i_T(0) = A\sin\phi = 0$, so, $\sin\phi = 0$, $\phi = 0^\circ$.

$$\frac{di}{dt} = \frac{di_T}{dt} = -A\delta e^{-\delta t}\sin(\omega_T t + \phi) + A\omega_T e^{-\delta t}\cos(\omega_T t + \phi),$$

$$\frac{di(0)}{dt} = \frac{di_T(0)}{dt} = -A\delta\sin\phi + A\omega_T\cos\phi = A\omega_T = V / L.$$

Therefore, $A = V/(L\omega_T)$, $i = i_T = \dfrac{V}{L\sqrt{\omega_0^2 - \delta^2}} e^{-\delta t}\sin(\sqrt{\omega_0^2 - \delta^2}\, t)$.

Example for task 1. The circuit parameters are: $V = E = 50V$, $R_1 = R_2 = R_3 = 10\,\Omega$, $L = 0.9H$, $C = 70\,\mu F$, $i(0) = 0$.

Before commutation the following is true: $i_1(0) = i_3(0) = 0$, $i_2(0) = 0$, $v_C(0) = E = 50V$.

The characteristic equation is got from the impedance of the circuit after commutation mode and it has to be equal to zero ($j\omega = p$):

$$Z(p) = \frac{R_2(R_3 + 1/(pC))}{R_2 + R_3 + 1/(pC)} + R_1 + pL = \frac{10 \cdot (10 + 1/(p \cdot 70 \cdot 10^{-6}))}{10 + 10 + 1/(p \cdot 70 \cdot 10^{-6})} + 10 + 0.9p = 0.$$

It follows as:

$1.26 \cdot 10^{-3} p^2 + 0.921 p + 20 = 0$, with the roots: $p_1 = -22.42, p_2 = -708{,}53$.

The steady state currents and voltage are: $i_{1SS} = i_{2SS} = 50/(R_1 + R_2) = 2.5\,A$, $i_{3SS} = 0\,A$ and $v_{CSS} = v_{R2} = 25V$.

The general solution of homogeneous second order differential equation for real roots $p_1 \neq p_2$ is: $i_T = A_1 e^{p_1 t} + A_2 e^{p_2 t}$. The initial condition for the current (according to the first commutation law) is $i(0) = i(0_-) = 0$. Therefore, $i(0) = i_T(0) + i_{SS}(0) = A_1 + A_2 + 2.5 = 0$, $A_1 = -A_2 - 2.5$.

The general solution of homogeneous second order differential equation for real roots is $p_1 \neq p_2$, $v_{CT} = B_1 e^{p_1 t} + B_2 e^{p_2 t}$. The initial condition for the voltage (according to the second commutation law) is $v_C(0) = v_C(0_-) = 50$. Therefore, $v_C(0) = v_{CT}(0) + v_{CSS}(0) = B_1 + B_2 + 25 = 0$, $B_1 = -B_2 - 25$.

The initial conditions for current and voltage derivatives at the moment $t = 0$ are: $\frac{di(0)}{dt} = \frac{di_T(0)}{dt} = A_1 p_1 + A_2 p_2$, $\frac{dv_C(0)}{dt} = \frac{dv_{CT}(0)}{dt} = B_1 p_1 + B_2 p_2$.

These initial conditions can be defined from the system of equations according to the first and second Kirchhoff's laws for after commutation mode:

$$\begin{cases} R_1 i_1(0) + L\frac{di_1(0)}{dt} + R_2 i_2(0) = 50 \\ R_3 i_3(0) + v_C(0) - R_2 i_2(0) = 0 \\ i_3(0) + i_2(0) = i_1(0) \end{cases}, \quad \begin{cases} 10 \cdot 0 + 0.9\frac{di_1(0)}{dt} + 10 i_2(0) = 50 \\ 10 i_3(0) + 50 - 10 i_2(0) = 0 \\ i_3(0) + i_2(0) = 0 \end{cases},$$

The solutions are: $i_2(0) = -i_3(0) = 2.5\,A$, $di_1(0)/dt = 27{,}78$.

$$\frac{di_1(0)}{dt} = A_1(-22.42) + A_2(-708{,}53) = 27{,}78,$$

$A_2 = 0{,}0412$, $A_1 = -A_2 - 2.5 = -2.54$.

From the equation $i_3(0) = i_1(0) - i_2(0) = C\frac{dv_C(0)}{dt}$ the initial conditions are defined:

$$i_3(0) = i_1(0) - i_2(0) = \frac{dv_C(0)}{dt} = (0 - 2.5)/70 \cdot 10^{-6} = -35714,$$

$$\frac{dv_C(0)}{dt} == B_1(-22.42) + B_2(-708.53) = -35714, \; B_2 = 52.878,$$

$B_1 = -B_2 - 25 = -77.87$

So, $i_1(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t} + i_{SS} = -2.54 e^{-22.42t} + 0.0412 e^{-708.53t} + 2.5\,A$,
$v_C(t) = B_1 e^{p_1 t} + B_2 e^{p_2 t} + v_{C\,SS} = -77.87 e^{-22.42t} + 52.878 e^{-708.53t} + 25V$,
$i_3(t) = C\dfrac{dv_C}{dt} = C(B_1 p_1 e^{p_1 t} + B_2 p_2 e^{p_2 t}) = 0.122 e^{-22.42t} - 2.62 e^{-708.53t} A$,
$i_2(t) = i_1(t) - i_3(t) = -2.66 e^{-22.42t} + 2.63 e^{-708.53t} + 2.5\,A$.

Operator method

This method is used to calculate the transient processes at any circuit. The essence of the method is:

1. The time function *f(t)* is replaced by its image *F(p)*, the function of operator ***p.*** *f(t)* is transformed into *F(p)* by the Laplace direct conversion. Thus, simple linear equation is obtained instead of the differential ones.

2.The images of unknown currents, voltages, etc. are received after solution of the linear equations.

3.F(p) is back-transformed into *f(t)* according to the decomposition theorem. The image of constant is $A \equiv A/p$,

The exponential function image is: $e^{\alpha t} \equiv 1/(p-\alpha)$, thus, $e^{j\omega t} \equiv 1/(p - j\omega)$.

The complex-form-presented current image is: $I_m e^{j(\omega t+\psi i)} = \underline{I}_m e^{j\omega t} \equiv \underline{I}_m/(p - j\omega)$.

The derivative image is $di(t)/dt \equiv pI(p) - i(0)$.

The voltage image is: $v_L = Ldi(t)/dt \equiv LpI(p) - Li(0)$.

The integral image is $\int_0^t i(t)dt \equiv I(p)/p$.

The voltage image is $v_C = v_C(0) + \dfrac{1}{C}\int_0^t i(t)dt \equiv I(p)/Cp + v_C(0)/p$.

For the circuit with connected in series *RLC* and DC source operator scheme is as

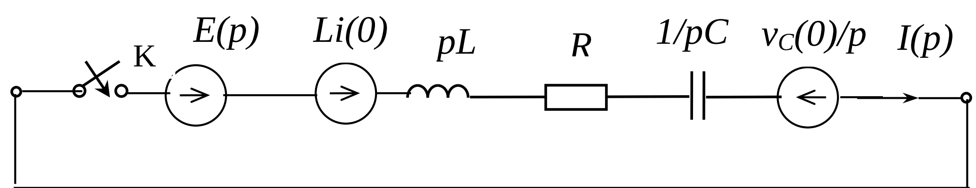


Fig. 5.4

in the fig. 5.4. For this circuit the electrical state equation for instantaneous values is: $e(t) = v_R + v_L + v_C$, $e(t) = Ri(t) + Ldi(t)/dt + v_C(0) + 1/C\int i(t)dt$.

The same equation for the images is:

$$E(p) = RI(p) + LpI(p) - Li(0) + I(p)/Cp + v_C(0)/p.$$

So $I(p) = (E(p) + Li(0) - v_C(0)/p)/Z(p)$, where $Z(p) = R + pL + 1/(pC)$ is an operating impedance of the circuit.

Then decomposition theorem is used to replace image *F(p)* by time function *f(t)*:

$F(p) = \dfrac{N(p)}{M(p)} \equiv f(t) = \sum_{k=1}^{m} \dfrac{N(p_k)}{M'(p_k)} e^{p_k t}$, where p_k - are roots of equation $M(p) = 0$.

Example for task 1 (operator method). The system of equations according to the Kirchhoff's laws for images that correspond to the originals (circuit shown on fig. 5.1) is

:
$$\begin{cases} R_1 I_1(p) + LpI_1(p) - Li(0) + R_2 I_2(p) = E(p), \\ R_3 I_3(p) + v_C(0)/p + I_3(p)/(Cp) - R_2 I_2(p) = 0, \\ I_3(p) + I_2(p) = I_1(p) \end{cases}$$

$$\begin{cases} 10 I_1(p) + 0.9 pI_1(p) + 10 I_2(p) = 50, \\ 10 I_3(p) + 50/p + I_3(p)/(7 \cdot 10^{-5} p) - 10 I_2(p) = 0, \\ I_3(p) + I_2(p) = I_1(p) \end{cases}$$

When resolved: $I_1(p) = (100p + 714.55)/(1.8p^2 + 1315.74p + 28572) = N(p)/M(p)$,

The roots of equation $M(p) = (1.8p^2 + 1315.74p + 28572) = 0$ are $p_1 = -22.42$ and $p_2 = -708.53$. Then $M'(p) = 3.6p + 1315.74$, $N(p) = 100p + 714.55$.

And finally, according to decomposition theorem:

$$i_1(t) = \frac{N(p_1)}{M'(p_1)} e^{p_1 t} + \frac{N(p_1)}{M'(p_1)} e^{p_1 t} = -2.54 e^{-22.42t} + 0.0412 e^{-708.53t} + 2.5\ A.$$

Task for individual work

1. Define the duration of the transient process $t = 3\tau$ in the circuit with connected in series elements $L = 4\,H$ and $R = 100\,\Omega$ that is switched on to the DC source.

2. Define the duration of the transient process $t = 3\tau$ in the circuit with connected in series elements $C = 100\,\mu F$ and $R = 100\,\Omega$ that is switched on to the DC source.

3. How transient process time constant is changed in the circuit with connected in series elements $L = 4\,H$ and $R_1 = 100\,\Omega$ when resistance $R_2 = 100\,\Omega$ is connected in parallel to R_1?

4. How transient process time constant is changed in the circuit with connected in series elements $C = 100\,\mu F$ and $R_1 = 100\,\Omega$ when resistance $R_2 = 100\,\Omega$ is switched on in parallel to R_1?

5. The circuit with connected in series elements $R = 5\Omega$ and $L = 0.05\,H$ is switched off from a DC source $V = 110\,V$ and short-circuited. Define the transient current and voltage v_L.

6. The circuit with connected in series elements $R = 5\Omega$ and $C = 100\,\mu F$ is switched on to a DC source $V = 110\,V$. Define the transient current and voltage v_C.

7. The connected in series capacitance $C = 100\,\mu F$ and resistance $R = 50\,\Omega$ are switched on to a DC source $E = 50\,V$. Define the capacitance voltage at moment $t = 3\tau$.

8. The coil with parameters $L = 12.6\,mH$ and resistance $R = 4\Omega$ is switched on to a DC source $E = 40\,V$. Define the coil current at the moment $t = 2\tau$.

6. Magnetic circuits calculation

Task 1. Calculate the magneto-motive force *F*, the magnetic field tension *H*, the magnetic flux *Φ* and the magnetic resistance R_m. Calculate the coil induction *L* for a coil with toroidal ferromagnetic core with a given medial line of magnetic circuit length of *l=0.15m*, cross-section area of $S = 0.5 \cdot 10^{-4} m^2$, coil current *I=1.1A*, number of winding turns *w=100*, steel relative magnetic permeability μ_r=850 and vacuum magnetic permeability $\mu_0 = 4\pi \cdot 10^{-6}$.

Magneto-motive force is: $F = Iw = 1.1 \cdot 100 = 110\ A$.

Magnetic field tension is: $H = F / l = 110 / 0.15 = 733{,}3 A/m$.

Magnetic induction value is obtained from the main magnetic curve: $B = \mu_r \mu_0 H = 850 \cdot 4\pi \cdot 10^{-6} \cdot 733{,}3 = 7.83 T$.

Magnetic flux is: $\Phi = BS = 7.83 \cdot 0.5 \cdot 10^{-4} = 3.92 \cdot 10^{-4} Wb$.

Magnetic resistance is: $R_M = F / \Phi = 110 / 3.92 \cdot 10^{-4} = 28.06 \cdot 10^4$.

Coil induction is: $L = w\Phi / I = 100 \cdot 3.92 \cdot 10^{-4} / 1.1 = 0.0356\ H$.

Task for individual work

1. A coil with toroidal ferromagnetic core has a cross-section area of $S = 28\, sm^2$, medial line magnetic circuit length of $l = 5.6\, sm$ and inductivity of $L = 0.4\, H$. Define the number of winding turns necessary to create magnetic flux of $\Phi = 0.02\, Wb$.

2. The coil consumes power of $P = 500\ W$ at current $I = 5\ A$. The same coil with a core consumes power $P = 300\ W$ at current $I = 3\ A$ and voltage $V = 120\ V$. Define the electrical, magnetic losses and power factor.

3. A ferromagnetic core has a magnetic permeability of $\mu_r = 1000$, cross-section area of $S = 20\, sm^2$ and magnetic induction $B = 1.2 T$. Define the magnetic flux and the magnetic field tension.

4. A coil with a core consumes power of $P = 300\, W$ at current $I = 3\ A$ and voltage $V = 120\ V$. It has a DC resistance of $R = 20\ \Omega$. Define the electrical and magnetic losses, the power factor and the reactive power.

Task 2. Calculate the transformation ratio *k*, the primary and secondary windings' currents I_1 and I_2, the primary and secondary windings' powers P_1 and P_2, the transformer losses ΔP and the efficiency factor η at given total power of *S=900VA*, primary and secondary windings' voltages V_1*=220V* and V_2*=22V*, power factor $\cos\varphi_1 = 0.8$ and load impedance *Z=10*+j10Ω.

Transformation ratio is: $k = V_1 / V_2 = 220 / 22 = 10$.

Primary winding current is: $I_1 = S / V_1 = 900 / 220 = 4{,}1 A$.

Secondary winding current is: $I_2 = kI_1 = 10 \cdot 4.1 = 41 A$.

Primary winding impedance is: $Z_1 = V_1 / I_1 = 220 / 4.1 = 53.66\ \Omega$.

Transformer input active power is: $P_1 = S \cos\varphi_1 = 900 \cdot 0.8 = 720\ W$.

Secondary winding power factor is: $\cos\varphi_2 = R_2 / \sqrt{R_2^2 + X_2^2} = 5/\sqrt{10^2 + 10^2} = 0.71$.

Transformer output active power is: $P_2 = S\cos\varphi_2 = 900 \cdot 0.71 = 639\ W$.

Transformer losses are: $\Delta P = P_1 - P_2 = 720 - 639 = 81\,W$.

Efficiency factor is: $\eta = P_2 / P_1 = 639/720 = 0.89$.

Task for individual work

1. A transformer that is fed by industrial frequency voltage has a cross-section area $S = 20\,sm^2$, magnetic induction $B = 1.2\,T$, primary and secondary windings turns' number are $w_1 = 400$ and $w_2 = 50$ correspondingly. Define the e.m.f. of a one-turn winding, primary and secondary windings' e.m.f. and transformation ratio.

2. A transformer that is fed by voltage with frequency $400\ Hz$ has a number of primary winding turns 1000 and core magnetic flux $\Phi_m = 1.25 \cdot 10^{-4}\ Wb$. Define the one winding e.m.f.

3. Define the transformer number of primary and secondary windings' turns, that is fed by voltage $220\,V$ of industrial frequency and which core's magnetic flux is $\Phi_m = 2 \cdot 10^{-3}\ Wb$.

4. The idle and short circuit mode powers of transformer are $400\,W$ and $150\,W$ correspondingly. Total power is $20\ kVA$ and $\cos\varphi_2 = 0.87$. Define the efficiency factor at nominal mode.

5. Idle mode transformer voltage is $220\,V$, current is 1 A and consumer power is $150\,W$. Define the resistance, reactance and impedance of idle mode substitutional scheme.

6. Define the transformer's number of primary winding turns and transformation ratio at given voltage $220\,V$ of industrial frequency, $w_2 = 40$, core cross-section area $S = 7{,}6\ sm^2$ and magnetic induction $B = 0.95\,T$.

7. Electric motors calculation

Task 1. The parameters of an induction motor with the short circuit rotor and the connected WYE stator windings are given in table below (nominal voltage V_{NOM}, nominal power P_{NOM}, number of pole pares p, nominal slip s_{NOM},, nominal efficiency factor η_{NOM}, power factor $\cos\varphi$, $\lambda = M_{MAX}/M_{NOM}$, $\gamma = M_{ST}/M_{NOM}$ and $\beta = I_{ST}/I_{NOM}$).

Define the motor nominal and starting currents, the nominal, maximum and starting moments. Define the motor starting moment, when the voltage is 15 % less than nominal. Make a conclusion, if the motor starts up at this voltage with a nominal load. Draw the speed *n(s)*, the moment *M(s)* and the mechanical *n(M)* curves of motor characteristics.

Var.	V_{NOM}	P_{NOM}	*p*	$s_{NOM,}$	η_{NOM}	$cos\varphi$	$\frac{M_{MAX}}{M_{NOM}}$	$\frac{M_{ST}}{M_{NOM}}$	$\frac{I_{ST}}{I_{NOM}}$
№	*V*	*kW*		*%*			λ	γ	β
01	380	75,0	2	3,0	0,925	0,92	2,0	1,1	7,0
02	380	0,8	1	3,0	0,780	0,86	2,2	1,9	7,0
03	380	0,1	1	3,0	0,795	0,87	2,2	1,9	7,0
04	380	1,5	1	4,0	0,805	0,88	2,2	1,8	7,0
05	380	2,2	1	4,5	0,830	0,89	2,2	1,8	7,0
06	380	3,0	1	3,5	0,845	0,89	2,2	1,7	7,0
07	380	4,0	1	2,0	0,855	0,89	2,2	1,7	7,0
08	380	5,5	1	3,0	0,860	0,89	2,2	1,7	7,0
09	380	7,5	1	3,5	0,870	0,89	2,2	1,6	7,0
10	380	10,0	1	4,0	0,880	0,89	2,2	1,5	7,0
11	380	13,0	1	3,5	0,880	0,89	2,2	1,5	7,0
12	380	17,0	1	3,5	0,880	0,90	2,2	1,2	7,0
13	380	22,0	1	3,5	0,880	0,90	2,2	1,1	7,0
14	380	30,0	1	3,0	0,890	0,90	2,2	1,1	7,0
15	380	40,0	1	3,0	0,890	0,91	2,2	1,0	7,0
16	380	55,0	1	3,0	0,900	0,92	2,2	1,0	7,0
17	380	75,0	1	3,0	0,900	0,92	2,2	1,0	7,0
18	380	100,0	1	2,5	0,915	0,92	2,2	1,0	7,0
19	380	10,0	2	3,0	0,885	0,87	2,0	1,4	7,0
20	380	13,0	2	3,0	0,885	0,89	2,0	1,3	7,0

Task 2. The parameters of an induction motor with the phase rotor and the connected WYE stator windings are given in table below (nominal voltage V_{NOM}, number of pole pares *p*, nominal slip $s_{NOM,}$, stator phase winding turns number w_1, rotor phase winding turns number w_2, stator and rotor phase resistances and reactances R_1, R_2, X_1, X_2 correspondingly).

Define the stator and rotor starting currents, the power factor, the starting moment without starting rheostat, the starting rheostat resistance and the maximum starting moment.

Draw the speed *n(s)*, the moment *M(s)* and the mechanical *n(M)* curves of motor characteristics.

Var.	V_{NOM}	*p,*	s_{NOM}	R_1,	X_1,	R_2,	X_2,	w_1,	w_2,
№	*V*		*%*	Ω	Ω	Ω	Ω		
01	220	2	3,0	0,46	1,52	0,07	0,22	190	64
02	220	2	3,5	0,58	2,32	0,06	0,35	260	82
03	380	2	3,5	0,62	1,84	0,04	0,42	362	72

04	380	3	2,5	0,74	3,52	0,07	0,37	216	48
05	380	3	2,5	0,78	4,12	0,06	0,62	424	74
06	220	3	4,0	0,36	3,62	0,045	0,48	358	62
07	220	2	4,5	0,42	2,82	0,05	0,34	184	42
08	220	2	5,0	0,64	3,12	0,06	0,65	412	82
09	220	2	5,0	0,82	3,82	0,07	0,48	362	65
10	380	3	3,0	0,84	4,24	0,06	0,52	254	46
11	380	3	3,0	0,78	3,64	0,04	0,48	228	42
12	380	2	2,5	0,86	3,48	0,05	0,78	316	54
13	380	2	2,5	0,76	2,24	0,065	0,54	272	78
14	220	2	2,5	0,48	3,48	0,03	0,62	458	92
15	220	2	3,0	0,52	2,94	0,055	0,36	162	43
16	220	3	3,0	0,56	4,42	0,045	0,64	288	54
17	380	3	3,0	0,62	3,54	0,06	0,46	204	62
18	380	3	5,0	0,76	3,72	0,045	0,54	356	72
19	380	2	5,0	0,66	2,92	0,05	0,64	384	68
20	220	2	5,0	0,58	2,56	0,035	0,48	452	82

Example for task 1. The parameters of an induction motor with the short circuit rotor and the connected WYE stator windings are: nominal voltage $V_{NOM}=380\ V$, nominal power $P_{NOM}=1\ kW$, number of pole pares $p=2$, nominal slip $s_{NOM,}=3\%$, nominal efficiency factor $\eta_{NOM}=0.84$, power factor $cos\varphi=0.84$, $\lambda=M_{MAX}/M_{NOM}=1.8$, $\gamma=M_{ST}/M_{NOM}=1.1$ and $\beta= I_{ST}/I_{NOM}=6.5$.

Define the motor nominal and starting currents, the nominal, maximum and starting moments. Define the motor starting moment, when the voltage is 15 % less than nominal. Make a conclusion, if the motor starts up at this voltage with a nominal load. Draw the speed, the moment and the mechanical curves of motor characteristics.

Motor electrical (input) power is: $P_{1NOM} = P_{NOM} / \eta_{NOM} = 10000 / 0.84 = 11900\ W$.

Motor nominal losses are: $\Delta P = P_{1NOM} - P_{NOM} = 11900 - 10000 = 1900\,W$.

Motor nominal current is:

$$I_{NOM} = P_{1NOM} /(\sqrt{3}U_{NOM} \cos\varphi_{NOM}) = 11900 /(\sqrt{3} \cdot 380 \cdot 0.85) = 21.28\ A.$$

Motor starting current is: $I_{ST} = \beta\, I_{NOM} = 6{,}5 \cdot 21{,}28 = 138.32\ A$.

Motor synchronous frequency (electromagnetic field rotation frequency) is:

$$n_1 = 60 f / p = 60 \cdot 50 / 2 = 1500\ rpm .$$

Motor asynchronous frequency (rotor rotation frequency) is:

$$n_2 = n_1 (1 - s_{NOM}) = 1500 \cdot (1 - 0.03) = 1455\ rpm .$$

Angular asynchronous frequency is:

$$\omega_2 = 2\pi\, n_2 / 60 = 2 \cdot 3{,}14 \cdot 1455 / 60 = 152.4\,rad/\text{s} .$$

Motor nominal moment is: $M_{NOM} = P_{NOM} / \omega_2 = 10000 / 152.4 = 65.63\, N \cdot m$

Motor maximum moment is: $M_{MAX} = \lambda M_{NOM} = 1.8 \cdot 65.63 = 118.13\ N \cdot m$.

Motor starting moment is: $M_{ST} = \gamma\ M_{NOM} = 1.1 \cdot 65.63 = 72.2\ N \cdot m$

The following alignments are to be used to define the motor starting moment, when the voltage is 15% less than nominal: $M_{ST}(0{,}85) / M_{ST} = (0{,}85 V_{NOM})^2 / V_{NOM}^2$. Thus, $M_{ST}(0{,}85) = 0{,}85^2 M_{ST} = 0{,}85^2 \cdot 72.2 = 52.16\ N \cdot m$.

Thus, nominal moment should be at least $M_{NOM} = 65.63 N \cdot m$. But the starting moment at 15% less than nominal voltage is $M_{ST}(0.85) = 52.16\ N \cdot m$. This is less than nominal. Therefore, the motor can't start up.

Critical slip is: $S_{CR} = S_{NOM}(\lambda + \sqrt{\lambda^2 - 1}) = 0.03 \cdot (1.8 + \sqrt{1.8^2 - 1}) = 0.1 \cdot$.

Several slip values are to be set to draw the curves of motor characteristics. They are to range within $s = 0 \div 1$, including s_{NOM} and s_{CR}. Define the rotor rotation frequency $n_2 = n_1(1 - s)$ and the motor mechanical moment for every slip value: $M = 2 \cdot M_{MAX} / (s_{CR} / s + s / s_{CR})$.

Fill in the table with the results of calculations:

s,	0.0	0.03	0.1	0.2	0.3	0.40	0.60	0.80	1.00
n, rpm	1500	1455	1350	1300	1200	1050	900	600	0
M, N×m	0	65.6	118.1	78.75	70.88	55.6	38.28	29.1	23.39

Draw the speed $n_2(s)$ (fig.7.1), the moment $M(s)$ (fig.7.2) and the mechanical $n_2(M)$ (fig.7.3) curves of motor characteristics out from data in the table.

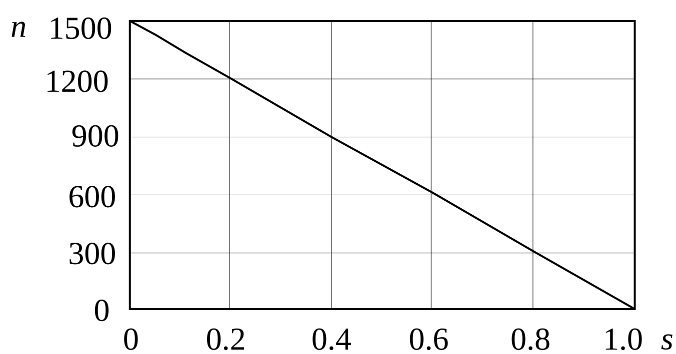


Fig. 7.1

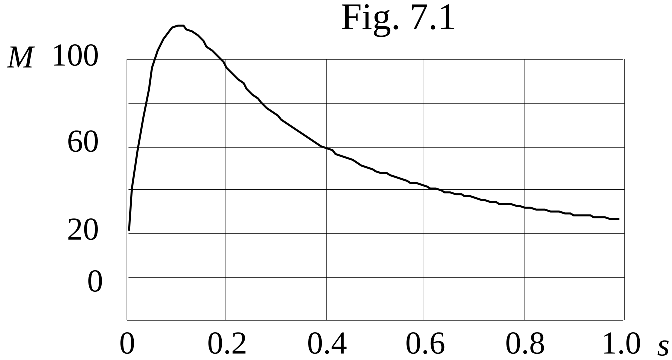


Fig. 7.2

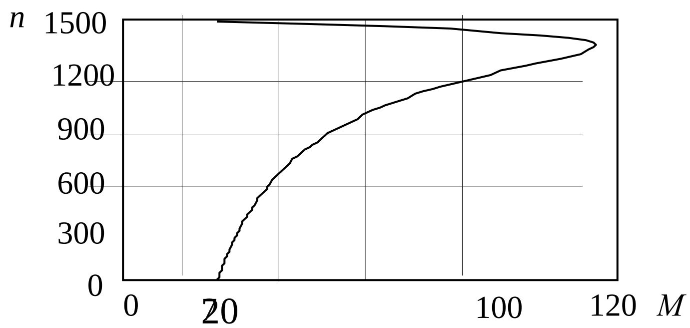


Fig. 7.3

Example for task 2. The parameters of an induction motor with the phase rotor and the WYE connected stator windings are: nominal voltage $V_{NOM}=220\ V$, number of pole pares $p=2$, nominal slip $s_{NOM,}=3\%$, stator phase winding turns number $w_1=187$, rotor phase winding turns number $w_2=36$, stator and rotor phase resistances and reactances $R_1=0.46\ \Omega$, $R_2=0.28\ \Omega$, $X_1=2.24\ \Omega$, $X_2=0.08\ \Omega$.

Define the stator and rotor starting currents, the power factor, the starting moment without starting rheostat, the starting rheostat resistance and the maximum starting moment.

Draw the speed, the moment and the mechanical curves of motor characteristics.

Phase transformation ratio is: $K = w_1 / w_2 = 187/36 = 5.19$.

Brought resistance and reactance of rotor phase are accordingly:

$R_2^{'} = R_2 \cdot K^2 = 0.02 \cdot 5.19^2 = 0.54\Omega$, $X_2^{'} = X_2 \cdot K^2 = 0.08 \cdot 5.19^2 = 2.155\Omega$.

Resistance, reactance and impedance of the short circuit rotor are:

$R_{SC} = R_1 + R_2^{'} = 0{,}46 + 0{,}54 = 1{,}0\Omega$, $X_{SC} = X_1 + X_2^{'} = 2{,}24 + 2{,}159 = 4{,}4\Omega$,

$$Z_{SC} = \sqrt{R_{SC}^2 + X_{SC}^2} = \sqrt{1^2 + 4{,}4^2} = 4{,}5\Omega.$$

Stator starting current is: $I_{1ST} = U_{NOM} / Z_{SC} = 220/4.5 = 48{,}88\ A$.

Rotor starting current is: $I_{2ST} = K \cdot I_{1ST} = 5.19 \cdot 48.88 = 253.357\ A$.

Power factor for starting without starting rheostat is:

$$\cos\varphi_{ST} = R_{SC} / Z_{SC} = 1/4.51 = 0.22.$$

Motor synchronous frequency is: $\omega_1 = 2\pi f / p = 2 \cdot 3.14 \cdot 50/3 = 104.73\ rad/s$.

Motor starting moment without starting rheostat is:

$$M_{ST} = 3R_2 I_{2ST}^2 / \omega_1 = 3 \cdot 0.02 \cdot 253.37^2 / 104.72 = 36.77\ N \cdot m.$$

Critical slip is: $S_{CR} = R_2^{'} / X_{SC} = 0.54/4.4 = 0.12 \cdot$.

Brought resistance of the starting rheostat is defined from the expression:

$$S_{CR} = (R_2^{'} + R_R^{'}) / X_{SC} = 1,\quad R_R^{'} = X_{SC} - R_2^{'} = 4.4 - 0.54 = 3.86\ \Omega.$$

Resistance of the starting rheostat is: $R_R = R_R^{'} / K^2 = 3.86/5.19^2 = 0.14\ \Omega$.

Motor maximum starting moment with the starting rheostat is:

$$M_{MAX} = 3V_{NOM} /(2\omega_1) \cdot (R_1 + R_R^{'} + \sqrt{(R_1 + R_R^{'})^2 + X_{SC}^2}) =$$

$$= 3 \cdot 220 /(2 \cdot 104,72) \cdot (0.46 + 3.86 + \sqrt{(0.46 + 3.86)^2 + 4.4^2}) = 33.04 \ N \cdot m \ .$$

Several slip values ranging within $s = 0 \div 1$ are to be set to draw the curves of motor characteristics. They should include s_{NOM}, s_{CR}. Define the rotor rotation frequency $n_2 = n_1(1 - s)$ and the motor mechanical moment for every slip value: $M = 2 \cdot M_{MAX} /(s_{CR} / s + s / s_{CR})$.

Fill in the table with the obtained results:

s,	0,00	0,03	0,12	0,20	0,40	0,60	0,80	1,00
n, rpm	1000	970	880	800	600	400	200	0
M, N·m	0	15,54	33.04	29,11	18,2	12.6	9,69	7,08

Draw the speed $n_2(s)$ (fig.7.4), the moment $M(s)$ (fig.7.5) and the mechanical $n_2(M)$ (fig.7.6) curves of motor characteristics out of table data.

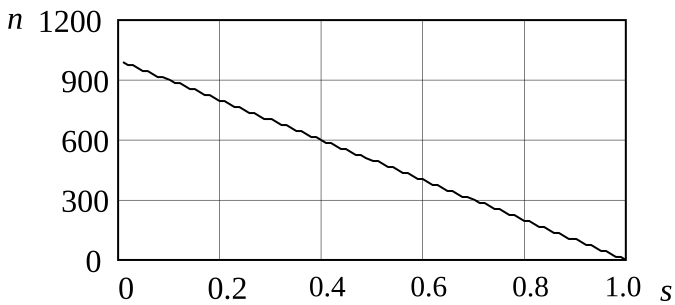


Fig. 7.4

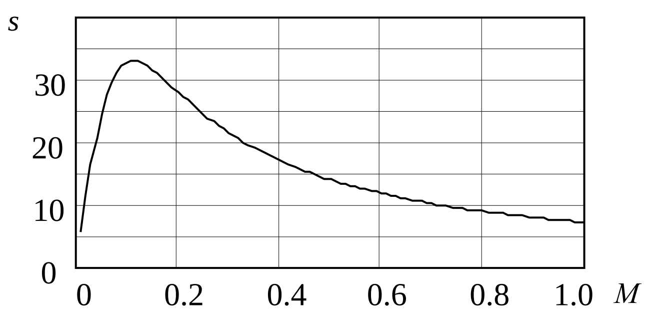


Fig. 7.5

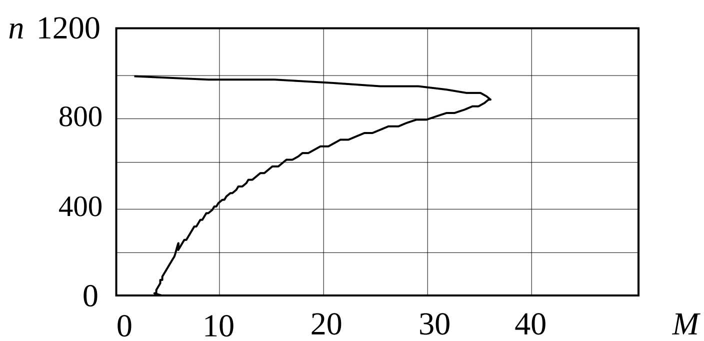


Fig. 7.6

Tasks for individual work

1. An induction motor rotor is rotating with a frequency $n_2 = 1440\ rpm$, an input power is $P_1 = 5.5kW$ and losses are $\Delta P = 55\ W$. Define the motor power and the moment.

2. An induction motor moment is $M = 58\ N \cdot m$, the rotating frequency is $n_2 = 585\ rpm$. The efficiency factor is 0.95 and the power factor is 0.8. Define the motor power, the input active and total power.

3. Define the power of an induction motor with the phase rotor and stator currents when windings are connected WYE and DELTA correspondingly. Nominal motor parameters are: the power $P_2 = 300\ W$, the input voltage $V_1 = 380/220\ V$, the efficiency factor 0.9 and the power factor 0.85.

4. Define the rotating frequency of a six poles synchronous motor that is fed with the industrial frequency voltage. What is the maximum frequency when motor is fed with the industrial frequency voltage?

5. The four poles induction motor power is $P_2 = 4kW$, the moment is $M = 27\ N \cdot m$ and the efficiency factor is 0.9. Define the input electro-magnetic powers and the slip.

6. Define the frequency of the magnetic field rotation, the number of pole pairs and the power of an asyncronous three-phase motor, if its electromagnetic power is $P_{EM} = 4kW$, and the shaft mechanical moment is $M = 27\ N \cdot m$ at rotor rotation frequency of $n_2 = 1440\ rpm$.

7. An asyncronous three-phase six-poled motor has a shaft moment of $M = 27\ N \cdot m$ at rotor rotation frequency of $\omega_2 = 99.4\ rad/s$. Define the slip, the electromagnetic power and the electrical losses of motor's rotor.

Task 3. The parameters of a direct current motor with a parallel excitation are given in table below (nominal voltage V_{NOM}, nominal current I_{NOM}, open circuit mode current I_0, nominal rotation frequency n_{NOM},, excitation winding resistance R_E, and armature winding resistance R_A).

Define the nominal electrical power, the armature winding current, the excitation winding current, the armature winding electromotive force, the input current, the armature and excitation windings losses, the excitation and armature windings' resistances, the nominal moment, the starting moment, the armature winding starting current, the open circuit mode frequency rotation and the starting rheostat resistance for the condition $I'_{AST} = 2.5 \cdot I_{ANOM}$. The mechanical and magnetic losses in open circuit and nominal modes are the same. Draw the mechanical *n(M)* curve of motor characteristic.

Var	V_{NOM}	I_{NOM},	I_0,	R_A,	R_E,	n_{NOM},
№	*V*	*A*	*A*	Ω	Ω	*rpm*
01	220	53,0	6,3	0,212	33	1225
02	115	100,0	9,5	0,11	50	1000

03	110	267,0	30,0	0,04	27,5	1100
04	220	16,3	1,78	1,16	75	1025
05	110	7,8	0,7	0,80	210	1240
06	220	19,9	2,0	1,50	150	960
07	110	35,0	3,2	0,60	60	1400
08	220	32,0	2,8	0,94	120	1600
09	220	34,0	3,0	0,45	110	1100
10	110	9,5	0,9	1,90	200	850
11	110	20,0	1,8	0,70	80	940
12	220	15,0	1,5	0,82	200	1350
13	110	8,2	0,8	1,40	220	1450
14	220	20,5	2,35	0,74	258	1025
15	220	40,0	4,2	0,52	190	1420
16	110	10,5	1,2	1,20	160	960
17	110	18,6	2,0	0,90	120	825
18	220	16,0	1,8	0,60	270	1600
19	220	32,0	3,5	0,62	200	1350
20	110	28,0	3,2	0,55	80	875

Task 4. The parameters of a direct current motor with the parallel excitation are given in table below (nominal voltage V_{NOM}, nominal power P_{NOM}, nominal rotation frequency $n_{NOM,,}$ nominal efficiency factor $\acute{\eta}_{NOM}$, excitation winding losses ΔP_E and armature winding losses ΔP_A).

Define the nominal electrical power, the armature winding current, the excitation winding current, the armature winding electromotive force, the input current, the excitation and armature windings resistances, the nominal moment, the starting moment, the armature winding starting current, the starting rheostat resistance for the condition $I'_{AST} = 2.5 \cdot I_{ANOM}$ and the open-circuit mode rotation frequency. Draw the mechanical *n(M)* curve of motor characteristic.

Var.	V_{NOM}	P_{HOM}	ΔP_A	ΔP_E,	n_{NOM},	η_{NOM},
№	*V*	*kW*	*%*	*%*	*об/хв*	*%*
01	110	60,0	5,2	4,8	980	86,5
02	220	10,0	5,0	4,8	2250	86,0
03	220	4,0	6,2	4,2	1025	82,2
04	220	6,6	6,2	4,1	2400	85,5
05	220	4,4	6,5	4,8	2100	84,5
06	220	2,5	5,8	4,8	1000	85,0
07	220	10,0	5,3	4,4	2250	83,0
08	110	77,0	5,0	4,2	1050	85,5

09	110	80,0	5,4	4,5	1150	85,8
10	110	92,0	5,3	4,1	970	86,5
11	110	66,0	6,2	5,0	1050	85,5
12	110	35,0	6,3	5,2	2200	84,5
13	110	45,0	5,7	4,6	1500	85,0
14	220	15,0	5,0	4,0	1000	84,5
15	220	10,0	5,2	4,2	970	85,5
16	220	5,8	6.0	5,0	2200	84,0
17	220	19,0	4.8	4,5	980	86,5
18	220	29,0	5.0	4,3	2520	86,0
19	220	46,5	5,4	4,8	1025	82,2
20	220	14,0	4.0	4,6	2400	84,0

Example for task 3. The parameters of a DC motor are: nominal voltage $V_{NOM}=220V$, nominal current $I_{NOM}=20.5A$, open circuit mode current $I_0=2.35A$, nominal rotation frequency $n_{NOM}=1025\ rpm$, excitation winding resistance $R_E=258\ \Omega$, and armature winding resistance $R_A=0.75\ \Omega$.

Define the nominal electrical power, the mechanical power, the rotation frequency, the moment, the efficiency factor, the armature current; the losses in excitation and armature windings, the armature starting current without starting rheostat, the starting rheostat resistance and the armature starting current with a starting rheostat. Draw the mechanical *n(M)* curve of motor characteristic.

Motor nominal electrical power is: $P_{1NOM} = V_{NOM} I_{NOM} = 220 \cdot 20.5 = 4510\,W$.

Excitation current is: $I_E = V_{NOM} / R_E = 220/258 = 0.853\,A$.

Armature nominal current is: $I_{ANOM} = I_{NOM} - I_E = 20.5 - 0.853 = 19.65\,A$.

Excitation winding losses are: $\Delta P_E = R_E I_E^2 = 258 \cdot 0.853^2 = 187.72\,W$.

Armature winding losses are: $\Delta P_A = R_A I_{ANOM}^2 = 0.75 \cdot 19.65^2 = 289.5\,W$.

Open circuit mode power is: $P_0 = V_{NOM} I_0 = 220 \cdot 2.35 = 517\ W$.

Armature winding losses for open-circuit mode are:

$$\Delta P_{0A} = R_A (I_0 - I_E)^2 = 0.75 \cdot (2.35 - 0.853) = 1.123\ W .$$

Mechanical and magnetic losses are:

$$\Delta P_{MM} = P_0 - \Delta P_{0A} - \Delta P_{E,} = 517 - 1.123 - 187.72 = 328.16\ W .$$

Motor losses are:

$$\Delta P = \Delta P_A + \Delta P_E + \Delta P_{MM} = 289.5 + 187.72 + 328.16 = 805.38\ W .$$

Mechanical power is: $P_{2NOM} = P_{1NOM} - \Delta P = 4510 - 805.38 = 3705\ W$.

Efficiency factor is: $\eta = P_{2NOM} / P_{1NOM} = 3705/4510 = 0.82 \cdot$.

Motor nominal angular rotation frequency is:

$$\omega_{NOM} = 2 \cdot \pi \cdot n_{NOM} / 60 = 2 \cdot 3.14 \cdot 1025 / 60 = 107.3\ rps .$$

Motor nominal moment is: $M_{NOM} = P_{2NOM} / \omega_{NOM} = 3705/107.3 = 34.53\ N \cdot m$.

Armature starting current without the starting rheostat is:

$$I_{AST} = V_{NOM} / R_A = 220/0.75 = 293.33 \cdot A.$$

Armature starting current with the starting rheostat is:

$$I'_{AST} = 2.5 \cdot I_{AST} = 2.5 \cdot 19.65 = 49.13 \cdot A.$$

Starting rheostat resistance can be defined from the expression:

$$I'_{AST} = V_{NOM} / (R_A + R_{ST}),$$

$$R_{ST} = (V_{NOM} / I'_{AST}) - R_A = (220/49.13) - 0.75 = 3.73\Omega.$$

Starting moment can be defined from the expression:

$$M_{ST} / M_{NOM} = I'_{AST} / I_{ANOM} = 2.5,$$

$$M_{ST} = 2{,}5 \cdot M_{NOM} = 2.5 \cdot 34{,}53 = 86{,}3\ N \cdot \mathrm{m}.$$

Mechanical characteristics calculation

Armature winding electromotive force is:

$$E_{ANOM} = V_{NOM} - R_A I_{ANOM} = 220 - 0.75 \cdot 19.65 = 205.26\ V.$$

Open-circuit mode rotation frequency is:

$$n_0 = V_{NOM} \cdot n_{NOM} / E_{NOM} = 220 \cdot 1025 / 205.26 = 1100\ rpm.$$

Draw the curve of mechanical characteristics *n(M)* – the dependence of rotation frequency from the moment, – by using two points: the first point corresponds to open circuit mode $M = 0$ and $n_0 = 1100\ rpm$; the second point corresponds to nominal mode (fig. 7.7) $M_{NOM} = 34.52\ N \cdot m$, and $n_{NOM} = 1025\ rpm$.

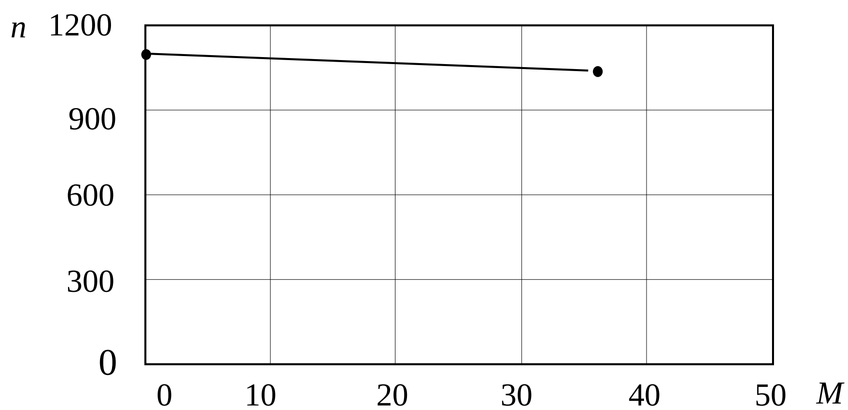


Fig. 7.7

Example for task 4. The parameters of a DC motor are: nominal voltage V_{NOM}*=220 V*, nominal power P_{NOM}*=6kW*, nominal rotation frequency n_{NOM}*=1025rpm*, nominal efficiency factor $\acute{\eta}_{NOM}$*=82.2%*, excitation winding losses ΔP_E*=4.4%* and armature winding losses ΔP_A*=5%*.

Define the nominal electrical power, the armature winding current, the excitation winding current, the armature winding electromotive force, the input current, the excitation and armature windings resistances, the nominal moment, the starting moment, the armature winding starting current and the open-circuit mode rotation frequency. Draw the mechanical *n(M)* curve of motor characteristic.

Motor electrical power is: $P_{1\,NOM} = P_{NOM} / \eta_{NOM} = 6000 / 0{,}822 = 7299\ W$.

Excitation winding losses are: $\Delta P_{E,} = \Delta P_{E,}(\%)\, P_{1\,NOM} / 100 = 4.4 \cdot 7299 / 100 = 321\ W$.

Excitation current is: $I_{\mathrm{E}} = \Delta P_E / V_{NOM} = 321 / 220 = 1.46\, A$.

Input nominal current is: $I_{NOM} = P_{1\,NOM} / V_{NOM} = 7299 / 220 = 33.18\, A$.

Armature winding nominal current is:

$$I_{ANOM} = I_{NOM} - I_E = 33.18 - 1.46 = 31.72\, A .$$

Excitation winding resistance is: $R_E = V_{NOM} / I_E = 220 / 1.46 = 150.7\,\Omega$

Armature winding losses are: $\Delta P_{A,} = \Delta P_{A,}(\%)\, P_{1\,NOM} / 100 = 5 \cdot 7299 / 100 = 365\ W$

Armature winding resistance is: $R_A = \Delta P_A / I^2_{ANOM} = 365 / 31.72^2 = 0.363\ \Omega$.

Motor angular rotation frequency is:

$$\omega_{NOM} = (2\pi n_{NOM}) / 60 = 2 \cdot 3.14 \cdot 1025 / 60 = 107.3\ rps .$$

Nominal moment is: $M_{NOM} = P_{NOM} / \omega_{NOM} = 6000 / 107.3 = 55{,}9\, Nm$.

Armature starting current is: $I_{AST} = V_{NOM} / R_A = 220 / 0.363 = 606\, A$.

Armature starting current with the starting rheostat is:

$$I'_{AST} = 2.5 \cdot I_{ANOM} = 2.5 \cdot 31.98 = 79.95 \cdot A .$$

Starting rheostat resistance can be defined from the expression:

$$I'_{AST} = V_{NOM} / \left(R_A + R_{ST}\right),$$

$$R_{ST} = (V_{NOM} / I'_{AST}) - R_A = (220 / 79.95) - 0.363 = 2.39\,\Omega .$$

Starting moment can be defined from the expression:

$$M_{ST} / M_{NOM} = I'_{AST} / I_{ANOM} = 2.5 ,$$

$$M_{ST} = 2{,}5 \cdot M_{NOM} = 2.5 \cdot 55{,}9 = 139{,}75\ N \cdot \mathrm{m} .$$

Mechanical characteristics calculation

Armature winding electromotive force is:

$$E_{ANOM} = V_{NOM} - R_A I_{ANOM} = 220 - 0.363 \cdot 31.72 = 208.58\ V .$$

Open-circuit mode rotation frequency is:

$$n_0 = V_{NOM} \cdot n_{NOM} / E_{NOM} = 220 \cdot 1025 / 208.58 = 1081\ rpm .$$

Draw the curve of mechanical characteristics *n(M)* – the dependence of rotation

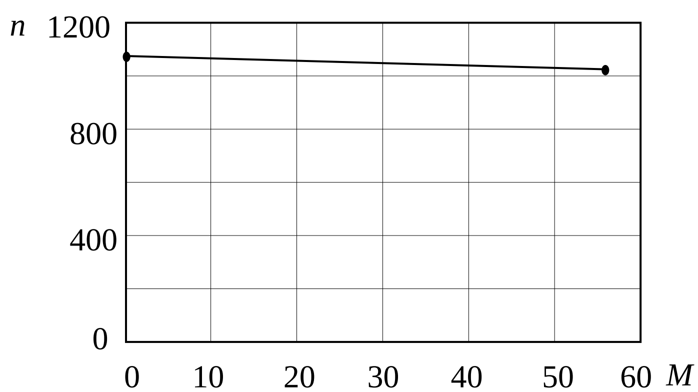


Fig. 7.8

frequency from the moment, – by using two points: the first point corresponds to open-circuit mode $M = 0$ and $n_0 = 1081\ rpm$; the second point corresponds to nominal mode (fig. 7.8) $M_{NOM} = 55.9\ N \cdot m$, and $n_{NOM} = 1025\ rpm$.

Tasks for individual work

1. DC generator with an independent excitation has a nominal power of *10kW*, a nominal voltage of *110V*, a rotating frequency of *1450 rpm* and an armature winding resistance of 0.05*Ω*. Define the e.m.f., the electromagnetic moment, the load current and the excitation winding current if $I_E = 5\,\%\ I_{A\ NOM}$.

2. DC generator with a parallel excitation has a nominal voltage of *230V*, a load current of *18A* and an excitation winding resistance of 20*Ω*. Define the armature winding current.

3. DC generator with a mixed excitation has a nominal power of *15kW*, a nominal voltage of *110V* , an armature winding resistance of 0.05*Ω* and its excitation winding resistances are 9.5*Ω* (for parallel winding) and 0.2*Ω* (for series winding) correspondingly. Define the generator efficiency factor.

4. DC generator with a parallel excitation has a nominal voltage of *220V*, a load resistance of 10*Ω*, an armature winding resistance of 2*Ω* and an excitation winding resistance of 110*Ω*. Define the e.m.f. and the generator efficiency factor.

5. DC generator with a parallel excitation has a load current of 22*A*, an excitation winding resistance of 110*Ω*, an armature winding resistance of 2*Ω* and a load resistance of 10*Ω*. Define the generator output power and efficiency factor.

6. DC generator with a parallel excitation has an e.m.f. of 268*V*, an output power of 6430*W*, an efficiency factor of 0.89 and an armature winding resistance of 2*Ω*. Define the generator voltage and the excitation winding resistance.

7. DC motor with a parallel excitation armature has winding losses of 200*W*, excitation winding losses of 110*W*, a feeding voltage of 110*V* and an input power of 1210*W*. Define the armature winding resistance and the motor efficiency factor.

8. DC motor with a parallel excitation has an armature winding resistance of 3*Ω*, an excitation winding resistance of 100*Ω* and an input power of 1210*W* at voltage 110*V*. Define the mechanical power and the motor efficiency factor.

9. DC motor with a serial excitation has an output power of 900*W* at voltage 110*V*, an efficiency factor of 0.82 and an excitation winding resistance of 0.5*Ω*. Define the armature winding resistance.

10. DC motor with a parallel excitation rotor rotates with a frequency 3000*rpm*, is fed by voltage 220*V*, has an armature winding resistance of 0.5*Ω*, an excitation winding resistance of 60 Ω, an efficiency factor of 0.87. Motor's nominal power is 32kW. Define the input power, the e.m.f. and the motor moment.

11. The power of a DC motor with a parallel excitation is 900W at voltage 110V, an armature current is 10A and an excitation winding resistance is 110Ω. Define the motor current, the efficiency factor and the armature winding resistance.

12. DC motor with a serial excitation has an input power of 1100W at voltage 110V, an efficiency factor of 0.82 and an excitation winding resistance of 5Ω. Define the armature winding resistance, the armature and the excitation windings' losses.

Attachment

№	***Greek letters***		
1	*Α*	*α*	*alfa*
2	*Β*	*β*	*beta*
3	*Γ*	*γ*	*gamma*
4	*Δ*	*δ*	*delta*
5	*Ε*	*ε*	*epsilon*
6	*Ζ*	*ζ*	*dzeta*
7	*Η*	*η*	*eta*
8	*Θ*	*θ,*	*teta*
9	*Ι*	*ι*	*jota*
10	*Κ*	*κ*	*kapa*
11	*Λ*	*λ*	*lambda*
12	*Μ*	*μ*	*miu*
13	*Ν*	*ν*	*niu*
14	*Ξ*	*ξ*	*ksi*
15	*Ο*	*ο*	*micron*
16	*Π*	*π*	*pi*
17	*Ρ*	*ρ*	*ro*
18	*Σ*	*σ,ς*	*sigma*
19	*Τ*	*τ*	*tau*
20	*Υ*	*υ*	*ipsilon*
21	*Φ*	*φ*	*fi*
22	*Χ*	*χ*	*hi*
23	*Ψ*	*ψ*	*psi*
24	*Ω*	*ω*	*omega*

Physical values designation and units

Value	***Designatio n***	*Dimension*
Resistance	*R*, Ω	*Om*
Reactance	*X*, Ω	*Om*
Impedance	*Z*, Ω	*Om*
Conductance	*G, Sm*	*Simens*
Susceptance	*B, Sm*	*Simens*
Admittance	*Y, Sm*	*Simens*
Capacity	*C, F*	*Farada*
Inductance	*L, H*	*Henry*
Inductance mutual	*M, H*	*Henry*
Electromotive force	*E, V*	*Volt*
Potential	*φ, V*	*Volt*
Voltage	*V, V*	*Volt*
Current	*I, A*	*Amper*
Active power	*P, W*	*Watt*
Reactive power	*Q, VAr*	*Volt-Amper reactive*
Total power	*S, VA*	*Volt-Amper*
Magnetomotive force	*F, A*	*Amper*
Magnetic induction	*B, T*	*Tesla*
Magnetic field tension	*H, A/m*	*Amper per meter*
Magnetic stream	*Φ, Wb*	*Weber*
Linkage	*ψ, Wb*	*Weber*
Magnetic permeability (absolute)	μ_a,, *Гн/м*	*Henry per meter*
Magnetic permeability (relative)	μ	
Magnetic constant	μ_0, *Гн/м*	$4\pi\cdot10^{-7}$
Frequency	*f, Hz*	*Herz*
Angular frequency	*ω, rad/s*	*radian per second*
Length	*1, m*	*meter*
Hight, depth	*h, m*	*meter*
Layer	*δ, d, m*	*meter*
Arial	*S*, m^2	*square meter*
Magnetic resistance	R_m	
Number of turns	*w*	
Force	*F, N*	*Newton*
Work (energy)	*W, J*	*Joule*
Charge	*Q, C*	*Coulomb*

Content

ДЛЯ НОТАТОК

ДЛЯ НОТАТОК

Навчально-методична література

Куземко Н.А.

Методичні вказівки
для іноземних студентів з дисципліни

«Electrical Engineering. PRACTICUM»

Комп'ютерне макетування та верстка *А.П. Катрич*

Формат 60х90/16. Обл. вид. арк. 1,73. Тираж 30 прим. Зам. № 2988.

Тернопільський національний технічний університет імені Івана Пулюя.
46001, м. Тернопіль, вул. Руська, 56.
Свідоцтво суб'єкта видавничої справи ДК № 4226 від 08.12.11.