Вісник Тернопільського національного технічного університету
Scientific Journal of the Ternopil National Technical University
2017, № 3 (87)
ISSN 1727-7108. Web: visnyk.tntu.edu.ua

UDC 539.3

THE FLOW OF CONCENTRATED FORCE THROUGH OPEN ELASTIC ROD TO THE CONTOUR OF THE CURVED HOLE IN INFINITE PLATE

Andrii Siaskyi; Volodymyr Siaskyi; Nataliia Shevtsova

Rivne State University of Humanities, Rivne, Ukraine

***Summary**. The new approach to the problem of the flow to the contour of the curved hole in an infinite isotropic plate of concentrated force, applied to the end of reinforcing open elastic bar of constant rectangle cross section, has been proposed. The boundary conditions of the problem at the section of bonding of the rib with contour of the hole in the plate have been formulated as equalities of deformations. The mathematical model of the problem has been constructed as the system of singular integral and differential equations to determine the contact forces between the plate and the rib and internal forces and moments in the reinforcement. The structure of unknown functions at the ends of area of reinforcement is determined. The approximate solution of the problem has been founded by the method of mechanical quadrature and collocation.*

***Key words:** reinforcing rod, infinite plate, integral equations, curved hole, stress state.*

Received 05.09.2017

Introduction. Thin-wall plates with the technological or construction holes are widely applied in all branches of engineering. To decrease high concentration of stresses near holes their contour are reinforced by thin elastic rods. At the same time there are areas at the hole contour, which cannot be reinforced because of low stress concentrations. That is why partial reinforcement of them by the open elastic rods is the most economical from the point of view of the metal consuming.

Being dissipators of the stress concentrations in the area of reinforcement, the reinforced rods make possible to transmit concentrated force and moment load through them to the plate. Such tasks are the theoretic basis for strength and rigidity estimation of such machine units as gear drives, splined joints, as well as microlite tools (cutters, circular saws, etc.). Because of that the investigation of the stress state of the plate-open rod system is an urgent task for modern mechanical engineering.

Analysis of the available investigations and publications. The tasks on the transition of concentrated forces to the curved hole contour of the infinite isotropic plate have been thoroughly investigated only for the case, when the reinforced rod is modelled by the elastic line of the constant or varying tension (compression) rigidity [1 – 4].

Using the model of elastic line in [5, 6] a number of tasks on the transmition of concentrated force and moment load to the curved hole contour of the plate through open rods asymmetric relatively the midplate, have been analysed. In this case, irrespective of the outside load the plate undergoes the conditions of generalized plane stress state and cylindric bending.

Taking advantage of the boundary integral equations [7, 8] the solution of the task on the transition of concentrated forces to the circular hole contour of the infinite plate and circular disk through the system of elastic rods, which are moduled by the open rods without taking into account transverse forces, has been built.

For the curved holes, which are different of these circular ones, such tasks as specified more exactly, have not been analysed.

Statement of the problem. Let us analyse the $2h$ thick infinite isotropic plate with the curved hole limited by the smooth cylinder surface. The plate midplane treated as the Cartesian

 ……………………………………………... *Corresponding author: Nataliia Shevtsova; e-mail: shevtsova_nv@ukr.net*

(x_1, x_2) and polar $(\tilde{r}, \delta)$ hole-centre system of refernce, crosses the hole surface along the smooth curve.

Let us assume that in the area $\left[\alpha_0^*, \beta_0^*\right]$ the hole surface is reinforced by the symmetric relatively the plane (Ox_1x_2) open elastic rod of constant cross-section $2h_0 \times 2\eta$ (Fig. 1), where α_0^*, β_0^* – polar angles of the reinforced area ends; $2h_0$, 2η – the reinforced rod height and width; θ – the angle, which forms axis Ox_1, with the normal to the rod axis.

Uniformly distributed normal load with the main vector N_0 acting in the plate midplane is applied to the reinforcement end, which is determined by the polar angle $\delta = \alpha_0^*$. Another load on the plate and reinforced rod are not available.

The solution of the task deals with the finding of contact forces between the plate and reinforced rod, circular forces on the hole surface in the plate, the rod stress state and the investigations of the physical-geometric reinforcement parameters impact on these values.

Mathematical model of the task. As the plate and the reinforced rod are under generalized plane stress state (all values depend only on x_1, x_2), that is why we can analyse only the points located in the mutual plate midplane and the rod. Having separated the plate from the reinforcement conventionally and changing the action of one body on the other with unknown normal T_ρ and tangential $S_{\rho\lambda}$ contact forces, acting in midplane, let us solve the first main task for the infinite plate with the smooth curved hole and elastic open rod (Fig. 1).

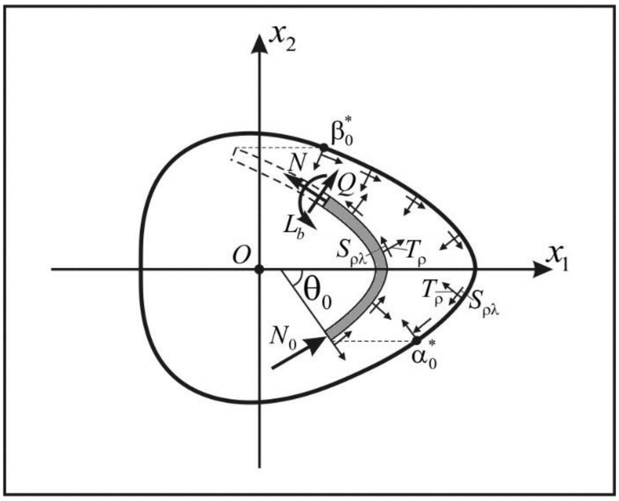

Figure 1. Loading diagram of the plate and the reinforcing rod

The plate undergoes contact forces in the area $\left[\alpha_0^*, \beta_0^*\right]$ of the contour Γ and is in equilibrium, and the elastic rod is under contact forces and concentrated force N_0.

Isotropic plate with curved hole. Let the hole shape in the infinite plate be determined by the function [3],

$$z = x_1 + ix_2 = \omega(\xi) = R_0\left(\xi + \sum_{j=1}^{M} \frac{\varepsilon_j}{\xi^j}\right), \tag{1}$$

which realises the conform reflection of the external side S^- of the unit circle γ in the area

$\xi = \tilde{\rho} e^{i\lambda}$ on the area occupied by the plate midplane. Here R_0 – specified hole size (without breaking generalisation we consider $R_0 = 1$); $\varepsilon_1, \varepsilon_2, ..., \varepsilon_M$ – parameters specifying the contour Γ deflection from the circle; $i = \sqrt{-1}$; $(\tilde{\rho}, \lambda)$ – polar coordinates in the plane ξ.

Under condition $|\varepsilon_1| + 2|\varepsilon_2| + ... + M|\varepsilon_M| < 1$ function (1) specifies the contour Γ as the combination of the circle and regular (j+1) – triangles with circled angles.

The plate deformation tensor components in the points of the contour Γ under given load are found according to the formulas [3, 4]

$$\varepsilon_\lambda = \frac{1}{2Eh(\alpha^2+\beta^2)}\left\{(1-\nu)(\alpha^2+\beta^2)T_\rho(\lambda) - \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[H(\lambda,t) - G(\lambda,t)\operatorname{ctg}\frac{\lambda - t}{2}\right]T_\rho(t)\,dt + \right.$$

$$\left. + \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[G(\lambda,t) + H(\lambda,t)\operatorname{ctg}\frac{\lambda - t}{2}\right]S_{\rho\lambda}(t)\,dt + \alpha\varepsilon_\lambda^0 + \beta V^0\right\};$$

$$V = \frac{1}{2Eh(\alpha^2+\beta^2)}\left\{(1-\nu)(\alpha^2+\beta^2)S_{\rho\lambda}(\lambda) - \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[H(\lambda,t) - G(\lambda,t)\operatorname{ctg}\frac{\lambda - t}{2}\right]S_{\rho\lambda}(t)\,dt - \right.$$

$$\left. - \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[G(\lambda,t) + H(\lambda,t)\operatorname{ctg}\frac{\lambda - t}{2}\right]T_\rho(t)\,dt - \beta\varepsilon_\lambda^0 + \alpha V^0 + C_0(\alpha^2+\beta^2)\right\}, \qquad (2)$$

where $[\alpha_0;\ \beta_0]$ – image of the reinforced area $[\alpha_0^*,\ \beta_0^*]$ at reflection (1); E, ν – the Young's modulus and the Poisson's ratio of the plate material; ε_λ, V – relative extension of the contour Γ and the normal reversement to it; C_0 – the constant specifying the plate rotation at infinity;

$$H(\lambda,\ t) = \alpha(\lambda)\alpha(t) + \beta(\lambda)\beta(t);\quad G(\lambda,\ t) = \alpha(\lambda)\beta(t) - \beta(\lambda)\alpha(t);\quad \alpha(\lambda) + i\beta(\lambda) = \omega'(\sigma);$$

$$\varepsilon_\lambda^0 + iV^0 = \frac{1}{2\pi R_0}\left[\frac{(3-\nu)(X_1^0 + iX_2^0)}{\sigma} - (1+\nu)(X_1^0 - iX_2^0)\left(\frac{\varepsilon_2}{\sigma^2} + \frac{2\varepsilon_3}{\sigma^3}\right)\right] - 4\frac{\bar{a}_1\varepsilon_3}{R_0\sigma^2};\ \kappa = \frac{3-\nu}{1+\nu};\ \sigma = e^{i\lambda};$$

$$a_1 = -\frac{1}{2\pi}\int_{\alpha_0}^{\beta_0}\left[T_\rho(t) + iS_{\rho\lambda}(t)\right]\omega'(\tau)\tau^2 dt + \bar{a}_1\varepsilon_3 + \frac{X_1^0 - iX_2^0}{2\pi(1+\kappa)}\varepsilon_2;\quad \tau = e^{it}; \qquad (3)$$

$X_1^0 = -N_0\sin\theta_0$; $X_2^0 = N_0\cos\theta_0$ – the outside load main vector components transmitted from the rod to the plate. It should be noted that expressions (3) are written for the case $M = 3$.

Circular forces T_λ at the contour Γ on the plate are found according to the relation [3, 4]

$$T_\lambda = \nu T_\rho + 2Eh\varepsilon_\lambda. \qquad (4)$$

The reinforced rod is modelled by the elastic curved bar, the connection surface of which with the plate does not coincide with its midsurface [9, 10].

Inside longitudinal N and transverse Q forces and bending moment L_b, acting in the bar cross-sections and displaced to its axis, are connected with the contact forces T_ρ, $S_{\rho\lambda}$, which are transmitted from the plate, by the balance differential equations

$$T_\rho = \frac{N}{r_1} - \frac{dQ}{ds}; \quad S_{\rho\lambda} = -\frac{Q}{r_1} - \frac{dN}{ds}; \quad \eta S_{\rho\lambda} = -\frac{dL_b}{ds} + Q\left(1 - \frac{\eta}{r_1}\right). \tag{5}$$

Here r_1 – the curvature radius of the bar fiber, connected with the plate; s – the contour arc parameter Γ; s_0 – the arc parameter, which corresponds to the free end of reinforcement; $ds = |\omega'(\sigma)|\, d\lambda$.

Traditionally calculation of the stress-strain state of the bar is based on the plane cross-section hypothesis. In this case the deformations of the bar longitudinal fiber with the curvature ρ (relative extension $\varepsilon_\lambda^{(c)}$ and the angle of the normal reversement θ_b), located in the plate midplane, are found according to the formulas

$$\varepsilon_\lambda^{(c)} = \frac{1}{E_0 F_0}\left[N + \left(1 - \frac{r_0}{\rho}\right)\frac{RL_b}{\omega_0}\right]; \quad \theta_b = \frac{1}{E_0 F_0}\left[\int_{s_0}^{s}\left(N + \frac{RL_b}{\omega_0}\right)\frac{ds}{\rho}\right] + C_2, \tag{6}$$

in which such signs are used: $\omega_0 = J_0 / F_0 = \eta^2 / 3$; $E_0 F_0$, $E_0 J_0$ – of the bar tension (compression) and bending rigidity relatively the main axis cross-section; R, r_0 – curvature radii of the axial and neutral reinforcement for the pure bending of the longitudinal fibers; E_0 – the Young's modulus of the reinforcement material; $F_0 = 2h \cdot 2\eta$ – its cross-section square; $C_2 = \theta_b(s_0)$.

Taking into account the real available shearing strain, which causes the deplantating of the plane cross-sections and their additional rotation can be performed by introducing the medium for the shearing cross-section γ_3, which is found as the additional reversement angle of the cross-section relatively the normal to the bar [10]

$$\gamma_3 = -\frac{2(1+\nu_0)}{E_0 F_0}\mu Q,$$

where μ – constant (for the bar rectangular cross-section $\mu = 1.2$); ν_0 – the Poisson's ratio.

Then

$$\theta_b = \frac{1}{E_0 F_0}\left[\int_{s_0}^{s}\left(N + \frac{RL_b}{\omega_0}\right)\frac{ds}{\rho} - 2(1+\nu_0)\mu Q\right] + C_2. \tag{7}$$

If the values N, Q, L_b will be known, the normal stresses $\sigma^{(c)}$ in the longitudinal fibers are found according to the Hooke's law in its the most simple one-dimensional form, and maximal tangential stresses $\tau_{\max}^{(c)}$ in the axis fiber – according to the Zhuravskii formula [9]

$$\sigma^{(c)} = \frac{1}{F_0}\left[N + \left(1 - \frac{r_0}{\rho}\right)\frac{RL_b}{\omega_0}\right]; \quad \tau_{\max}^{(c)} = \frac{3}{2}\frac{Q}{F_0}. \tag{8}$$

Boundary conditions of the task in the reinforcement area are chosen as the deformation uniformity of mutual fibers of the plate and the rod

$$\varepsilon_\lambda = \varepsilon_\lambda^{(c)}; \quad V = \theta_b; \quad \lambda \in [\alpha_0; \beta_0]; \quad \rho = r_1. \tag{9}$$

Substitution of (2), (3), (6), (7) into the boundary conditions (9) taking into account (5) will result in the system of two singular integral with Hilbert kernels and three differential equations for determination of the contact forces in the area of connecting of the plate and the reinforcement rod, as well as the values N, Q, L_b in the reinforcement

$$\frac{E_0F_0}{2Eh\left(\alpha^2+\beta^2\right)}\left\{(1-\nu)\left(\alpha^2+\beta^2\right)T_\rho(\lambda) - \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[H(\lambda,t) - G(\lambda,t)\operatorname{ctg}\frac{\lambda-t}{2}\right]T_\rho(t)\,dt + \right.$$

$$\left. + \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[G(\lambda,t) + H(\lambda,t)\operatorname{ctg}\frac{\lambda-t}{2}\right]S_{\rho\lambda}(t)\,dt + \alpha\varepsilon_\lambda^0 + \beta V^0\right\} = N + \left(1 - \frac{r_0}{r_1}\right)\frac{RL_b}{\omega_0};$$

$$\frac{E_0F_0}{2Eh\left(\alpha^2+\beta^2\right)}\left\{(1-\nu)\left(\alpha^2+\beta^2\right)S_{\rho\lambda}(\lambda) - \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[H(\lambda,t) - G(\lambda,t)\operatorname{ctg}\frac{\lambda-t}{2}\right]S_{\rho\lambda}(t)\,dt - \right.$$

$$\left. - \frac{1}{\pi}\int_{\alpha_0}^{\beta_0}\left[G(\lambda,t) + H(\lambda,t)\operatorname{ctg}\frac{\lambda-t}{2}\right]T_\rho(t)\,dt - \beta\varepsilon_\lambda^0 + \alpha V^0 + C_0\left(\alpha^2+\beta^2\right)\right\} =$$

$$= \int_{\beta_0}^{\lambda}\left(N + \frac{R}{\omega_0}L_b\right)\frac{|\omega'(\tau)|}{r_1}dt - 2(1+\nu_0)\mu Q + C_2E_0F_0; \quad T_\rho = \frac{N}{r_1} - \frac{dQ}{|\omega'(\sigma)|d\lambda};$$

$$S_{\rho\lambda} = -\frac{Q}{r_1} - \frac{dN}{|\omega'(\sigma)|d\lambda}; \quad \eta S_{\rho\lambda} = -\frac{dL_b}{|\omega'(\sigma)|d\lambda} + Q\left(1 - \frac{\eta}{r_1}\right); \quad \lambda \in [\alpha_0; \beta_0]. \tag{10}$$

Let us provide the system (10) with the condition

$$N(\alpha_0) = -N_0, \tag{11}$$

which is necessary for finding the constant $2EhC_2 - C_0$.

Interrelations (10), (11) specify the mathematic model of the task in question.

At $E_0 = E$, $\nu_0 = \nu$, $h_0 = h$ the plate with the partially reinforced hoe contour can be treated as homogeneous plate, on the smooth contour of which with curvature radius $\rho = r_1 - 2\eta$ outside the area $\left[\alpha_0^*; \beta_0^*\right]$ the constant depth 2η recess is available.

Such tasks for the circular holes appear while calculating the splined joints in the rotary motion drives.

Approximate solution of the task. The accurate solution of the system (10), (11) cannot be found. For its approximate solution let us find the structure of these functions at the ends of the reinforcement area. With this purpose let us come to (10), (11) in the integration range $[-1;\,1]$. It can be done by changing the variables

$$\lambda = 2arctg\left(a_0 x + b_0\right);\;\; t = 2arctg\left(a_0 y + b_0\right), \qquad (12)$$

where

$$2a_0 = tg\frac{\beta_0}{2} - tg\frac{\alpha_0}{2};\;\; 2b_0 = tg\frac{\beta_0}{2} + tg\frac{\alpha_0}{2};$$

$$d\lambda = \frac{2a_0 dx}{1+\left(a_0 x + b_0\right)^2};\;\; dt = \frac{2a_0 dy}{1+\left(a_0 y + b_0\right)^2};\;\; ctg\frac{\lambda - t}{2}dt = \frac{2dy}{x-y} + \frac{2a_0\left(a_0 y + b_0\right)dy}{1+\left(a_0 y + b_0\right)^2}.$$

As the result of substitution of (12) in (10), (11) after some transformations we will obtain

$$\frac{E_0 F_0}{2Eh\left(\alpha^2+\beta^2\right)}\left\{(1-v)\left(\alpha^2+\beta^2\right)T_\rho\left(x\right)+\right.$$

$$\left.+\frac{1}{\pi}\int_{-1}^{1}\left[G(x,y)T_\rho\left(y\right)+H(x,y)S_{\rho\lambda}\left(y\right)\right]\frac{2}{x-y}dy+\alpha\varepsilon_\lambda^0+\beta V^0\right\}+F_1(x)=N(x)+\left(1-\frac{r_0}{r_1}\right)\frac{RL_b(x)}{\omega_0};$$

$$\frac{E_0 F_0}{2Eh\left(\alpha^2+\beta^2\right)}\left\{(1-v)\left(\alpha^2+\beta^2\right)S_{\rho\lambda}\left(x\right)-\right.$$

$$\left.-\frac{1}{\pi}\int_{-1}^{1}\left[H(x,y)T_\rho\left(y\right)-G(x,y)S_{\rho\lambda}\left(y\right)\right]\frac{2}{x-y}dy-\beta\varepsilon_\lambda^0+\alpha V^0+C_0\left(\alpha^2+\beta^2\right)\right\}+F_2(x)=$$

$$=\int_{1}^{x}\left(N\left(y\right)+\frac{R}{\omega_0}L_b\left(y\right)\right)\frac{\left|\omega'(\tau)\right|}{r_1}\frac{2a_0 dy}{1+\left(a_0 y+b_0\right)^2}-2\left(1+\nu_0\right)\mu Q\left(x\right)+C_2E_0F_0;$$

$$T_\rho\left(x\right)=\frac{N\left(x\right)}{r_1}-\frac{1+\left(a_0 x+b_0\right)^2}{\left|\omega'(\sigma)\right|2a_0}\frac{dQ}{dx};\;\; S_{\rho\lambda}\left(x\right)=-\frac{Q\left(x\right)}{r_1}-\frac{1+\left(a_0 x+b_0\right)^2}{\left|\omega'(\sigma)\right|2a_0}\frac{dN}{dx};$$

$$\eta S_{\rho\lambda}(x)=-\frac{1+\left(a_0 x+b_0\right)^2}{\left|\omega'(\sigma)\right|2a_0}\frac{dL_b}{dx}+Q(x)\left(1-\frac{\eta}{r_1}\right);\;\; N(-1)=-N_0;$$

$$\lambda \in [\alpha_0; \beta_0]; \quad x \in [-1; 1]. \qquad (13)$$

Here $F_1(x)$, $F_2(x)$ – the functions containing regular integrals from T_ρ, $S_{\rho\lambda}$.

Characteristic part of singular integral equations of the system (13) are similar to those as the corresponding part of the paper [13]. Because of this the contact forces in the reinforcement area can be presented as

$$T_\rho(x) + iS_{\rho\lambda}(x) = \frac{1}{\sqrt{1-x^2}}\left[\Phi_1(x) + i\Phi_2(x)\right]. \qquad (14)$$

The structure of the internal forces N and Q and the bending moment L_b is determined by the boundary conditions at the ends of the reinforcement rod

$$N(-1) = -N_0; \quad Q(-1) = L_b(-1) = N(1) = Q(1) = L_b(1) = 0. \qquad (15)$$

Due to (15) the functions $N(x)$, $Q(x)$, $L_b(x)$ in the range $[-1; 1]$ are chosen as

$$N(x) = \sqrt{1-x}\,\Phi_3^0(x); \quad Q(x) = \sqrt{1-x^2}\,\Phi_4^0(x); \quad L_b(x) = \sqrt{1-x^2}\,\Phi_5^0(x). \qquad (16)$$

In the interrelations (14), (16) $\Phi_1(x)$, $\Phi_2(x)$, $\Phi_3^0(x)$, $\Phi_4^0(x)$, $\Phi_5^0(x)$ – are limited and continuous in $[-1; 1]$ functions.

Having known the structure of the determined functions (14), (16), the approximate solution of the task can be built taking advantage of the method of mechanical quadrature and collocation. Quadrature formulas of this method for singular and regular integrals and those with the variable upper level are presented in the papers [2, 3, 4].

Results and their discussions. For the plate with the egg-like hole ($\varepsilon_1 = 0.2$; $\varepsilon_2 = 0.1$; $\varepsilon_3 = 0$; $\nu = 0.3$; $h = 0.1R_0$) and reinforcing rod of such parameters $h_0 = h$; $E_0 = E$; $\nu_0 = 0.3$; $3\alpha_0 = -\pi$; $3\beta_0 = \pi$ the impact on the strain state of the plate and reinforcement of its relative width η / h_0 have been investigated. The results of numerical calculation of equivalent efforts $T = \sqrt{\left(T_\lambda - T_\rho\right)^2 + T_\rho T_\lambda + 3S_{\rho\lambda}^2}$ on the contour Γ of the plate according to the energy theory of strength [9] and strains $\sigma^{(c)}$, $\tau_{\max}^{(c)}$ in the reinforcing rod are presented in Fig. 2 – 3. The full lines are for the case $\eta = h_0$; dot – $\eta = 0.5h_0$; dot-and-dash – $\eta = 0.25h_0$.

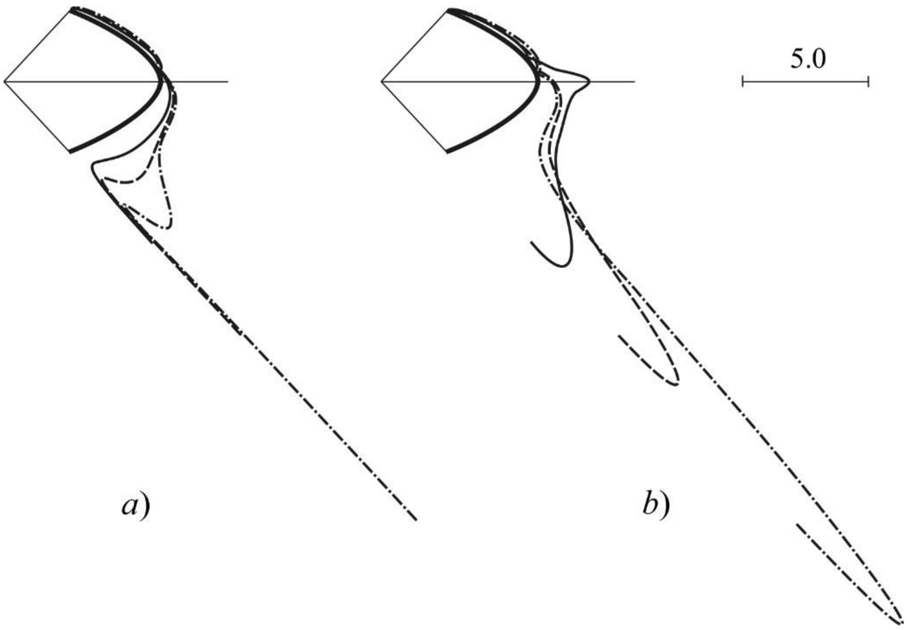

Figure 2. Distribution of normal stresses $\sigma^{(c)} 2h_0 R_0 / N_0$ in the external extreme (a) and in the internal extreme (b) fibers of the rod

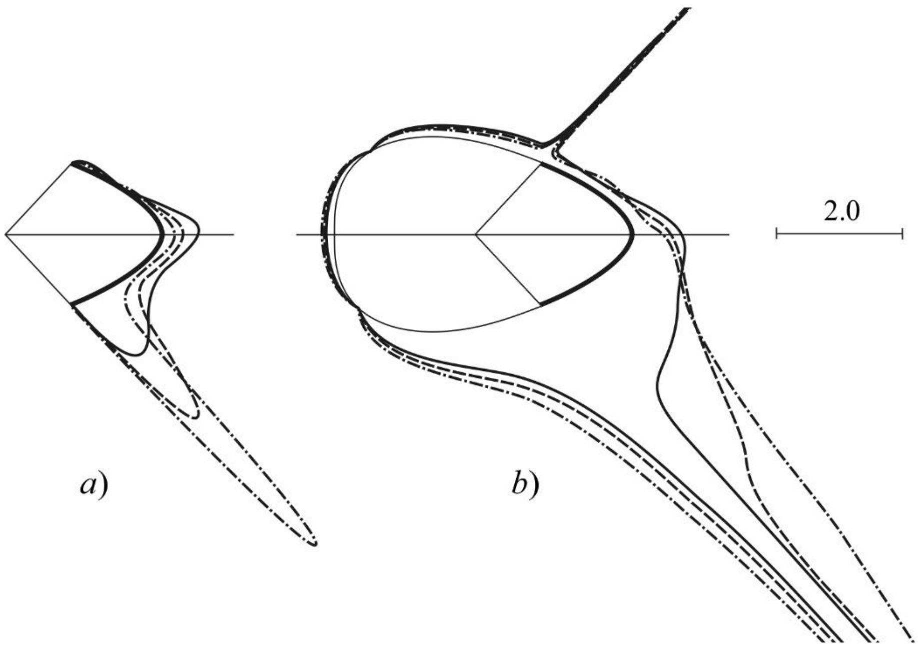

Figure 3. Distribution of tangential stresses $\tau^{(c)}_{max} 2h_0 R_0 / N_0$ in the axial fiber of the rod (a) and equivalent efforts TR_0 / N_0 on the contour of the hole in the plate (b)

Conclusions. As the result of the obtained numerical calculations it has been found that:

– normal stresses in the extreme longitudinal fibers of the rod increase sharply at the end tip, to which the external force is applied, but they are unsufficient at the end itself;

– the impact of transverse forces on the strain state of long curved bars of constant cross-section is unsufficient, which confirms the known fact in the course of strength of materials;

– the plate stress state components at the ends of the reinforcing area are unlimited values, which is caused by the initiation of plastic deformations in these areas;

– the prosed method of solving the task can be generalised for the case of the system of similar rods, through which the moment load is transmitted to the hole contour in the plate.

References

1. Aleksandrov V.M., Mxitaryan S.M. Kontaktnye zadachi dlya tel s tonkimi pokrytiyami i proslojkami, Moscow, Nauka, 1983, 488 p. [In Russian].
2. Siaskyi A.O., Batyshkina Yu.V. Peredacha syl do kryvoliniinoho otvoru neskinchennoi ortotropnoi plastynky stryzhniamy zminnoi zhorstkosti, Visnyk Ternopilskoho derzhavnoho tekhnichnoho universytetu. Vol. 9, no. 4, 2004, pp. 5 – 11. [In Ukrainian].
3. Siaskyi A.O., Batyshkina Yu.V. Peredacha obertalnoho momentu do kryvoliniinoho otvoru neskinchennoi plastynky cherez systemu stryzhniv zminnoi zhorstkosti, Naukovi notatky, Lutsk, LTDU, Vypusk 12, 2004, pp. 331 – 339. [In Ukrainian].
4. Siaskyi A.O., Batyshkina Yu.V. Peredacha syl do kryvoliniinoho otvoru neskinchennoi plastynky stryzhniamy zminnoi zhorstkosti, Mashynoznavstvo, no. 6(84), 2004, pp. 21 – 26. [In Ukrainian].
5. Siaskyi A.O., Shevtsova N.V. Peredacha syl do masyvnykh tsylindrychnykh til cherez nesymetrychni rozimkneni rebra zminnoi zhorstkosti, Naukovi notatky, Mizhvuzivskyi zbirnyk za napriamom „Inzhenerna mekhanika“, Lutsk, Vypusk 23, 2008, pp. 345 – 351. [In Ukrainian].
6. Shevtsova N., Siaskyi A. Peredacha syl i par syl do tsylindrychnykh til cherez systemu nesymetrychnykh tonkostinnykh elementiv zminnoi zhorstkosti, Visnyk Ternopilskoho derzhavnoho tekhnichnoho universytetu, vol. 13, no. 3, 2008, pp. 15 – 20. [In Ukrainian].
7. Siaskyi A.O., Kombel S.M. Matematychna model odniiei zadachi trybomekhaniky dlia shlitsevoho ziednannia, Problemy trybolohii, no. 2, 2004, pp. 53 – 58. [In Ukrainian].
8. Siaskyi A.O., Kombel S.M. Teoretychni osnovy utochnenoho rozrakhunku shlitsovykh ziednan, Mashynoznavstvo, no. 4(70), 2003, pp. 27 – 33. [In Ukrainian].
9. Pysarenko H.S. Kvitka O.L., Umanskyi E.S. Opir materialiv, Kiev, Vyshcha shkola, 2004, 655 p. [In Ukrainian].
10. Filin A.P. Tananajko O.D, Cherneva I.M., Shvarc M.A. Algoritmy postroeniya razreshayushhix uravnenij mexaniki sterzhnevyx system, Leningrad, Strojizdat, 1983, 232 p. [In Russian].
11. Musxelishvili N.I. Singulyarnye integral'nye uravneniya, Moscow, Nauka, 1968, 511 p. [In Russian].
12. Hrylitskyi D.V. Lutsyshyn R.M. Napruzhennia v plastynkakh z kolovoiu liniieiu rozmezhuvannia hranychnykh umov, Lviv,Vyshcha shkola, 1975, 116 p. [In Ukrainian].
13. Siaskyi A.O. Shevtsova N.V. Peredacha pary syl do konturu kryvoliniinoho otvoru plastynky cherez systemu odnakovykh pruzhnykh reber, Visnyk Khmelnytskoho natsionalnoho universytetu. Tekhnichni nauky, no. 3(249), 2017, pp. 7 – 13. [In Ukrainian].

Список використаної літератури

1. Александров, В.М. Контактные задачи для тел с тонкими покрытиями и прослойками [Текст] / В.М. Александров, С.М. Мхитарян. – М.: Наука, 1983. – 488 с.
2. Сяський, А.О. Передача сил до криволінійного отвору нескінченної ортотропної пластинки стрижнями змінної жорсткості [Текст] / А.О. Сяський, Ю.В. Батишкіна // Вісник ТДТУ. – 2004. – Т. 9, № 4. – С. 5 – 11.
3. Сяський, А.О. Передача обертального моменту до криволінійного отвору нескінченної пластинки через систему стрижнів змінної жорсткості [Текст] / А.О. Сяський, Ю.В. Батишкіна // Наукові нотатки. – Луцьк: ЛТДУ, 2004. – Випуск 12. – С. 331 – 339.
4. Сяський, А. Передача сил до криволінійного отвору нескінченної пластинки стрижнями змінної жорсткості [Текст] / А. Сяський, Ю. Батишкіна // Машинознавство. – 2004. – № 6 (84). – С. 21 – 26.
5. Сяський, А.О. Передача сил до масивних циліндричних тіл через несиметричні розімкнені ребра змінної жорсткості [Текст] / А.О. Сяський, Н.В. Шевцова // Наукові нотатки. Міжвузівський збірник за напрямом «Інженерна механіка». – Луцьк, 2008. – Випуск 23. – С. 345 – 351.
6. Шевцова, Н. Передача сил і пар сил до циліндричних тіл через систему несиметричних тонкостінних елементів змінної жорсткості [Текст] / Н. Шевцова, А. Сяський // Вісник Тернопільського державного технічного університету. – 2008. – Т. 13, № 3. – С. 15 – 20.
7. Сяський, А.О. Математична модель однієї задачі трибомеханіки для шліцевого з’єднання [Текст] / А.О. Сяський, С.М. Комбель // Проблеми трибології. – 2004. – № 2. – С. 53 – 58.
8. Сяський, А.О. Теоретичні основи уточненого розрахунку шліцьових з’єднань [Текст] / А.О. Сяський, С.М. Комбель // Машинознавство. – 2003. – № 4 (70). – С. 27 – 33.
9. Писаренко, Г.С. Опір матеріалів [Текст] / Г.С. Писаренко, О.Л. Квітка, Е.С. Уманський. – К.: Вища школа, 2004. – 655 с.
10. Алгоритмы построения разрешающих уравнений механики стержневых систем [Текст] / А.П. Филин, О.Д. Тананайко, И.М. Чернева, М.А. Шварц. – Л.: Стройиздат, 1983. – 232 с.
11. Мусхелишвили, Н.И. Сингулярные интегральные уравнения [Текст] / Н.И. Мусхелишвили. – М.:

Наука, 1968. – 511 с.
12. Гриліцький, Д.В. Напруження в пластинках з коловою лінією розмежування граничних умов [Текст] / Д.В. Гриліцький, Р.М. Луцишин. – Львів: Вища школа, 1975. – 116 с.
13. Сяський, А.О. Передача пари сил до контуру криволінійного отвору пластинки через систему однакових пружних ребер [Текст] / А.О. Сяський, Н.В. Шевцова // Вісник Хмельницького національного університету. Технічні науки. – 2017. – № 3 (249). – С. 7 – 13.

УДК 539.3

ПЕРЕДАЧА ЗОСЕРЕДЖЕНОЇ СИЛИ ЧЕРЕЗ РОЗІМКНЕНИЙ ПРУЖНИЙ СТРИЖЕНЬ ДО КОНТУРУ КРИВОЛІНІЙНОГО ОТВОРУ НЕСКІНЧЕННОЇ ПЛАСТИНКИ

Андрій Сяський; Володимир Сяський; Наталія Шевцова

Рівненський державний гуманітарний університет, Рівне, Україна

Резюме. *Запропоновано новий підхід до розв'язання задачі про передачу до контуру криволінійного отвору нескінченної ізотропної пластинки зосередженої сили, прикладеної до торця розімкненого пружного стрижня сталого прямокутного поперечного перерізу. Крайові умови задачі на ділянці сполучення стрижня з контуром отвору пластинки сформульовано у вигляді рівності їх деформацій. Математичну модель задачі побудовано у вигляді системи сингулярних інтегральних і диференціальних рівнянь для визначення контактних зусиль між пластинкою та стрижнем, а також внутрішніх сил і моментів у підсиленні. Встановлено структуру шуканих функцій на кінцях ділянки підсилення. Наближений розв'язок задачі знайдено методом механічних квадратур і колокації.*

Ключові слова: *підсилювальне ребро, нескінченна пластинка, інтегральні рівняння, криволінійний отвір, напружений стан.*

Отримано 05.09.2017