INTERRELATION BETWEEN INCIDENCE ANGLE AND ROLL ANGLE OF CONCAVE DISKS OF SOIL TILLAGE IMPLEMENTS

ВЗАЄМОЗВЯЗОК МІЖ КУТАМИ АТАКИ ТА КРЕНУ СФЕРИЧНИХ ДИСКІВ ГРУНТООБРОБНИХ ЗНАРЯДЬ

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ABSTRACT

A holistic analytical model of a concave disk, which is arranged in a three-dimensional coordinate system with a given direction of unit motion along OY-axis, has been developed. It includes the design parameters of a disk and the depth of soil tillage. An interrelation between an incidence angle and a roll angle of a disk under a set value of a back angle at field surface level has been determined.

РЕЗЮМЕ

Розроблено цілісну аналітичну модель сферичного диска, розташованого в просторовій системі координат із заданим напрямом руху агрегату вздовж осі ОУ. До неї входять конструктивні параметри диска та глибина обробітку ґрунту. Встановлено взаємозв'язок між кутами атаки і крену диска для заданого значення затильного кута на рівні поверхні поля.

INTRODUCTION

The design parameters of a disk and the angles of its installation influence the technological process of unit operation (turning and breaking up soil, cutting nutrient residues and their mixing up with soil, disk width etc.). Each parameter has its specific influence on the technological process flow. For example, an incidence angle increase results in the improvement of mixing up soil with nutrient residues and a disk width increase, however, angular velocity of rotation can be decreased due to disk drawing and, as a result, clogging of space between disks with soil and nutrient residues. If disks are installed upright, soil takes up mainly tearing and shearing strains, the height of its lift is small and as a result it does not mix well with nutrient residues. If there is a deflection in the upright direction of a disk blade through the so called roll angle, the moving is improved, but it is the case only to a certain increase in the value of this angle.

Main geometrical characteristics of a disk of specified dimensions and setting angles, which determine its operating capacity at a certain depth of tilling, are the following: a back angle and a slope angle of generators of a cone of grinding (*Zaika P.M., 2001*). A back angle is changed according to the depth of soil tillage: it is a maximum at a furrow bottom and its value decreases as field surface is approached. At field surface level its value must range from 3^0 to 5^0 , in some cases it may equal to zero (*Zaika P.M., 2001*). In upper sections, that is above the field surface, this angle will decrease still further and its smallest value will be reached in diametric section, but all these sections are outside the soil and are inactive. Under a negative value of this angle in the operating area, a disk, leaning with its back on a furrow bottom, rolls out to the field surface, thus the depth of soil tillage decreases. Disk setting angles (incidence angle and roll angle) influence the value of a back angle. It is important to choose their correct combination in order to provide a specified value of a back angle at field surface level.

Designing of disk soil tillage implements has been considered in the paper (*Zaika P.M., 2001*) and other research works (*Sineokov G.N. and Panov I.M., 1977; Hevko R.B. et. al., 2012; Hevko R.B. and Klendiy O.M., 2014; Hevko R.B. et. al., 2014; Hevko R.B. et. al., 2015*). There are scientific works fully devoted to this subject (*Strelbitskii V.F., 1978; Tsimmerman M.Z., 1978; Blednyx V.V., 2010; Posmetyev V.I. et. al., 2013;* Demydenko A.I. et. al., 2009). More specified works investigate different aspects of the improvement of the quality of soil tillage by means of such implements (*Hrynenko O. and Lebedev S., 2011; Haponenko O.I., 2012; Soxt K.A., 2014*). Prospects of further improvement of disk and other soil tillage implements have been considered in the paper (*Heruk S.M. and Petrychenko E.A., 2014*).

MATERIAL AND METHOD

A disk is characterized by the following design parameters (Fig.1, a): diameter *D*, radius of a sphere *R*, angle σ of slope of generators to a cone axis with its vertex in the middle of a sphere and which leans against the cutting edge of a disk, angle δ of slope of generators of a cone of grinding to cutting edge area and a cutting-point angle *i*. There are relations between them:



Fig.1 - Picture of a soil tillage disk: a) crosscut; b) axonometry

If OY-axis is taken as a direction of unit motion, taking into consideration an incidence angle α and a roll angle β in a three-dimensional coordinate system OXYZ, concave disk parametric equations can be written down as follows (*Pylypaka S.F. and Klendii M.B., 2016*):

$$X = R(\cos u \cos \alpha \cos \beta - \sin u \sin v \sin \alpha + \sin u \cos v \cos \alpha \sin \beta);$$

$$Y = R(\cos u \sin \alpha \cos \beta + \sin u \sin v \cos \alpha + \sin u \cos v \sin \alpha \sin \beta);$$

$$Z = R(-\cos u \sin \beta + \sin u \cos v \cos \beta),$$

(2)

where v and u – independent variables of the surface; v being an angle of rotation of a variable point of a sphere about OX-axis along the parallel ($v=0...2\pi$); u being an angular coordinate of this point along the meridian, the reading of which begins at the cross point of OX-axis and the surface of a sphere ($u=0...\sigma$).

Analogically, the equations of the surface of a cone of grinding can be written down as follows (*Pylypaka S.F. and Klendii M.B., 2016*):

$$X = r \left[u \operatorname{tg} \delta \cos \alpha \cos \beta - (1 - u) \sin v \sin \alpha + (1 - u) \cos v \cos \alpha \sin \beta \right];$$

$$Y = r \left[u \operatorname{tg} \delta \sin \alpha \cos \beta + (1 - u) \sin v \cos \alpha + (1 - u) \cos v \sin \alpha \sin \beta \right];$$

$$Z = r \left[-u \operatorname{tg} \delta \sin \beta + (1 - u) \cos v \cos \beta \right],$$
(3)

where v and u – independent variables of the surface; v being an angle of rotation of a variable point of a cone about *OX*-axis ($v=0...2\pi$); u being a rectilinear coordinate of this point along the generator of a cone, the reading of which begins at its base; r-radius of the base of a cone.

Fig.1b shows the back surface of a concave disk, constructed from the equations (2), and the surface of a cone of grinding under $\alpha = \beta = 0$, constructed from the equations (3). In this case a cutting edge is in the plane of *ZOY*. For better visualization, the surface of a cone of grinding is darkened. If a disk is arranged in such a way, it does not perform its task when moving along *OY*-axis; it will only rotate and leave a mark in the soil. In order to make it cut and turn the soil, it is necessary to select an incidence angle by rotating the cutting edge plane about *OZ*-axis in such a way as to build the angle α between it and the plane of *ZOY*. This angle has its limits, because as it increases, there is a point at which the angular velocity of disk rotation can decrease due to its drawing. In order to have better visualization of disk installation, taking into consideration the back angle, a nonworking incidence angle is selected to be $\alpha = 90^\circ$, such that the disk does not rotate.

If the cutting edge plane is positioned at a right angle to the direction of unit motion along *OY*-axis (α =90°, Fig. 2), soil undergoes deformation due to its simple shearing away from the direction of unit motion without disk rotation. In this case the back angle is invariable at a specified depth of soil tillage. In case of a disk that is deepened into the soil at the depth of the radius r=D/2, the back angle ε on the field surface will be equal to ($90^\circ - \delta$) on both sides of a disk (Fig.2, side view). It is built as a result of the section of a cone by the field surface plane, which in this case passes through the vertex of a cone. The back angle increases as the cutting plane approaches a furrow bottom. For example, at distance *a* from a furrow bottom it accounts for ε_a and is built by a tangent to a hyperbola – a correspondent horizontal plane section of a cone – with the direction of unit motion. Thus, if the depth of soil tillage is specified by *a*, the least value of the back angle from top to bottom of a furrow will be at field surface level and will account for ε_a . Difference with the angle $90^\circ - \delta$ will be $\Delta \varepsilon$ (Fig.2).



Fig.2 – Three projections of a cone of grinding under the nonworking incidence angle $\alpha = 90^{\circ}$

Now, an operating incidence angle is selected to be equal to the angle of disk grinding: $\alpha = \delta$. In this case, the outside generator of a cone coincides with the direction of unit motion, which can be seen in the side view (Fig.3). If a disk is deepened to a half of its height, in other words if a field surface plane passes through its vertex, the back angle at field level will be equal to zero, that is to say there might be friction of soil against the surface of a cone of grinding. But if a disk tills the soil at a depth of *a* under the same incidence angle $\alpha = \delta$, the back angle at field surface level will be equal to $\Delta \varepsilon$ (Fig. 3).

It is necessary to determine the value of this angle, since the specified value of a back angle must be 3^0-5^0 , otherwise as the back angle increases, the incidence angle α and the cutting angle γ ($\gamma = \epsilon + i$) will increase as well. The determination of the angle $\Delta \epsilon$ does not depend on the value of the incidence angle α . But in order to simplify analytical calculation, it is to the purpose to select $\alpha = 90^0$, which corresponds to Fig. 2. Under the value of $\alpha = 90^0$ and $\beta = 0$ (roll angle is nonexistent), parametric equations of the surface of a cone of grinding (3) can be written down as follows:

$$X = -r[(1-u)\sin v];$$

$$Y = ur \operatorname{tg} \delta;$$

$$Z = r[(1-u)\cos v],$$
(4)

Where: $r=D/2=R \sin \sigma$ according to (1).



Fig.3 – Side view of a cone of grinding under roll angle $\alpha = \delta$

We shall find the curve of the surface section of a cone of grinding (4) by the horizontal plane of a field under the specified depth *a* of soil tillage. In this case a coordinate *Z* of a field surface level will be equal to Z=-a+r-const. We shall equate this value to the last equation (4) and we obtain:

$$r\left[(1-u)\cos v\right] = -r + a \tag{5}$$

The equation (5) is an intrinsic equation of a hyperbola – a cone section, which establishes interdependence between variables u and v of the surface of a cone. It can be solved as u=u(v) or v=v(u). Both variants are given below:

$$u = \frac{r(1+\cos v)-a}{r\cos v}; \qquad v = \operatorname{Arc} \cos \frac{a-r}{r(1-u)}. \tag{6}$$

The substitution of one of the expressions (6) in the equation (4) gives parametric equations of hyperbola –section of a cone of grinding at a height of *a* from a furrow bottom. In order to build it, it is better to use the first expression (6), which gives two half-hyperbolas, whereas the second one gives one. But it is to the purpose to use the second one when determining the angle $\Delta \varepsilon$, which has one side that is tangent to a hyperbola at its tie point with a blade, that is to say under the value of u=0, whereas in the first expression (6) in the cone equation (4), we obtain the equation of its section by field level plane under the specified depth of tillage *a*:

$$x = \sqrt{r^2 (1-u)^2 - (a-r)^2};$$
 $y = ur \operatorname{tg} \delta.$ (7)

The direction of a tangent line can be determined by the differentiation of the expressions (7) in variable *u*:

$$x' = \frac{r^2 (1-u)}{\sqrt{r^2 (1-u)^2 - (a-r)^2}}; \qquad y' = r \operatorname{tg} \delta.$$
(8)

We determine the angle between OY-axis and the tangent (Fig. 2) from the expression:

$$\varepsilon_{a} = \operatorname{Arctg} \frac{x'}{y'} = \operatorname{Arctg} \frac{r(1-u)\operatorname{ctg}\delta}{\sqrt{r^{2}(1-u)^{2}-(a-r)^{2}}}.$$
(9)

The expression (9) under *u*=0 provides the value of the angle of the tangent to a hyperbola at its tie point with a blade. The angle $\Delta \varepsilon$ is determined as the difference between the angles (Fig. 2): $\Delta \varepsilon = \varepsilon_a - (90^\circ - \delta)$. Taking into consideration the above mentioned, we can write down the following:

$$\Delta \varepsilon = \varepsilon_a - \left(90^0 - \delta\right) = \operatorname{Arctg} \frac{r \operatorname{ctg} \delta}{\sqrt{r^2 - \left(a - r\right)^2}} - \left(\frac{\pi}{2} - \delta\right). \tag{10}$$

For example, we shall take a disk with the specified design parameters of a cone of grinding and the depth of soil tillage (taken from the scientific work (Zaika P.M., 2001): δ=39,2°; r=225 mm; a=80 mm. The substitution of these data into the expression (10) results in (the value of the angle δ must be substituted in radians): $\Delta \varepsilon = 0.13 \text{ rad} = 7.2^{\circ}$. Thus, if a disk is arranged under the incidence angle $\alpha = \delta$ (Fig. 3), the back angle at field surface level is $\varepsilon_a = \Delta \varepsilon = 7.2^{\circ}$. Accordingly, the incidence angle can be adjusted in such a way that the back angle obtains a specified value (for instance $\varepsilon_a=3^{\circ}$). If we reduce the incident angle by $\Delta \varepsilon$, the back angle ε_a at field surface level will be equal to zero. Thus, in order to obtain a specified value of the back angle, the incidence angle must be reduced by the value of $\Delta \varepsilon \cdot \varepsilon_a$. In his case the incidence angle is obtained from the expression: $\delta - (\Delta \varepsilon - \varepsilon_a)$, since the initial incidence angle is $\alpha = \delta$ according to Fig.3. In our case the incidence angle reaches $\alpha = \delta - \Delta \varepsilon + \varepsilon_a = 39.2^{\circ} - 7.2^{\circ} + 3^{\circ} = 35^{\circ}$. The same result is given in the paper [1], but it has been obtained not on the basis of a holistic three-dimensional model, but on the basis of projections. The analyzed procedure of obtaining the incidence angle a under the specified parameters of a cone of grinding, the depth of soil tillage a and the back angle ε_a is demonstrative but long. It would be easier to obtain the angle between the tangent to a hyperbola - the section of a cone of grinding turned though the incidence angle by horizontal plane of field level - and the direction of unit motion. It should be obtained similarly to the analyzed case, in which we took $\alpha = 90^{\circ}$, $\beta = 0$ in the equations (3). Under the specified value of the angles α and $\beta=0$ we obtain the equation that is similar to the equation (9), which includes the angle α , that is to say one can find the back angle ε_a under the specified α , or vice versa, one can find the incidence angle α at vertical mounting of a disk under the specified back angle ε_a .

The developed three-dimensional model gives the opportunity to determine not only the incidence angle α , but also the roll angle β of a disk, under which the availability of the specified back angle ε_a is provided, since if there is an additional vertical deviation of the blade plane of a disk through the angle β , the back angle ε_a will change. Thus, using the analogical approach, one can determine the dependence (9), which interrelates the angles α , β and ε_a (that is to say under $\beta \neq 0$). This expression will certainly be more cumbersome than (9). Its closed form (at u=0) is the following:

$$tg\varepsilon_{a} = -\frac{\sqrt{a(2r\cos\beta - a)}\cos\alpha\,tg\delta - \sin\alpha[r\cos\beta\sec\delta\cos(\beta + \delta) + a\sin\beta\,tg\delta]}{\sqrt{a(2r\cos\beta - a)}\sin\alpha\,tg\delta + \cos\alpha[r\cos\beta\sec\delta\cos(\beta + \delta) + a\sin\beta\,tg\delta]} \cdot \quad (11)$$

RESULTS

The expression (11) determines a dependence of the design parameters of a cone of grinding (*r* and δ) and the angles α , β and ε_a at the specified depth of soil tillage *a*. On substituting the values of the disk angles $\alpha = 35^{\circ}$, $\beta = 0^{\circ}$, the parameters of a cone of grinding *r*=225 mm, $\delta = 39.2^{\circ}$ and the depth of soil tillage *a*=80 mm into it, we get the value of the back angle $\varepsilon_a \approx 3^{\circ}$, that is to say the result, which was previously obtained. By the expression (11) one can determine a dependence $\alpha = \alpha(\beta)$ with the known design parameters and the specified back angle ε_a and the depth of tillage *a*. Fig.4 graphically illustrates such a dependence.



Fig.4 – Graphic illustration of disk installation with design parameters *r*=225 mm, δ=39.2⁰, depth of soil tillage *a*=80 mm and specified back angle ε_a=3⁰:
a) graph α=α(β) of incidence angle versus roll angle;
b) horizontal plane section lines at field level under different disk angles

As the graph suggests, vertical deviation of a disk plane results in the increase of the incidence angle α . If the roll of a disk is nonexistent ($\beta = 0^{0}$), the minimum value of the incidence angle, which provides the specified back angle $\varepsilon_{a}=3^{0}$, must be $\alpha=35^{0}$. If we set the roll angle of a disk $\beta=15^{0}$, the back angle must be increased by 5^{0} (Fig. 4.a). We can determine the precise value of the incidence angle in this case - $\alpha=40.2^{0}$. Accordingly, if $\beta=30^{0}$, the incidence angle is $\alpha=49.4^{0}$. For better visualization horizontal plane sections of a cone of grinding at field level for all three cases have been prepared. Fig. 5 illustrates such a section for disk installation under angles $\beta=30^{0}$, $\alpha=49.4^{0}$.



Fig.5 – Rectangular projections of a cone of grinding: a) front projection and side view; b) additional view in direction A, when a blade plane is projected on a straight line

The symbols are the following: *V* – direction of speed motion of a unit, *d* –grip of a disk at field level (front projection of an intercept of soil entry points of a blade). If the incidence angle β is present, the angle between the lower generator of a cone and a furrow bottom decreases by its value. In our case it is equal to 90° - $(30^{\circ}+39.2^{\circ})=20.8^{\circ}$. The back angle at the lower point of a disk depends on it.

Fig.4.b combines three sections of a cone of grinding for three values of the roll angle ($\beta=0^0$, $\beta=15^0$ and $\beta=30^0$) with the corresponding incidence angles and under the specified value of a back angle $\varepsilon_a=3^0$. It shows, that the distance *d* increases with the increase of the roll angle (and the correspondent increase of the incidence angle), which results in the increase in the grip of a disk. However, this is inferred based only on providing the value of the back angle, which does not prevent a disk from being deepened into the soil without taking into consideration the requirements for cutting and mixing crop residues with soil. Uncontrolled increase of the incidence angle under the increase of the roll angle may cause disk drawing.

CONCLUSIONS

Due to the developed holistic three-dimensional model of disk arrangement one can calculate all the necessary data of disk installation for soil tillage with the specified design parameters. It allows getting the values of an incidence angle and a roll angle under which a specified back angle is provided and vice versa, it allows determining the value of a back angle under specified values of an incidence angle and a roll angle under specified values of an incidence angle and a roll angle. Besides, one can get the curves of horizontal plane section of a cone of grinding at any level of tillable soil. The indicated parameters are merely geometrical and are not connected with the technological process flow. In order to interrelate geometrical and technological parameters it is necessary to use the developed model when investigating the interaction between a disk and soil.

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