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INFLUENCE OF THE INITIAL DEFORMATION OF THE THICK PLATE ON CONTACT INTERACTION WITH PARABOLIC PUNCH

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***Summary**. Calculating the strength of structural elements and mechanisms is one of the most important stages in the process of their design. Residual deformation is almost always available in the structural elements and machine parts. Resultant stress can cause fracture and accelerate some phase transitions, corrosion in particular. To improve the accuracy of calculations the residual deformations must be taken into account. In the article the solution of axisymmetric contact problem of pressure a parabolic punch for an elastic isotropic half-space, taking into account preliminary stresses is described. Besides distribution function of contact stresses and displacements for the plane boundary of semi-space was created. The authors have shown influence of residual stress on the distribution of contact stresses under the punch.*

***Key words:** contact stresses, residual deformations, parabolic punch, thick plate, semi-space.*

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Introduction. To calculate the strength of structural elements and mechanisms is one of the most important stages during their design. It is necessary to compute contact stresses strains to determine the strength and endurance of contacting bodies. Maximum number of factors that influence their interactions should be considered in order to minimize any errors. A key factor is considering the residual deformation which directly affects the contact stress.

Analysis of recent research. Many scientists both foreign and home ones studied the interaction of hard punches and elastic bodies with residual deformations. Contact problems for bodies with initial stresses in the concrete form of elastic potential were studied by Artiunian N.Kh., Alexandrov V.M., Smetanin B.I., Filipova L.M. and others. In general, the formulation of such problems requires the involvement of nonlinear elasticity, but with a rather large initial deformations it can be limited to its linearized version. Basic results of linearized theory of elasticity were obtained by a Ukrainian scientist, academician of Ukraine, prof. O.M. Huz [1]. The theory of contact interaction of bodies with initial stresses was further researched in the works by S.Yu. Babich, V.B. Rudnytskyi, B.H. Shelestovskyi [2 – 11] and other home and foreign scientists.

Continuous increase of the number of studies on the contact interaction of bodies of pre-stressed state can be explained by its topicality for the development of basic research of contact interaction of bodies and for use in various industries. However, tasks on the problem of parabolic pressure and ring-parabolic freeform punches in pre-stressed semi-space or layer has not been resolved within the framework of linearized elasticity theory for compressible and incompressible bodies at arbitrary structure of the stressed elastic potential.

Research objective is, firstly, to demonstrate the developed technique for constructing solutions of axisymmetric problems of determining the stress state in a thick prestressed slab in its contact interaction with rigid punch; and, secondly, to investigate the effect of residual deformations on the distribution of contact stresses and vertical displacements at the contact interaction of punches and plates.

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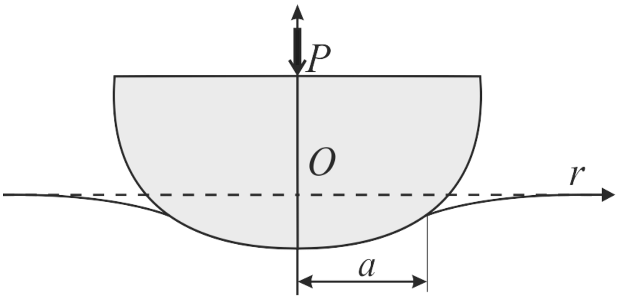

Figure 1. Contact interaction scheme

Problem setting. We will analyze the problem of constant force punch P of the stiff punch into previously strengthened hard thick plate which is modeled as semi-space. We choose a cylindrical coordinate system $Or\theta z$ so that the coordinate plane $rO\theta$ coincides with the boundary plane of the semi-space and axis Oz coincides with a line of force P (Fig. 1).

Problem solution. The punch touches the semi-space in point O. The bigger the force P is, the bigger the size of the contact area is. We assume that the radius a is known, then the value of the applied force is determined by the condition

$$P = -2\pi \int_0^a r\sigma_{zz}(r,0)\,dr. \quad (1)$$

The boundary conditions of the problem are the following

$$\sigma_{rz}(r) = 0,\ 0 \le r < \infty; \quad (2)$$

$$\sigma_{zz}(r) = 0,\ a \le r; \quad (3)$$

$$u_z(r) = f(r),\ 0 \le r \le a. \quad (4)$$

Function $\omega(r)$ corresponds the shape of the surface that limits the punch. We choose it as

$$f(r) = u_z(a) + \omega(r).$$

To solve this problem we use the key ratios of the linearized elasticity theory [1]. Satisfying the boundary conditions (2), we get the ratio between the unknown functions F_1 та F_2:

$$F_1 = -s_0 F_2. \quad (5)$$

Considering (5) the expressions for the normal stresses and displacements vector have the following formula:

$$\sigma_{zz}(r) = c_{44}(1+m_1)(s-s_0)l_1 \int_0^\infty \alpha^3 F_2 J_0(\alpha r)\,d\alpha; \quad (6)$$

$$u_z(r) = \frac{m_1(s_1-s_0)}{\sqrt{n_1}} \int_0^\infty \alpha^2 F_2 J_0(\alpha r)\,d\alpha. \quad (7)$$

In the relationships (6) and (7) constants c_{44}, m_1, n_1, l_1, s, s_0, s_1 depend on the nature of the elastic capacity and are chosen in each case [2].

Having satisfied the boundary condition (3), we have:

$$c_{44}\left(1+m_1\right)\left(s-s_0\right)l_1\int_0^\infty \alpha^3 F_2 J_0\left(\alpha r\right)d\alpha = 0, \quad a \le r . \tag{8}$$

We introduce unknown function $x(r)$, $0 \le r \le a$ by which we continue ratio (8) onto the period $0 \le r < \infty$:

$$c_{44}\left(1+m_1\right)\left(s-s_0\right)l_1\int_0^\infty \alpha^3 F_2 J_0\left(\alpha r\right)d\alpha = x(r)\eta(a-r), \; 0 \le r < \infty , \tag{9}$$

where $\eta(r)$ – the Heaviside step function.

The function $x(r)$ determines the distribution of contact stresses under the punch.

Taking into account their continuity and absence on the border area of contact (at $r = a$) we show $x(r)$ as the form of a segment of the generalized Fourier series for functions $J_0\left(\frac{\lambda_n}{a} r\right)$

$$\sigma_{zz}(r) = x(r) = \sum_{n=1}^{N} a_n J_0\left(\frac{\lambda_n}{a} r\right), \quad 0 \le r \le a , \tag{10}$$

where λ_n, $n = \overline{1,N}$ – positive roots of Bessel functions $J_0\left(\lambda_n\right) = 0$, a_n – unknown coefficients.

Applying the inversion formula of Hankel integral transformation to ratio (9), we get the expression

$$\alpha^2 F_2 = \frac{1}{c_{44}\left(1+m_1\right)\left(s-s_0\right)l_1}\sum_{n=1}^{N} a_n \int_0^a r J_0\left(\frac{\lambda_n}{a} r\right) J_0\left(\alpha r\right)dr, \; 0 \le \alpha < \infty . \tag{11}$$

Using the relation (7), (11) and boundary condition (4) we will have

$$k_1 \sum_{n=1}^{N} a_n \int_0^\infty \Psi_n(\alpha)\left[J_0(\alpha r) - J_0(\alpha a)\right]d\alpha = \omega(r), \; 0 \le r \le a , \tag{12}$$

$$k_1 = \frac{m_1\left(s_1 - s_0\right)}{c_{44}\left(1+m_1\right)\left(s-s_0\right)l_1\sqrt{n_1}} \, ; \; \Psi_n(\alpha) = \int_0^a r J_0\left(\frac{\lambda_n}{a} r\right) J_0\left(\alpha r\right)dr . \tag{13}$$

Multiplying relation (12) by $r J_0\left(\frac{\lambda_q}{a} r\right)$ and upon integration of the expressions r from 0 to a, we get:

$$\sum_{n=1}^{N} a_n \int_0^{\infty} \Psi_n(\alpha)\left[\Psi_q(\alpha) - K_q J_0(\alpha a)\right] d\alpha = \frac{w_q}{k_1},\ q = \overline{1, N}\,, \tag{14}$$

where $K_q = \int_0^a r J_0\left(\frac{\lambda_q}{a} r\right) dr$; $w_q = \int_0^a r\omega(r) J_0\left(\frac{\lambda_q}{a} r\right) dr$.

Correlations (14) define system N of linear algebraic equations for the unknown a_n .

Taking into account (10), we obtain the law of distribution of contact stresses under the punch

$$\sigma_{zz}(r) = -\frac{P}{2\pi} \frac{\sum_{n=1}^{N} a_n J_0\left(\frac{\lambda_n}{a} r\right)}{\sum_{n=1}^{N} a_n K_n}\,. \tag{15}$$

From the correlations (7), (11) and (13) we set formula for determining vertical displacements of the boundary semi-space plane

$$u_z(r) = -\frac{k_1 P}{2\pi} \frac{\sum_{n=1}^{N} a_n \int_0^{\infty} \Psi_n(\alpha) J_0(\alpha r) d\alpha}{\sum_{n=1}^{N} a_n K_n}\,. \tag{16}$$

While solving problems (2) – (4) radius a of the contact area was considered to be known and force P of the punch into semi-space was determined by (1). To study the effect of initial strain on the distribution of contact stress and vertical displacement in the semi-space it is required to determine the area of contact for fixed punch form and given power P . In most cases, the radius a is calculated from the equation $\sigma_{zz}(x,0) = 0$, in which one of the numerical methods is used, e.g. semi-division method [12]. Its root determines the value at which contact stresses are converted to zero, ie limit areas of contact. However, to solve the problem of contact that matches the form of the punch, this method is not appropriate because the selection of the desired contact stresses of the distribution function in the form (10) automatically provides performance of the condition $\sigma_{zz}(a,0) = 0$. We describe a slightly different approach to determining the radius a of the area.

We choose a punch formed by the rotation line $W(r)$ round axis Oz (fig. 2).

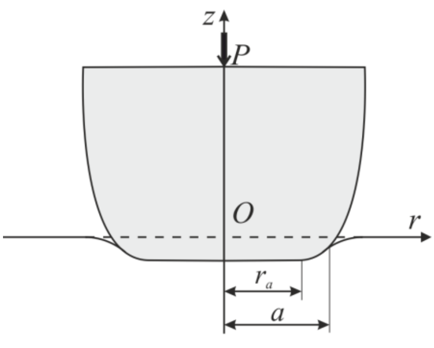

Figure 2. Contact interaction scheme

$$W(r) = \begin{cases} 0, & 0 \le r \le r_a; \\ \frac{1}{2R}(r - r_a)^2, & r_a < r. \end{cases}$$

Then function $\omega(r)$ is

$$\omega(r) = \begin{cases} -\frac{1}{2R}(r_a - a)^2, \; 0 \le r \le r_a; \\ \frac{1}{2R}\left[(r_a - r)^2 - (r_a - a)^2\right], \; r_a < r \le a. \end{cases}$$

In this case (14) it is advisable to replace $a_n = \frac{1}{2k_1 R} a_n^*$ while solving system (14).

The relationship between focal parameter of parabola R and the value of the applied force is established by (1) where

$$R = \frac{-\pi}{2k_1 P} \sum_{n=1}^{N} a_n^* K_n. \tag{17}$$

Let us consider a specific example. Let punch $R = 2$ and $r_a = 0$ punches into semi-space with Bartenev-Khazanovych potential.

We choose 5 key values for the parameter a with step 0.1 and find the corresponding values R for different values λ_1 (табл. 1).

Table 1

The value of R for different λ_1

a	R / $\lambda_1 = 1$	R / $\lambda_1 = 1.2$	R / $\lambda_1 = 0.8$
0.8	1.035500	1.144685	0.606100
0.9	1.474167	1.629607	0.862862
1.0	2.021983	2.235185	1.183511
1.1	2.691076	2.974829	1.575145
1.2	3.493553	3.861921	2.044853

$\lambda_1 = 1$ – corresponds to the absence of residual deformation in the semi-space, $\lambda_1 = 1.2$ – the presence of residual deformation stretches, and $\lambda_1 = 0.8$ – available in semi-space squeezing residual deformations.

For each of the cases examined, using the approximation of the cubic spline environment Mathcad, we build graphs dependences $R = R(a)$ (fig. 3).

From the resulting equations we can easily find options of the contact area according to the set R, force P and characteristics of the remaining deformations field k_1, when $\lambda_1 = 1$, $a = 0.996$, when $\lambda_1 = 1.2$, $a = 0.963$ and when $\lambda_1 = 0.8$, $a = 1.191$. The difference between calculated R using (17), which match calculated a, and pre-selected R is not more than 1 percent. The resulting accuracy is sufficient for most engineering calculations, but it can be improved by increasing the number of key parameter values a.

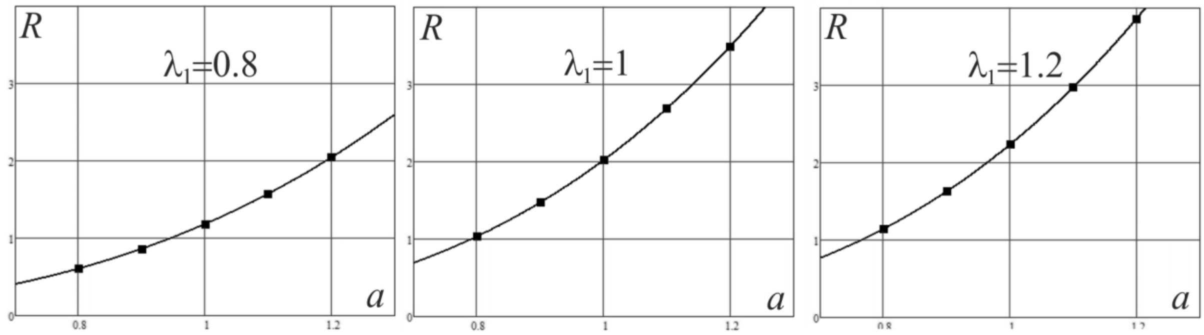

Figure 3. Determining the radius of the contact area

Built for calculated a in (15) and (16) the functions determine the distribution of contact stress and vertical displacements for fixed R and different λ_1 and allow to analyze the impact of residual deformations field.

Fig. 4 shows the graph of the function that describes the distribution of contact stresses under the punch for the harmonic type of the elastic potential, and Fig. 5 shows the case for Bartenev-Khazanovych potential.

The graph for u^* that describes the vertical displacement of the points of the semi-space limit plane for elastic potential of the harmonic type is built in Fig. 6, and for Bartenev Khazanovych potential is in Fig. 7.

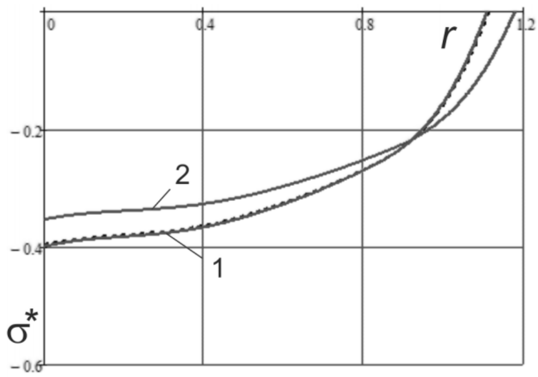

Figure 4. Contact stress for the harmonic potential

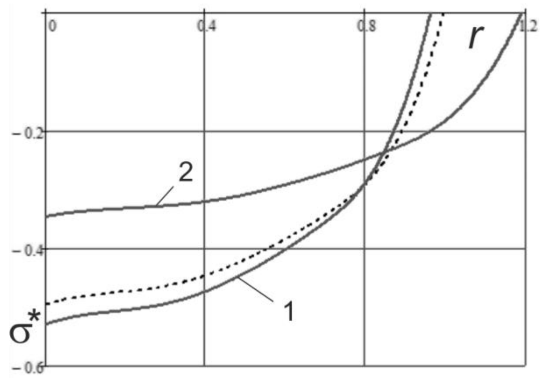

Figure 5. Contact stress for Bartenev-Khazanovych potential

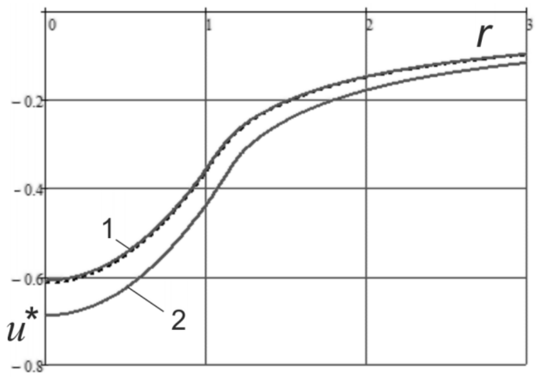

Figure 6. Vertical displacements for the harmonic potential

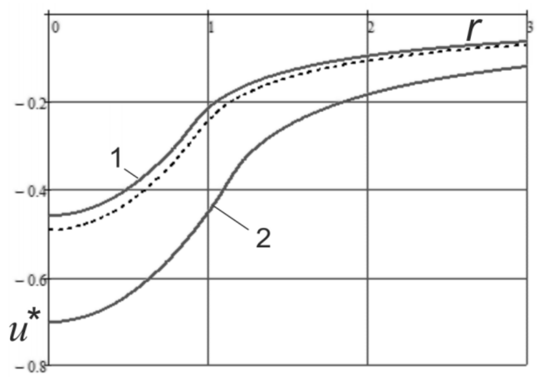

Figure 7. Vertical displacements for Bartenev-Khazanovych potential

In Fig. 4 – 7 dotted curve corresponds to the absence of residual deformations in the semi-space ($\lambda_1 = 1$), curve 1 – available strain deformations ($\lambda_1 = 1.2$) and curve 2 – compression deformation ($\lambda_1 = 0.8$).

Conducted numerical analysis makes it possible to argue that stretching of residual deformations causes constriction of the contact area, increase in the absolute value of contact stresses and reduction of vertical movement. The emergence of compression strains causes expansion of the contact area, reduction of the absolute value of contact stresses and increase in vertical movements.

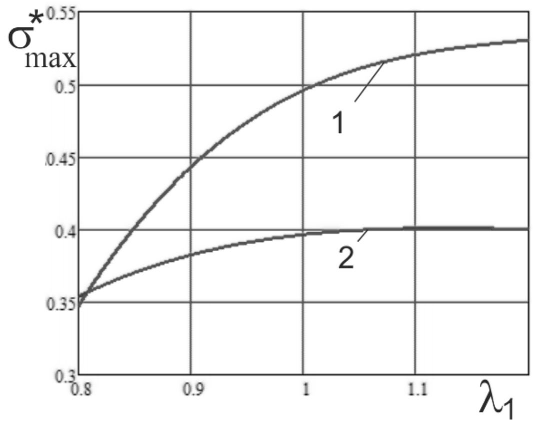

Figure 8. Extreme values of contact stresses

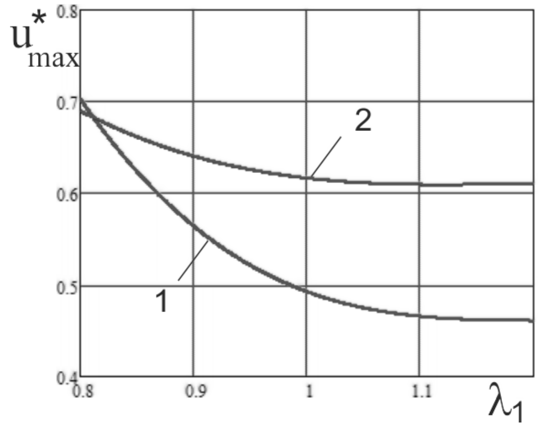

Figure 9. Extreme values of vertical displacements

Fig. 8 – 9 illustrate the dependence of extreme values of contact stresses and vertical displacements of the boundary plane of the semi-space under pressure of parabolic punch. Curves 1 correspond to incompressible semi-space with the available elastic capacity of Bartenev-Khazanovych potential and curves 2 correspond to compressible semi-space with the harmonic potential. Fig. 10 illustrates the dependence of the radius a of the contact area in the case of a parabolic punch parameter λ_1, i.e. on the characteristics of the original deformations field. All curves are built for the case $\frac{P}{E} = 1$, $R = 2$, $r_a = 0$.

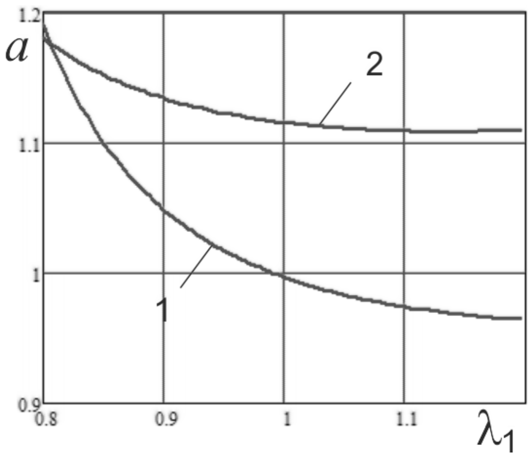

Figure 10. Dependence of the radius of contact area from λ_1

Conclusions. Data in fig. 8 – 10 confirm earlier findings that regardless of the structure of the elastic potential (both compressible and incompressible bodies) residual strain deformations cause narrowing of the contact areas, increase in the absolute value of contact stresses and reduction in vertical movement. The presence of the semi-space residual deformations of compression s causes the expansion of the contact area, reduction of the absolute value of contact stresses and increase of vertical movements.

The reliability of the findings confirms their agreement with the results obtained by other authors [3].

The results can be used in the construction of the experimental methods for determining the nature of the existing body of initial deformation based on punch with a certain force in the test plate of the parabolic or ring-parabolic punch. Thus, the values of vertical displacement and size of the contact area can be used to calculate λ_1.

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ВПЛИВ ПОЧАТКОВИХ ДЕФОРМАЦІЙ ТОВСТОЇ ПЛИТИ НА ЇЇ КОНТАКТНУ ВЗАЄМОДІЮ ІЗ ПАРАБОЛІЧНИМ ШТАМПОМ

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Резюме. *Розрахунок міцності елементів конструкцій і механізмів є одним з найважливіших етапів у процесі їх розроблення. Початкові деформації майже завжди присутні в елементах конструкцій та деталях машин. Напруження, що при цьому виникають, можуть призвести до їх руйнування та прискорити певні фазові переходи, зокрема корозію. Для підвищення точності розрахунків початкові деформації повинні бути враховані. У статті наведено розв'язання осесиметричної контактної задачі про тиск параболічного штампа на попередньо напружену товсту плиту, що моделюється пружним півпростором. Отримано функції розподілу контактних напружень і переміщень для граничної площини півпростору. Проаналізовано вплив початкових напружень на розподіл контактних напружень під штампом.*

Ключові слова: *контактні напруження, початкові деформації, параболічний штамп, товста плита, півпростір.*

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